Potential Distribution on the Surface of Multi-Layer Earth Structure

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Grounding systems consist one of the most important part of building's protection systems. The behavior of any grounding system greatly depends on the earth structure in which the grounding system is buried. The objective of this paper is to compare various analytical and simulation methods for the behavior of grounding systems regarding the surface potential rise. For this purpose the potential distribution in the soil around a driven rod is computed by using analytical equations. Moreover two software packages have been used in order for a driven rod grounding system to be simulated. The results, derived from the analytical formulae, are compared to the simulation results. This comparison demonstrates the influence of the earth structure to the potential distribution on the surface of the earth and the limitations of the analytical equations. Additionally the simulation computer programs are proved to be a useful, effective and reliable tool for studying the response of grounding systems.

***Index Terms*—Finite Element Methods, Grounding Electrodes, Soil, Electric Field, Simulation**

I. Introduction

The grounding system comprises an essential part of the protection system of any power system, building etc. Moreover, an efficient grounding system can dissipate the stroke current into the soil and reduce the damages to electrical and electronic equipment and to personnel.

The commonly used grounding structures are single horizontal grounding wires, vertical rods, ring conductors or a suitable combination of the above mentioned structures. The measurements of the soil resistivity have shown that the soil consists of different layers, having different characteristics (resistivity, depth). Furthermore, closed-form mathematical formulae for the calculation of the distribution of the surface potential [1]-[4] are limited to vertical or horizontal electrodes. Arithmetic methods such as Finite Element Method [5]-[7], Method of Moments [8], Boundary Element Method [9], [10] or hybrid methods [11] overcome such geometrical limitations and provide a successful implemented method for the calculation of the potential distribution. The software packages Opera PC uses FEM.

II. Potential Distribution on soil surface

A. Analytical Expressions

Assuming homogenous and isotropic soil with resistivity ρ_o, the potential at any point on the surface of the ground due to a point current source I, situated on the surface, is given by the following formula [1]:

$$V_o(x) = \frac{\rho_o \cdot I}{2 \cdot \pi} \int_0^{\infty} \mathrm{J}_0(\lambda \cdot x) \cdot \partial\lambda \tag{1}$$

where J_o is the Bessel function of the first kind of zero order and x is the distance from the current source.

However, the soil is almost always non-homogenous [2]. Tagg [1] proposed a model of a two-layer soil consisting of a surface layer of resistivity ρ_1 of thickness h_1 overlaying a second layer of resistivity ρ_2. The potential due to a current flow at a point on the surface is given by the equation [1]:

$$V(x) = \frac{\rho_1 \cdot I}{2 \cdot \pi \cdot x} \cdot [1 + 2 \cdot x \cdot \int_0^{\infty} \frac{k_1 \cdot e^{-2 \cdot \lambda \cdot h}}{1 - k_1 \cdot e^{-2 \cdot \lambda \cdot h}} J_o(\lambda \cdot x) \cdot \partial\lambda] \tag{2}$$

k_1 is the coefficient of reflection given by:

$$k_1 = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1} \tag{3}$$

Takahashi and Kawase [3] have developed a method for the calculation of the surface potential considering a multi-layer structure for the earth. The structure of N-layer earth model is shown in Fig. 1. The first layer has thickness h_1 and soil resistivity ρ_1, the second layer has thickness h_2 and soil resistivity ρ_2, the thickness of the last (N[th]) layer is infinity and its soil resistivity is ρ_N.

