#### Journal of Constructional Steel Research 67 (2011) 462-470

Contents lists available at ScienceDirect







journal homepage: www.elsevier.com/locate/jcsr

# Modeling of curved composite I-girder bridges using spatial systems of beam elements

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#### ARTICLE INFO

Article history: Received 25 May 2010 Accepted 25 September 2010

Keywords: Curved bridges Modeling Buckling Grillage analysis Composite bridges Beam elements I-girders Warping

#### ABSTRACT

A new way of modeling steel composite bridges has been presented by Vayas et al. (in press, 2010) [3,4]. The proposed model is based on the representation of steel I-girders by equivalent trusses. The concrete slab is suitably represented by a set of bar elements, and the bearings by appropriate springs. Diaphragms and stiffeners may also be taken into account. In comparison to the grillage model, which is usually used for the analysis of bridges, the proposed three-dimensional model allows a more reliable prediction of deformations, internal forces, and stresses. Curved bridges display unique behavior characteristics, and for this reason a grillage analysis is not always suitable. The new way of modeling composite bridges, using a spatial system of beam-like structural elements, is applied in this paper for the modeling of curved composite bridges. Worked examples are provided to illustrate the set-up procedure of the proposed modeling and to compare its results with those of corresponding finite element models.

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#### 1. Introduction

Plane grillage models are widely used for the design of steelconcrete composite bridges. Grillage analysis is used both for the analysis and the design of the bridge for the most common design situations, as well as for the construction stages [1,2]. This method is based on idealization of the slab and the I-girders using beam elements. The longitudinal composite girders are represented by beam elements with equivalent cross-sectional properties that include the steel beam and the concrete flange. The deck slab is idealized by a series of transverse beams.

Although this model is generally accepted as sufficiently accurate and it has the advantage of generality, it is associated with some drawbacks. Eccentricities among the structural elements of a bridge cannot be taken into account in the model, and inevitably additional internal forces and possible load distributions are ignored. Torsion and distortional warping effects are difficult to be taken into account, and buckling phenomena of the steel girders during erection stages cannot be easily investigated.

On the other hand, the finite element (FE) analysis that is widely used in bridge engineering also has some limitations and needs more time and effort in modeling than a grillage analysis. In addition, the quantity of computations and output can be enormous,

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and the engineer may not always check the large amount of computer data and the results. Furthermore, there are various sources of error that can contribute to incorrect results, like the choice of element type, its shape, or the meshing of the structure elements.

### 2. Bridge analysis using a three-dimensional (3D) model

A bridge analysis model should be based on the following criteria.

- (a) It should reflect the structural response in terms of deformation, strength, and local and global stability.
- (b) It should include as many structural elements and parts (crossframes, stiffeners, bearings, etc.) as possible, and their possible eccentric connections.
- (c) It should cover all construction stages and loading cases.
- (d) Loads should be easily introduced.
- (e) It should allow the performance of dynamic analysis and include the most important modes.
- (f) It should run with a common analysis and design software.

To overcome the difficulties of grillage and FE method analyses and to fulfil the above criteria, a 3D truss model was proposed in [3] and [4] by Vayas et al. The intention of this model was to better represent the 3D behavior of composite bridges using a new method that would be neither complicated nor time consuming compared to the grillage analysis, providing at the same time useful results that would probably require an FE analysis.

<sup>0143-974</sup>X/\$ – see front matter 0 2010 Elsevier Ltd. All rights reserved. doi:10.1016/j.jcsr.2010.09.008



Fig. 1. Truss idealization for a single steel and composite girder Vayas et al. [3].

#### Table 1

Comparison of the proposed model, a simply supported beam model and an FE model (simply supported beam, L = 25 m, q = 15 kN/m), [3].

