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Robust design with Variability Response Functions; an alternative approach

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ABSTRACT

In this work a different Robust Design Optimization (RDO) approach is proposed implementing the concept of Variability Response Function (*VRF*), which is a function that when combined with the power spectral density of the stochastic field that models the system's uncertainty, formulates an integral expression for the variance of the system's response. The basic idea is to exploit a very well-known property of the *VRF*, which is its independence of the stochastic system parameters, in order to obtain global optima that depend only on the deterministic parameters of the problem. This way, optimal structural designs are achieved which are globally insensitive to uncertainties, that is to say they are free of the spectral-distribution characteristics of the stochastic fields modeling the uncertainties. This is achieved by setting in addition to the total material cost, the maximum *VRF* value as an objective function. The advantages of using the proposed methodology over traditional Robust Design Optimization are illustrated through an application to a frame-type structure where it is demonstrated that the designs achieved through classical RDO for a given stochastic field description are not optimal for a variation on the spectral properties of the random field modeling the system uncertainty, while optimal designs obtained with the *VRF*-based RDO remain optimum for the worst case scenario stochastic fields.

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1. Introduction

The concept of Robust Design Optimization (RDO) has been introduced in order to deal with intrinsic uncertainties in physical systems that drive the system performance to deviate from the deterministically expected performance into sub-optimal designs, thus neutralizing the effort of the optimization procedure itself. In RDO the analyst is taking into account the stochastic properties of the system variables/parameters and/or system constraints and effectively reaches a safer optimum design which should be less sensitive to random system parameter variations. Various methodologies have been proposed in recent years regarding RDO and its applications to various problems. In classical RDO formulation the goal of minimizing objective function(s) is achieved by considering the mean and/or the standard deviation of a response quantity and trying to establish the designs that minimize the aforementioned quantities considering deterministic or reliability constraints [1,2]. In Reliability-based Robust Design Optimization (RRDO) [3–5] usually care is taken to address the influence of probabilistic constraints as a limit on the probability of failure in the context of RDO of structures. Vulnerability-based Robust Design Optimization (VRDO) [6] is a special case of RRDO where intermediate limit states approaching the probabilistic constraints are also taken into account thus providing possibly crucial information regarding structural behavior and operational integrity.

All previously mentioned RDO formulations are to be carried out in a stochastic finite element method (SFEM) framework so as to efficiently estimate the required quantities associated with system variations. This consideration of system randomness however, for it to be reliable, requires a precise knowledge of probabilistic characteristics (marginal *pdf's* and correlation structures) of the respective random fields modeling system parameters acquired only through corresponding experimental surveys or otherwise careful assumption/selection of various statistical properties describing the system variables/parameters uncertainty. Furthermore it increases substantially the analysis computational cost as any candidate design requires full stochastic analysis for the estimation of various statistical quantities. In the frequent case that such conditions are not met, similar analyses are implemented based on sensitivity analyses with respect to the aforementioned parameters resulting in a significant further increase of the overall computational cost.

In the present paper an alternative RDO procedure is proposed utilizing Variability Response Functions (*VRF*) concept [7–15] in an

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effort to provide an answer in aforementioned known issues while optimizing a frame structure involving stochastic field material properties with respect to its total weight and robustness of its displacement response. It is reminded here that system response variance, as originally proposed in [7] and then extended and further developed in [8–14], can be expressed in the following integral form expression:

$$Var(\mathbf{u}) = \int_{-\infty}^{\infty} \mathbf{VRF}(\kappa, \sigma_{ff})\, S_{ff}(\kappa) d\kappa \tag{1}$$

In the above expression σ_{ff} is the uncertain system variable standard deviation, $S_{ff}(\kappa)$ is the stochastic field spectral density and κ the spatial frequency (rad/m). **VRF**'s product and integration with the spectral density function $S_{ff}(\kappa)$ of the stochastic field that models the uncertain system variable(s) amounts to system response variance vector $Var(\mathbf{u})$. In the above expression **VRF**, which is a vector comprised of a *VRF* for each degree of freedom of the FE system, is assumed to be deterministic, an assumption proven rigorously only for statically determinate beam-type structures. For a number of other applications this assumption has been demonstrated numerically while further evidence has been provided with the introduction of the so called Generalized *VRF* (*GVRF*) which is a *VRF* calculated from a family of spectral density functions and various *pdfs*. What is really beneficial under this assumption is the ability to establish spectral- and *pdf*-free upper bounds in a straightforward manner described in the following equation as it has been explained in [12]:

$$Var(\mathbf{u}) \leqslant \mathbf{VRF}(\kappa^{\max}, \sigma_{ff})\, \sigma_{ff}^2 \tag{2}$$

where $\mathbf{VRF}(\kappa^{\max},\sigma_{ff})$ is the maximum value of the *VRF* attained at some wave number $\kappa^{\max}$. Therefore, setting maximum *VRF* value as an objective function accounting for system response robustness, in addition to the total weight, the system is ensured to exhibit, for a given weight class, the lowest possible variance response under conditions imposed by the worst possible stochastic field. The worst possible stochastic field for a particular design candidate is determined by means of Eq. (2) i.e. it is a stochastic field with a monochromatic *SDF* concentrated at $\kappa^{\max}$ [12]. The optimum design candidate for this particular weight class is the one that minimizes the respective $\mathbf{VRF}(\kappa^{\max},\sigma_{ff})$ value. Repeating this process for all possible weight classes one a two dimensional Pareto front is created for two objective functions: the weight and the system variance response accruing from Eq. (2).

