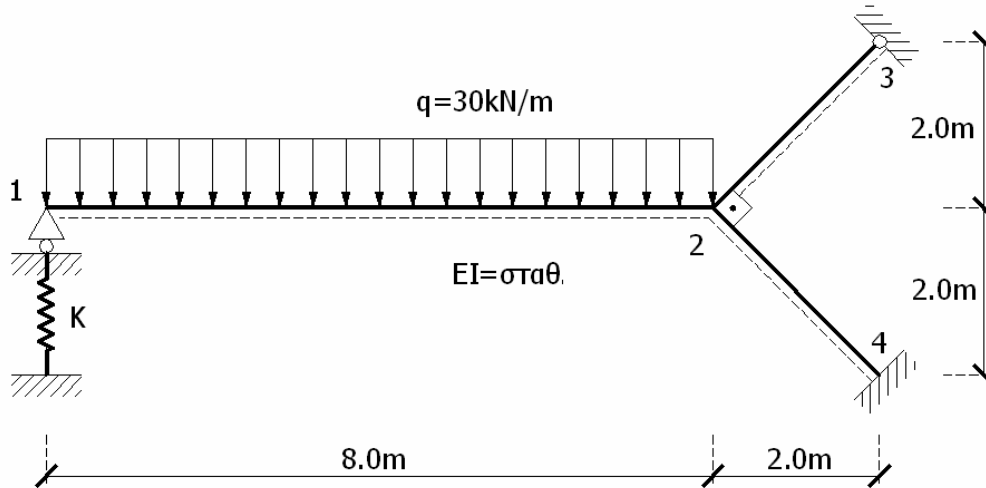


### ΑΣΚΗΣΗ 18

**ΔΕΔΟΜΕΝΑ:**

Στο φορέα του σχήματος ζητούνται να χαραχθούν τα διαγράμματα  $M$ ,  $Q$ ,  $N$  (3 μονάδες)  
 Δίνονται:  $E = 2 \times 10^8 \text{ kN/m}^2$ ,  $I = 100000 \text{ cm}^4$ ,  $k = 6000 \text{ kN/m}$

**ΕΠΙΛΥΣΗ:**

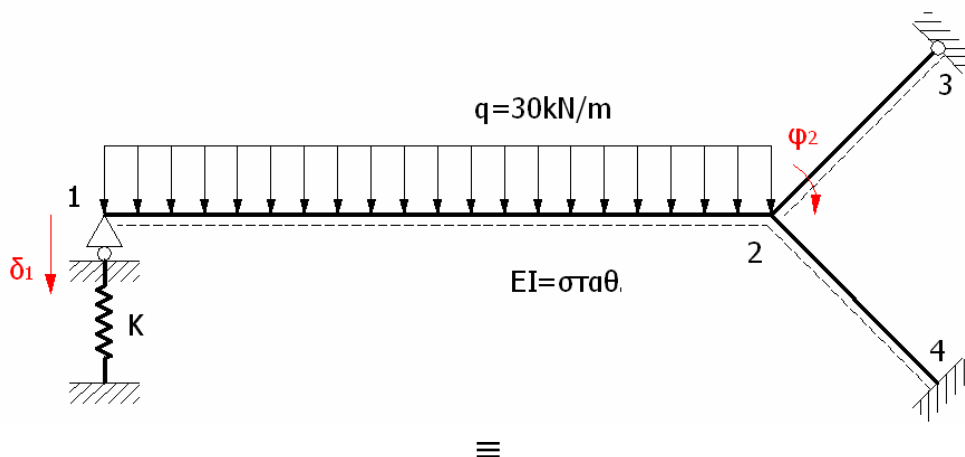
$$E = 2 \times 10^8 \text{ kN/m}^2, \quad I = 100000 \text{ cm}^4 = 10^{-3} \text{ m}^4, \quad EI = 2 \times 10^8 \text{ kN/m}^2 \times 10^{-3} \text{ m}^4 = 2 \times 10^5 \text{ kNm}^2$$