	Steel girder 1			Steel gir	Steel girder 2			Composite girder 1			Composite girder 2		
$b_c \times h_c^{a}$	_			-	_			3000 × 200			2000 × 350		
$b_{fo} \times t_{fo}^{b}$	$300 \times 3$	0		$300 \times 3$	$300 \times 30$			$300 \times 30$			$300 \times 30$		
$h_w \times t_w^c$	$1000 \times 12$		$1500 \times$	$1500 \times 12$		$1000 \times 12$	$1000 \times 12$			$1500 \times 12$			
$b_{fu}  imes t_{fu}{}^{\mathrm{d}}$	400  imes 40		$300 \times 3$	300 × 30		400  imes 40	$400 \times 40$		300 × 30				
	3D	1D	FEM	3D	1D	FEM	3D	1D	FEM	3D	1D	FEM	
$w^{e}$	51.0	49.4	53.7	27.1	26.1	27.7	17.6	16.6	19.6	10.1	9.1	10.5	
$\sigma_{c}^{f}$	-	-	-	-	-	-	-0.26	-0.26	<b>-0.28</b>	-0.19	-0.19	<b>-0.20</b>	
$\sigma_{s}{}^{g}$	7.07	7.02	7.21	6.66	6.57	6.65	5.22	5.18	5.43	4.68	4.41	4.56	
n <sub>cr</sub> <sup>h</sup>	0.72	0.64	0.68	0.51	0.43	0.52	-	-	-	-	-	-	
$f_{ m dyn}{}^{ m i}$	2.51	2.59	2.43	3.46	3.57	3.36	4.28	4.48	4.00	5.69	6.04	5.37	

All dimensions are given in mm.

For the critical load factors of the 1D model, beam elements with seven degrees of freedom were used.

Modulus of elasticity: for concrete,  $E_c = 3350 \text{ kN/cm}^2$ ; and for structural steel,  $E_s = 21000 \text{ kN/cm}^2$ .

<sup>a</sup> Concrete plate.

<sup>b</sup> Upper flange.

<sup>c</sup> Web.

<sup>d</sup> Lower flange.

<sup>e</sup> Maximum deflection, in mm.

<sup>f</sup> Stress at the top of the concrete slab, in  $kN/cm^2$ .

 $^{g}$  Stress at the bottom of the lower flange, in kN/cm<sup>2</sup>.

<sup>h</sup> Critical load factor for lateral torsional buckling

<sup>i</sup> Eigenfrequency for vertical bending, in Hz.

According to the proposed model, the steel I-girders are modeled by equivalent trusses based on the classical "tension field method". The deck slab is idealized by a grillage of concrete beams. The set-up procedure of the model is explained further below, and numerical investigations are demonstrated for simply supported orthogonal and curved composite bridges. A comparative analysis of the proposed model, a grillage system, and an "exact" FE model for a simply supported straight composite bridge has been presented and discussed in [3].

#### 2.1. Representation of steel I-girders

The structural system must reproduce the 3D behavior of a bridge as accurately as possible. This is achieved through the representation of the steel I-girders by equivalent trusses. The deck slab is idealized by a grillage of concrete beams. The main concept is based on the set-up of a global model, which will be easy to modify during the different construction stages, including stages of erection or deck concreting.

Fig. 1 illustrates the modeling of a composite girder through the use of an equivalent truss. The model is developed in such a way that the final model has the same behavior as the prototype cross-section. The flanges of the truss are beam elements with a cross-section composed of the flange and part (1/3) of the web of the steel girder. The geometry of the T-sections was chosen so as to lead to similar vertical deformations and stresses as in the original cross-section, and to predict the buckling shapes during a buckling

analysis. The flanges are connected by a "hybrid combination" of truss and beam elements that represent the web of the steel girder, ensuring the stability of the truss. For a better representation of the behavior of the web, a distance between the vertical struts equal to or smaller than 5% of the total span length is chosen. The concrete section is represented by another beam element, connected with the upper flange of the truss, as shown in Fig. 1, through the appropriate offset.

