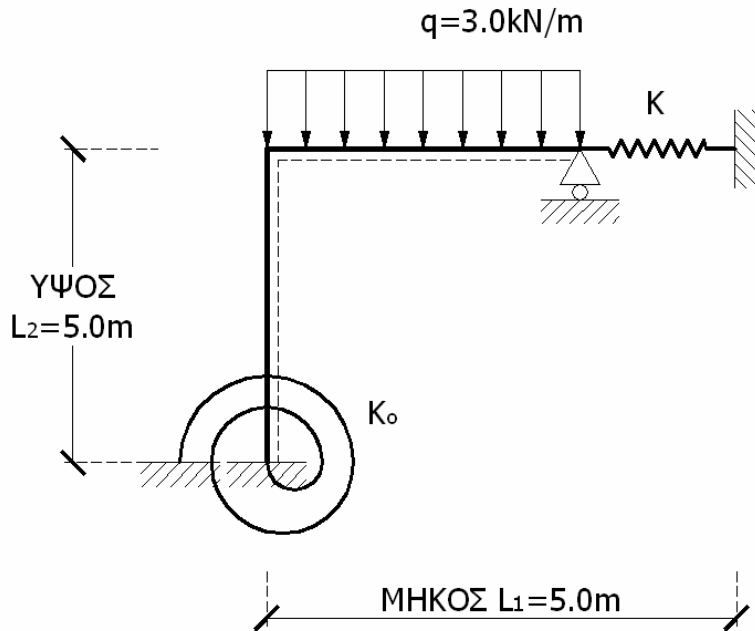


ΑΣΚΗΣΗ 12

ΔΕΔΟΜΕΝΑ:

Να μορφωθούν τα διαγράμματα M,Q του φορέα. που ακολουθεί:



Δίνονται:

$$K_{\phi} = EI/50 \text{ kNm}, K = 40EI$$

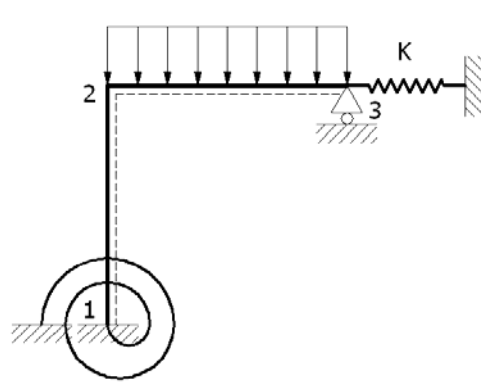
$$EI = 100.000 \text{ kNm}^2$$

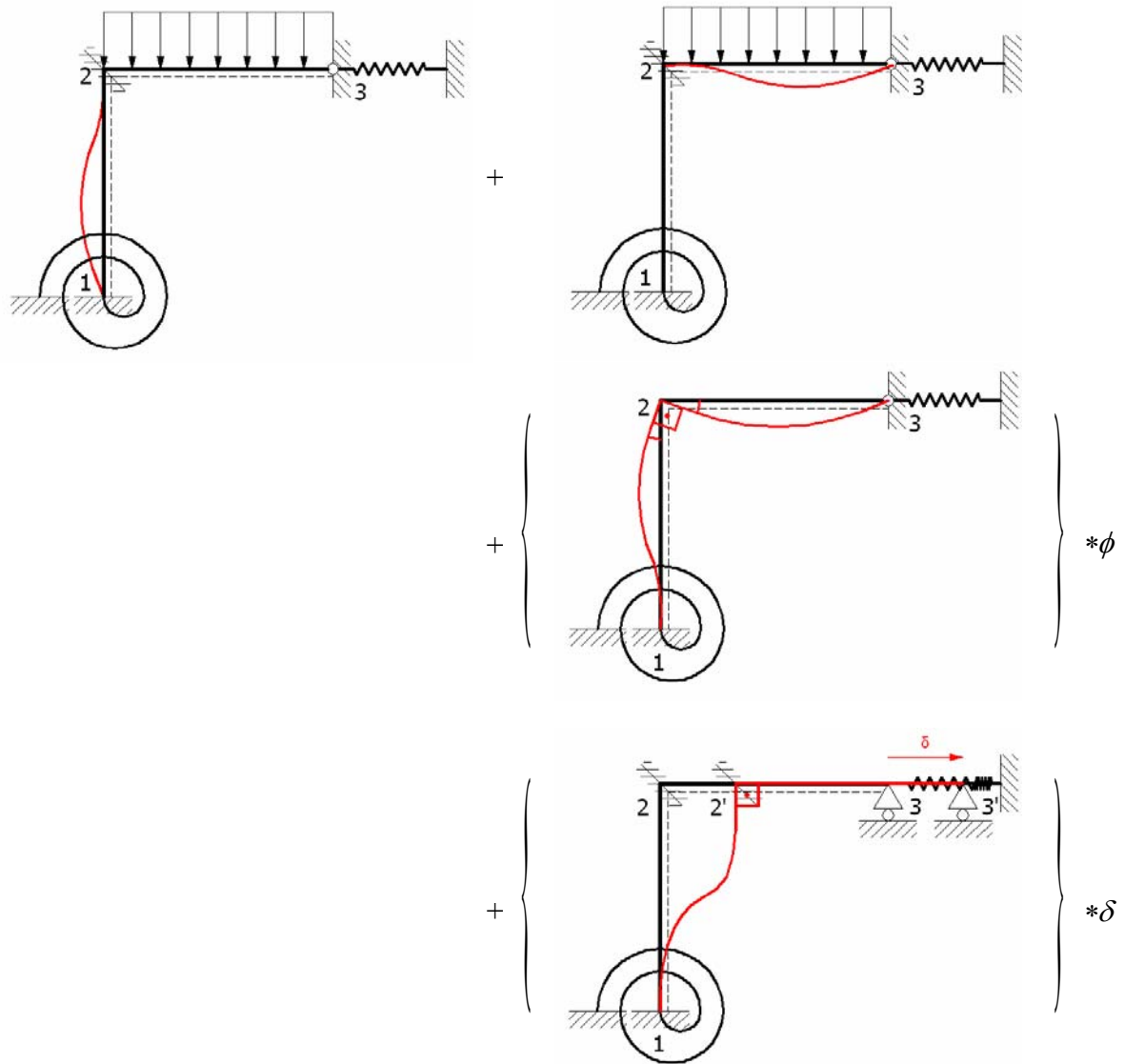
ΕΠΙΛΥΣΗ:

Εύρεση κινηματικής αοριστίας

$$K.A. = 3: \phi_1, \phi_2 \text{ και } \delta = \delta_{x,2} = \delta_{x,3}$$

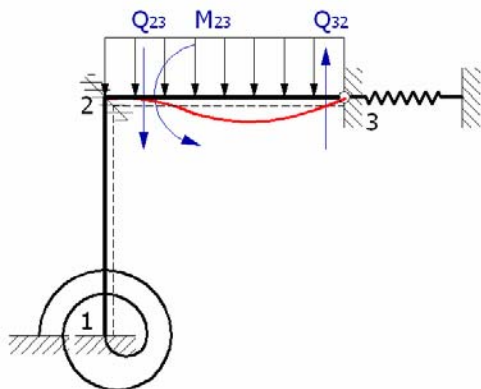
Σύμφωνα με την αρχή της επαλληλίας, έχω:





**Παγιωμένος φορέας ($\phi=\delta=0$)
Μόρφωση ελαστικών γραμμών και υπολογισμός των M,Q**

α) για εξωτερικό φορτίο q

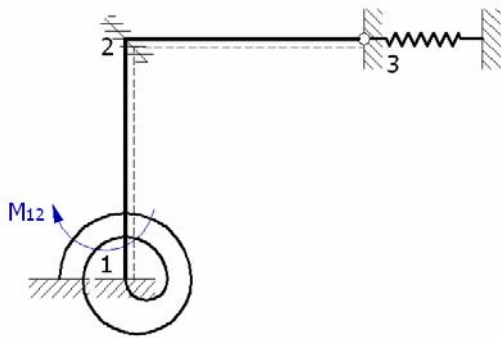


$$M_{23}^{(1)} = ql^2 / 8 = 3 \cdot 5^2 / 8 = 9,375 \text{KNm}$$

$$Q_{23}^{(1)} = 5ql / 8 = 5 \cdot 3 \cdot 5 / 8 = 9,375 \text{KN}$$

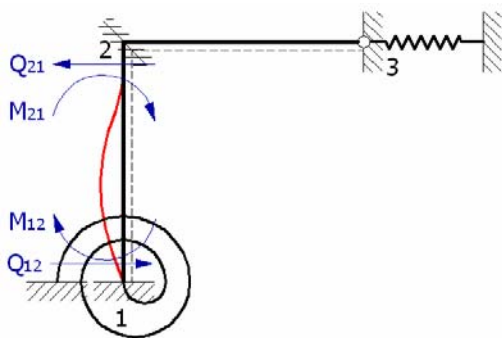
$$Q_{32}^{(1)} = 3ql / 8 = 3 \cdot 3 \cdot 5 / 8 = 5,625 \text{KN}$$

β) λόγω στροφικού ελατηρίου



$$M_{12}^{(2)} = K_{\phi} \cdot \phi_1$$

Παραμορφωσιακή κατάσταση για $\phi_1=1$

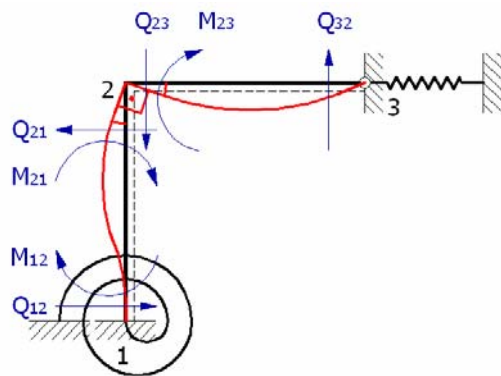


$$M_{12}^{(3)} = \frac{4EI}{L} \phi_1 = \frac{4EI}{5} \phi_1$$

$$M_{21}^{(3)} = \frac{2EI}{L} \phi_1 = \frac{2EI}{L} \phi_1$$

$$Q_{12}^{(3)} = Q_{21}^{(3)} = \frac{6EI}{L^2} \phi_1 = \frac{6EI}{25} \phi_1$$

Παραμορφωσιακή κατάσταση για $\phi_2=1$



$$M_{21}^{(4)} = \frac{4EI}{L} \phi_2 = \frac{4EI}{5} \phi_2$$

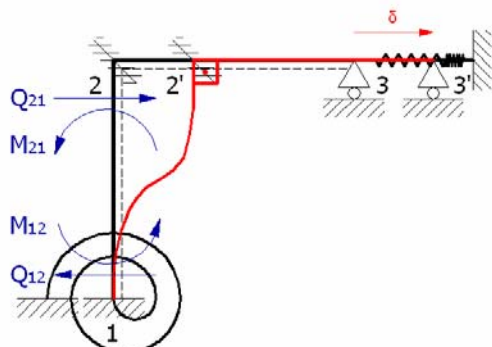
$$M_{12}^{(4)} = \frac{2EI}{L} \phi_2 = \frac{2EI}{L} \phi_2$$

$$Q_{12}^{(4)} = Q_{21}^{(4)} = \frac{6EI}{L^2} \phi_2 = \frac{6EI}{25} \phi_2$$

$$M_{23}^{(4)} = \frac{3EI}{L} \phi_2 = \frac{3EI}{5} \phi_2$$

$$Q_{23}^{(4)} = Q_{32}^{(4)} = \frac{3EI}{L^2} \phi_2 = \frac{3EI}{25} \phi_2$$

Παραμορφωσιακή κατάσταση για $\delta=1$

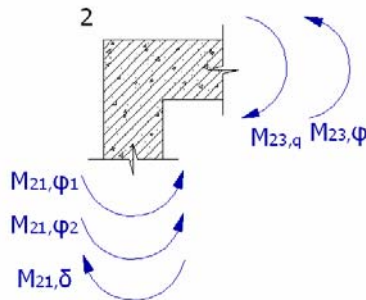


$$M_{12}^{(5)} = M_{21}^{(5)} = \frac{6EI}{L^2} \delta = \frac{6EI}{25} \delta$$

$$Q_{12}^{(5)} = Q_{21}^{(5)} = \frac{12EI}{L^3} \delta = \frac{12EI}{125} \delta$$