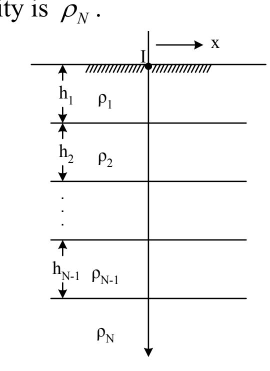

Fig. 1. Multi-layer earth model

According to their model the potential at any point x on the earth surface for an injected current I is described by the following equation [3]:

$$V_N(x) = \frac{\rho_1 \cdot I}{2 \cdot \pi \cdot x} \cdot [1 + 2 \cdot x \cdot \int_0^{\infty} \frac{K_{N1} \cdot e^{-2 \cdot \lambda \cdot h_1}}{1 - K_{N1} \cdot e^{-2 \cdot \lambda \cdot h_1}} J_o(\lambda \cdot x) \cdot \partial\lambda] \tag{4}$$

For $1<i<N-1$ the coefficient of reflection k_i for two sequential layers is given by the formula:

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$$k_i = \frac{\rho_{i+1} - \rho_i}{\rho_{i+1} + \rho_i} \quad (5)$$

Additionally, for $N>2$ and $1<S<N-2$ the factor K_{NS} is given by:

$$K_{NS} = \frac{k_S + K_{NS+1} \cdot e^{-2\cdot\lambda\cdot h_{S+1}}}{1 + k_S \cdot K_{NS+1} \cdot e^{-2\cdot\lambda\cdot h_{S+1}}} \quad (6)$$

and

$$K_{NN-1} = k_{N-1} \quad (7)$$

B. Software Packages

Opera-2d is a software package for electromagnetic field analysis, which solves a wide range of electromagnetic and electrostatic applications in 2-dimensional xy and axisymmetric coordinates. This package uses the finite element method to obtain solutions to partial differential equations (Poisson's, Helmholtz and Diffusion equations) that cannot be solved by analytic methods. Additionally non-linear materials can be modeled by this program.

Since much information is required before the analysis has been performed, data entry is carried out using a powerful interactive pre-processor. Using the graphical interaction within the pre-processor, the space is divided into a contiguous set of triangular elements.

Once the model has been prepared, the solution is achieved using a suitable analysis module. Several modules exist for analysis of the different types of electromagnetic excitation conditions, e.g. static, transient, steady state. The analysis program iteratively determines the correct solution, including non-linear parameters if these are modelled [12].

The result may then be examined using a versatile interactive postprocessor. As with the pre-processor, this is predominantly controlled by interaction through a graphical menu system. Many system variables are available for examination, including potentials, currents, fields, forces, temperature. Numerical errors due to non-successful mesh definition are also analysed, so that the mesh can be refined to achieve the required accuracy [12].

In contrary, CDEGS is a software package developed especially for grounding systems' analysis. The program solves grounding problems based on the equations of IEEE Std 80-2000 [13] and is subjected to limitations regarding the geometry of the earth structure.

The advantage of the proposed simulation can be estimated by comparing the results of the field analysis, using the CDEGS and PC Opera package, to the results of other methodologies. From this comparison, it is concluded that this software package can be a useful tool in the simulation of grounding systems and in the accurate calculation of the potential on the ground surface around the location, where the grounding system is installed.

III. Driven Rod In Multi-layer earth

The simulated grounding systems consisting of a 1m single driven rod buried in different soil types and subjected to 1000 V voltage are simulated by using the software packages PC Opera and CDEGS.

The different soil structures, that have been used for the estimation of the surface potential by using analytical expressions and the above mentioned simulation programs, are presented in Tables I-III. For 1-Layer earth structure the soil's resistivity is equal to average resistivity

$$\rho_{aveg} = \frac{\sum_{i=1}^{N} \rho_i}{\sum_{i=1}^{N} h_i} \quad (8)$$

where ρ_i and h_i are the values for the resistivity and layer's depth of multi-layer soil respectively.