In classical RDO formulation, optimization is performed for an a priori selected stochastic field. In real life applications however correlation structure of the uncertain system parameter is rarely known thus rendering such an optimization procedure redundant. Consequently the designer is obliged to conduct multiple such optimization procedures to shield the designed system from all possibilities. By using the proposed methodology this problem is overcome because each design candidate is evaluated based on its performance under the worst case scenario determined for the specific design. Effectively the designer is ensured that the system will have the best possible performance at the worst possible conditions.

The advantages of using the proposed methodology over traditional Robust Design Optimization are illustrated through an application to a frame-type structure where it is demonstrated that the designs achieved through classical RDO for a given stochastic field description are not optimal if a variation on the spectral properties of the random field modeling the system uncertainty occurs. On the other hand optimal designs obtained with the VRF-based RDO remain optimum for the worst case scenario stochastic fields. In order to demonstrate this, a bi-objective function is formulated taking into account uncertainties in the material properties modeled as random fields. Deterministic constraints of maximum stress and displacement response are applied. A Pareto front is initially constructed through a classical RDO formulation and multi-objective Genetic Algorithm solver for the two conflicting objective functions, namely the total structural weight and the system response variability, for a given stochastic field with a classical Robust Design Optimization formulation. Then, maximum possible variances of the selected designs are computed from the respective maximum values (see Eq. (2)) of the corresponding Variability Response Functions characteristic to these designs. The resulting front is then compared to a new Pareto front in which the second objective function is the maximum possible system variance which can be readily obtained by minimizing the maximum value of the Variability Response Function min$VRF(\kappa^{\max},\sigma_{ff})$. The former classical RDO front proves to be, as expected, sub-optimal to the VRF-based one since the latter is by definition independent of the probability distribution and the spectral density used to model system's uncertainty. It is mentioned that the generated front and the respective proposed designs are referring to a variety of stochastic fields in contrast to the classical RDO. It is also clarified that the proposed designs are not necessarily optimal when examined under the scope of only one predesignated stochastic field. In the case that an optimization is carried out for a specific correlation structure the resulting design selection will be suboptimal with respect to any other correlation structure.

2. Classical RDO formulation

A general formulation of an optimization problem can be stated as:

$$\left.\begin{array}{llll} \text{optimize}: & f(\mathbf{x}), & & (\mathrm{a}) \\ \text{subject to}: & g_i(\mathbf{x}) \leqslant 0, & i = 1,\ldots,I, & (\mathrm{b}) \\ & h_j(\mathbf{x}) = 0, & J = 1,\ldots,J, & (\mathrm{c}) \end{array}\right\} \tag{3}$$

where (3b) represents the set of inequality constraints and (3c) the set of equality constraints. In our case objective function $f(\mathbf{x})$ is a bi-objective function taking under consideration total material cost i.e. total structural weight and system variance response. Thus our problem falls into the category of multi-objective optimization with structural cost and robustness of the response being the focus of our design. So the RDO formulation in our example for demonstrative purposes can be stated as follows:

$$\min_{\mathbf{s}\in\mathbf{F}} \quad f = [C(\mathbf{s},\mathbf{x}), \mathrm{var}(\mathbf{u})]^T \tag{4}$$

subjected to deterministic constraints:

$$g_j(\mathbf{x}) \leqslant 0 \quad J = 1,\ldots,k \tag{5}$$

where f are the objective functions related to the material cost C and system variance response $var(\mathbf{u})$. Vector $\mathbf{s}$ represents the design variable vectors and $\mathbf{x}$ is the position vector. F is the feasible region where all the deterministic constraint functions g_j are satisfied. It is mentioned here that an alternative second objective function could be selected as opposed to var($\mathbf{u}$) i.e. $(\varepsilon[\mathbf{u}] + 3\sigma_{\mathbf{u}})$ where $\sigma_{\mathbf{u}} = \sqrt{\mathrm{var}(\mathbf{u})}$, that would also be a very valid conceptually selection as well. However this would lead to an identical selection of design variables as with the methodology followed in the current work since $\varepsilon(\mathbf{u})$ is almost constant with respect to different *SDF* as shown in [11] and very close to the deterministic displacement $u_{\det}$. Therefore, $\min\{\varepsilon(\mathbf{u}) + 3\sigma_u\}\varepsilon(\mathbf{u}) + 3\min(\sigma_u)$. Apart from this, it is quite common that coefficient of variation *COV* alone is selected as the second counterpart of a bi-objective function in a robust design problem [1,2,6].

3. Analysis of response variability using VRFs

Without loss of generality consider the linear stochastic FE system of Fig. 1 which is a fixed–fixed beam/frame structure. The inverse combined bending stiffness parameter EI is assumed to vary randomly as a stochastic field along the length of the beam according to Eq. (6) implying that both E and I are random parameters. Beam section area A is assumed deterministic since it is a parameter essentially not contributing to the bending dominated response of the frames.

$$\frac{1}{(EI)(x)} = F_0\ (1 + f(x)) \tag{6}$$

where E is the elastic modulus, I is the moment of inertia, F_0 is the mean value of the inverse of EI, and $f(x)$ is a zero-mean homogeneous stochastic field modeling the variation of $1/(EI)$ around its mean value.

