Transformers Made of Composite Magnetic Cores: An Innovative Design Approach

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Abstract: Nowadays, transformers are made of conventional magnetic cores which are constructed of a single grain-oriented or amorphous, magnetic steel. Even though, the transformer is the most efficient of electrical machines, with efficiencies typically above 90%, it is possible to improve transformer performance by using composite magnetic cores. Patents related to this simple and effective technique can be traced back to 1929. The specific technique can be applied to wound core distribution transformers. By using wound cores constructed with a combination of conventional and high permeability grain-oriented steel the total owning cost (TOC) of the transformer can be reduced effectively. This paper presents a brief review of patents on wound and composite magnetic cores and introduces a generalized technique for the determination of the optimum design variables of a new composite wound core design.

Keywords: Composite magnetic cores, distribution transformers, finite element methods, grain-oriented steel, no load losses, optimization methods, total owning cost, wound cores.

1. INTRODUCTION

The design optimization of a wound core distribution transformer consists in minimizing its total owning cost (TOC), where TOC is defined as the first cost of the transformer plus the calculated present value (PV) of its future losses [1]. The transformer manufacturer must minimize TOC while satisfying transformer ratings, design constraints, and technical specifications imposed by international standards, utilities, and users [2].

Based on experimental evidence concerning the non-uniformity of the wound core flux density distribution [3, 4], the TOC of the transformer can be reduced effectively by using composite wound cores constructed with a combination of different grades of grain-oriented magnetic steel. The multiple grade lamination wound core technique introduces only two design variables, it can be applied after the design optimization of the transformer, or it can be integrated directly in the design optimization scheme, resulting in this way in the generalization of the transformer optimization procedure [5, 6]. If applied after the transformer design optimization a significant reduction of the sum of magnetic steel cost and PV of future no load loss is achieved. This is very important considering that the PV of future no load loss constitutes more than 60% of the PV of total future losses and of the various materials required to manufacture a wound core transformer the magnetic steel comprises the largest investment [1]. In the case where the multiple grade lamination technique is integrated in the design optimization scheme the resultant optimum transformer designs tend to possess a reduced TOC in comparison with the optimum designs of conventional wound core transformers.

In order to evaluate the optimum design variables of a multiple grade lamination wound core, the accurate computation of the peak flux density distribution and no load loss is needed. Furthermore, an optimization problem such as this presents multiple optima in the feasible domain. These two problems were addressed by combining anisotropy finite element (FEM) models of very low computational cost, with three stochastic optimization algorithms. From the considered optimization algorithms, an improvement of the simulated annealing (SA) for continuous problems, the simulated annealing with restarts (SAR) [7], is proven to be the most effective in the solution of the optimization problem under consideration.

2. BRIEF REVIEW OF PATENTS ON WOUND AND COMPOSITE MAGNETIC CORES

2.1. Wound Cores

In order to transmit and distribute electrical energy over large distances economically, it is necessary to minimize Joule losses in the transmission lines by using a high voltage. The required increase and decrease of the voltage is performed by the electrical transformer which in its simplest form consists of two coils of conductive wire wound around a magnetic core of soft iron. In practice two types of magnetic cores are used, the stack core and the wound core. The wound core is comprised from long continuous strips of sheet steel wound around the coils. The main advantages include reduction of joints and the use of the grain direction of the steel for the flux path. There have been several patents on wound core design and manufacturing [8-19]. One of the most successful wound core design, used extensively at present Fig. (1) was disclosed in 1960 by Treanor [20].
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The aforementioned wound core design is of simple structure and highly efficient performance and it is used in the construction of distribution transformers with ratings that range from a few kVA to a few MVA.

Despite the fact that the transformer is the most efficient of all electrical machines, present energy costs are forcing transformer manufacturers and utilities to reduce transformer life-cycle costs while maintaining the manufacturing cost within acceptable limits. A number of patents on new transformer topologies [21], novel core joint types [22], and improved amorphous and grain-oriented magnetic steels presenting superior magnetization and low loss characteristics [23-27], have been developed recently in order to reduce the manufacturing and operational cost of the transformer. Nevertheless, there exists an alternative technique with which it is possible to improve transformer efficiency without the need to revolutionize transformer design or magnetic materials. By using composite magnetic cores constructed with a combination of different grades of magnetic steels the transformer designer can effectively reduce cost and losses.

