Electric Power Systems Research 81 (2011) 580–588

Contents lists available at ScienceDirect

Electric Power Systems Research

journal homepage: www.elsevier.com/locate/epsr

A fuzzy logic optimization methodology for the estimation of the critical flashover voltage on insulators

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ARTICLE INFO

Article history:
Received 28 November 2008
Received in revised form 6 October 2010
Accepted 8 October 2010
Available online 23 November 2010

Keywords:
Critical flashover voltage
Confidence interval
Fuzzy logic
High voltage insulators
Polluted insulators

ABSTRACT

The objective of this paper is to present a new methodology for predicting the critical flashover voltage of polluted insulators based on fuzzy logic. The prediction contains not only the estimated value, but also the respective confidence interval based on the re-sampling method. Various parameters, such as the number and the base width of the triangular membership functions used for the fuzzification process, etc., are assigned different values in order to optimize the estimation of the critical flashover voltage. Additionally, different methods for training the fuzzy system are applied and compared for their appropriateness in accurately predicting the critical flashover voltage.

1. Introduction

Insulators are one of the most important components that greatly affect the reliability of the electric system. As, during the last years, the demand on energy consumption increases, higher standards regarding the insulators' strength are set.

An electrolytic layer on the surface of the insulator is formed when the insulator is under heavy atmospheric pollution. When combined with fog or rain, it causes a leakage current to flow the conducting layer. Additionally, surface pollution and non-uniform potential distribution along the insulator surface cause glow discharges or quasi-stable arcs to appear, which are elongated through successive root formation over the polluted insulator surface until the flashover causes the complete bridging. Therefore, it is important that the insulator's condition is monitored so as to ensure that the maintenance takes place in due time.

Several researches concerning the insulators' performance under pollution conditions have been conducted, in which mathematical or physical models have been used, experiments have been carried out or simulation programs have been developed. Since the experiments concerning the critical flashover voltage U_c are time-consuming and have further obstacles, such as high cost and the need for special equipment, approaches based on circuit models for the calculation of the analytical mathematical relationship for either dc or ac flashover voltage on polluted insulators have been developed [1–3]. Furthermore, the complexity of arcing phenomena taking place on the surface of a polluted insulator renders necessary simplifying assumptions during the development of mathematical models describing the aforementioned phenomena. However such assumptions need not be made in applying artificial intelligence methods, such as artificial neural networks and fuzzy logic.

Specifically, artificial neural networks are developed for the qualitative control of the insulators by determining important parameters (such as leakage current or the critical flashover voltage) [4–7].

Furthermore, an adaptive network based fuzzy inference system is applied for the estimation of the flashover voltage on insulators [8]. Additionally, a fuzzy logic algorithm has already been developed for detecting electrical trees in polymeric insulation systems [9], since an electrical tree is the ultimate breakdown mechanism for this kind of insulation systems. In [10], a self-organizing fuzzy inference system is designed for electrode optimization.

In this paper, a methodology was developed for the estimation of the critical flashover voltage of polluted insulators by using fuzzy logic, selecting the optimum training method and the respective

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doi:10.1016/j.epsr.2010.10.024

parameters. The contribution of this paper is focused in its basic features:

- the optimization process to determine the number of the triangular membership functions and their base width,
- the comparison process between different methods for building the fuzzy logic model (see Section 3),
- the use of a small experimental training set through the extension of the training set using the elder mathematical model [3],
- the use of three sets for the total optimization process: the training set, which is used for the training of each fuzzy logic algorithm, the evaluation set, which is used for the selection of the parameters that provide us with the biggest correlation index R between the experimental and estimated values of the critical flashover voltage, and the test set, which is the final under estimation set and proves the generalization ability of the proposed methodology,
- the calculation of the confidence intervals using the re-sampling method [11], so that the width of the confidence interval of the critical flashover voltage for each insulator is calculated beyond the estimated value.

The main advantage of any fuzzy logic method against other methods like regression models and neural networks is that the values of the respective input parameters can be linguistic or can be approached approximately. The advantages of the proposed methodology against previous classical fuzzy models are the optimization process of the characteristics of the triangular membership functions, the selection of the training methods and the calculation of the confidence intervals.

The methodology is successfully implemented for the estimation of the critical flashover voltage of 24 artificially polluted insulators, while the training and evaluation sets are formed by 140 vectors from the mathematical model [3] and other 4 experimental vectors (different from the test set).

In Section 2, the basic principles of fuzzy logic are presented, while in Section 3, the proposed fuzzy logic methodology for the estimation of the critical flashover voltage is analyzed. In Section 4, the implementation of the methodology is shown analytically. Section 5 concludes the fuzzy methodology performance, while in Appendix A the experimental data used as test set are presented.

2. Basic principles of fuzzy logic

The mathematical foundation of fuzzy logic is based on the theory of fuzzy sets, which may be considered as a generalization of the classic theory of sets [12]. Fuzziness is a language attribute. Its main origin is the ambiguity that exists in the definition and use of symbols. The switch from the classic theory of sets, where a strict sense of the participation of an object in a set exists, to the application of fuzzy logic is achieved by the use of the membership functions and the logical rules, which compose the means of realization of the classic fuzzy logic models. These models consist of four elements: the fuzzification, the development of rule basis, the deduction mechanism and the defuzzification, as it is presented in Fig. 1.