Following a procedure similar to the one presented in [11], it is possible to express the variance of the response variability of a stochastic finite element system in the integral form expression of Eq. (1). The numerical estimation of **VRF** in Eq. (1) involves a FEM-based fast Monte Carlo simulation (FEM-FMCS) whose idea is to consider the random field in Eq. (6) as a random sinusoid [12,13] and plug its monochromatic power spectrum into Eq. (1), in order to compute the respective mean and variance response at various wave numbers. The steps of the FEM-FMCS approach are the following:

(i) Generate N (5–10) sample functions of the below random sinusoid with standard deviation σff and wave number $\bar{\kappa}$ modeling the variation of the inverse of the combined stiffness parameter 1/(EI) around its mean F_0:

$$f_j(x) = \sqrt{2}\sigma_{ff} \cos\ (\bar{\kappa}x + \varphi_j) \tag{7}$$

where j = 1, 2,..., N and φ_j varies randomly under uniform distribution in the range [0, 2π]. These samples are generated by dividing the range [0, 2π] at 5–10 equally spaced distances and selecting the centers of these distances as values of random phase angles φj's.

(ii) Using these N generated sample functions it is straightforward to compute their respective response variance, $Var[\mathbf{u}]_{\bar{\kappa}}$, by solving the corresponding FEM system under the applied loading.

(iii) The value of the VRF at wave number $\bar{\kappa}$ can then be computed as follows:

$$\mathbf{VRF}(t, \bar{\kappa}, \sigma_{ff}) = \frac{Var[\mathbf{u}]_{\bar{\kappa}}}{\sigma_{ff}^2} \tag{8}$$

The previous equation is a direct consequence of the integral expression in Eq. (2) in the case that the stochastic field becomes a random sinusoid.

(iv) Get VRF as a function of wave number κ by repeating previous steps for various wave numbers. The entire procedure can be repeated for different values of the standard deviation σ_{ff} of the random sinusoid.

(v) Identify maximum VRF value and then apply Eq. (2) to calculate spectral- and distribution-free upper bounds for each degree of freedom of the FE system.

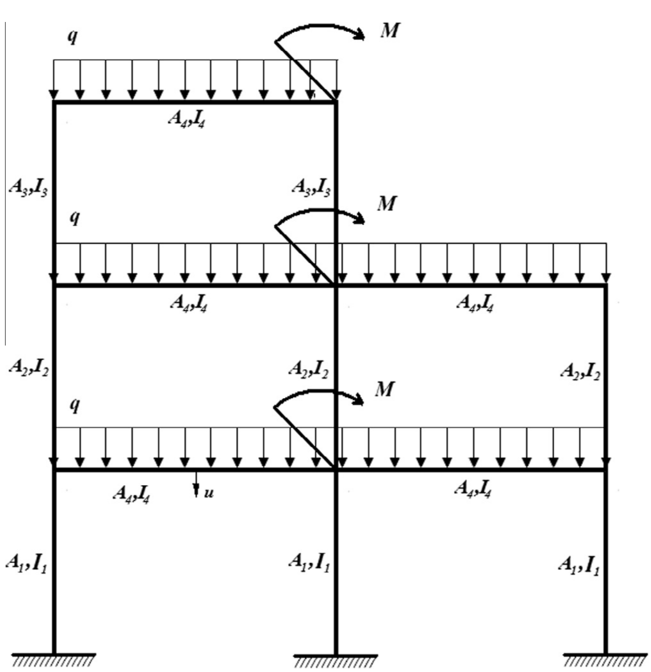

Fig. 1. Geometry and loading of the fixed–fixed frame discretized with 60 beam elements.

3.1. Numerical validation of the VRF with GVRF

In the context of this work and in order to validate our findings we have utilized the recently established concept of *GVRF* [15] in order to further evidence the assumption of independence of the **VRF** of the stochastic parameters of the problem. For this purpose a *GVRF* was calculated for a family of moving *SDFs* and then compared to the *VRF* computed via FEM-FMCS.

For this purpose Eq. (1) can be rewritten in the following discretized form:

$$Var[\mathbf{u}(x)] = 2[S_f(\kappa_1)\ \ S_f(\kappa_2)\ \ \cdots\ \ S_f(\kappa_N)] \times \begin{bmatrix} \mathbf{VRF}(x,\kappa_1) \\ \mathbf{VRF}(x,\kappa_2) \\ \vdots \\ \mathbf{VRF}(x,\kappa_N) \end{bmatrix} \Delta\kappa \tag{9}$$

Having assumed that **VRF** is independent of the power spectral density and the marginal *pdf*, it is natural to assume that the same **VRF** values can be used to estimate system variance for various *SDFs*. Therefore the following relation should also be true, only now that *VRF* is named *Generalized Variability Response Function* (**GVRF**)

$$\begin{bmatrix} Var[\mathbf{u}(x)_1] \\ Var[\mathbf{u}(x)_2] \\ \vdots \\ Var[\mathbf{u}(x)_N] \end{bmatrix} = 2\begin{bmatrix} S_{f_1}(\kappa_1) & S_{f_1}(\kappa_2) & \cdots & S_{f_1}(\kappa_N) \\ S_{f_2}(\kappa_1) & S_{f_2}(\kappa_2) & \cdots & S_{f_2}(\kappa_N) \\ \vdots & \vdots & \ddots & \vdots \\ S_{f_N}(\kappa_1) & S_{f_N}(\kappa_2) & \cdots & S_{f_N}(\kappa_N) \end{bmatrix} \times \begin{bmatrix} \mathbf{GVRF}(x,\kappa_1) \\ \mathbf{GVRF}(x,\kappa_2) \\ \vdots \\ \mathbf{GVRF}(x,\kappa_N) \end{bmatrix} \Delta\kappa \tag{10}$$