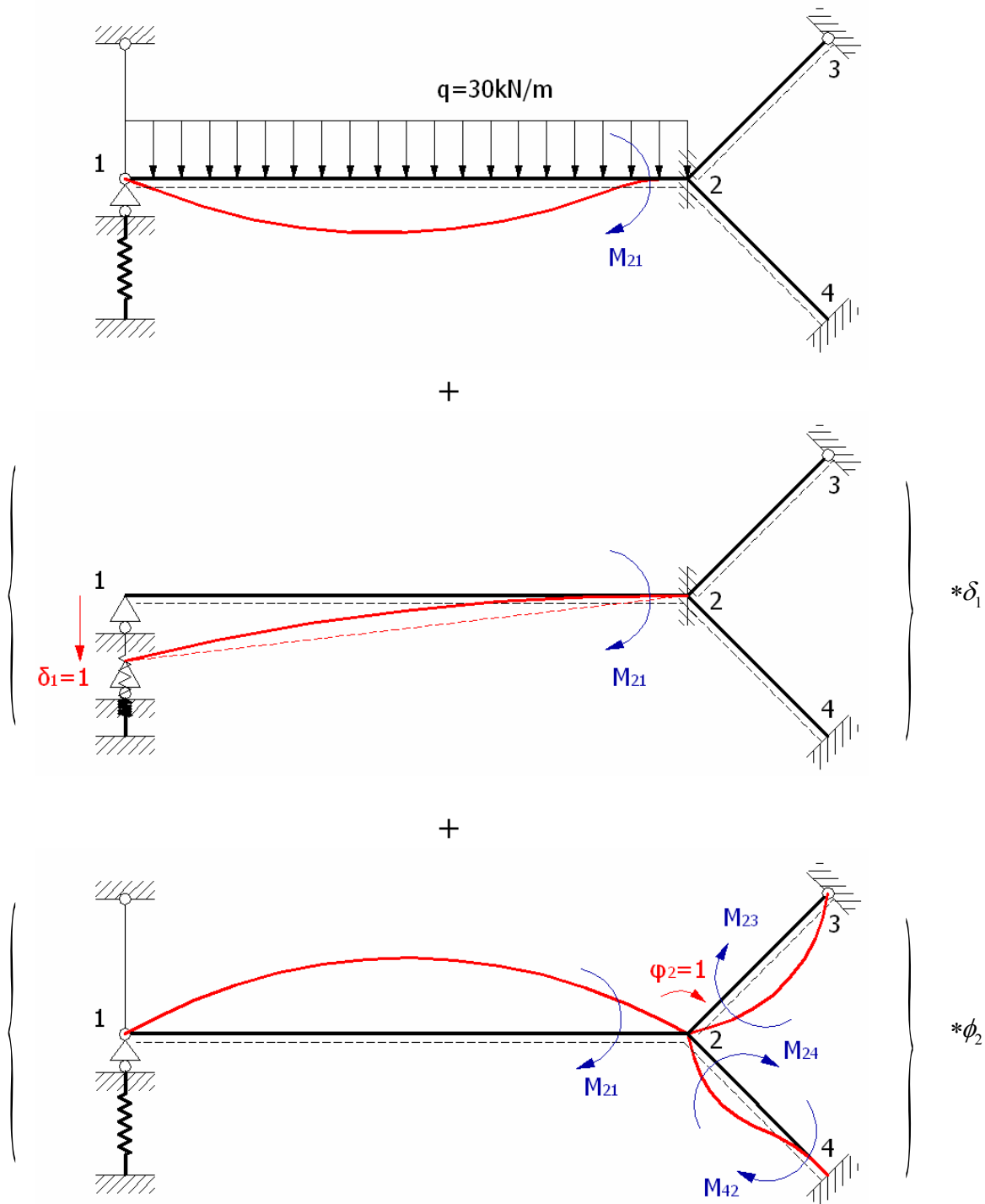
$$l_{23} = l_{24} = 2\sqrt{2} \text{ m}$$

Κινηματική Αοριστία: 2,  $(\delta_1, \phi_2)$

**Μέθοδος των Μετατοπίσεων:**

- Οι εξισώσεις συμβιβαστού ικανοποιούνται
- Απαίτησης ικανοποίησης και των εξισώσεων ισορροπίας