Specifically:

- *Fuzzification*: The process through which a non-fuzzy set is converted to a fuzzy set (or through which the fuzziness of the latter merely increases). A linguistic variable is a variable whose arguments are fuzzy numbers and more generally words represented by fuzzy sets. For example, the arguments of the linguistic variable *diameter* may be *small*, *medium* and *large*. We call such arguments *fuzzy values*. Each and every one of them

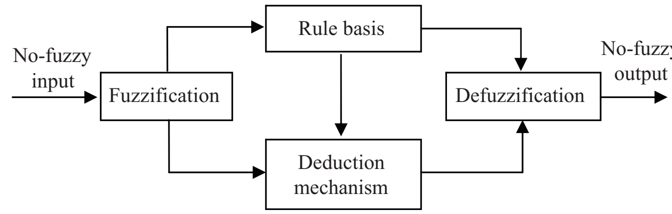

Fig. 1. Basic structure of fuzzy logic [12].

is modelled by a respective membership function. The fuzzy values *small*, *medium* and *large* may be modelled as shown in Fig. 2, where three continuous membership functions, $m_{small}(D)$, $m_{medium}(D)$, $m_{large}(D)$ modelling the arguments *small*, *medium* and *large* respectively, are illustrated. Any value of diameter, e.g. 23 cm has a unique degree of membership to each fuzzy value of *diameter*. In Fig. 2, for example, diameter 23 cm is *small* to a degree *0.40*, *medium* to a degree *0.60* and *large* to a degree zero.

- *Rule base*: It is a set of fuzzy rules describing the dependence between several linguistic variables. These rules are described by the following pattern:

$$\text{IF } A_1 \text{ is } x_1 \text{ AND} \ldots \text{AND } A_N \text{ is } x_N \text{ THEN } B \text{ is } y \tag{1}$$

where $A_1, \ldots, A_N$ are the input variables, $x_1, \ldots, x_N$ are the respective fuzzy values of the input variables, B is the output variable and y is the fuzzy value of the output.

- The *deduction mechanism* is comprised by three sequential steps [12]:
  i. The Larsen–Max Product Implication, which for every rule of one input–one output implies the membership function from the input to the output.
  ii. The degree of fulfillment (DOF), which extends the previous step for more than one variable for each rule. The gth rule for the kth vector is determined:

$$dof_g = m_{A_1,l_{1,g}}(x_{1k}) \cdots m_{A_N,l_{N,g}}(x_{Nk}) \tag{2}$$

  iii. The *border method* forms the final function of the output variable. Fig. 3 illustrates the application of this method in the case of three neighboring activated triangles, where $f_b(x)$ is the respective function of the output variable.
- Defuzzification, which converts the fuzzy output values into real non-fuzzy values. The most common methods are the maximum, the mean value of the maximum and the centre of area (COG, centre of gravity; or COA, centre of area). When the DOF is being used, the criterion of the centre of area is the most suitable [12]:

$$\tilde{b}_0 = \frac{\sum_{j=1}^{n} m_B(w_j) \cdot w_j}{\sum_{j=1}^{n} m_B(w_j)} \tag{3}$$

where $\tilde{b}_0$ is the centre, n is the number of intervals of width d_w, dividing the axis of the output variables, m_B is the membership

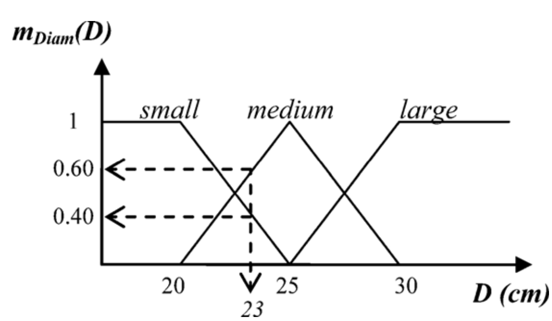

Fig. 2. Membership functions $m_{Diam}(D)$ describing the primary values *small*, *medium* and *large*, of the linguistic variable *diameter*.

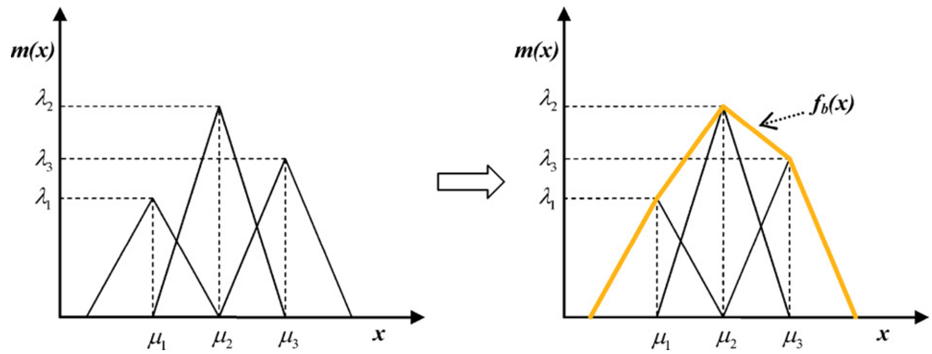

Fig. 3. Formation of the final function of the output variable using the border method.

function of the variable B, w_j is the value for which the membership function becomes $m_B(w_j)$. This method provides mean square error smaller than the maximum method [12].

3. Proposed fuzzy logic methodology for the estimation of the critical flashover voltage

A fuzzy logic methodology is applied in order to estimate the critical flashover voltage of the polluted insulators. This methodology includes the process of selecting the proper training algorithm and of optimizing the respective parameters. The basic steps of the methodology are presented in the flowchart of Fig. 4 and analyzed in the following subsections.