For the computation of the **GVRF** the left hand side vector, which is the vector of different system variances, is calculated by respective brute-force Monte Carlo simulations. The matrix on the right hand side is the matrix of *SDF* values for various corresponding spectral density types $S_{f_i}(\kappa),\ i = 1, 2, \ldots, N$. For the purposes of this work a parent *SDF* S_P of exponential form has been used given by:

$$S_P\ (\kappa) = \sigma_{ff}^2 \exp\ (-2|\kappa|) \tag{11}$$

In each row of Eq. (10) corresponds a different *SDF* of the S_P family. The *i*th *SDF* in the *i*th row of Eq. (10) is defined as follows:

$$S_{P_i}\ (\kappa) = \begin{cases} S_P(\kappa + \kappa_u - i\Delta\kappa + \Delta\kappa), & 0 \leqslant \kappa \leqslant (i-1)\Delta\kappa \\ S_P\ (\kappa - i\Delta\kappa), & i\Delta\kappa \leqslant \kappa \leqslant \kappa_u \end{cases} \tag{12}$$

Four different *SDFs* of the S_P family are depicted in Fig. 2. Effectively, Eq. (10) describes a system of N linear equations with N unknowns, thus providing a unique solution for the *GVRF* vector.

4. RDO using Variability Response Functions

RDO using *VRFs* (VRF-RDO) implements a bi-objective function involving maximum *VRF* value and total structural weight. The constraints of this function can be either stress-and/or displacement-related. *VRF* is a function characterizing variability response of the system regardless of the spectral density function of the stochastic field modeling the inverse of the combined bending stiffness parameter *EI*. Thus, minimizing its maximum value selects a design candidate for the system that has the optimal performance with respect to the worst case scenario.

A general formulation of the VRF-RDO can be stated as follows:

$$\min_{\mathbf{s}\in\mathbf{F}} \quad f = [C(\mathbf{s},\mathbf{X}), \mathbf{VRF}(\kappa^{\max},\sigma_{ff})]^T \tag{13}$$

subjected to deterministic constraints:

$$g_j\ (\mathbf{X}) \leqslant 0 \quad J = 1,\dots,k \tag{14}$$

where f are the objective functions related to the material cost C and the vector that contains the maximum values of selected Variability Response Function quantities $\mathbf{VRF}(\kappa^{\max},\sigma_{ff})$. Material cost C is an obvious selection as an objective function in most structural design problems. Maximum attained *VRF* value $\mathbf{VRF}(\kappa^{\max},\sigma_{ff})$, is chosen as the second objective function to minimize, accounting for system variability and effectively dealing with existing uncertainty in a structural probabilistic environment. Vector $\mathbf{s}$ represents the design variable vectors and $\mathbf{X}$ is the position vector. F is the feasible region where all the deterministic constraint functions g_j are satisfied. The $\mathbf{VRF}(\kappa^{\max},\sigma_{ff})$ is qualified as an objective function because it provides with more general system inherent information independent of the stochastic field correlation structure. Therefore under the VRF-RDO formulation the design candidate is selected so that it attains the lowest possible variability response when the worst case scenario, in terms of stochastic field spectral density, is applied.

5. Multi-objective optimization using Genetic Algorithms

The solution of a multi-objective optimization problem is given in the form of a so-called Pareto front as opposed to a single-objective problem where the solution is singular. Several methods have been proposed for multi-objective optimization such as the weighted sum method [16], goal programming [17], physical programming [18,19], compromise programming [20], as well as recently developed evolutionary algorithms such as Strength Pareto Evolutionary Algorithm 2 (SPEA-2) [21], simulated annealing [22], particle swarm optimization [23,24] and Non-dominated Sorting Genetic Algorithm II (NSGA-II) [25]. In the current work the multi-objective optimization is conducted implementing the NSGA-II which is established as a standard approach in identifying the 'Pareto front'. Multiobjective evolutionary algorithms are preferred to classical optimization methods primarily due to their ability to find multiple Pareto-optimal solutions in one single run.

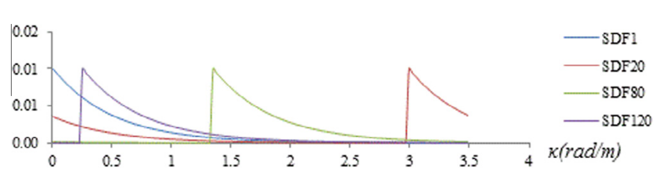

Fig. 2. Plots of different spectral density functions of the S_p family for a discretization of 128 steps in the frequency domain.

However, they have been mainly criticized for issues like high computational complexity and non-elitistic approach.

NSGA-II which is adopted as an optimization method in this study has dealt efficiently with these issues. It uses an elitist principle and an explicit diversity preserving mechanism and it emphasizes on non-dominated solutions. A simple flowchart of the algorithm is shown in Fig. 3. At any generation t, utilizing the standard genetic operators (selection, crossover, mutation), the offspring population Q_t is created from the parent population P_t. Population R_t is formed combining the two populations. The new population is now of total size $2N$. Then, the population R_t is classified into different non-domination classes. Thereafter, the new population is filled by points of different non-domination fronts, one at a time. The filling starts with the first non-domination front (of class one) and continues with points of the second non-domination front, and so on. Since the overall population size of R_t is $2N$, not all fronts can be accommodated in N slots available for the new population. All fronts which could not be accommodated are deleted. To deal with diversity-preservation issues of the estimated Pareto front the NSGA-II utilizes the crowding distance d_i. This quantity d_i is the perimeter of the cuboid formed by using the nearest neighbors in the objective space as the vertices and it is a measure of the objective space around i which is not occupied by any other solution in the population. The optimal Pareto front points are selected as those individuals of the population that demonstrate the non-domination property and have the highest possible crowding distance d_i.