Ροπές Κάμψεως

$$M_{12} = 0$$

$$M_{21} = -240 - \frac{3EI}{8^2} \delta_1 - \frac{3EI}{8} \phi_2$$

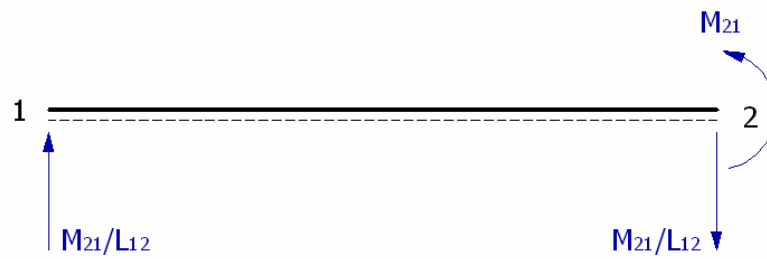
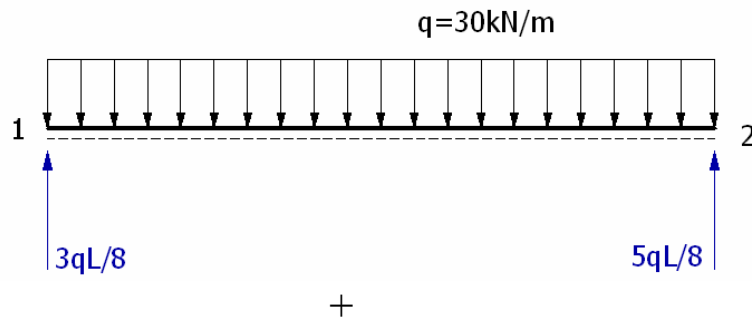
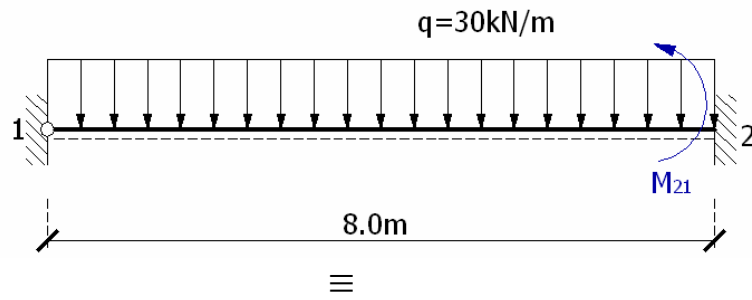
$$M_{23} = \frac{3EI}{2\sqrt{2}} \phi_2$$

$$M_{32} = 0$$

$$M_{24} = \frac{4EI}{2\sqrt{2}} \phi_2$$

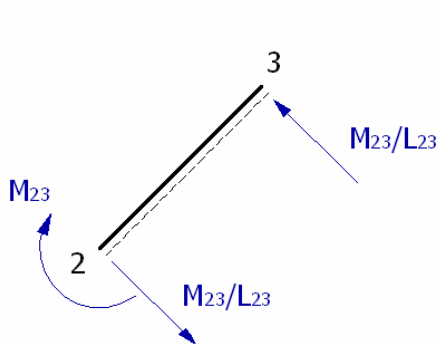
$$M_{42} = -\frac{2EI}{2\sqrt{2}} \phi_2$$