TABLE I
The Soil Parameters Used For Analytical Computations And Simulation For Case 1

i	1 Layer	2 Layer		Multi-Layer		
	ρ_{aveg} [Ωm]	ρ_i [Ωm]	h_i [m]	ρ_i [Ωm]	h_i [m]	ε_i
1	190	908	2.5	1000	1	9
2		73		550	2	14
3				250	3	19
4				100	4	36

TABLE II
The Soil Parameters Used For Analytical Computations And Simulation For Case 2

i	1 Layer	2 Layer		Multi-Layer		
	ρ_{aveg} [Ωm]	ρ_i [Ωm]	h_i [m]	ρ_i [Ωm]	h_i [m]	ε_i
1	210	946	3.2	1000	2	9
2		40		500	2	15
3				300	2	19
4				200	2	20
5				100	2	36

TABLE III
The Soil Parameters Used For Analytical Computations And Simulation For Case 3

i	1 Layer	2 Layer		Multi-Layer		
	ρ_{aveg} [Ωm]	ρ_i [Ωm]	h_i [m]	ρ_i [Ωm]	h_i [m]	ε_i
1	240	135	1.5	100	2	36
2		2000		500	2	15
3				800	2	11
4				1000	4	9

The arrangement is axisymmetric. Mesh generation is of essential importance for simulation by using PC Opera. Various meshes have been examined in order to define the most appropriate for the problem. A more dense mesh has been used on the common surface of different materials and in the area around the grounding rod. Fig 2. displays the mesh within the examined regions for Case 3.

In Fig. 3 - 5 the filled zone contour of the potential as derived from the PC Opera for the examined cases are presented.

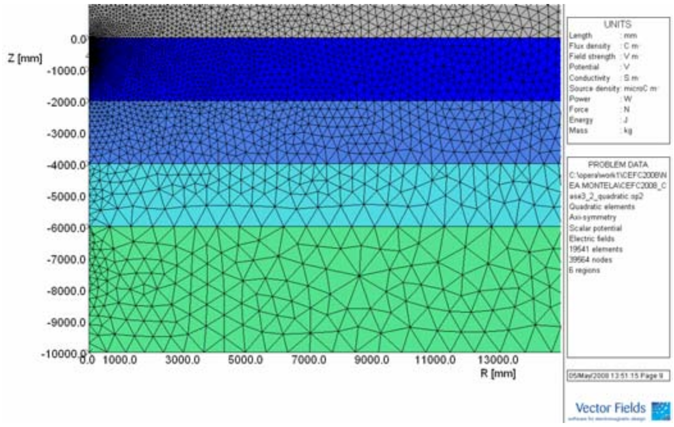

Fig. 2. Mesh within the background region for Case 3.

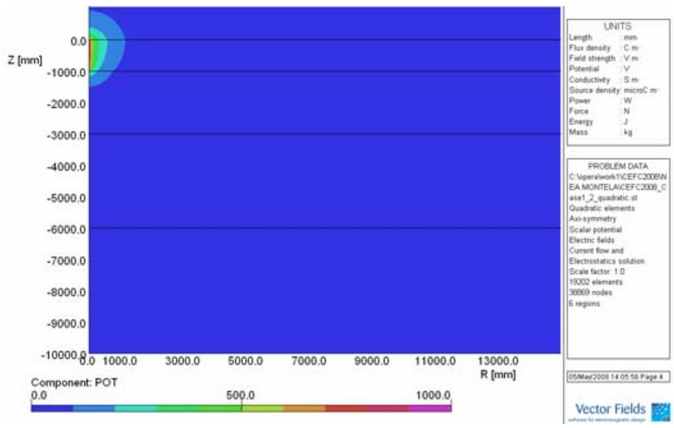

Fig. 3. Filled zone contours of potential for Case 1.

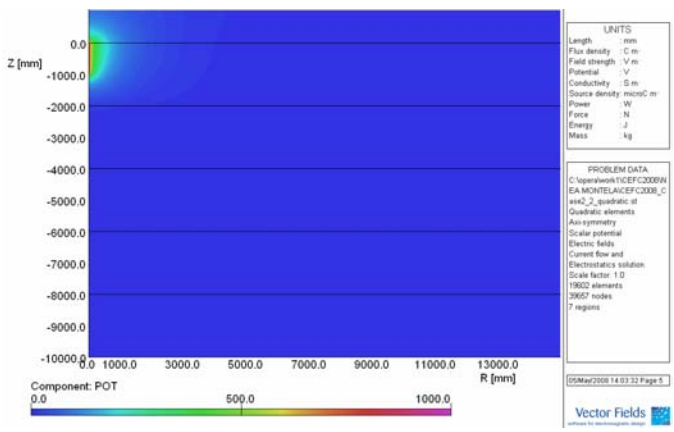

Fig. 4. Filled zone contours of potential for Case 2.