2.2. Composite Magnetic Cores

The concept of composite magnetic cores was disclosed in 1929 by Johannesen [28]. Johannesen claimed that decided savings in material and improvement in efficiency results from the use of nickel-iron alloy, having high permeability or low specific core losses, in the core part surrounded by windings and from the use of silicon steel in the rest part of the core [28]. Since then, there have been a number of patents related to composite magnetic cores where inventors proposed the combination of silicon steel and nickel iron alloy [29-31], hot and cold rolled silicon steels [32], grain-oriented and amorphous steels [33-35], grain-oriented 3% silicon and non oriented 6.5% silicon steels [36, 37], as well as high and low permeability magnetic materials [38-41] for the construction of wound and stack cores of low cost, core loss, and noise.

In 1980 Lin, Ellis, and Burkhardt disclosed a composite wound core having two parallel magnetic circuits of unequal mean lengths as shown in Fig. (2) [42]. The short magnetic circuit, i.e. inner part of the core, is constructed of a high permeability grain-oriented steel, having low loss but high cost unit, and the long magnetic circuit, i.e. outer part of the core, is constructed of a conventional grain-oriented steel, having higher loss but low cost unit [42].

The specific composite wound core exhibits comparable core losses with a core constructed of the high cost, high permeability grain-oriented steel even when the high permeability material represents only a fraction of the total weight of the wound core with the rest part of the core being a low cost, conventional grain-oriented steel. This is attributed to the wound core flux density non-uniformity and the fact that the core loss is a function of the flux density.

In the next sections a new composite wound core design is presented and a generalized optimization procedure is introduced with which the optimum design variables of the composite core, that minimize $TOC$, i.e. the sum of manufacturing and operational cost, are determined.

3. DESCRIPTION OF THE MULTIPLE GRADE LAMINATION WOUND CORE TECHNIQUE

The multiple grade lamination wound core technique is based on experimental evidence concerning the flux density distribution non-uniformity of strip wound cores [3, 4]. According to [3, 4] the peak flux density is low in the inner steel sheets of the wound core, then it increases to a value higher than the core mean flux density, and finally it decreases until the outer sheets. Fig. (3) depicts experimental curves of the peak flux density distribution, across the limb of a wound core consisting of 90 steel sheets, for different magnetization levels. The flux density distribution non-uniformity is attributed to a number of factors like the step-lap joints, the difference in magnetic path length of the steel sheets, harmful stresses induced in the inner steel sheets during core formation, and flux leakage in the inner and outer steel sheets. Specific core loss, i.e. core loss per unit mass, is a function of the flux density modulus. As a result, for an arbitrary magnetization level the specific core loss of
the inner and outer part of the wound core is lower than that of the rest part of the core.

High permeability grain-oriented steels present improved magnetization and specific core loss characteristics in comparison with the conventional grain-oriented steel as illustrated in Fig. (4 & 5). On the other hand the cost unit of high permeability steel is higher than that of conventional grain-oriented steel. Thus, by using conventional grain-oriented steel for the inner and outer part of the wound core and high permeability grain-oriented steel for the rest part of the core, the transformer manufacturer may achieve an optimum tradeoff between first cost and PV of future no load loss.

A multiple grade lamination wound core is shown in Fig. (6) where \( x_1, x_4, x_5, x_6 \) are the geometric parameters of the wound core and \( x_1, x_2 \) are the variables of the multiple grade lamination wound core technique. Figure 7 illustrates the three parts from which an actual multiple grade lamination wound core consists of.

The manufacturing processes used for the construction of the conventional wound core, invented in 1960 by Treanor [20], can be used for the construction of the multiple grade lamination wound core as the only difference is that the multiple grade lamination wound core is constructed of at least two grades of grain-oriented silicon steel with the same lamination thickness.

The composite wound core disclosed in 1980 by Lin, Ellis, and Burkhardt [42], is also constructed of two different grades of grain-oriented steel and it consists of two parallel magnetic circuits of unequal path lengths. However, the multiple grade lamination wound core technique exhibits three significant differences.
1. A multiple grade lamination wound core consists of at least three parallel magnetic circuits in contrast with the composite wound core disclosed in [42] which consists of two parallel magnetic circuits.

2. The multiple grade lamination technique can be extended so that the composite wound core is constructed of more than two different grades of grain-oriented steel and more than three parallel magnetic circuits.

3. Finally, the multiple grade lamination technique is a generalized optimization procedure which enables the minimization of the TOC and not just the reduction of core losses or manufacturing cost.

4. NO LOAD LOSS EVALUATION OF MULTIPLE GRADE LAMINATION WOUND CORES

Accurate no load loss evaluation of wound core distribution transformers present significant difficulties and analytical relationships usually do not suffice [43]. A satisfactory estimation can be achieved however, by using field analysis numerical techniques in conjunction with the detailed modeling of the magnetic material properties of the core [44]. The methodology applied in the present paper is to combine the experimentally evaluated local specific core losses with the computed peak flux density distribution of the wound core. The local specific core losses are expressed as a function of peak flux density and the computation of the peak flux density distribution of the wound core is performed using the finite element method.