Generally, the cross-sectional area of the diagonals can be calculated through the principle of virtual work and the formulas included in [5], based on the shear stiffness of the web. However, in bridge structures, the spans are sufficiently long, and as a consequence the influence of the work of shear is low. In this way, in the present paper, the modeling of the steel section is based on a different simulation, in which the cross-sectional area of the diagonals does not depend on the shear stiffness of the web. The following modeling has been chosen after various examples that have been carried out for steel and composite sections of different geometries and spans, under uniformly distributed or concentrated loads, the results of which are in very good agreement with those of FE models [4].

In order to verify the validity of the proposed model, numerical investigations for deformations, stresses, buckling, and dynamical modes are performed for a simply supported beam with either steel or composite cross-sections. Table 1 compares the results of the proposed 3D model, a beam model (1D), which would be introduced in a grillage analysis, and an FE model (FEM). For these

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Comparison of results at mid-span for the proposed 3D beam model, the grillage model and the FE model (simply supported bridge of Fig. 3).

	3D beam model				Grillage m	Grillage model				FE model		
	$\sigma_c$	$\sigma_{s}$	w	R	$\sigma_c$	$\sigma_{\rm s}$	w	R	$\sigma_c$	$\sigma_{s}$	w	R
Load case 1	-0.37	7.86	26.7	312.8	-0.37	8.19	26.0	339.9	-0.37	8.58	30.2	351.1
Load case 2	-0.48	7.02	22.8	156.9	-0.32	6.95	21.3	158.8	-0.51	6.20	19.8	167.0
Load case 3	0.27	1.21	30.2	108.4	0.36	1.22	30.6	72.6	0.29	1.30	31.8	112.4

 $\sigma_c$ : Minimum stress in concrete, in kN/cm<sup>2</sup>.

w: Maximum deflection at mid-span, in mm.

 $\sigma_s$ : Maximum stress in structural steel, in kN/cm<sup>2</sup>.

R: Maximum support reaction, in kN.

examples the modulus of elasticity for concrete and structural steel are 3350 kN/cm<sup>2</sup> and 21 000 kN/cm<sup>2</sup>, respectively. For the beams of the steel section, the critical load factor for lateral torsional buckling is calculated using some appropriate buckling analysis software.

#### 2.2. Grillage representation of the slab

According to the grillage analysis by Hambly [1], the concrete slab of a composite bridge can be represented by a grillage of interconnected beams. The grillage is connected to the upper flange of the truss, as shown in Fig. 1. Attention must be paid so that the grillage has its longitudinal members coincident with the centre lines of the steel sections.

The longitudinal beams, which are located inside the effective width of the slab, are considered at mid-spans with their uncracked properties. At internal supports, the cross-sectional area of the longitudinal beams is equal to the total reinforcement amount, which can be assumed at the centre of the slab. The effect of *tension stiffening* can be taken into account with the help of the following equation [6]:

$$A = \frac{A_{s,\text{tot}}}{1 - \frac{0.5 \cdot f_{\text{ctm}}}{\rho_{\text{s} \text{ tor}} \cdot f_{\text{s} \text{k}}}},\tag{1}$$

where  $f_{\text{ctm}}$  is the mean tensile strength of concrete;  $A_{\text{s,tot}}$  is the total amount of reinforcement in the slab;  $\rho_{\text{s,tot}}$  is the total reinforcement ratio;  $f_{\text{sk}}$  is the characteristic yield strength of steel.

The longitudinal beams, which are located outside the effective width  $b_{\text{eff}}$ , do not participate in the distribution of the normal stresses. Therefore, their cross-sectional area *A* is equal to zero.

The slab reinforcement can be calculated from the bending moment diagrams of both the transverse and the longitudinal beams. In cases of pre-stress, the actual sections must be replaced by transformed sections, in which the pre-stress steel is included [7].

The grillage mesh depends generally on the geometry of the slab. The spacing of the beams should not be less than twice the slab depth. If the local dispersion of concentrated loads has to be considered, then smaller values have to be adopted.

It has been also recommended by Hambly [1] that the row of longitudinal beams at each edge of the grillage should be located at a distance of  $0.3h_c$  from the edge of the slab, where  $h_c$  is the slab depth. More information about the grillage mesh of slabs is given in [1].