Τέμνουσες Δυνάμεις



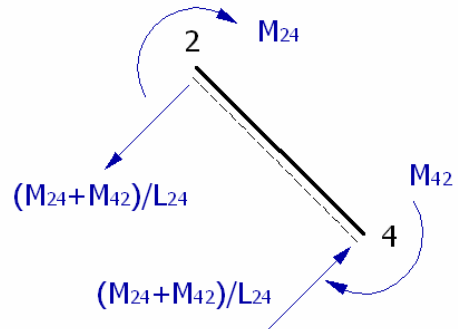
$$Q_{12} = 90 + \frac{M_{21}}{8}$$

$$Q_{21} = -150 + \frac{M_{21}}{8}$$



$$Q_{23} = -\frac{M_{23}}{2\sqrt{2}}$$

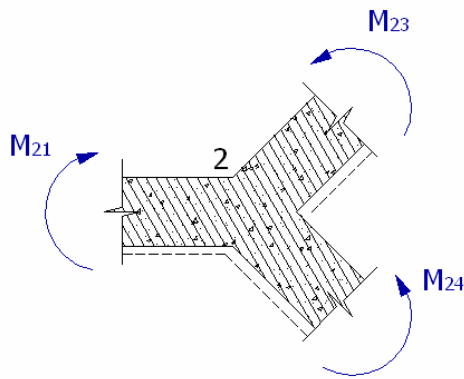
$$Q_{32} = -\frac{M_{32}}{2\sqrt{2}}$$



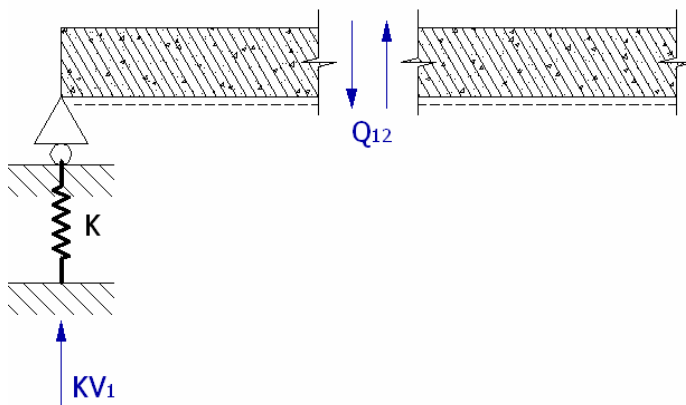
$$Q_{24} = -\frac{M_{24} + M_{42}}{2\sqrt{2}}$$

$$Q_{42} = -\frac{M_{24} + M_{42}}{2\sqrt{2}}$$

## Εξισώσεις Ισοροπίας



$$\begin{aligned}
 M_{21} &= M_{23} + M_{24} \Rightarrow \\
 \Rightarrow -240 - \frac{3EI}{8^2} \delta_1 - \frac{3EI}{8} \phi_2 &= \frac{3EI}{2\sqrt{2}} \phi_2 + \frac{4EI}{2\sqrt{2}} \phi_2 \\
 \Rightarrow -240 - 9375\delta_1 - 75000\phi_2 &= 494974.7468\phi_2 \\
 \Rightarrow \boxed{569974.7468\phi_2 + 9375\delta_1 = -240} &\quad (A)
 \end{aligned}$$



$$\begin{aligned}
 K\delta_1 = Q_{12} = 90 - 30 - 1171.875\delta_1 - 9375\phi_2 &\Rightarrow 6000\delta_1 = 60 - 1171.875\delta_1 - 9375\phi_2 \\
 \Rightarrow \boxed{7171.875\delta_1 + 9375\phi_2 = 60} &\quad (B)
 \end{aligned}$$

Η λύση του συστήματος εξισώσεων (A) και (B) είναι:

$$\begin{aligned}
 \phi_2 &= -5.7095 \times 10^{-4} \text{ rad} \\
 \delta_1 &= +9.1124 \times 10^{-3} \text{ m}
 \end{aligned}$$

Επομένως,

$$M_{12} = 0 \quad M_{21} = -240 - \frac{3EI}{8^2} \delta_1 - \frac{3EI}{8} \phi_2 = -240 - 85.42875 + 42.82125 = -282.61 \text{ kNm}$$