By means of the nature of this particular problem, objectives and constraints are regarded as non-linear functions. The population size is set equal to 50 for each generation. Migration and crossover fractions are set equal to 0.5. Maximum number of generations was set equal to 150.

6. Numerical example

The three-story frame shown in Fig. 1 is selected in order to showcase the potential of the VRF-RDO formulation. For this structure, the inverse of (*EI*) is assumed to vary randomly along its length according to Eq. (1) with $F_0 = (1.35 \times 10^8\ \text{kN m}^3)^{-1}$. Additionally, each story is 4 m long and 3 m high. For the analysis of the frame structure we used 220 beam elements, 15 for each column and 20 for each beam resulting in 654 d.o.f.'s.

A concentrated moment is applied at the middle of each storey equal to $M = 70$ kN m and a distributed load q = 3.2 kN/m along all beams (see Fig. 1). Assuming full statistical dependence, the stochastic field $f(x)$ in Eq. (6) is considered to vary across the length of the columns and the beams of the frame as follows: x is assumed to run first along the columns from left to right and from bottom to top in the first story; then along the beams of the first floor from left to right. Following the same pattern for stories 2 and 3 a continuous field is formed. The *SDF* of the field is assumed to be exponential and given as:

$$S_{ff}\ (\kappa) = \frac{1}{4}\sigma_{ff}^2 b^3 \kappa^2 e^{-b|\kappa|} \tag{15}$$

Two different values of the correlation length parameter were examined, b = 10, 70 with a standard deviation σ_{ff} = 0.1. Plots of the *SDF* with respect to the frequency κ (rad/m) for the selected values of b can be seen in Fig. 4.

The geometric properties of the columns and the beams at each storey of the frame are considered to be the four distinct design variables for the VRF-RDO formulation (see Fig. 1). The selection of the geometric properties of the columns and the beams of the frame has been defined within the set of the Eurocode-8 HEB

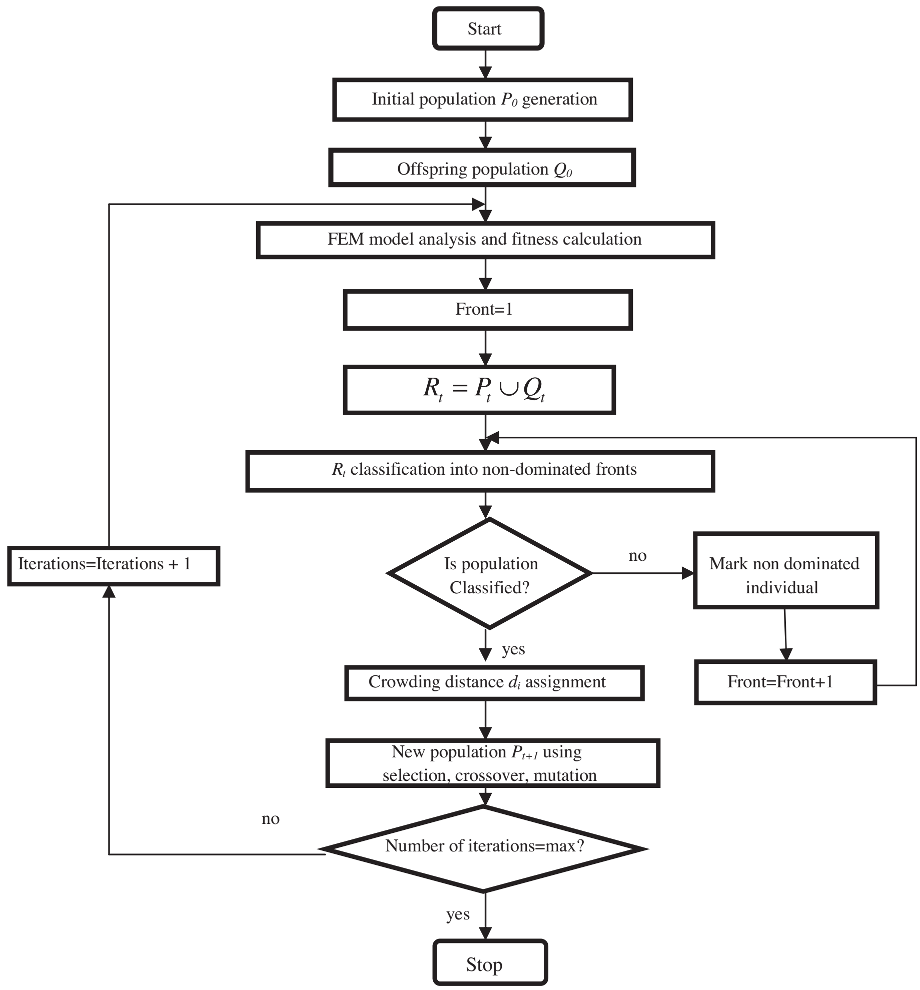

Fig. 3. Schematic flowchart of the NSGA-II as implemented.