Since the torsion stiffness of the deck slab is much lower than the bending stiffness of the composite girders, a torsionless approach can be adopted ( $I_T = 0$ ).

The creep of concrete can be taken into account through the use of an age-adjusted modulus of elasticity,  $E_{c,eff}$ :

$$E_{c,\text{eff}} = \frac{E_c(t_0)}{1 + \varphi_t},\tag{2}$$

where  $E_c(t_0)$  is the modulus of elasticity of concrete at age  $t_0$ , the time of application of the loading;  $\varphi_t$  is the creep coefficient.

#### 2.3. Comparative analysis for a simply supported orthogonal bridge

In Fig. 3, three different models (proposed model, grillage and FE model) for three load cases are compared. The grillage model was set up according to the recommendations of Unterweger [8] and Hambly [1]. The results for the flexural stresses, the support reactions, and the deflections are given in Table 2. In all models, the bearings are represented by springs of equivalent stiffness [3]. Table 2 shows that the 3D model accommodates the stress and deformation behavior of the composite structure very well, comparing its results with those of the FE model or classic grillage theory. It is worth mentioning that in all models the deck slab is represented in a different way. In particular, shell elements are used in the FE model while beam elements are used for the representation of the slab in both the grillage and the 3D models. Besides, in the grillage model, the concrete slab is incorporated in the longitudinal beams using the inertia of an equivalent composite section. In the proposed 3D model this is not the case. For these reasons, the different girders in all three models share the loads differently, and that leads to some deviations in the results, especially in the case of the reactions.

#### 3. Curved bridges

Curved composite bridges display unique behavior characteristics, some of which are not immediately obvious. The presence of curvature affects the geometry, and, as a consequence, the behavior of the structure. Curved bridges are subjected to coupled torsion and bending because of the curvature, and their analysis is more complex than that of straight bridges. In addition to simple vertical flexure behavior, there can be significant torsional loading and twisting of the girders that cause lateral stresses to the flanges [9].

Due to the complexity of the curved structure and to its complicated 3D response, different methods have been developed for the static and dynamic analysis of curved bridges, while many technical papers and books have been published as well [10,11]. Grillage models that are commonly used in bridge analysis treat curved members as straight members, while it is difficult to simulate properly the bracing effect between the bottom flanges and the interconnection between the main girders. In addition, special phenomena like twisting and out of plane rotations of the steel main girders are difficult to investigate [11]. For this reason, a 3D computer analysis is recommended for the analysis of horizontally curved bridges.

In what follows, a worked example of analysis of a curved composite bridge for different values of the radius of curvature is given. The curved composite bridge is modeled using the proposed 3D model described above, while a comparative analysis is performed between the curved structure and a straight equivalent bridge of the same transversal geometry. The results of the proposed model are compared to those of FE models that have been developed for this reason. Through the comparative analysis between the straight and the curved structure, the effect of curvature on the bridge behavior is demonstrated.



Fig. 2. Grillage idealization for the slab of a steel-concrete composite beam Vayas et al. [4].



Fig. 3. Bridge cross section, load cases and structural systems, Vayas et al. [3].

#### 3.1. Simply supported curved composite bridge of three main girders

In order to verify the ability of the 3D model to simulate a composite curved bridge properly, the single-span curved bridge shown in Fig. 4 is studied under a uniformly distributed and an eccentric linear load, applied on the composite structure.

As is shown in Fig. 4, two different curved structures are investigated according to the 3D method discussed above. The total length of each bridge, measured at mid-width, is 30.48 m. In the first case a higher curvature is adopted (R = 60.96 m), while in the second case the curvature is smoother (R = 101.5 m).

In order to compare the results of the curved structures with each other, and also with those for an orthogonal bridge of the same geometry, a straight composite bridge of the same transversal section and a length of 30.48 m is also considered. For all the three cases, the bridges are modeled using the proposed 3D model and an FE model.