Υπολογισμός των μεγεθών ϕ και δ

1) Ισοροπία κόμβου 2: $M_{21}=M_{23}$

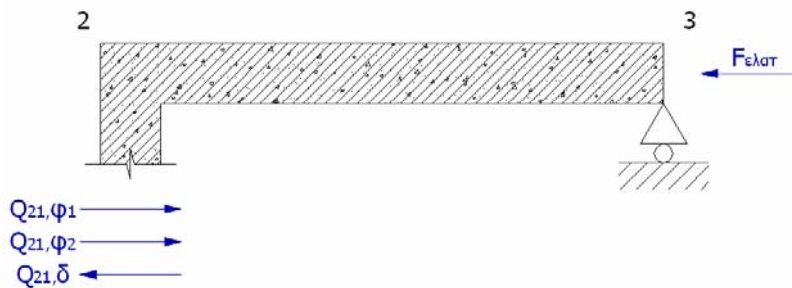


$$-M_{21}^{(3)} - M_{21}^{(4)} + M_{21}^{(5)} = -M_{23}^{(1)} + M_{23}^{(4)} \rightarrow$$

$$-\frac{2EI}{5}\phi_1 - \frac{4EI}{5}\phi_2 + \frac{6EI}{25}\delta = -9,375 + \frac{3EI}{5}\phi_2 \rightarrow$$

$$\boxed{0,4EI\phi_1 + 1,4EI\phi_2 - 0,24EI\delta = 9,375} \quad (1)$$

2) Ισοροπία ζυγώματος (2-3): $Q_{21}+F_{ελαστ.}=0$



$$-Q_{21}^{(3)} - Q_{21}^{(4)} + Q_{21}^{(5)} + F_{ελαστ.} = 0 \rightarrow -\frac{6EI}{25}\phi_1 - \frac{6EI}{25}\phi_2 + \frac{12EI}{125}\delta + K_{ελαστ.}\delta = 0 \rightarrow$$

$$\boxed{0,24EI\phi_1 + 0,24EI\phi_2 - 40,096EI\delta = 0} \quad (2)$$

Από (1),(2) και (3) προκύπτει: $\phi_1=-3,9/EI$, $\phi_2=7,8148/EI$ και $\delta=0,0234/EI$.

Εύρεση εντατικών μεγεθών

$$\left\{ \begin{array}{l} \text{Στύλος 1-2:} \\ M_{12} = -\frac{1}{K_{\phi}}\phi_1 = -\frac{EI}{50}\left(-\frac{3,9}{EI}\right) = 0,078\text{KNm} \\ M_{21} = -M_{21}^{(3)} - M_{21}^{(4)} + M_{21}^{(5)} = -\frac{2EI}{5}\phi_1 - \frac{4EI}{5}\phi_2 + \frac{6EI}{25}\delta = -4,69\text{KNm} \\ Q_{12} = -Q_{12}^{(3)} - Q_{12}^{(4)} + Q_{12}^{(5)} = -\frac{6EI}{25}\phi_1 - \frac{6EI}{25}\phi_2 + \frac{12EI}{125}\delta = -0,94\text{KN} \\ Q_{21} = -Q_{21}^{(3)} - Q_{21}^{(4)} + Q_{21}^{(5)} = -\frac{6EI}{25}\phi_1 - \frac{6EI}{25}\phi_2 + \frac{12EI}{125}\delta = -0,94\text{KN} \end{array} \right.$$

$$\left\{ \begin{array}{l} \text{Ζυγώμα 2-3:} \\ M_{23} = -M_{23}^{(1)} + M_{23}^{(4)} = -9,375 + \frac{3EI}{5}\phi_2 = -4,69\text{KNm} \\ Q_{23} = Q_{23}^{(1)} - Q_{23}^{(4)} = 9,375 - \frac{3EI}{25}\phi_2 = 8,44\text{KN} \\ Q_{32} = -Q_{32}^{(1)} - Q_{32}^{(4)} = -5,625 - \frac{3EI}{25}\phi_2 = -5,56\text{KN} \end{array} \right.$$

Μόρφωση των διαγραμμάτων M,Q

