

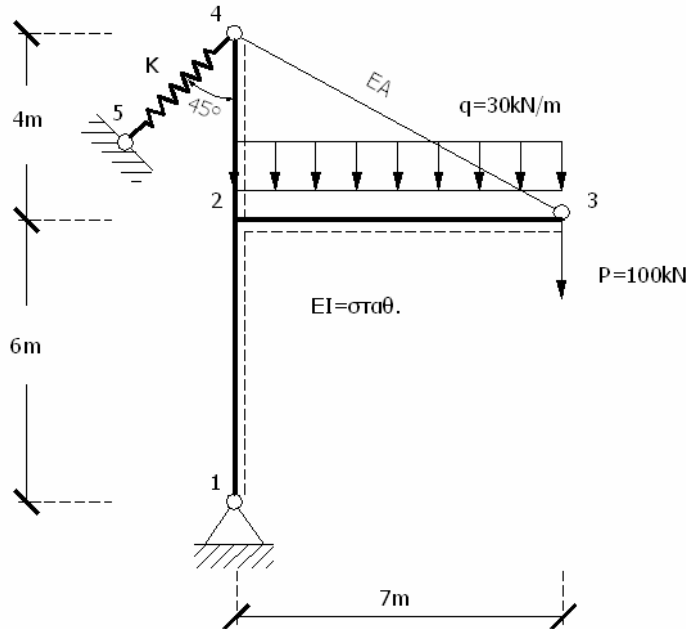
ΑΣΚΗΣΗ 15

ΔΕΔΟΜΕΝΑ:

Στο φορέα του σχήματος ζητούνται:

α) να χαραχθούν τα διαγράμματα M, Q, N (2.5 μονάδες)

β) να υπολογιστεί το μέτρο και η φορά της κατακόρυφης μετατόπισης του κόμβου 3 (1 μονάδα)



Δίνονται:

$$E = 2 \times 10^8 \text{ kN/m}^2$$

$$I = 120000 \text{ cm}^4$$

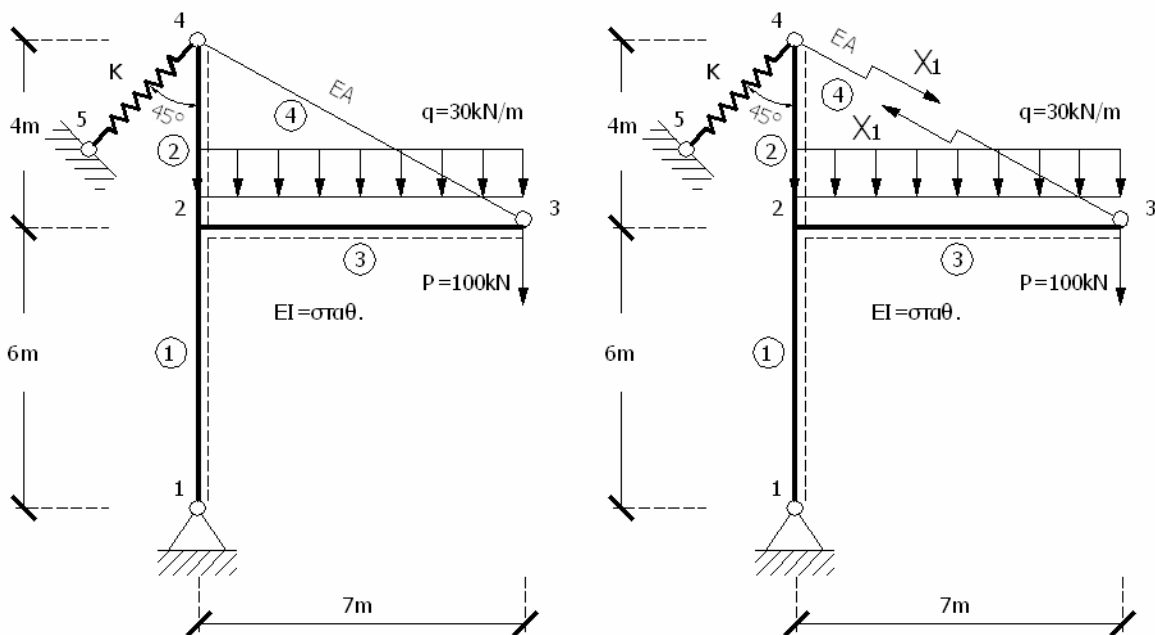
$$A = 10 \text{ cm}^2$$

$$k = 20000 \text{ kN/m}$$

ΕΠΙΛΥΣΗ:

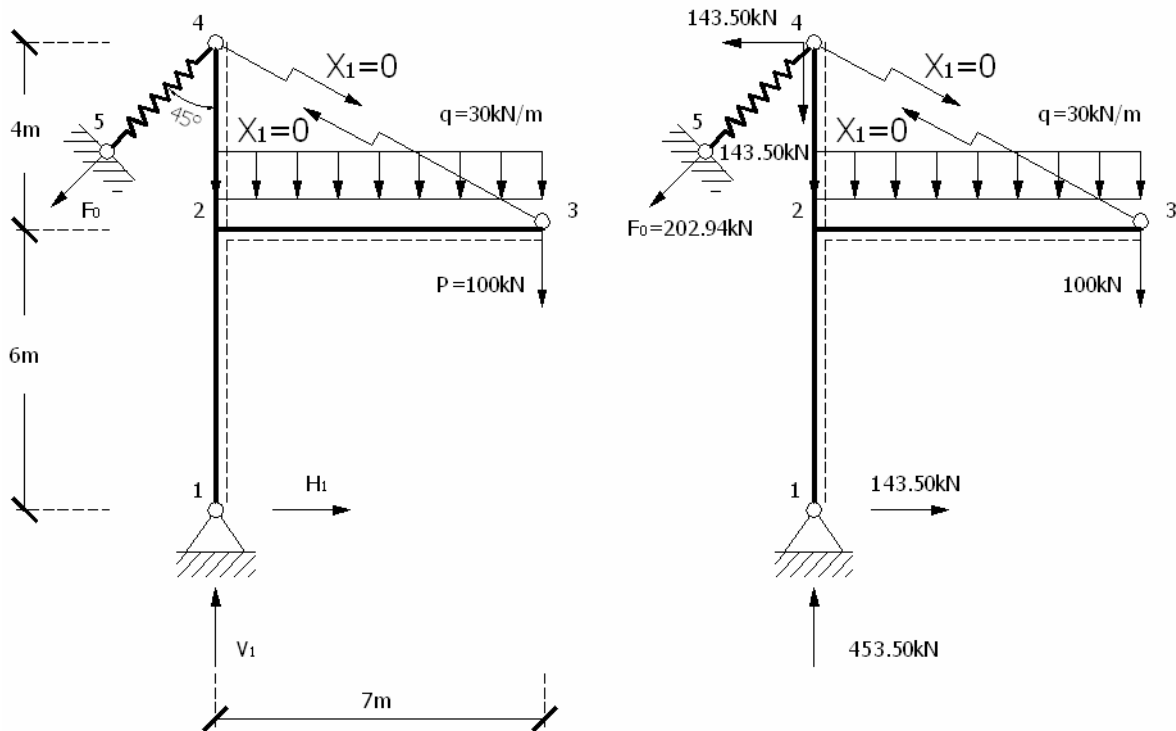
$$E = 2 \times 10^8 \text{ kN/m}^2, I = 120000 \text{ cm}^4 = 1.2 \times 10^{-3} \text{ m}^4, EI = 240000 \text{ kNm}^2$$

$$A = 10 \text{ cm}^2 = 10^{-3} \text{ m}^2, EA = 200000 \text{ kNm}^2/\text{m}^2, k = 20000 \text{ kN/m}, f = 1/k = 5 \times 10^{-5} \text{ m/kN}$$



Στατική Αοριστία = 1 (Αξονική Δύναμη Ράβδου 4)

Θεμελιώδης Φορέας ($X_1 = 0$)