In order to compute the flux density distribution of a wound core, each individual steel sheet and the varnish-air composite between successive sheets must be modeled. This approach was followed recently for the simulation of toroidal wound cores consisting of a small number of steel sheets [45]. The disadvantage of the aforementioned approach is its extremely high computational cost that limits the effectiveness of the method to two-dimensional (2D) applications [45], despite the recent advancements in computing. Furthermore, typical wound cores, used for the construction of distribution transformers, consist of hundreds of grain-oriented steel sheets and even the 2D field analysis is practically impossible due to the inherent large mesh size required to represent the laminated core.

In the present paper the accurate representation of the wound core with a low computational cost is achieved by considering the iron-laminated material as homogeneous media and by developing an elliptic anisotropy model specifically formulated for wound cores [5, 6, 46]. In this manner the mesh size is kept to a minimum and the computational cost of the FEM analysis is reduced drastically. The elliptic anisotropy model takes into account the directional dependence of the $B-H$ characteristic due to the iron laminations and the grain orientation of the magnetic steel. The material modeling of the wound core with the aforementioned technique makes practical not only the 2D FEM analysis but also the 3D FEM analysis and it can be applied both to conventional and multiple grade lamination wound cores.

4.1. 2D FEM Analysis and Formulation

In 2D FEM analysis the Poisson’s equation is solved which is a function of the magnetic vector potential and the reluctivity

$$\nabla \cdot \mathbf{v} \nabla A_z = -J_z,$$

where $\mathbf{v}$ is the reluctivity tensor and it is represented as a function of the flux density, $A_z$ is the $z$-th component of the magnetic vector potential, and $J_z$ is the $z$-th component of the current density.

The representation of the nonlinear characteristics of the core materials is achieved by cubic splines interpolation of the $v(B)$ characteristic and a Newton-Raphson iterative scheme is used for the solution of the particular nonlinear problem.

The elliptic anisotropy model, for the 2D FEM analysis, is based on the assumption that the flux density $\mathbf{B}$ has an elliptic trajectory for the modulus of magnetic field intensity constant [6]. Therefore, if $v_p$ is the reluctivity tangential to the lamination rolling direction, $v_q$ is the reluctivity normal to the lamination rolling direction, and $r$ is the ratio of the ellipse semi-axes then

$$v_q = rv_p, \quad r > 1.$$  

4.2. 3D FEM Analysis and Formulation

The 3D FEM analysis is based on a modified magnetic scalar potential formulation necessitating no prior source field calculation [46, 47]. According to this formulation the magnetic field intensity $\mathbf{H}$ is partitioned as follows

$$\mathbf{H} = \mathbf{K} - \nabla \Phi$$

where $\Phi$ is a scalar potential extended all over the solution domain and $\mathbf{K}$ is a fictitious field distribution. The distribution of $\mathbf{K}$ is easily determined analytically or numerically by the conductors shape with little computational effort [47]. The solution of the problem is obtained by discretizing (4) that ensures the solenoidality of the total field

$$\nabla \cdot (\mathbf{\mu} (\mathbf{K} - \nabla \Phi)) = 0$$

where $\mathbf{\mu}$ is the magnetic permeability tensor which is a function of the magnetic field intensity. The advantages of the latter formulation are its simplicity and computational efficiency in contrast with conventional magnetic scalar potential formulations.

The nonlinearity of the grain-oriented magnetic steel is taken into account by cubic splines approximation of the $\mu(H)$ characteristic and again a Newton-Raphson iterative scheme is adopted for the solution of the nonlinear problem.

In the case of the 3D FEM analysis and the particular magnetic scalar potential formulation, the elliptic anisotropy model is based on the assumption that the field intensity $\mathbf{H}$
has an elliptic trajectory for the modulus of flux density constant [46]. Thus, if $\mu_r, \mu_q$ are the magnetic permeability tangential and normal to the lamination rolling direction, and $r$ is the ratio of the ellipse semi-axes then

$$\mu_q = r\mu_r, \quad 0 < r < 1.$$  \hspace{1cm} (5)

Based on the formulations of Subsections 4.1 - 4.2 the author developed 2D, 3D FEM software and also a graphics postprocessor. For the 2D and 3D mesh generation the Triangle and TetGen open-source software were used. The postprocessor program for the no load loss computation has been developed separately from the 2D and 3D FEM calculation. Once the peak flux density distribution is obtained, it is combined with the experimentally determined local specific core losses which are expressed as a function of the peak flux density and are approximated by cubic splines. In this way the loss in each element can be calculated. The overall no load loss is obtained by summing up the losses in individual elements.