3.1. Data selection and preprocessing

In the first step the input variables are selected, which are the following: the maximum diameter D_m (in cm), the height H (in cm), the creepage distance L (in cm), the form factor F of the insulator and the layer conductivity σ_s (in μS), while the output variable is the critical flashover voltage U_c (in kV). The dataset was built using data acquired from experiments and the application of a mathematical model. In particular, the experiments were carried out in an insulator test station installed in the High Voltage Laboratory of Public Power Corporation's Testing, Research and Standards Center in Athens [13] according to the IEC standard 507:1991 [14]. Following the application of artificial pollution on the insulators, the critical flashover voltage was measured. This set of measurements was enriched by measurements from experiments performed by Sundararajan et al. [15] and Zhicheng and Renyu [2]. In addition, the mathematical model of an equivalent circuit for the evaluation of the critical flashover voltage presented by Topalis et al. [3] is used for the enlargement of the available dataset, as it has already happened for the construction of the ANN model [6], of the ANN optimization methodology [7] and of the adaptive-network based fuzzy inference system [8] for the estimation of the critical flashover voltage. A DC model such as the one applied for the enrichment of the dataset is often used to account for alternating-voltage test results as indicated by Rizk [1].

Afterwards the data preprocessing follows, where data are examined for normality, in order to modify or delete the values that are obviously wrong (*noise suppression*).

3.2. Main procedure

The target of the main procedure is the selection of the most proper training algorithm and its suitable parameters calibration. It is mentioned that the training algorithms differ to the deduction mechanism of the fuzzy value of the output, as it is presented in Section 3.2.2. For each training algorithm the respective parameters of the fuzzy logic model (such as the number of membership functions, the triangle's base width, etc.) are optimized through a set of trials. For each combination of the parameters the fuzzy model is actualized using the training set. After the algorithm execution, the respective fuzzy model for the evaluation set is applied and the R index between the experimental and the estimated values of critical flashover voltage is calculated and the biggest index is chosen as the best one with its parameters. It is noted that:

$$R^2 = r^2_{y-\hat{y}} = \frac{\left(\sum_{i=1}^{n}((y_i - \bar{y}_{real}) \cdot (\hat{y}_i - \bar{y}_{est}))\right)^2}{\sum_{i=1}^{n}(y_i - \bar{y}_{real})^2 \cdot \sum_{i=1}^{n}(y_i - \bar{y}_{est})^2} \tag{4}$$

where y_i is the experimental value of the critical flashover voltage, $\bar{y}_{real}$ the mean experimental value of the respective dataset (training, evaluation or test), $\hat{y}_i$ the estimated value, $\bar{y}_{est}$ the mean estimated value of the dataset, n the population of the respective dataset.

3.2.1. Fuzzy logic procedure 1: the optimization of the parameters of the fuzzy logic model

The main steps of the fuzzy logic procedure 1 (see Fig. 4), which estimates the best combination of the number of membership functions of each variable and the respective triangle's base width according to the performance of the R index for the evaluation set of a specific training algorithm, are presented in Fig. 5 and are as follows:

(1) For each variable, the odd number of the memberships functions t (3, 5, 7 ή 9) is selected.
(2) The centre c_j of the middle triangle of x_jth variable is given by the following expression:

$$c_j = \frac{\sum_{k=1}^{N} x_{jk}}{N} \tag{5}$$

The initial value of the triangle's base width b_{jI} is calculated as:

$$b_{jI} = 2\frac{\left(\max\limits_{k=1,\ldots,N} x_{jk} - \min\limits_{k=1,\ldots,N} x_{jk}\right)}{(t_j - 1)} \tag{6}$$

where N is the number of the training data.

Alternatively, the centre c_j of the middle triangle can be determined by user defined values as:

$$c_j = \frac{(\max\{x_{jk}\} + \min\{x_{jk}\})}{2} \tag{7}$$

Next, the base width of the triangle is modified from $-\alpha$% to α% with step s%, while the centre of the middle triangle remains constant. Thus, the number of possible triangles h to be exam-