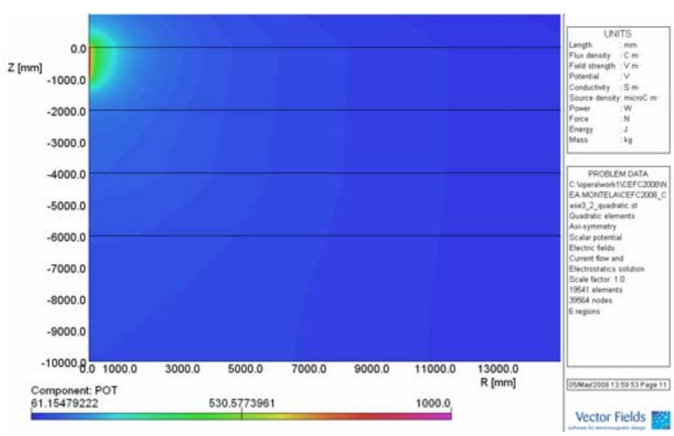

Fig. 5. Filled zone contours of potential for Case 3.

Moreover, the determination of the boundary conditions is of equal importance. In our simulation, not only the current flow equation was solved by using PC Opera, but also the electrostatic equation. For that reason the relative permittivity (ε_i) was used. In Tables I-III the values of ε_i are presented [14].

In Fig. 6 the simulation results of the potential distribution for multi-layer earth structure of Case 1 by using CDEGS are presented.

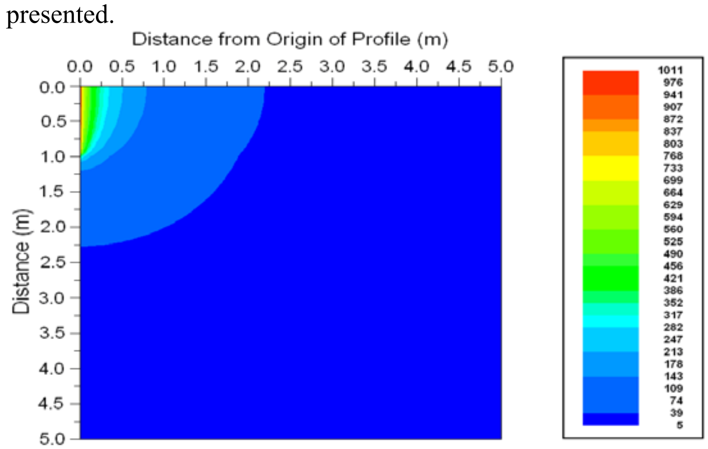

Fig. 6. Filled zone contours of potential for Case1 by using CDEGS.

IV. Calculation of Surface Potential

The result of computations and simulations is the variation of surface potential. The results obtained by the simulations are compared with results from the application of equation: a) (1), considering that the soil is homogeneous, b) (2), considering that the soil has two layers with parameters ρ_1 and h_1 for the first layer and ρ_2 for the second layer, these parameters have been estimated by using the genetic algorithm developed in [2], c) (4), considering a multi-layer earth structure.

The variation of the surface potential versus the horizontal distance from the rod for the examined grounding systems is shown in Fig. 7-9. The decrease of the surface potential in Cases 1 and 2 is steeper than in Case 3, as depicted in Fig. 7-9. This can be attributed to the lower values of resistivity of the upper earth layers, leading to higher values of step voltage.