**5. WOUND CORE DISTRIBUTION TRANSFORMER OPTIMIZATION PROCEDURE**

A practical method of transformer selection and specification based on economic reality and fair to both transformer manufacturers and utilities is to make the purchase decision on the basis of calculated TOC [1]. TOC is defined as the first cost plus the calculated PV of future losses and is given by

$$TOC = BP + A_{factor} P_{NL} + B_{factor} P_{LL}$$  \hspace{1cm} (6)

where $BP$ is the first cost (bidding price), $A_{factor}$ is the PV of 1 W of no load loss over the life of the transformer, $P_{NL}$ is the no load loss, $B_{factor}$ is the PV of 1 W of load loss over the life of the transformer, and $P_{LL}$ is the load loss.

The first cost is expressed by

$$BP = \frac{\sum_i C_i M_i(x) + C_r + C_i}{SM}$$  \hspace{1cm} (7)

where $C_i, M_i$ is the cost unit ($$/kg) and mass (kg) of the $i$-th material of the transformer, $x$ is the design variables vector, $C_r$ is the labor cost ($), $C_i$ is the installation cost ($), $SM$ is the sales margin, and $n$ is the number of the materials.

The equation for $A_{factor}$ is

$$A_{factor} = PV \cdot EL \cdot HPY$$  \hspace{1cm} (8)

where $PV$ is the present-value multiplier for selected project life ($PL$) and discount rate ($DR$), $EL$ is the cost of electricity ($$/Wh) expressed as fixed number, and $HPY$ is hours of operation per year (8,760 h).

A simple method for calculating PV of future annual expenses is the following, where the term $DR$ includes all capital costs for the business entity i.e. interest, insurance, taxes etc.

$$PV = \frac{\mu_{P-1}}{1 + DR}$$  \hspace{1cm} (9)

The equation for $B_{factor}$ is given by

$$B_{factor} = A_{factor} P^i$$  \hspace{1cm} (10)

where $P$ is the per unit load.

Electric utilities and users derive $A_{factor}, B_{factor}$ values from a number of utility specific-parameters, such as system cost, avoided cost for new facilities etc. Based on these values, transformer manufacturers determine the optimum design by minimizing TOC while satisfying a number of constraints imposed by international standards and customer needs.

In the following two subsections it is shown how the multiple grade lamination wound core technique can be applied after and integrated into the transformer optimization procedure.

**5.1. Applying Multiple Grade Lamination Technique after the Transformer Design Optimization**

This case is not only of practical importance, but it also allows one to study the effect of the multiple grade lamination wound core technique thoroughly as it is isolated from the design optimization procedure. The wound core design variables $x_1, x_i, x_3$, and $x_4$, shown in Fig. (6), the magnitude of the mean flux density $B$, and the number of turns of the primary and secondary windings $N_1, N_2$ are constants, predetermined by a typical industrial design optimization scheme [2, 48]. The multiple grade lamination design variables $x_1, x_3$ must be determined which are subject to constraints (11) - (13), where $x_i$ is the thickness of the core section.

$$0 \leq x_i \leq x_3$$  \hspace{1cm} (11)

$$0 \leq x_i \leq x_3$$  \hspace{1cm} (12)

$$0 \leq x_i + x_3 \leq x_4$$  \hspace{1cm} (13)

The evaluation of the multiple grade lamination wound core design parameters consists in the minimization of an objective function $f(x)$, where $x$ is the vector of the core variables $(x_1, x_3)$ illustrated in Fig. (6).

The objective function has to take into account the PV of the future no load loss and the cost of the conventional and high permeability grain-oriented steel. During the optimization process, the no load loss is calculated with the use of the FEM anisotropy models described in Section 4. The mass of the high permeability and conventional grain-oriented steel, $M_{IM}$ and $M_{IM}$, are calculated by (14) and (15) respectively, where $d_{in}$ is the magnetic steel density.
constant and also the cost of the winding material must 
amforementioned it follows that in this case the load loss is not 
respectively. Substituting (16) into (6) yields 
conventional grain-oriented steel cost unit ($/kg)
objective function to be minimized is the following 
in the particular case the load loss is a constant. Thus, the 
considered here for the minimization of 
where

\[
M_{nt} = d_w c_D \left\{ \pi x_2^3 x_3 + 2x_2 x_4 \left( \pi x_3 + x_4 + x_5 \right) \right\} 
\]

(14)

\[
M_{sm} = d_w c_D \left\{ \pi x_1 (x_2^3 - x_1^2 - 2x_2 x_4 \right\} 
+ 2x_2 (x_1 - x_5) (x_2 + x_4) \right\} 
\]

(15)