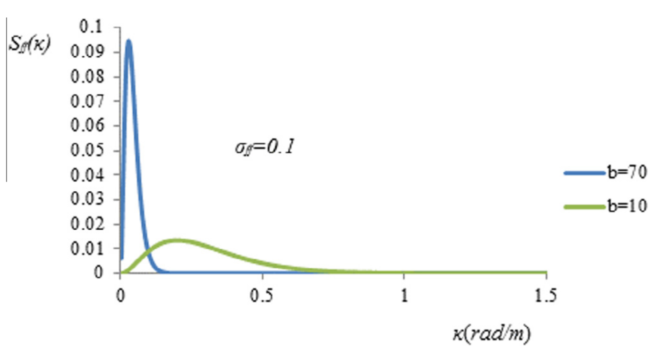

Fig. 4. Spectral density functions for stochastic field $f(\mathbf{x})$ standard deviation $\sigma_{ff} = 0.1$ for two different values of the correlation length parameter.

sections from HEB100 to HEB1000. The formulation of the VRF-RDO problem is as follows:

$$\begin{aligned} \min_{\mathbf{s}\in\mathbf{F}} \quad f &= [VOL(\mathbf{s}), VRF(\kappa^{\max}, \sigma_{ff})]^T \\ \mathbf{s} &= [\mathbf{X}_1, \mathbf{X}_2, \mathbf{X}_3, \mathbf{X}_4]^T \\ \mathbf{X}_i &= [A_i, I_i] \end{aligned} \tag{16}$$

subjected to

$$\begin{aligned} &\mathbf{X}_i \in \Omega \\ &\max(\sigma) \leqslant \sigma_y/1.10, \quad \sigma_y = 235 \text{ Mpa} \end{aligned} \tag{17}$$

where $VRF(\kappa^{\max}, \sigma_{ff})$ is the maximum VRF value corresponding to vertical displacement u in Fig. 1, Ω is the discrete set containing the geometric properties of the EC-8 sections from HEB100 to HEB1000, $F \subseteq \Omega^4$ is the feasible region for the design variable $\mathbf{s}$ where all constraints are satisfied, A_i, I_i are the mean values of cross-section and moment of inertia respectively of the structural members, max (σ) is the maximum deterministically derived effective stress for each design $\mathbf{s}$ appearing in the model and σ_y is the material yield stress. It is mentioned here that this methodology is fully extendable and able to facilitate multiple displacements of the structure with no further implications and additional cost in a straightforward manner.

An initial classical RD procedure was performed for the case that a given stochastic field with a SDF with $b = 10$ in Eq. (7) describes the inverse of the combined stiffness parameter in Eq. (6).

Fig. 5 presents the calculated Pareto front where, as expected, the heaviest designs exhibit the superior performance i.e. the minimum response variability. Likewise, lighter designs trade off less cost, in terms of total material volume, with increased variability. The same figure presents also a derivative plot which was produced by calculating the upper bound on the response variability by means of Eq. (2) for each of the designs of the previously calculated Pareto front from the classical RD procedure. As shown in Fig. 5 the derivative plot shifts clearly to the right which means that at least one field can be found, namely a random sinusoid at $\kappa = \kappa_{\max}$ of the VRF of each candidate design, in which the variance

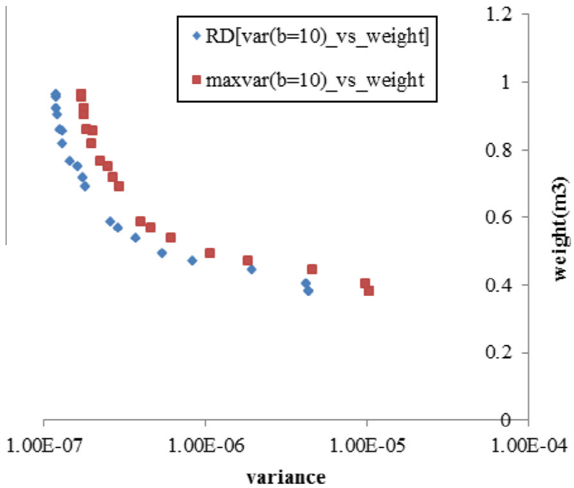

Fig. 5. Pareto front for classical RDO for a given field with $b = 10$ and total weight as objective functions and maximum possible variance for the selected designs. Variance axis in logarithmic scale.

is significantly higher that the on computed for the given stochastic field with $b = 10$.

In order to determine the upper bound on the variability response for each design we evaluate their corresponding *VRF*s. Fig. 6 depict some typical *VRF*s for the respective designs of this system. Specifically, in Fig. 6(a) the graphs of three conveniently selected designs are depicted; while the first design s_1 is the heaviest one, it demonstrates almost identical performance, as far as *VRF* values are concerned, with the last design s_3 which generates considerably lower structural weight. On the other hand for the design s_2 with yet identical resulting weight as design s_3, *VRF* is substantially augmented thus establishing it as an inferior design with respect to design s_3. In Fig. 6(b) two similar *VRF* graphs are depicted for two designs of unequal total accruing weight while in Fig. 6(c) two designs with equal total weight produce two disparate *VRF* graphs. In Fig. 6(d) two designs of equal total weight 0.74 m^3, namely s_2 and s_3, result in totally different *VRFs* while s_1 with lower total weight exhibits similar performance to s_3. Lastly in Fig. 6(e) two designs of substantially different total weights exhibit similar performance making it clear that there is plenty