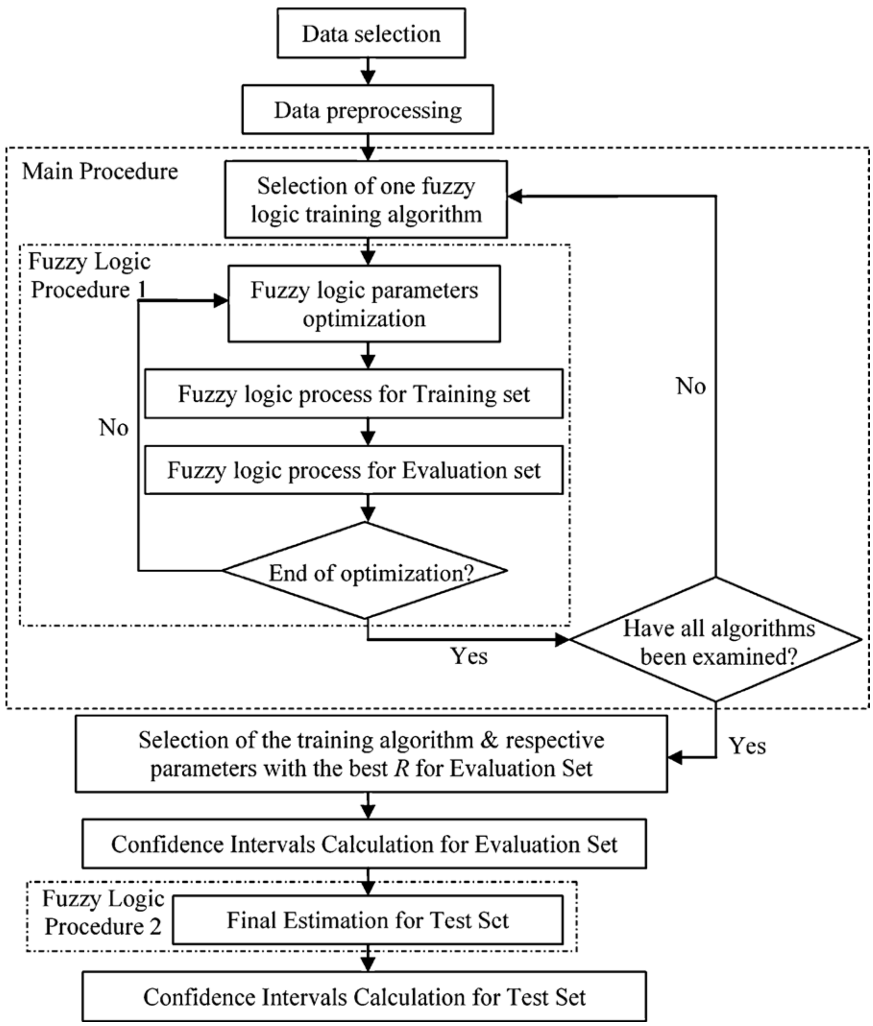

Fig. 4. Flowchart of the fuzzy logic optimization methodology for the estimation of the critical flashover voltage of insulators.

ined per variable equals to:

$$h = 2\left[\frac{a}{s}\right] + 1 \tag{8}$$

Therefore, for n variables, the number of possible combinations is h^n.

(3) For the training set the fuzzification process for all variables is realized.
(4) For the training set the rule base is created via the weight process: Assuming that for each fuzzy output value of the model there is a corresponding weight, i.e. −2, −1, 0, 1, 2 are used for "Very Negative", "NEgative", "ZEro", "PoSitive", "Big Positive" respectively. For instance, in a certain rule the output values may appear with the following frequencies: VN(1), NE(3), ZE(2), PS(2), BP(2), then based on the maximum frequency, the output value would have been "NEgative". By applying the weights method, the output is: $1\cdot(-2)+3\cdot(-1)+2\cdot0+2\cdot1+2\cdot2/(1+3+2+2+2)=0.1$, that is "ZEro". Therefore, for each rule the output with the greater significance according to the training process is selected.
(5) The left part of the rules is determined using the training set and the corresponding output values are produced. By applying the deduction mechanism and the COA method, the non-fuzzy output values are acquired, the respective errors and the R index are calculated.
(6) For the evaluation set the fuzzification process (step 3), the deduction mechanism and the defuzzification process (step 5) are realized. The respective R index is finally calculated, which synopsizes the comparison between the estimated and the actual values.
(7) Steps 1–6 are repeated for all possible combinations of the number of triangles of each variable and of the respective base width forming the respective optimization process. The combination with the biggest R index provides us with the most satisfactory results for the evaluation set.

3.2.2. *Fuzzy logic training algorithms*

In order to deduce the fuzzy value of the output, three cases are examined. The final estimation for the output is made by:

(1) selecting the one with the maximum frequency,
(2) calculating a rounded weighted mean value:

$$y = \left[\frac{\sum f_i y_i}{\sum f_i}\right] \tag{9}$$

where f_i is the frequency by which the output value y_i appears and
(3) calculating a no-rounded weighted mean value:

$$y = \frac{\sum f_i y_i}{\sum f_i} \tag{10}$$

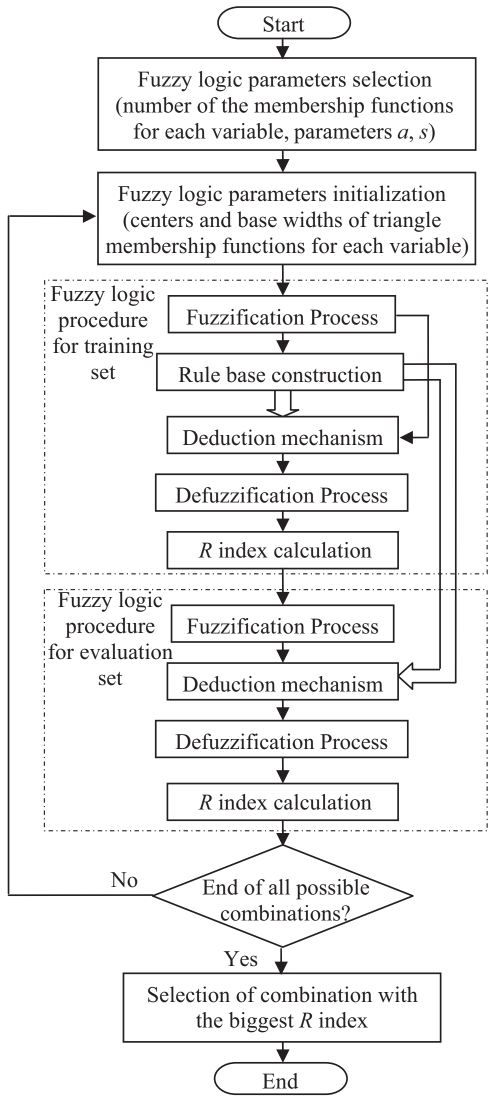

Fig. 5. Flowchart of the fuzzy logic procedure 1 (see Fig. 4) for the estimation of the best combination of the number of membership functions of each variable and the respective triangle's base width according to the performance of the *R* index for the evaluation set of a specific training algorithm.

which means the use of two triangles without rounding. That is, if the fuzzy output values “VN”, “NE”, “ZE”, “PS”, “BP” have the respective weights −2, −1, 0, 1, 2 and the calculated output by Eq. (10) is 0.75, then the second method gives as a result that it belongs to the fourth triangle “PS”, while in the third method the output values belongs by 75% to the fourth triangle “PS” and by 25% to the third “ZE”.