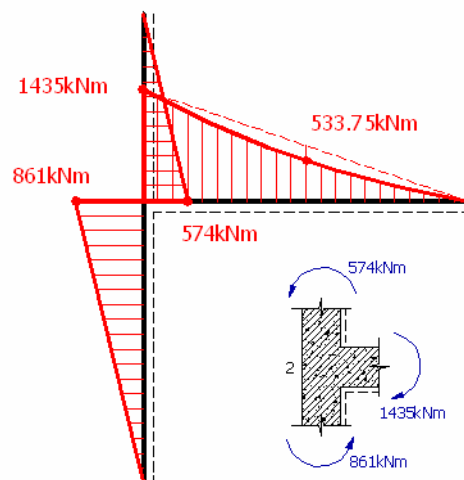


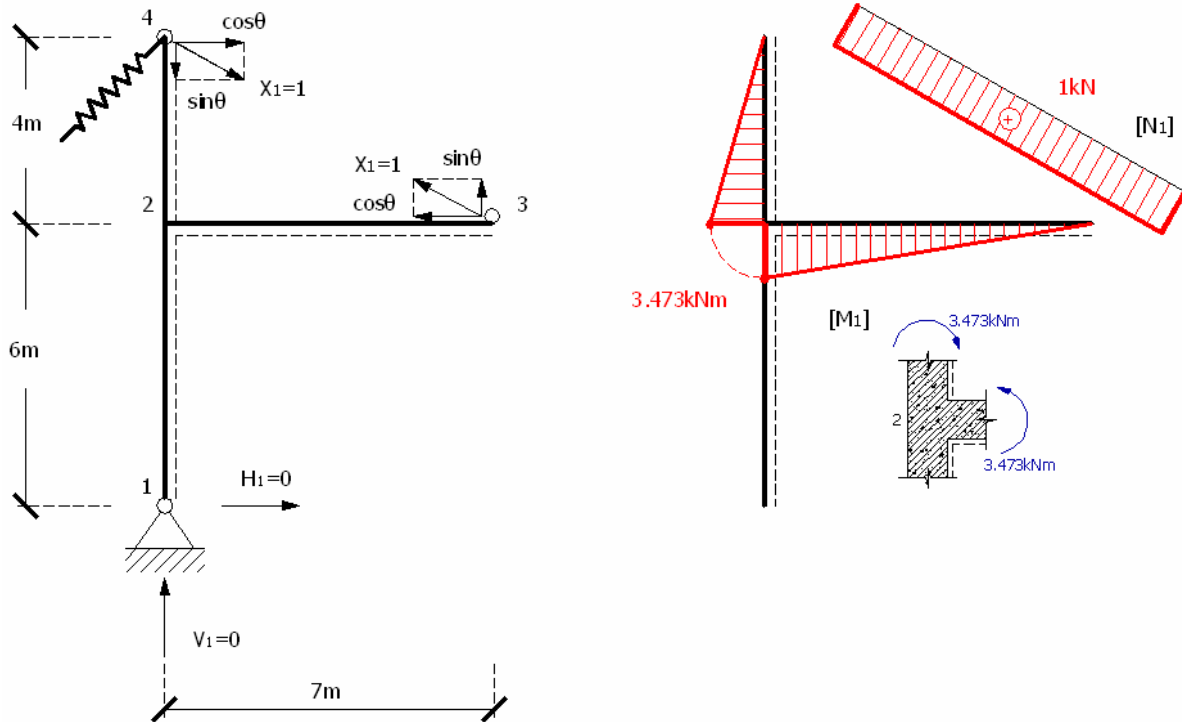
$$\sum F_x = 0 \rightarrow H_1 - 0.7071 \cdot F_0 = 0 \Leftrightarrow \boxed{H_1 = 0.7071 F_0} \quad (1)$$

$$\sum F_y = 0 \rightarrow V_1 - 0.7071 \cdot F_0 - 100 - 30 \cdot 7 = 0 \Leftrightarrow \boxed{V_1 = 310 + 0.7071 F_0} \quad (2)$$

$$\sum M_1 = 0 \rightarrow 100 \cdot 7 + 30 \cdot 7 \cdot 3.5 - 0.7071 \cdot F_0 \cdot 10 = 0 \Leftrightarrow \boxed{F_0 = 202.94 \text{ kN}} \quad (3)$$