$$M_{23} = \frac{3EI}{2\sqrt{2}} \phi_2 = -121.12 \text{ kNm}$$

$$M_{32} = 0$$

$$M_{24} = \frac{4EI}{2\sqrt{2}} \phi_2 = -161.49 \text{ kNm}$$

$$M_{42} = -\frac{2EI}{2\sqrt{2}} \phi_2 = 80.74 \text{ kNm}$$

$$Q_{12} = 90 + \frac{M_{21}}{8} = 90 - 35.32625 = 54.67 \text{ kN}$$

$$Q_{21} = -150 + \frac{M_{21}}{8} = -150 - 35.32625 = -185.33 \text{ kN}$$

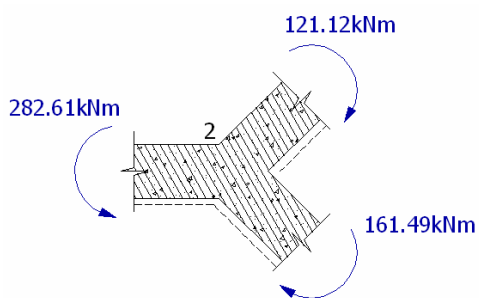
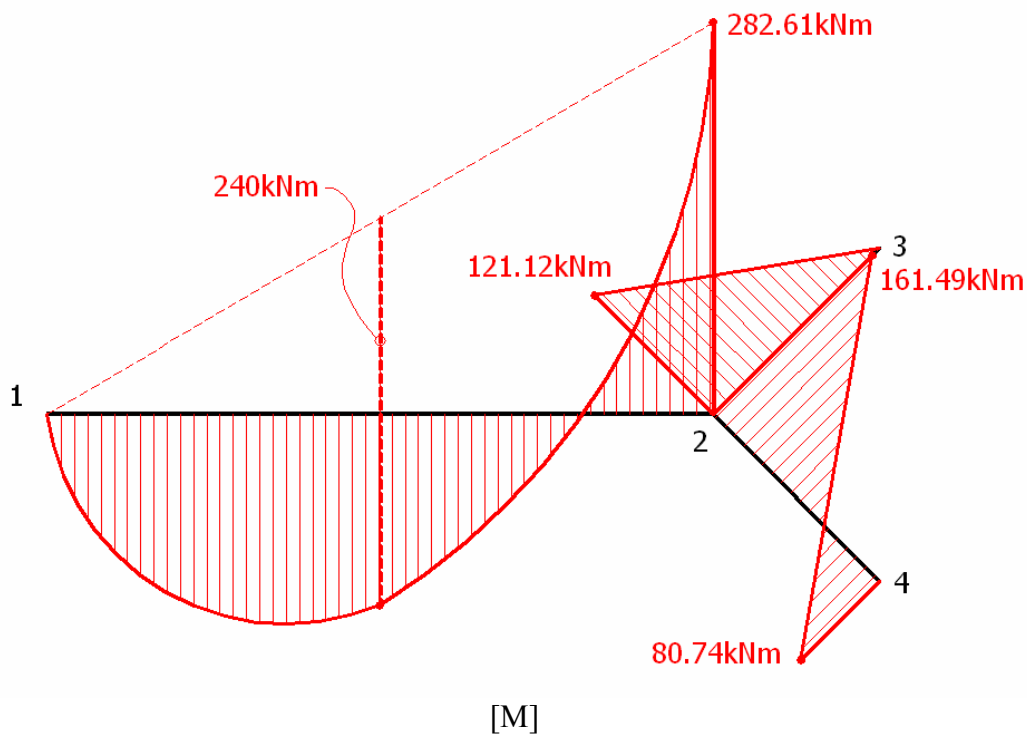
$$Q_{23} = -\frac{M_{23}}{2\sqrt{2}} = 42.82 \text{ kN}$$

$$Q_{32} = Q_{23} = 42.82 \text{ kN}$$

$$Q_{24} = -\frac{M_{24} + M_{42}}{2\sqrt{2}} = \frac{161.49 + 80.74}{2\sqrt{2}} = 85.64 \text{ kN}$$