For each one of the above-mentioned methods for deducing the fuzzy value of the output, two cases are examined regarding the centre of the middle triangle. It can be determined:

a. either as the mean value of the training set by Eq. (5), or
b. as the mean value of the upper and lower limits of the training set by Eq. (7).

In Table 1, the six training algorithms are summarized, which have come up from the combination of the three different ways of the final estimation for the output and from the two forms of calculation of the centre of the middle triangle.

3.3. Selection of the best R index from all the training algorithms

From all the training algorithms with the respective optimized parameters it is chosen the one that performs the biggest *R* index for the evaluation set. This algorithm is used for the next processes.

3.4. Calculation of the confidence intervals

The calculation of the confidence interval for the majority of the methods used for estimating unknown parameters is direct. For the fuzzy model, however, this calculation must be done according to one of the proposed methods for ANNs [11], such as multilinear regression adapted to ANNs, error output and re-sampling. In this case the third technique is applied, because the first one refers to ANNs exclusively and the second doubles the number of the fuzzy model's original outputs. According to the re-sampling method [11], the errors of the evaluation set are sorted in ascending order considering the signs and the cumulative sample distribution function of the prediction errors can be estimated as the following:

$$S_{m_1}(z) = \begin{cases} 0, & z < z_1 \\ r/m_1, & z_r \le z < z_{r+1} \\ 1, & z \le z \end{cases} \qquad (11)$$

Whenever m_1 is large enough, $S_{m_1}(z)$ is a good approximation of the true cumulative probability distribution $F(z)$. The confidence interval is estimated by keeping the intermediate z_r and discarding the extreme values, according to the desired confidence degree. The intervals are computed in order to be symmetrical in probability (not necessarily symmetric in z). The number of cases to discard in each tail of the estimation error distribution is $n{\cdot}p$, where p is the probability in each tail. If $n{\cdot}p$ is a fractional number, the number of cases to discard in each tail is $\lfloor n \cdot p \rfloor$ for safeness reasons. When the cumulative probability distribution $F(Z_p)$ is equal to p, there is a probability p that an error is less than or equal to Z_p, which indicates that Z_p is the lower confidence limit. Consequently, Z_{1-p} is the upper limit and there is a $(1-2p)$ confidence interval for future errors.

3.5. Final estimation for the test set

The critical flashover voltage for the polluted insulators of the test set is finally estimated by using the fuzzy logic procedure 2 (see Fig. 4) based on the training algorithm of the main procedure's step with the respective fuzzy model parameters. It is presented in Fig. 6 and it consists of the following four steps: (1) the fuzzification process, (2) the deduction mechanism based on the rule base formed by the training set, (3) the defuzzification process and (4) the *R* index calculation. Practically, it is similar to the fuzzy logic procedure for the evaluation set.

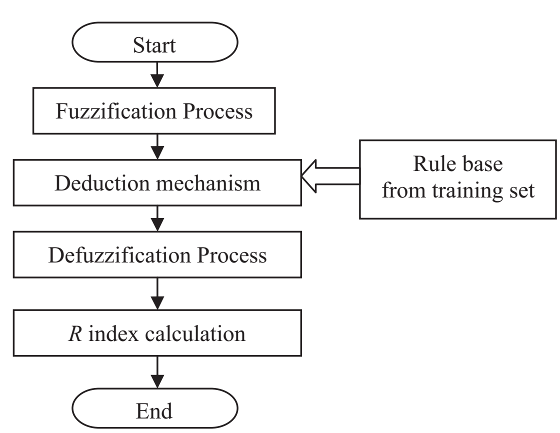

Fig. 6. Flowchart of the fuzzy logic procedure 2 (see Fig. 4) for the estimation of the critical flashover voltage for the polluted insulators of the test set.

Table 1
Description of fuzzy logic training algorithms.

No.	Description of fuzzy logic training algorithms
1	The final estimation for the output is made by maximum frequency and the middle triangle of each variable is estimated by the mean value of the training set.
2	The final estimation for the output is made by maximum frequency and the middle triangle of each variable is estimated by the mean value of the upper and lower limits of the training set.
3	The final estimation for the output is made by rounded weighted mean value and the middle triangle of each variable is estimated by the mean value of the training set.
4	The final estimation for the output is made by rounded weighted mean value and the middle triangle of each variable is estimated by the mean value of the upper and lower limits of the training set.
5	The final estimation for the output is made by no rounded weighted mean value and the middle triangle of each variable is estimated by the mean value of the training set.
6	The final estimation for the output is made by no rounded weighted mean value and the middle triangle of each variable is estimated by the mean value of the upper and lower limits of the training set.

Table 2
Synopsis of the consecutive executions followed for determining the optimal values for the base width and the number of triangles for each variable for the first training method of Table 1.