Since only the high permeability and conventional magnetic steels are examined in this case, (7) is reduced to 
the following

\[
BP = \frac{(C_{iat} M_{iat} + C_{sm} M_{sm}) + C_i + C_f}{SM} 
\]

(16)

where \( C_{iat} \) and \( C_{sm} \) are the high permeability and 
conventional grain-oriented steel cost unit ($/kg) 
respectively. Substituting (16) into (6) yields 

\[
f(x) = \frac{(C_{iat} M_{iat} + C_{sm} M_{sm}) + C_i + C_f}{SM} 
+ A_{factor} P_{nll} + B_{factor} P_{ll}.
\]

The labor and installation cost being constant may not be 
considered here for the minimization of \( f(x) \). Furthermore, 
in the particular case the load loss is a constant. Thus, the 
objective function to be minimized is the following

\[
f(x) = \frac{(C_{iat} M_{iat} + C_{sm} M_{sm}) + A_{factor} P_{nll}}{SM}.
\]

(17)

(18)

5.2. Integrating Multiple Grade Lamination Technique in 
the Transformer Design Optimization Procedure

In this case, the multiple grade lamination technique is 
integrated in a simple design optimization procedure. of a 
single-phase, core type wound core transformer. Even 
though there are many papers that give a complete treatment 
of the transformer optimization problem, the derivation 
given here is sufficient to develop and illustrate how the 
multiple grade lamination technique can be integrated in any 
state of the art transformer optimization procedure. 

The objective function must take into account not only 
the cost of the magnetic steel and the PV of future no load 
loss but also the cost of the winding material and the PV of 
future load loss. The variables \( x_i \), \( x_s \), \( x_t \), \( x_k \) shown in Fig. 
(6). \( B \), \( N_s \), and \( N_t \) are now to be determined. From 
the aforementioned it follows that in this case the load loss is not 
constant and also the cost of the winding material must be 
taken into account. Thus, (7) is written as follows

\[
BP = \frac{(C_{iat} M_{iat} + C_{sm} M_{sm} + C_{cs} M_{cs}) + C_i + C_f}{SM} 
\]

(19)

where \( C_{cs} \) is the winding material cost unit ($/kg) and \( M_{cs} \) 
is the winding material mass

Substituting (19) into (6) and omitting once again the 
labor and installation cost yields the following objective function

\[
g(x) = \frac{(C_{iat} M_{iat} + C_{sm} M_{sm} + C_{cs} M_{cs})}{SM} 
+ A_{factor} P_{sll} + B_{factor} P_{sl}.
\]

The optimum wound core transformer design and 
operational parameters are evaluated by minimizing (20). 
The no load loss is evaluated using the FEM models of 
Section 4. \( M_{cs} \) and \( P_{ll} \) are given by (21) and (22) 
respectively, where \( d_{cs} \) is the winding material density 
(\( kg/m^3 \)), \( c_g \) is the coil fill factor, \( \rho \) is the winding material 
resistivity (\( \Omega \cdot m \)), and \( J \) is the current density (\( A/m^2 \)). 

\[
M_{cs} = d_{cs} c_g x_1 x_3 \left( 2x_3 + \pi x_4 + 2x_5 \right) 
\]

(21)

\[
P_{ll} = c_g \rho J^2 x_1 x_3 \left( 2x_3 + \pi x_4 + 2x_5 \right)
\]

(22)

The objective function of (20) is subject to five nonlinear 
constraints. Two equality constraints, (23) and (24), 
representing the primary induced voltage (\( E_p \)) constraint, 
and the rated power (\( S_{rated} \)) constraint respectively, and three 
inequality constraints (25) - (27), where \( f \) is the frequency 
(Hz), \( k \) is the portion of the solid conductor area contributed 
by the primary winding, and \( P_{ll} , P_{sll} , P_{sl} \), are the guaranteed 
load and no load loss respectively (W), specified by 
international technical specifications and customer needs.

\[
E_p = \sqrt{2} \pi f c_g N_p x_3 x_5 B = 0
\]

(23)

\[
S_{rated} = \sqrt{2} \pi f c_g c_p k J x_1 x_3 x_4 x_5 B = 0
\]

(24)

\[
P_{ll} - 1.15 P_{ll}^* < 0
\]

(25)

\[
P_{sll} - 1.15 P_{sll}^* < 0
\]

(26)

\[
P_{ll} + P_{sll} - 1.10 \left( P_{ll}^* + P_{sll}^* \right) < 0
\]

(27)

Finally, the secondary winding turns are equal to 

\[
N_t = E_s N_t / E_p
\]

(28)

where \( E_s \) is the secondary induced voltage.