Από (2) $\xrightarrow{(3)}$ $\boxed{V_1 = 453.50 \text{ kN}}$ (4) και από (1) $\xrightarrow{(3)}$ $\boxed{H_1 = 143.50 \text{ kN}}$ (5)





Εξίσωση Ελαστικότητας:

$$\sum R_i \cancel{L_s} = 0 + 1 \cdot \cancel{\delta_{ελ}} = F_{10} + F_{11} \cdot x_1 \Rightarrow 0 = F_{10} + F_{11} \cdot x_1 \Rightarrow x_1 = -\frac{F_{10}}{F_{11}} \quad (6), \text{ όπου}$$

$$F_{10} = \sum \frac{N_1 \cdot N_0}{EA} \cdot l + \int \frac{M_1 \cdot M_0}{EI} dx + \sum F_i \cdot F_0 \cdot f = 0 \quad (7)$$

$$F_{11} = \sum \frac{N_1 \cdot N_1}{EA} \cdot l + \int \frac{M_1 \cdot M_1}{EI} dx + \sum F_i \cdot F_1 \cdot f = 0 \quad (8)$$

$$\int \frac{M_1 \cdot M_0}{EI} dx = \frac{1}{EI} \cdot \left[4 \cdot \frac{1}{3} \cdot 574 \cdot (-3.473) + 7 \cdot \frac{1}{6} \cdot 3.473 \cdot (-2 \cdot 533.75 - 1435) \right] =$$

$$= \frac{1}{EI} \cdot [-2658.00 - 10139.71] = -\frac{12797.71}{240000} = -0.0533238$$

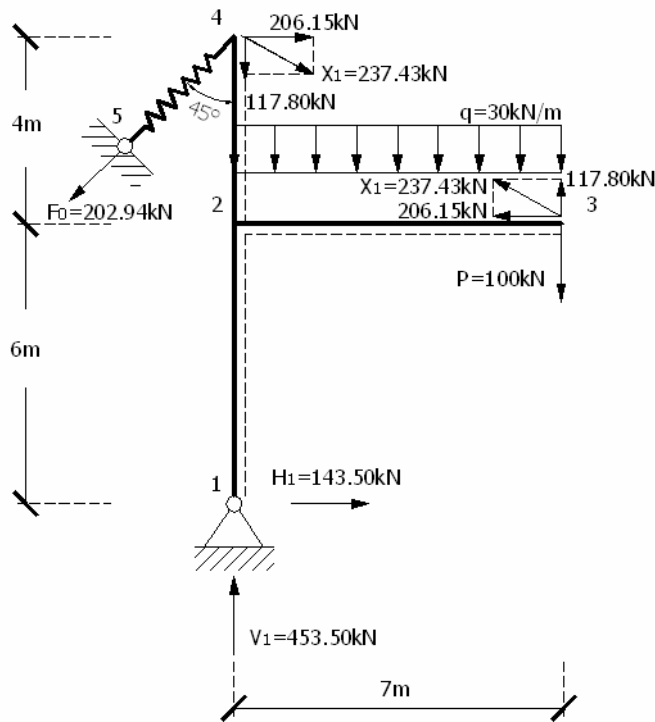
$$\int \frac{M_1 \cdot M_1}{EI} dx = \frac{1}{EI} \cdot \left[4 \cdot \frac{1}{3} \cdot (-3.473)^2 + 7 \cdot \frac{1}{3} \cdot 3.473^2 \right] = -\frac{44.226}{240000} = 1.842764153 \times 10^{-4}$$

$$\sum \frac{N_1 \cdot N_1}{EA} \cdot l = \frac{1 \cdot 1}{200000} \cdot \sqrt{7^2 + 4^2} = 4.03112887 \times 10^{-5}$$