Calibration of parameter	x_1	x_2	x_3
Parameters' range	$h_1=\{3,5,7,9\}$, $\alpha_1=50\%$, $s_1=5\%$, $h_2=3$, $h_3=3$, $h_4=9$, $h_5=9$, $h_6=5$, $\alpha_2=\alpha_3=\alpha_4=\alpha_5=\alpha_6=0$	$h_1=3$, $\alpha_1=5\%$, $h_2=\{3,5,7,9\}$, $\alpha_2=50\%$, $s_2=5\%$, $h_3=3$, $h_4=9$, $h_5=9$, $h_6=5$, $\alpha_3=\alpha_4=\alpha_5=\alpha_6=0$	$h_1=3$, $\alpha_1=5\%$, $h_2=3$, $\alpha_2=50\%$, $h_3=\{3,5,7,9\}$, $\alpha_3=50\%$, $s_3=5\%$, $h_4=9$, $h_5=9$, $h_6=5$, $\alpha_4=\alpha_5=\alpha_6=0$
Optimal choice	$h_1=3$, $\alpha_1=5\%$	$h_2=3$, $\alpha_2=50\%$	$h_3=3$, $\alpha_3=10\%$

Calibration of parameter	x_4	x_5	y
Parameters' range	$h_1=3$, $\alpha_1=5\%$, $h_2=3$, $\alpha_2=50\%$, $h_3=3$, $\alpha_3=10\%$, $h_4=\{3,5,7,9\}$, $\alpha_4=50\%$, $s_4=5\%$, $h_5=9$, $h_6=5$, $\alpha_5=\alpha_6=0$	$h_1=3$, $\alpha_1=5\%$, $h_2=3$, $\alpha_2=50\%$, $h_3=3$, $\alpha_3=10\%$, $h_4=7$, $\alpha_4=-30\%$, $h_5=\{3,5,7,9\}$, $\alpha_5=50\%$, $s_5=5\%$, $h_6=5$, $\alpha_6=0$	$h_1=3$, $\alpha_1=5\%$, $h_2=3$, $\alpha_2=50\%$, $h_3=3$, $\alpha_3=10\%$, $h_4=7$, $\alpha_4=-30\%$, $h_5=9$, $\alpha_5=-5\%$, $h_6=\{3,5,7,9\}$, $\alpha_6=50\%$, $s_6=5\%$
Optimal choice	$h_4=7$, $\alpha_4=-30\%$	$h_5=9$, $\alpha_5=-5\%$	$h_6=5$, $\alpha_6=-40\%$

The prospective confidence intervals are the respective ones of the confidence intervals' process. It is also possible to calculate the confidence intervals for the test set for comparative purpose.

4. Critical flashover voltage estimation using proposed fuzzy logic methodology

The developed methodology is applied for the critical flashover voltage estimation of polluted insulators. The data from the mathematical model and a set of the experimental data, containing the maximum and minimum values are used to train the fuzzy model, while the rest of the experimental data was used to test its performance. The training set consists of 144 patterns/vectors (of which 140 vectors are derived from the model and 4 vectors are real values), the evaluation set consists of 36 patterns/vectors and the fuzzy model is tested using 24 patterns (experimental data). The evaluation set is a subset of the training set with randomly chosen vectors and not an independent one because of the minimal population of vectors.

Following, the proposed methodology is applied, as it has been presented in Section 3. The parameters that need to be tuned are: the number of triangles used for the fuzzification process for each input variable (h_1 through h_6) and the percentage by which the initial value of the triangle's base is varied (α_1 through α_6) (which will ultimately determine the optimal selection for the base width).

For each one possible combination of the variables, the parameters of the fuzzy model need to be specified. In order to reduce the combinations that need to be examined, three steps are taken:

(1) In the first step, the optimal combination for the number of triangles for the five input variables and for the output variable is achieved, when the algorithm is executed while the number of triangles is simultaneously varied for the six parameters from 3 to 9 with step 2 and the triangles' base width

Table 3
Values interval during the optimization process of each parameter of every fuzzy logic training algorithm.

No.	Correlation index R			Triangles' number and base width
	Training set	Evaluation set	Test set	
1	0.936	0.945	0.931	$h_1=3$, $\alpha_1=5\%$, $h_2=3$, $\alpha_2=50\%$, $h_3=3$, $\alpha_3=10\%$, $h_4=7$, $\alpha_4=-30\%$, $h_5=9$, $\alpha_5=-5\%$, $h_6=5$, $\alpha_6=-40\%$
2	0.920	0.899	0.914	$h_1=5$, $\alpha_1=-50\%$, $h_2=3$, $\alpha_2=5\%$, $h_3=3$, $\alpha_3=50\%$, $h_4=5$, $\alpha_4=-35\%$, $h_5=9$, $\alpha_5=0\%$, $h_6=3$, $\alpha_6=-50\%$
3	0.978	0.974	0.964	$h_1=3$, $\alpha_1=50\%$, $h_2=3$, $\alpha_2=50\%$, $h_3=3$, $\alpha_3=15\%$, $h_4=3$, $\alpha_4=0\%$, $h_5=9$, $\alpha_5=0\%$, $h_6=9$, $\alpha_6=-35\%$
4	0.960	0.951	0.918	$h_1=3$, $\alpha_1=50\%$, $h_2=3$, $\alpha_2=0\%$, $h_3=3$, $\alpha_3=-25\%$, $h_4=2$, $\alpha_4=50\%$, $h_5=9$, $\alpha_5=50\%$, $h_6=9$, $\alpha_6=0\%$
5	*0.984*	*0.979*	*0.967*	$h_1=3$, $\alpha_1=50\%$, $h_2=3$, $\alpha_2=50\%$, $h_3=3$, $\alpha_3=-35\%$, $h_4=3$, $\alpha_4=15\%$, $h_5=9$, $\alpha_5=-5\%$, $h_6=9$, $\alpha_6=-45\%$
6	0.966	0.960	0.945	$h_1=3$, $\alpha_1=-10\%$, $h_2=3$, $\alpha_2=50\%$, $h_3=3$, $\alpha_3=50\%$, $h_4=7$, $\alpha_4=-5\%$, $h_5=9$, $\alpha_5=-10\%$, $h_6=9$, $\alpha_6=-35\%$

The optimal fuzzy logic training algorithm is No. 5 (denoted in italics).