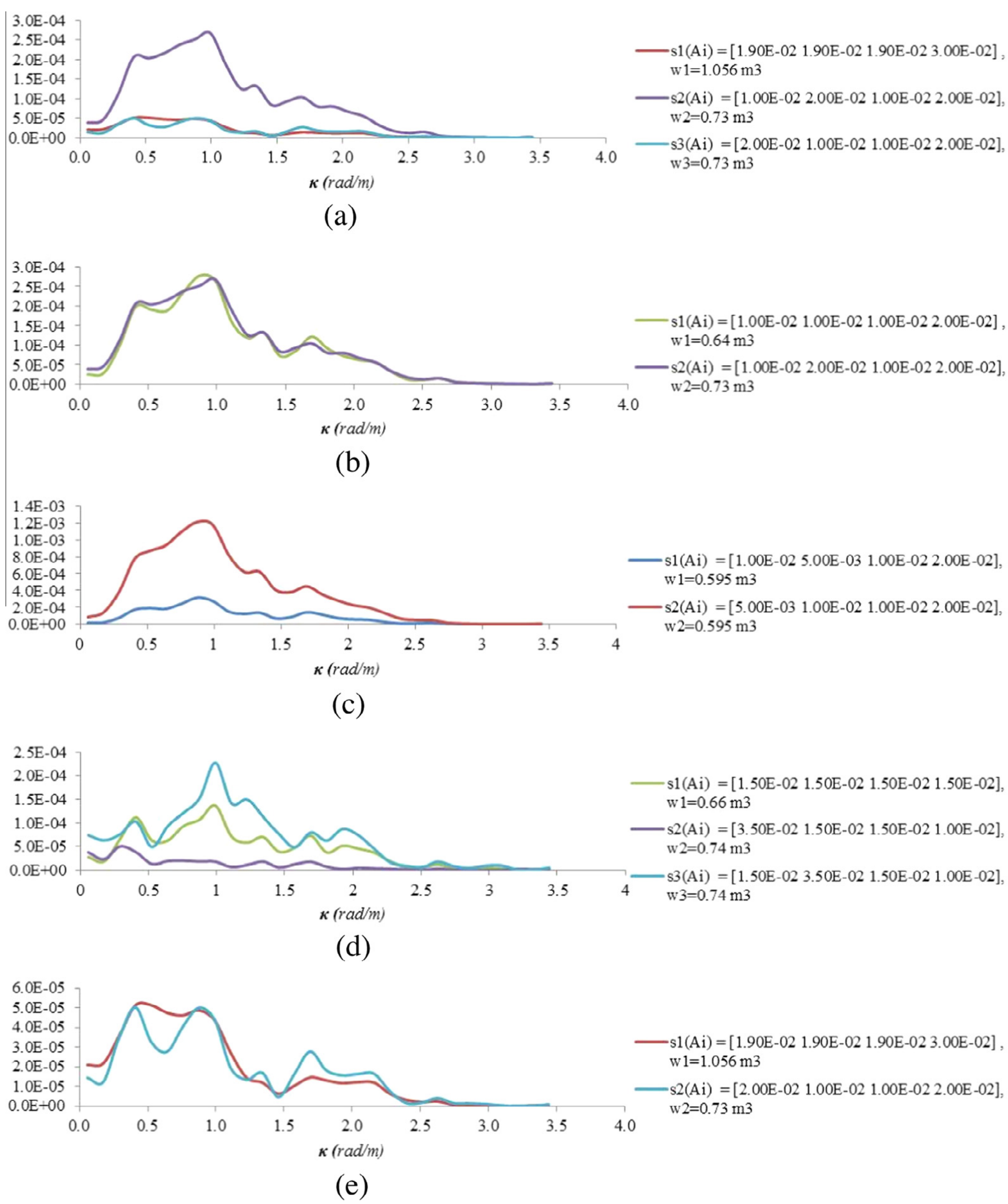

Fig. 6. Graphs of *VRF* for different total weight and structural members' cross sections included in design vector **s**.

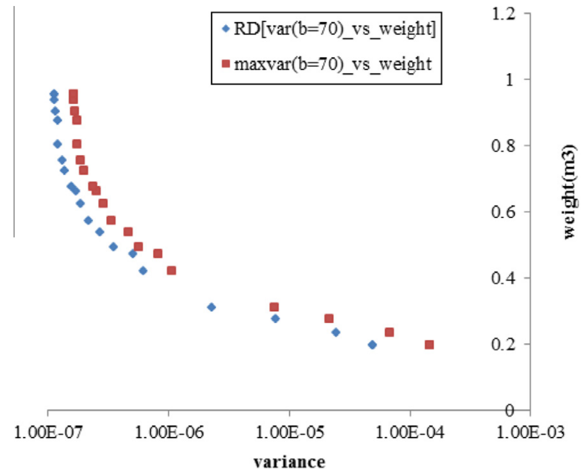

Fig. 7. Pareto front for classical RDO for a given field with $b = 70$ and total weight as objective functions and maximum possible variance for the selected designs. Variance axis in logarithmic scale.

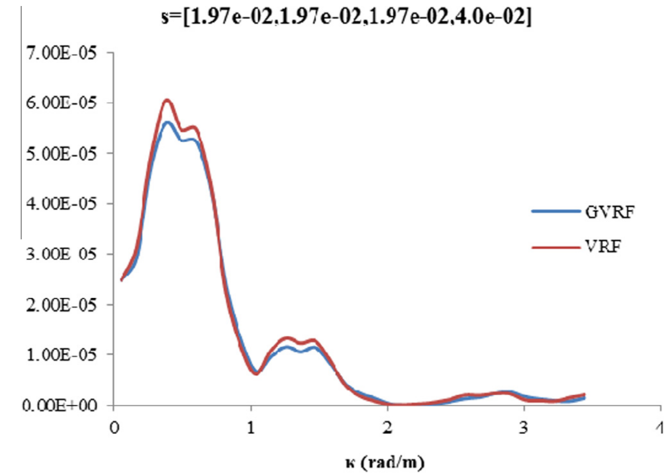

Fig. 9. Comparison of *GVRF* and *VRF* graphs for a specific design vector **s** with respective moments of inertia.

of room for optimization with respect to *VRF* maximum value depending on alterations on the design vector even for equal structural weights. From these *VRF* graphs it is evident that the wave number domains that are mostly contributing to the *VRF* and consequently to the response variance demonstrate a significant variation and strongly depend on the deterministic parameters of the problem. Therefore, if a classical RDO results in optimum system response variability for a given SDF this doesn't necessarily mean that this design is also optimum with respect to the response variability for a different SDF.