The optimization procedure developed herein is a 
generalization of the conventional design procedure as by 
setting \( x_i = 0 \), \( x_s = x_t \) the problem reduces to the design 
optimization of a transformer constructed of the conventional 
high permeability grain-oriented steel and by setting \( x_i = 0 \), \( x_s = x_t \) 
the problem reduces to the design optimization of a transformer 
constructed of the high permeability grain-oriented steel.

5.3. Optimization Algorithms

For the solution of the optimization problems presented 
in Subsections 5.1 - 5.2, five deterministic and three 
stochastic optimization algorithms have been tested. The 
deterministic algorithms used are three gradient-based, the 
Broydon-Fletcher-Goldfarb-Shanno (BFGS), the Davidson-
Fletcher-Powell (DFP), and the steepest descent, and two 
non-gradient methods, the downhill simplex method, and the
pattern search. The three stochastic optimization algorithms used are the genetic algorithm (GA), the simulated annealing (SA), and the simulated annealing with restarts (SAR). For continuous problems, e.g. optimization of electromagnetic devices, SA uses a modification of the downhill simplex method to generate random changes. However, premature convergence has been observed [7] that results in the pinning of the simplex in a local minimum. SA then uses a large number of evaluations to explore a small portion of the design space. SAR effectively reduces the objective function evaluations by forcing the simplex to start from a random point in the space, if it becomes too small [7].

6. EXPERIMENTAL SETUP

The experimental setup used for the local flux density and no load loss measurements of wound cores is illustrated in Fig. (8). It consists of a PC and a National Instruments (NI) 6143 data acquisition (DAQ) card (8 voltage differential inputs, 16 bit, 250 ksamples / s). The excitation coil terminals are connected to the input of the DAQ card via an active differential voltage probe (3 dB frequency of 18 MHz) and a current probe based on the Hall Effect (1 dB frequency of 150 kHz) is used for capturing the no load current. Both probes are depicted in Fig. (9). For the magnetization of the wound cores, a 23 turn excitation coil is supplied by a one-phase 230 V, 50 Hz source through a variable transformer. The no load loss was evaluated by the acquired voltage and current data with appropriate virtual instruments (VI) created with the use of LabVIEW.

![Experimental setup](image)

Two turn search coils wound around the total width of the sheet, were employed for determining the peak flux density distribution along the limb, yoke, and corner of the core for different magnetization levels. The voltages induced in the search coils were measured by connecting their terminals directly into the DAQ card voltage differential inputs. The local peak flux density $B_p$ is given by (29) where $\langle V \rangle$ is the average rectified voltage, $N$ is the number of turns of the search coils, and $S$ is the cross-sectional area of a single steel sheet. Furthermore, it was verified using Fourier analysis that the voltage applied to the excitation coil contained only odd harmonics. Thus, it was assumed that there are only odd harmonics in the local flux density waveforms.

$$B_p = \frac{\langle V \rangle}{4\pi N S}$$ (29)

![Current probe and differential voltage probe](image)

Figure 10 illustrates a detail of the wound core, the excitation coil, and the search coils. The wire used for the search coils is a solderable enameled copper wire 0.1 mm in diameter. A total of 30 search coils were inserted in the wound core for the experimental evaluation of its peak flux density distribution along the limb, yoke and corner. The overall loss of the core did not change noticeably after the coils were wound in position. Therefore, it was assumed that the flux distribution did not change much due to the search coils being introduced.

![Detail of the wound core, the excitation coil and the search coils](image)

7. COMPARISON BETWEEN COMPUTED AND EXPERIMENTAL RESULTS

The areas comprising the 2D FEM model of a conventional wound core are depicted in Fig. (11). One half...
of the geometry is modeled due to symmetry and in order to reduce the computational requirements. Also, a Dirichlet boundary condition \( A_z = 0 \) is imposed on the outer boundaries of the 2D model. The magnetic vector potential contours of a conventional wound core during no load test are shown in Fig. (12). Due to the negligible leakage flux during no load test, the contours are concentrated in the core area as can be seen from Fig. (12).

Figure 13 depicts the computed peak flux density nodal plot with the 2D FEM model, of a conventional wound core constructed of the high permeability grain-oriented steel M-OH. Figure 14 & 15 illustrate the computed peak flux density nodal plot with the 2D FEM model, of a multiple grade lamination wound core for two different \( x_1, x_2 \) configurations (Fig. (14): \( x_1 = 0 \) mm, \( x_2 = 10 \) mm, and Figure 15 \( x_1 = 5 \) mm, \( x_2 = 10 \) mm). The multiple grade lamination wound core is constructed of the conventional grain-oriented steel M4, and the high permeability grain-oriented steel M-OH. In all cases the wound core geometry parameters are \( x_1 = 24.3 \) mm, \( x_2 = 57 \) mm, \( x_3 = 183 \) mm, \( x_4 = 190 \) mm, and the magnetization level used for the FEM analysis is 1.5 T. An arbitrary level of magnetization is achieved by using magnetostatic FEM analysis and a systematic iterative procedure based on the bisection technique. Typically 20 iterations are enough in order to determine the current density which produces the desired magnetization level with an error of the order of \( 10^{-4} \).