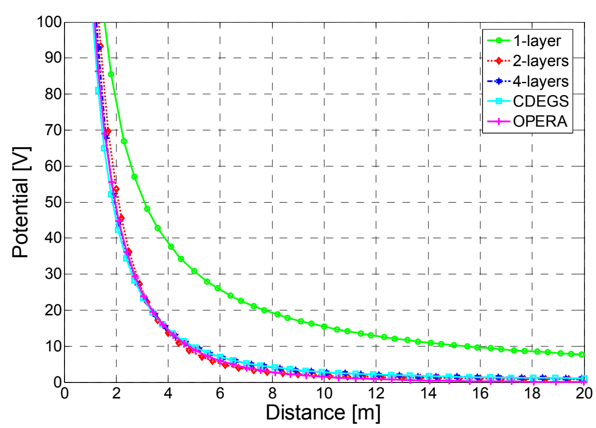

Fig. 7. The variation of surface potential versus the horizontal distance from the rod for Case 1.

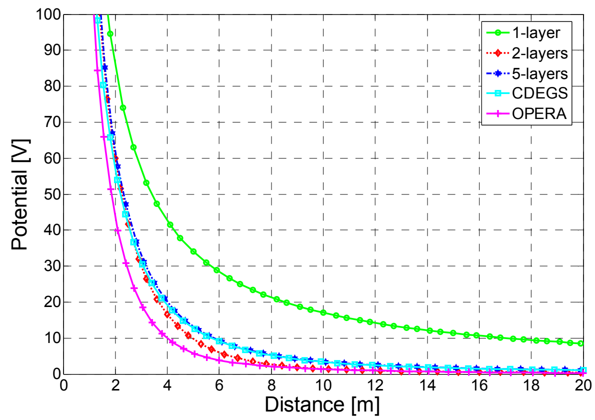

Fig. 8. The variation of surface potential versus the horizontal distance from the rod for Case 2.

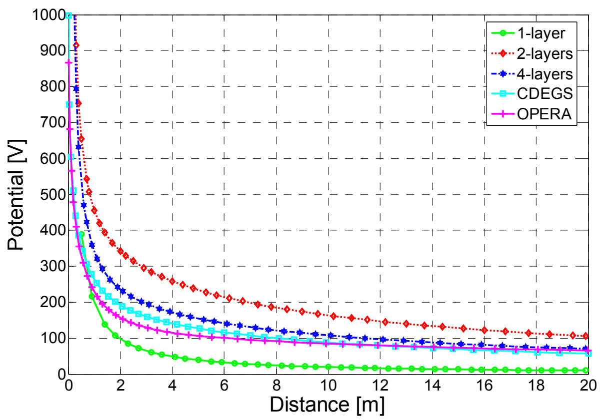

Fig. 9. The variation of surface potential versus the horizontal distance from the rod for Case 3.

Additionally, the results obtained by using (1) diverge from the results obtained by the rest methods. This proves that the approximation of 1 layer earth can lead to over- or under-estimation of the developed step voltage. Thus, the evaluation of surface potential considering multi-layer earth structure is mandatory. This is obvious in Fig. 9, where (4) approximates better the simulation results than (1) and (2). On the other hand analytical computations by using (4) are time consuming.

Furthermore the simulation programs provide us with more realistic values of potential because analytical equations are subjected to limitations. (1), (2), and (4) assume point electrode without any dimensions and horizontal stratification of the layers with different properties.

Regarding the simulation programs it can be said that PC Opera is advantageous in comparison with CDEGS as long as it can solve more complex geometries and allows better determination of the soil properties.

V. Conclusion

The non-uniformity of the soil affects the potential distribution on the surface of the earth. Thus the multi-layer earth structure must be taken into account during the design of a grounding system.

The usage of simulation packages can provide valuable assistance to the estimation of the surface potential developed on the ground, regardless of the grounding system's geometry, considering the fact that the closed-form mathematical formulae for multi-layer analysis are subjected to limitations regarding the earth structure, the grounding system's geometry and the computational time. Moreover, the simulation results are in good agreement with the results obtained by (4).

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