#### 3.2. Modeling of composite curved bridges

According to the proposed 3D representation of a composite bridge (Figs. 1 and 2), the deck of a curved bridge has to be represented through a grillage of beam-like elements. The decks that are curved in plan can be modeled through a grillage analogy based on the recommendations of the grillage theory [1,2] given also in Section 2.2.



Fig. 4. Curved bridge plan, cross section and load cases (Unterweger [8]).

Curved decks pose no particular problem for grillage modeling [1]. A curved bridge deck can be represented by a grillage of curved members or of straight members. Some computer programs support curved members but others do not. Generally, a grillage of straight beams with a very fine mesh is sufficiently accurate.

The following figures illustrate the procedure of the 3D representation of the composite bridge of Fig. 4, as described in Section 2.1. The composite main girders are represented by equivalent trusses according to Fig. 1. The beams that represent the slab are connected to the upper flanges of the truss through the appropriate offset.

The concrete slab is divided transversally into seven intermediate longitudinal members of 1016 mm width and two end members of 406 mm width (Fig. 5(c)). The longitudinal members have been placed along the lines of the steel girders, and extra members have been added between them (Fig. 5(b) and (c)). The longitudinal members, although straight, follow the curved layout closely. The transverse members have been placed generally at a spacing of 5% of the length of each girder. Their cross-section is defined by the spacing between them. As their spacing is not constant, an average width is considered. Transverse beams should have a spacing similar to that of the longitudinal beams. A ratio between one and three times the longitudinal spacing is generally accepted [1]. Since the bridge is curved and not straight, a "torsionless" approach is no longer applicable for the construction of the 3D model. Therefore, the torsional constants of the beams that represent the slab are calculated using grillage theory [1]. The torsion constant per unit width of slab is given by

$$j = \frac{h_c^3}{6}.$$
(3)

In all models, the bearings are represented by springs of equivalent stiffness, placed at the lower flange of the girders, as shown in Fig. 6.

For the straight orthogonal bridge, the same procedure is followed [3,4]. Finally, the 3D models for the curved and the straight bridge are shown in Fig. 6.

#### 3.3. Application of loads and static analysis

As shown in Fig. 4, two different load cases are applied on the three different bridge models. Fig. 6 shows the final 3D models; the intermediate cross-frames are not included for a better visualization of the structure.

Table 3 shows the results for the three different bridges, modeled using both the proposed model and the FE model. Maximum stresses and deformations for each main girder are included.

In Table 3, one can see that the results for deformations, stresses, and maximum reactions correlate very well between the 3D model and the FE model. For each girder, both models lead to almost identical results, with the divergence percentage always lower than 10% for the most unfavorable values. The main advantage of the 3D model over finite element analysis is its ability to simulate the whole structure using relatively fewer members, offering at the same time a faster solution and an easier interpretation of the results.

Figs. 7 and 8 show the maximum stresses and vertical deflections of the main girders of the three composite bridges in question, for both the 3D model and the FE model. Choosing the results of the 3D model, the following conclusions arise. For the first load case, the maximum stress of the steel section increases from 7.03 kN/cm<sup>2</sup>, for the straight bridge, to 14.55 kN/cm<sup>2</sup> for the curved bridge of low curvature (R = 101.5 m) and to 19.69 kN/cm<sup>2</sup> for the curved bridge of higher curvature (R = 60.9 m). The magnitude of stress increases significantly, by almost 100% in the first case and by 180% in the second case, respectively. This applies also to the maximum vertical displacements that change from



(a) Composite girder representation.



(b) Grillage mesh for slab: plan.



(c) Grillage mesh for slab: section.

Fig. 5. 3D representation of a simply supported curved bridge of 3 main girders-Modeling of the composite section and grillage mesh for the concrete slab.



Fig. 6. 3D beam model.

21.3 mm (for the orthogonal bridge) to 39.2 mm for the first case and 51.8 mm for the second case.