The computed peak flux density distribution across the limb of the core, line AB of Fig. (14 & 15), for the two aforementioned \( x_1, x_2 \) configurations and for two different magnetization levels is illustrated in Figs. (16-19). These distributions show clearly that the flux density is
Fig. (15). Peak flux density nodal plot of a multiple grade lamination wound core \((B = 1.5 \text{T}, x_1 = 5 \text{ mm}, x_2 = 10 \text{ mm})\).

considerably higher in the high permeability grain-oriented steel than in the conventional grain-oriented steel. Also the peak flux density decreases almost linearly both in the conventional and high permeability steel regions until the outer steel sheets.

Table 1 presents a comparison between the calculated no load loss with the FEM models of Section 4 and the respective experimental values for a number of \(x_1, x_2\) configurations. The considered wound core parameters are \(x_1 = 24.3 \text{ mm}, x_4 = 57 \text{ mm}, x_4 = 183 \text{ mm}, x_5 = 190 \text{ mm}\), and the mean flux density is \(B = 1.7 \text{ T}\). As can be seen from Table 1, the computed results are in good agreement with the measured ones and the error is less than 5\% in all cases. Also the difference between the computed no load loss with the 2D and 3D FEM models is insignificant and is less than 1.1\% in all cases.

Fig. (16). Peak flux density distribution along line AB for \(B = 1.4 \text{T} \text{ and } x_1 = 0 \text{ mm}, x_2 = 10 \text{ mm}\).

Fig. (17). Peak flux density distribution along line AB for \(B = 1.5 \text{T} \text{ and } x_1 = 0 \text{ mm}, x_2 = 10 \text{ mm}\).

Fig. (18). Peak flux density distribution along line AB for \(B = 1.4 \text{T} \text{ and } x_1 = 5 \text{ mm}, x_2 = 10 \text{ mm}\).

Fig. (19). Peak flux density distribution along line AB for \(B = 1.5 \text{T} \text{ and } x_1 = 5 \text{ mm}, x_2 = 10 \text{ mm}\).

8. APPLICATION OF THE MULTIPLE GRADE LAMINATION TECHNIQUE

8.1. Application after the Transformer Design Optimization Procedure

The multiple grade lamination technique was applied to the outer cores of a 100 kVA, 20 kV / 0.4 kV three-phase wound core distribution transformer after the design
optimization with a heuristic methodology which is already used by the transformer industry [48]. The parameters of the wound cores under consideration are \( x_1 = 51.3 \) mm, \( x_2 = 57 \) mm, \( x_3 = 183 \) mm, \( x_4 = 190 \) mm, and the mean flux density is 1.723 T. The optimum core material for the three-phase transformer, as predetermined by the heuristic procedure, is the conventional grade M4 0.27 mm. The multiple grade lamination wound cores are constructed of the conventional and high permeability grain-oriented steels, M4 0.27 mm and M-OH 0.27 mm respectively.

During the minimization of (18) a number of local minima with values very close to the global minimum were identified. Due to this fact the number of successes of the GA was lower than that of the SA and SAR algorithm, as the GA was trapped most of the times in local minima. The optimum distribution of the design parameters as calculated by the SA and SAR algorithms correspond to an objective function value of \( f(\mathbf{x}) = 677.5 \) $, i.e. a 3.2% reduction of the sum of magnetic steel cost and PV of future no load loss as it was pre-evaluated by the industrial optimization scheme [48]. Furthermore, the SAR algorithm exhibits a 20% reduction in objective function calls comparing to the classic SA algorithm. Figure 20 depicts the variation of the objective function with respect to the SAR iterations and Fig. (21) illustrates a detail of the computed flux density distribution with the 3D FEM model of the multiple grade lamination wound core with the optimum design variables \( x_1 \), \( x_2 \).

8.2. Application of the New Technique to a Single-Phase Wound Core Distribution Transformer

The transformer studied, is a 50 kVA, 20 kV / 231 V single-phase, core type wound core transformer with guaranteed no load and load loss, 130 W and 560 W respectively. The design optimization method of Subsection 5.2 was used in order to determine the optimum design of the transformer in three cases. In the first case the wound core is constructed of the conventional grain-oriented steel M4 0.27 mm, in the second case the wound core is constructed of the high permeability grain-oriented steel M-OH 0.27 mm, and in the last case the wound core is constructed with a combination of the two aforementioned magnetic steels. In the two first cases the multiple grade lamination wound core variables were constrained appropriately by setting \( x_2 = 0 \), \( x_3 = x_1 \) (M4) and \( x_2 = 0 \), \( x_4 = x_3 \) (M-OH).