$$(7) \rightarrow F_{10} = -0.0533238$$

$$(8) \rightarrow F_{11} = 4.03112887 \times 10^{-5} + 1.842764153 \times 10^{-4} = 2.24587704 \times 10^{-4}$$

$$(6) \rightarrow x_1 = 237.43 \text{ kN} \quad (9)$$



$$\sum F_x = 0 \rightarrow H_1 - 0.7071 \cdot F_0 = 0 \Leftrightarrow$$

$$H_1 = 0.7071 F_0 \quad (11)$$

$$\sum F_y = 0 \rightarrow$$

$$V_1 - 0.7071 \cdot F_0 - 100 - 30 \cdot 7 = 0 \Leftrightarrow$$

$$V_1 = 310 + 0.7071 F_0 \quad (12)$$

$$\sum M_1 = 0 \rightarrow$$

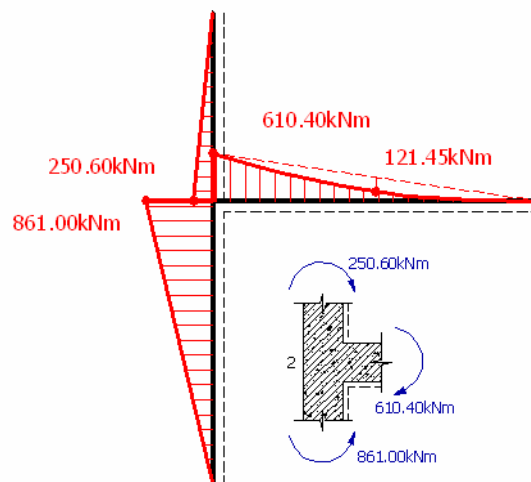
$$0.7071 \cdot F_0 \cdot 10 - 206.15 \cdot 10 + 206.15 \cdot 6 +$$

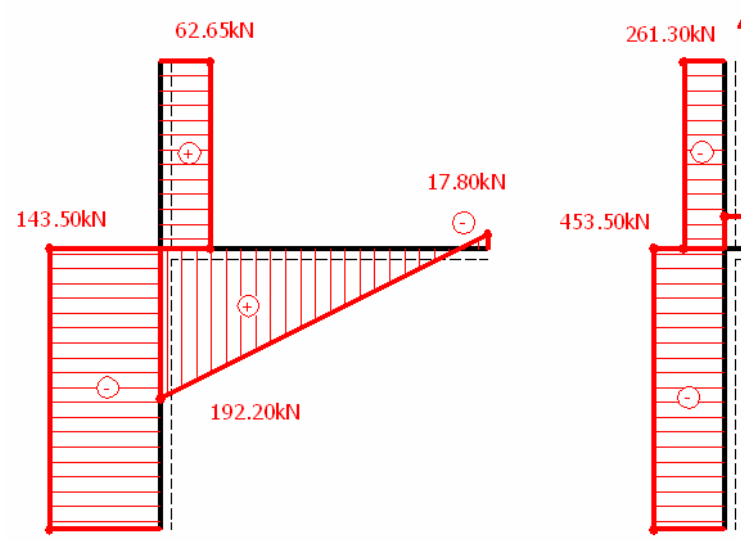
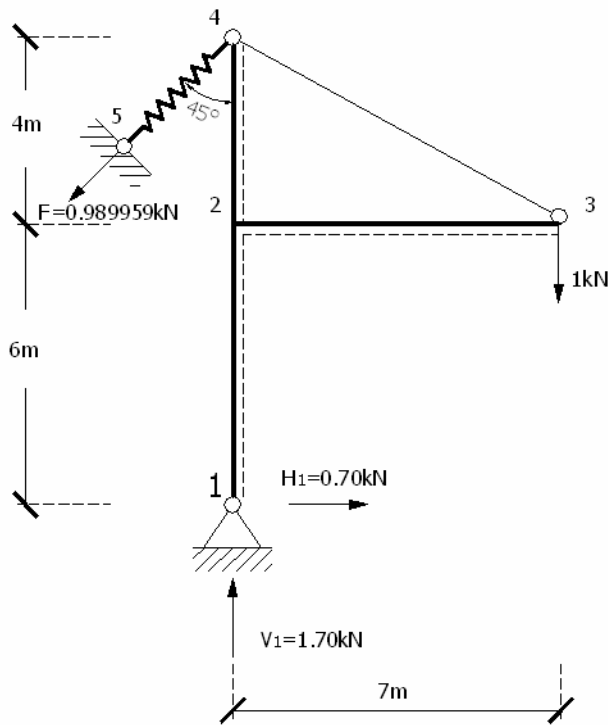
$$+ 117.8 \cdot 7 - 100 \cdot 7 + 30 \cdot 7 \cdot 3.5 = 0 \Leftrightarrow$$