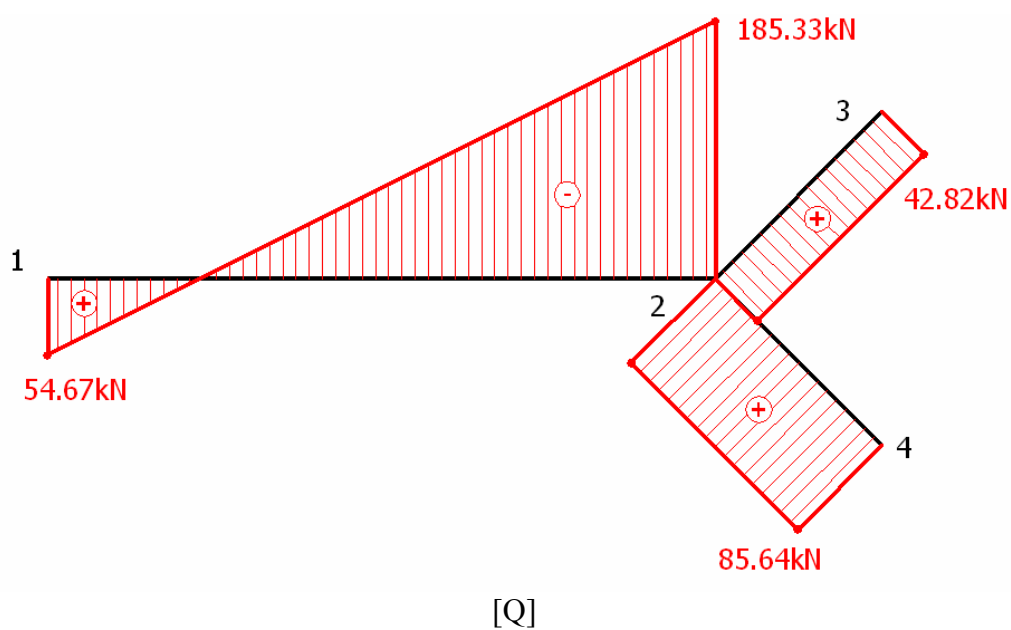
$$Q_{42} = Q_{24} = 85.64 \text{ kN}$$

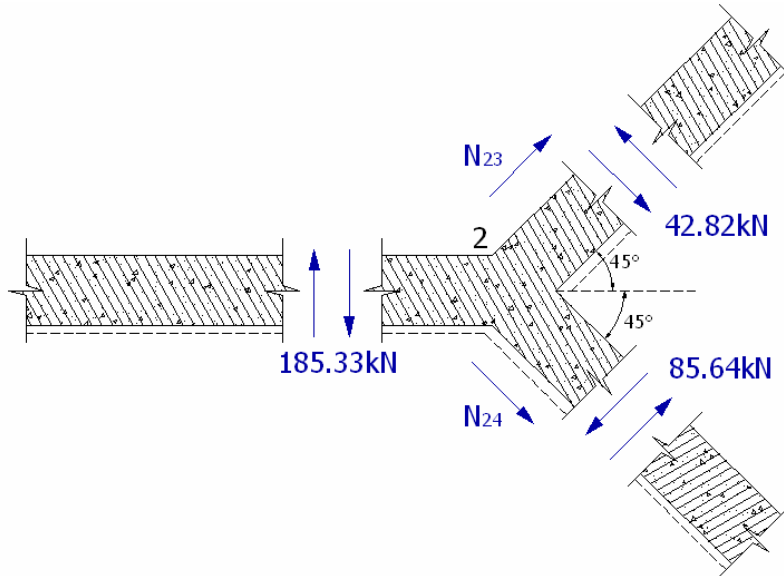
**Διαγράμματα Εντατικών Μεγεθών**



Έλεγχος:

$$121.12kNm + 161.49kNm = 282.61kNm \text{ ισχύει}$$





$$\sum F_x = 0 \Rightarrow 42.82 \cos 45^\circ + N_{23} \cos 45^\circ + N_{24} \cos 45^\circ = 85.64 \cos 45^\circ \Rightarrow$$

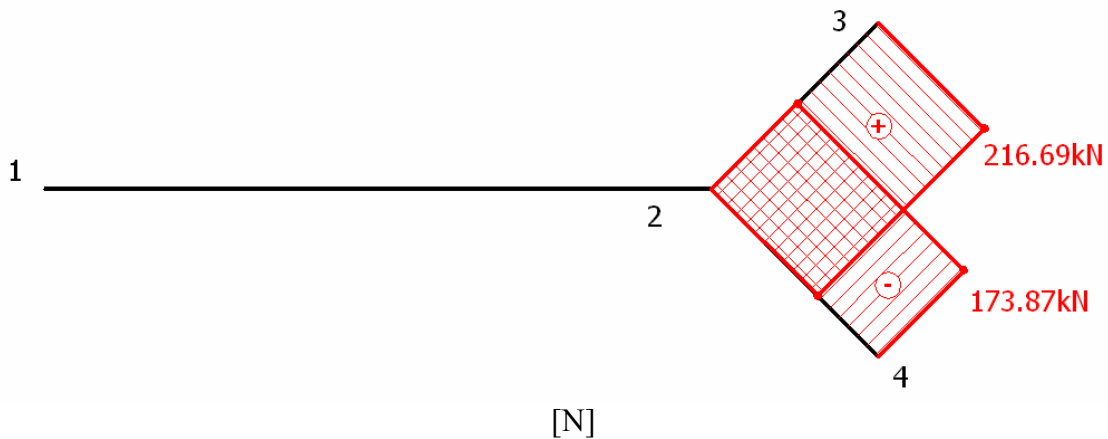
$$\boxed{N_{23} + N_{24} = 42.82} \quad (\Gamma)$$

$$\sum F_y = 0 \Rightarrow 185.33 + 42.82 \sin 45^\circ + 85.64 \sin 45^\circ + N_{24} \cos 45^\circ = N_{23} \sin 45^\circ \Rightarrow$$

$$\boxed{N_{23} - N_{24} = 390.56} \quad (\Delta)$$

Η λύση του συστήματος εξισώσεων (Γ) και (Δ) είναι:

$$\boxed{\begin{aligned} N_{23} &= 216.69 \text{ kN} \\ N_{24} &= -173.87 \text{ kN} \end{aligned}}$$



Έλεγχος:

$$\sum F_x = 0 \Rightarrow 216.69 \cos 45^\circ + 42.82 \sin 45^\circ = 85.64 \cos 45^\circ + 173.87 \sin 45^\circ \Rightarrow 259.51 = 259.51$$