Significant increase of stresses and vertical displacements is observed also for the second load case, as shown in Fig. 8. According to the results of the 3D model, the stress increases from 4.96 to 11.59 kN/cm<sup>2</sup> for the structure of high curvature, and the vertical displacement from 14.8 to 34.1 mm. The increase of stresses in curved bridges is caused by the distortion of the web, which leads to lateral bending of the flanges.

#### 3.4. Analysis of the main girders during concreting

During the bridge construction sequence, the girders deflect under their own weight and the weight of the deck. On straight bridges, the deflections across any section of the bridge due to the deck weight are almost identical. The point of maximum deflections for each steel girder will be at its mid-span. By contrast, on a curved bridge, the deflections are not the same across the width of the bridge since the girders do not have the same length,

#### Table 3

Maximum stresses, deformations and support reactions for the two different loading cases for the straight and the curved composite bridges: 3D model versus FE model.

	LC1			LC2				
	Straight	R = 101.5	R = 60.96	Straight	R = 101.5	R = 60.96		
w <sub>A</sub>	21.3	39.2	51.8	14.8	27.1	34.1		
$w_B$	21.3	27.9	33.5	7.0	12.9	17.4		
$w_{C}$	21.3	16.8	15.6	-0.5	-0.5	1.5		
$\sigma_A$	7.03	14.55	19.69	4.96	8.93	11.59		
$\sigma_B$	7.03	10.45	12.32	2.59	3.68	4.70		
$\sigma_{\rm C}$	7.03	5.82	3.72	-0.31	-2.38	-3.70		
<i>R</i> <sub>max</sub>	216.7	307.8	354.4	186.6	222.6	239.74		

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	LC1			LC2				
	Straight	R = 101.5	R = 60.96	Straight	R = 101.5	R = 60.96		
w <sub>A</sub>	<u>21.0</u>	<u>38.5</u>	<u>51.7</u>	<u>15.7</u>	<u>26.4</u>	<u>33.6</u>		
$w_B$	21.0	27.7	33.9	6.9	12.8	17.4		
$w_{C}$	21.0	17.1	16.5	-0.7	0.2	1.9		
$\sigma_A$	<u>6.77</u>	<u>14.21</u>	<u>20.1</u>	<u>5.13</u>	<u>8.56</u>	<u>11.60</u>		
$\sigma_{B}$	6.77	10.91	13.2	2.42	3.71	4.91		
$\sigma_{C}$	6.77	6.25	4.07	-0.50	-1.79	-2.97		
R <sub>max</sub>	217.5	308.4	354.2	186.7	223.1	240.38		

w: Maximum vertical deflection for each girder, in mm.

 $\sigma$ : Maximum stress of each girder, in kN/cm<sup>2</sup>.

R<sub>max</sub>: Maximum support reaction, in kN.









with the outer girders being longer than the inner ones. Generally, typical cross-frames are placed transversally in order to prevent lateral torsional buckling of the steel girders during concreting.

It is important for the design to predict the real deformations of the steel non-composite girders during construction, as any items connected to the top flange at these locations may also be a problem. Current research shows that considerable attention must be given to the construction methods in horizontally curved I-girder bridges and to the connection of the cross-frames with the main girders [12]. More details about construction issues for curved bridges are also given by the AASHTO/NSBA collaboration [13] and AASHTO [14]. In order to show the differential deflections that occur on a curved bridge during the concreting and the lateral displacement of the flanges, the curved bridge shown in Fig. 4 is investigated during concreting, using FE and 3D analyses. The curved bridge with R = 101.5 has been chosen for analysis. The steel girders are connected with intermediate cross-frames, shown in Fig. 4. A uniform load of 12.4 kN/m is applied on the two external girders while a uniform load of 15.5 kN/m is applied on the middle girder, representing the weight of fresh concrete.

Fig. 9 shows the differential vertical deflections of the steel girders for the 3D model and the FE model. The table in Fig. 9 shows the maximum vertical and lateral displacements for all three



Fig. 9. Vertical deformations for the three main girders-Maximum vertical and lateral deformations at midspan.



model.

(d) Stresses on flanges for the curved bridge and the orthogonal bridge: 3D model.