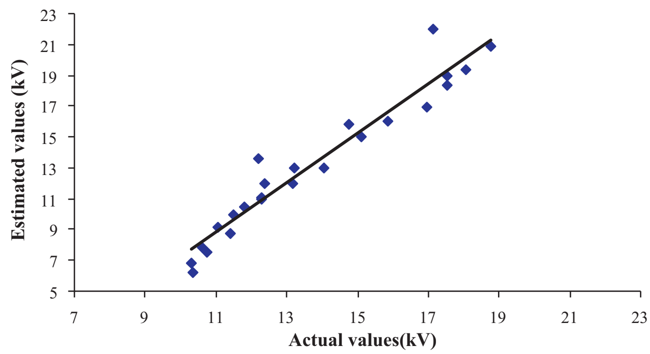

Fig. 7. Correlation between the actual and estimated values for the critical flashover voltage for 5th method (the final estimation for the output is made by no rounded weighted mean value and the middle triangle of each variable is estimated by the mean value of the training set.).

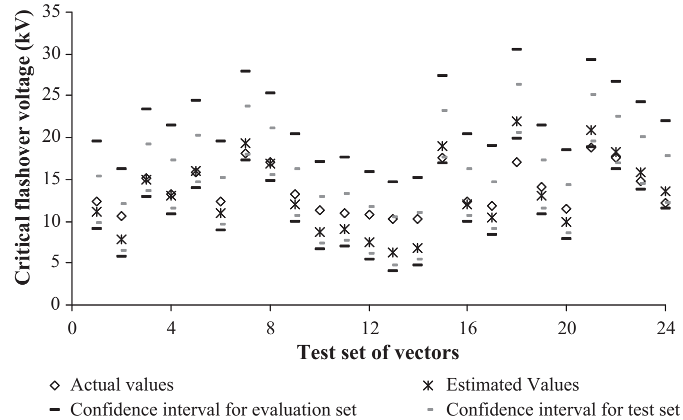

Fig. 8. Representation of the actual and the estimated values and of the confidence interval for the evaluation and test set with 5% probability in each tail for 5th method (the final estimation for the output is made by no rounded weighted mean value and the middle triangle of each variable is estimated by the mean value of the training set).

remains unchanged. $2^{12} = 4096$ repetitions of the execution of the algorithm are realized. During each execution, the correlation between the actual and the estimated values is calculated. The results are expanded by making the following comparison: firstly, the combination, for which the maximum correlation for the evaluation set is achieved, is determined. Following, several other combinations that give satisfactory correlation close to the optimal are examined, so that the stability of the best solution can be confirmed. By this process, the optimal combination for the number of triangles is determined.

(2) In the second step, the base width and the number of triangles for each parameter are determined separately, by giving fixed values for the parameters of all variables but one. Then, for each one of the remaining parameters the optimal value is similarly selected, while assigning to the rest of the parameters the optimal values that emerged from previous executions, if any. Table 2 summarizes these executions as described above for the 1st method of Table 1 (The final estimation for the output is made by maximum frequency and the middle triangle of each variable is estimated by the mean value of the training set.). At the same time, each parameter's "optimal" region, which gives similar results to the optimal choice, is determined.

(3) In the third step, the main process is repeated for the reduced number of combinations, in which all parameters can take any value of their respective "optimal" regions, as determined in the first and the second step. After this final execution of the algorithm, the optimization process comes to an end and the combination that presents the maximum R index in the estimation of the evaluation set is selected. This combination is used for the estimation of the critical flashover voltage of the test set.

A polarization of the values (systematic error) is observed resulting to non-zero mean error, because in fuzzy logic models the minimization of an error function is not realized as in regression and ANN models. In order to avoid this, the polarization of the training set is added to the estimated values. By this way, a zero mean error is achieved without affecting the correlation value.

Table 3 summarizes the optimal results for each training algorithm with the respective values of the parameters provided by the optimization process. Specifically, by comparing methods 1–3–5 and 2–4–6 we conclude that the best results are achieved when the estimation for the critical flashover voltage is done using the no

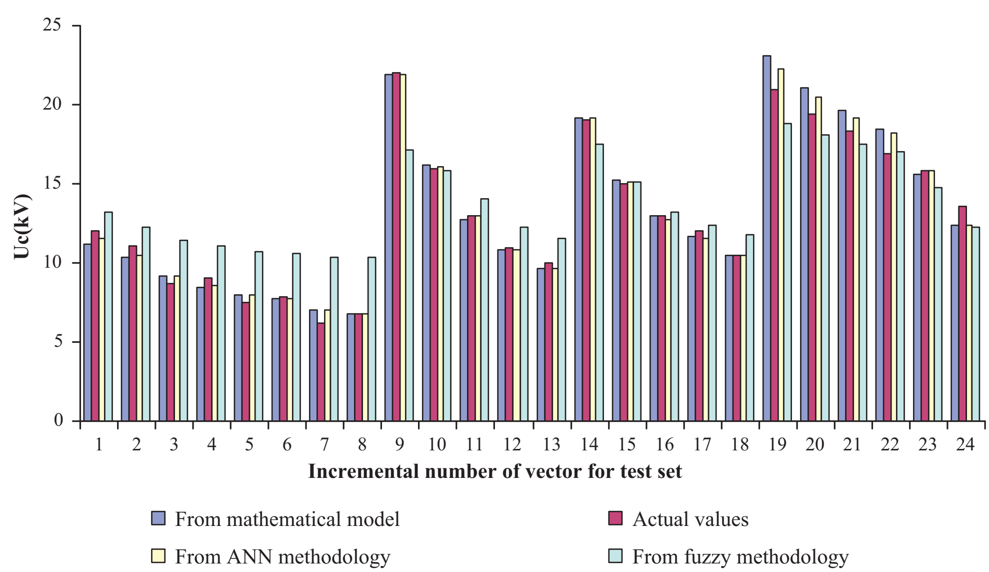

Fig. 9. Representation of the actual and the estimated values from the mathematical model [3], from the ANN methodology [7] and from the proposed fuzzy logic methodology (results from 5th method) for the test set.