$$F_0 = 202.94 \text{ kN} \quad (13)$$

$$(11) \xrightarrow{(13)} H_1 = 143.50 \text{ kN} \quad (14) \text{ και}$$

$$(12) \xrightarrow{(13)} V_1 = 453.50 \text{ kN} \quad (15)$$





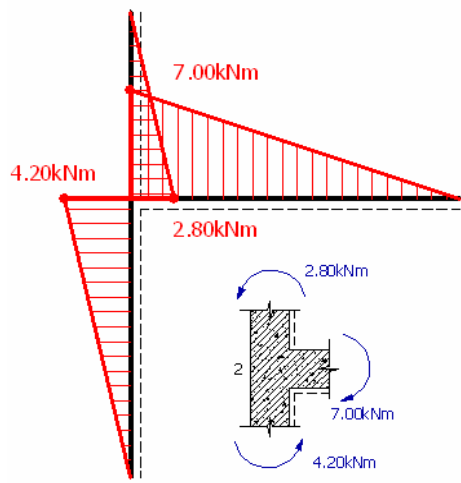
$$\sum F_x = 0 \rightarrow \bar{H}_1 - 0.7071 \cdot \bar{F} = 0 \Leftrightarrow \boxed{\bar{H}_1 = 0.7071\bar{F}} \quad (16)$$

$$\sum F_y = 0 \rightarrow \bar{V}_1 - 0.7071 \cdot \bar{F} - 1 = 0 \Leftrightarrow \boxed{\bar{V}_1 = 1 + 0.7071\bar{F}} \quad (17)$$

$$\sum M_1 = 0 \rightarrow 0.7071 \cdot \bar{F} \cdot 10 - 1 \cdot 7 = 0 \Leftrightarrow \boxed{\bar{F} = 0.989959kN} \quad (18)$$

$$(16) \xrightarrow{(18)} \boxed{\bar{H}_1 = 0.70kN} \quad (19) \text{ και}$$

$$(17) \xrightarrow{(18)} \boxed{\bar{V}_1 = 1.70kN} \quad (20)$$



Θεώρημα Μοναδιαίου Φορτίου (Εξίσωση Ελαστικότητας):

$$1 \cdot \delta_3 = \sum \frac{N_1 \cdot \bar{N}_1}{EA} \cdot l + \int \frac{M \cdot \bar{M}}{EI} dx + \sum F \cdot \bar{F} \cdot f$$

$$\int \frac{M \cdot \bar{M}}{EI} dx = \frac{1}{EI} \cdot \left[6 \cdot \frac{1}{3} \cdot (-861) \cdot (-4.20) + 4 \cdot \frac{1}{3} \cdot (-250.6) \cdot 2.80 \right. \\ \left. + 7 \cdot \frac{1}{6} \cdot (-7) \cdot (-2 \cdot 121.45 - 610.40) \right] = 0.05527268056$$

$$\sum F \cdot \bar{F} \cdot f = 202.94 \cdot 0.989959 \cdot 5 \cdot 10^{-5} = 0.0100451139$$

$$\text{Άρα, } 1 \cdot \delta_3 = \int \frac{M \cdot \bar{M}}{EI} dx + \sum F \cdot \bar{F} \cdot f = 0.06532m$$