Fig. 10. Deformed shape of the steelwork for the 3D model (a) and the FE model (b); max stresses for upper and lower flange for the curved bridge (c); comparison of max. stresses in upper and lower flange for the curved and straight bridge (d).

main girders. The lateral displacements correspond to the upper flange deformations. One can see that the values for the vertical displacements correlate very well between the two models. The lateral displacements of the 3D model are reduced by 12.5% because they correspond to the center of gravity of the upper T-section of the model. In contrast, the FE model results correspond to the upper fiber displacements of the upper flange of the steel section.

Generally, the webs of the I-girders are not stiff enough, resulting in web distortion associated with the flanges lateral bending between the cross-frame locations. In the composite structure, the deck slab provides a restraint to the twisting of the top flange, and only the bottom flange is sensitive to lateral bending. At construction stages, where there is no slab, both the bottom and the upper flanges are subjected to lateral bending between the cross-frame locations. Fig. 10 shows the deformed shape of the steelwork, along with the stresses on the upper and lower flanges of the steel section, for the 3D model and the FE model, under the weight of concreting. The results in Fig. 10 show that the two models give almost identical stresses for the upper and lower flanges, while the 3D model can predict the real deformed behavior of the structure.

In Fig. 10(d), the maximum stresses along the outer girder for the curved bridge and the orthogonal bridge are summarized in a diagram, for the upper and lower flanges, using the results of the 3D model. The difference between the two structures is significant. The maximum stress for the orthogonal bridge attains a value of  $8.64 \text{ kN/cm}^2$ , while the maximum stress for the curved beam is  $21.87 \text{ kN/cm}^2$ . For the curved bridge, the points of maximum stresses are the positions of the transverse bracing. The differential deflections that occur at these points (which would be much higher without the presence of transverse bracings) are restrained by the transversal bracing, and lateral stresses are developed on the upper and lower flanges.



Fig. 11. Modal shapes and critical load factors for the load case of concreting.

#### 3.5. Buckling of the main girders during concreting

During the construction stages, the upper flange of the steel section is under compression in the span regions. Plate girders have low torsion stiffness and at the same time a high ratio of major to minor axis second moment of area. For this reason they are sensitive to lateral-torsional buckling. When there is no slab, the girders can buckle in a lateral-torsional mode.

A separate analysis must be carried out during the construction stages in order to verify the resistance of the steel girders towards lateral-torsional buckling. A linear buckling analysis allows the determination of the buckling modal shapes and the critical load factors  $n_{\rm cr}$ . The critical load factor is the ratio by which the applied load must be increased to cause the structure to become unstable. Fig. 11 demonstrates the stability analysis for the bridge shown in Fig. 4 during the deck concreting for both the 3D model and the FE model.

From the results of Fig. 11, it is obvious that the buckling factors of the two different analyses correlate very well with each other, with the deviation percentage always under 5%, so we conclude that the 3D model can also be used for stability analysis.

#### 4. Conclusions

Finite element models that are usually used for the analysis of straight and curved bridges are time consuming, and their results depend on engineer choices. On the other hand, grillage models are not always capable of predicting the real 3D behavior of complicated structures. Curved bridges display unique behavior characteristics and a complicated 3D behavior. In spite of the vertical bending, there can be significant torsional loading and twisting of the girders.

In this paper, a new way for modeling steel-concrete composite bridges, using a spatial system of beam-like structural elements, has been presented. The implementation and validation of the new method has been studied through the use of worked examples. The results show that the 3D modeling can be as accurate as a relatively

fine mesh finite element model, while it has the advantages of being quicker and easier to set up.

The proposed model that is presented in this paper is part of a research project, which is being carried out in the National Technical University of Athens, for the modeling of steel and composite bridges. The project is always under development so as to be able to simulate the 3D structural behavior of bridges properly. Alternative techniques are being examined for an eventual refinement of the model, in order to obtain the best possible results for different types of bridge.

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## Buckling factors n<sub>cr</sub>