The same conclusion can be derived from the Pareto front of the classical RDO in Fig. 7 but for a correlation length parameter $b = 70$. From Figs. 5 and 7 it can be observed that in the case of $b = 10$, average 'shift' in variance is equal to 74% ranging from 44% to 140% while in the case of $b = 70$ the respective percentages are 86%, 42% and 226%. Fig. 8 presents the two previous results in comparison to the Pareto front produced by the VRF-RDO formulation. What is important to bear in mind in the VRF-RDO procedure is that optimal designs in the Pareto front of Fig. 8 exhibit the globally optimal performance when focusing on different possible stochastic fields of the uncertain system parameter. Specifically, comparing the VRF-RDO Pareto front with the maximum possible variance front for $b = 10$ case we notice that for a similar weight, i.e. the last point of each front (VRF-RDO point weight equal to 0.181 m^3 and $b = 10$ case point weight equal to 0.197 m^3) there is a 45% reduction in variance achieved. In another case for the weight class around 0.27 m^3 the reduction is almost 60%. When comparing VRF-RDO Pareto front with the maximum possible variance front for $b = 70$, reduction in variance can be even more dramatic reaching up to 80% (VRF-RDO point weight equal to 0.430 m^3 and $b = 70$ case point weight equal to 0.444 m^3). This can be explained by the following observation; in the specific static model it seems to be a standard feature of *VRF* (see Fig. 6) to attain maximum value far from the neighborhood of $\kappa = 0$ rad/m while the *SDF* that is used in our example, when the correlation length parameter b is equal to 70, concentrates 99% of its power at the proximity of 0 rad/m wavenumber i.e. for $\kappa \leqslant 0.13$ rad/m (see Fig. 4). Thus, the integral expression of Eq. (1) produces a deceivingly low variance for the case when $b = 70$ not taking into account the evolution of *VRF* for higher rad/m where practically *SDF* is zero and consequently the classical RDO procedure effectively focuses its selection process on designs that give low *VRF* values at low wave-numbers neglecting what the variance might be for an alternative stochastic field.

Finally, in Fig. 9(a) comparison of *VRF* with the respective *GVRF* generated with the methodology described in Section 3.1 is presented for a randomly selected design of the structural model. The agreement of the two curves validates the conjecture of

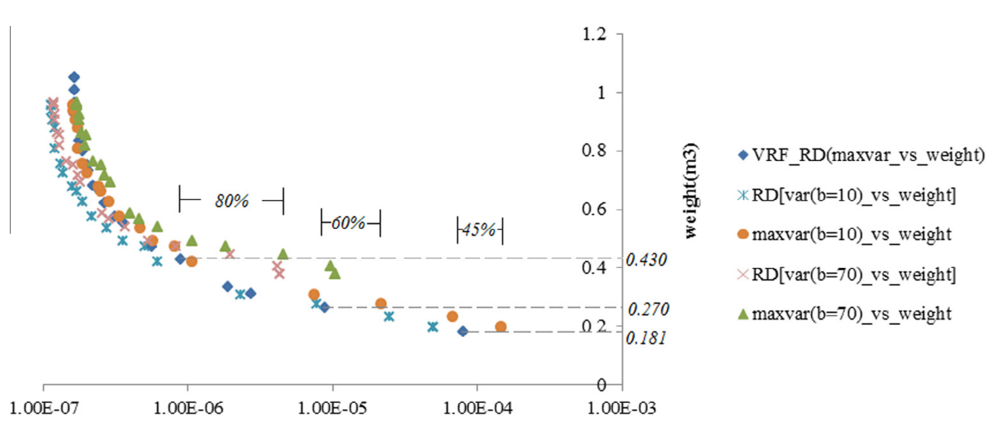

Fig. 8. Pareto fronts for classical RDO with variability response for a given field with $b = 10$, 70 and total weight as objective functions with respective maximum possible variances for the selected designs and Pareto front with *VRF* and total weight as objective functions. Variance axis in logarithmic scale.

independence of the *VRF* from the stochastic parameters of the problem.

7. Conclusions

In the present work, an alternative Robust Design Optimization is proposed based on the concept of Variability Response Function. Taking advantage of the *VRF*'s invariance to the stochastic field's correlation structure and probability distribution, an alternative Robust Design Optimization formulation is achieved that is dependent only upon deterministic parameters of the problem. The VRF-RDO derived Pareto front provides design candidates, through an essentially deterministic procedure, that have an optimal performance taking into account the worst possible stochastic field for the system response. The advantages of using the proposed methodology over traditional Robust Design Optimization are illustrated through an application to a frame-type structure where it is demonstrated that the designs achieved through classical RDO for a given stochastic field description are not optimal for a variation on the spectral properties of the random field modeling the system uncertainty, while designs obtained with the VRF-RDO achieve optimal performance for the worst case scenario stochastic fields.

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