rounded weighted mean value independently regarding the calculation form of the centre of the middle triangle and the kind of set (the inequalities $R_1 < R_3 < R_5$ and $R_2 < R_4 < R_6$ hold for training, evaluation and test sets). Similarly, by comparing methods 1–2, 3–4 and 5–6 we conclude that the best results are achieved when the estimation for the critical flashover voltage is done using the mean value of the training set as the centre of the middle triangle independently regarding the calculation form of the final estimation for the output and the kind of set (the inequalities $R_1 > R_2$, $R_3 > R_4$ and $R_5 > R_6$ hold for training, evaluation and test sets).

Totally, by comparing the R index of the evaluation and the training set of all the six methods, it is concluded that the best results are achieved when the final estimation for the output is done using the no rounded weighted mean value and the centre of the middle triangle is determined as the mean value of the training set (5th method – see Table 3). Then the correlation between the actual and the estimated values for the test set R is 96.7%.

Fig. 7 depicts the success of the 5th method (see Table 1) in estimating the critical flashover voltage. It is obvious that this method gives a satisfactory approach of the values for the critical flashover voltage. Fig. 8 illustrates the confidence interval of the evaluation and the test set. Obviously, the interval of the test set is narrower than the interval for the evaluation set because of the successive estimation. This suggests that, if the only given information is the confidence interval of the evaluation set, then the estimation of the critical flashover voltage will be satisfactorily accurate.

The respective results would have been improved if more vectors for the training set and the evaluation set were available.

The correlation index R achieved by the optimal selection of parameters is 96.7% when the final estimation for the output is done by no rounded mean value and the centre of the middle triangle is estimated by the mean value of the training set (5th method), which is quite satisfactory compared to the respective index of ANN methodology [6], which is equal to 99.3%, of the ANN optimization methodology [7], which is equal to 99.86%, of the adaptive-network-based fuzzy inference application [8], which is equal to 99.93%, of simple empirical models [3] and [16], which are equal to 98.7% and 96.4%, respectively. However, it should be noted that such a comparison is not justifiable due to the different datasets upon which each method is applied. The last one is confirmed in Fig. 9. Fig. 9 represents the actual and the estimated values derived from the mathematical model [3], from the ANN methodology [7] and from the proposed 5th fuzzy method for 24 experimental vectors of the test set.

5. Conclusions

A methodology for the estimation of the critical flashover voltage of polluted insulators using fuzzy logic was presented. It performs an extensive search in order to select the optimum training algorithm and the respective parameters such as the number and the base width of the triangles, etc. The generalization ability of the proposed methodology is achieved by the use of three sets (training, evaluation and test). Finally, the best correlation index R is achieved when the final estimation for the output is done by no rounded mean value and the centre of the middle triangle is estimated by the mean value of the training set. The respective results are quite satisfactory compared to the respective indexes of previous methods [3,6,7,8,14].

Currently, the estimation of the critical flashover voltage for each insulator contains not only the estimated value, but also the respective confidence interval based on the re-sampling method. The estimated value based on the selection of the training method with a high correlation index R is a necessary but not a unique criterion. The wide spread of the estimated values using the confidence interval for the evaluation set gives useful information about the estimation of the critical flashover voltage, because for each insulator there is not only one estimated value, but a values interval with known probability (in this case 90%), which gives an estimation of the error size (in this case there is probability of 10% the final estimated value to be out of the respective values interval). This leads to a more accurate, generalized and objective estimation of the respective critical flashover voltage and makes the proposed methodology a powerful and useful tool.

Appendix A.

In this section, the experimental data used in this work as a test set are presented [2,13,15].

D_m (cm)	H (cm)	L (cm)	F	C (mg/cm^2)	U_c (kV)
25.4	14.6	27.9	0.68	0.13	12.0
25.4	14.6	27.9	0.68	0.16	11.1
25.4	14.6	27.9	0.68	0.23	8.7
25.4	14.6	27.9	0.68	0.28	9.1
25.4	14.6	27.9	0.68	0.34	7.5
25.4	14.6	27.9	0.68	0.37	7.8
25.4	14.6	27.9	0.68	0.49	6.2
25.4	14.6	27.9	0.68	0.52	6.8
25.4	14.6	30.5	0.70	0.02	22.0
25.4	14.6	30.5	0.70	0.05	16.0
25.4	14.6	30.5	0.70	0.10	13.0
25.4	14.6	30.5	0.70	0.16	11.0
25.4	14.6	30.5	0.70	0.22	10.0
25.4	14.6	43.2	0.92	0.05	19.0
25.4	14.6	43.2	0.92	0.10	15.0
25.4	14.6	43.2	0.92	0.16	13.0
25.4	14.6	43.2	0.92	0.22	12.0
25.4	14.6	43.2	0.92	0.30	10.5
22.9	16.6	43.2	1.38	0.03	20.9
22.9	16.6	43.2	1.38	0.04	19.4
22.9	16.6	43.2	1.38	0.05	18.3
22.9	16.6	43.2	1.38	0.06	16.9
22.9	16.6	43.2	1.38	0.10	15.8
22.9	16.6	43.2	1.38	0.20	13.6

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