Table 2 summarizes the optimum configurations and the number of objective function evaluations for the three cases, obtained from the minimization of (20) by the SAR algorithm, i.e. the most effective of the three stochastic optimization algorithms tested, while satisfying the constraints (23) - (27). The parameters used for the optimization analysis are shown in Table 2. The \( A_{\text{factor}} \) used is equal to 8 $/W and it corresponds to a 20 years project life and a 10.429% discount rate. Furthermore, a 0.5 pu load is used, according to NEMA recommendations for medium-voltage transformers [1], which leads to a \( B_{\text{factor}} \) of 2 $/W.

Table 2 shows that the application of the multiple grade lamination technique results in a reduction (\( \Delta g = 2.43\% \) and \( \Delta g = 1.93\% \) respectively) of the sum of magnetic steel cost, winding material cost, and PV of total future losses, compared to the cases where the transformer is manufactured using only conventional (M4) or high permeability (M-OH) grade. The procedure used for constructing a multiple grade lamination wound core is the same with the one used for the conventional wound core. Thus, the method of assembling a

---

### Table 1. Comparison Between Measured and Computed no Load Loss (\( x_1 = 24.3 \) mm)

<table>
<thead>
<tr>
<th>( x_1 ) (mm)</th>
<th>( x_2 ) (mm)</th>
<th>No load loss 2D FEM (W)</th>
<th>No load loss 3D FEM (W)</th>
<th>No load loss Experiment (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>24.3</td>
<td>0</td>
<td>22.7</td>
<td>22.8</td>
<td>23.1</td>
</tr>
<tr>
<td>0</td>
<td>24.3</td>
<td>18.5</td>
<td>18.7</td>
<td>19.4</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>21.2</td>
<td>21.3</td>
<td>22.0</td>
</tr>
<tr>
<td>0</td>
<td>10</td>
<td>21.7</td>
<td>21.7</td>
<td>22.2</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>21.9</td>
<td>22.0</td>
<td>22.4</td>
</tr>
</tbody>
</table>

---

**Fig. (20).** Variation of the objective function with respect to SAR iterations.
closed core section about a preformed electrical winding coil is exactly the same and as a result the labor cost does not change. From the aforementioned it follows that by integrating the multiple grade lamination technique in a typical design optimization procedure, the transformer designer can effectively control not only the magnetic steel cost and the no load loss, but also the winding material cost, the load loss, and other design and operational parameters, in contrast with the method given in Subsection 5.1. The deterministic algorithms reported in Subsection 5.3 were also used for the minimization of (18) and (20), however they did not determine the global optimum but instead they converge to a local minimum near to the starting point. Table 2. Comparison Between Optimum Designs of Conventional and Multiple Grade Laminations Wound Core Transformers

<table>
<thead>
<tr>
<th>Quantity</th>
<th>M4 0.27 mm</th>
<th>M-OH 0.27 mm</th>
<th>Multiple grade wound core (M4 and M-OH)</th>
</tr>
</thead>
<tbody>
<tr>
<td>x₁ (mm)</td>
<td>107.6</td>
<td>0</td>
<td>4.2</td>
</tr>
<tr>
<td>x₂ (mm)</td>
<td>0</td>
<td>113.1</td>
<td>64.2</td>
</tr>
<tr>
<td>x₃ (mm)</td>
<td>107.6</td>
<td>113.1</td>
<td>110.8</td>
</tr>
<tr>
<td>x₄ (mm)</td>
<td>60</td>
<td>49.5</td>
<td>66</td>
</tr>
<tr>
<td>x₅ (mm)</td>
<td>268</td>
<td>300</td>
<td>231</td>
</tr>
<tr>
<td>x₆ (mm)</td>
<td>152</td>
<td>152</td>
<td>152</td>
</tr>
<tr>
<td>Nₛ</td>
<td>39</td>
<td>36</td>
<td>37</td>
</tr>
<tr>
<td>Nᵢ</td>
<td>3,377</td>
<td>3,117</td>
<td>3,203</td>
</tr>
<tr>
<td>B (T)</td>
<td>1.69</td>
<td>1.74</td>
<td>1.73</td>
</tr>
<tr>
<td>Pₛₑ (W)</td>
<td>143.8</td>
<td>140.4</td>
<td>133.4</td>
</tr>
<tr>
<td>Pᵢₑ (W)</td>
<td>609.4</td>
<td>545.3</td>
<td>598.4</td>
</tr>
<tr>
<td>Mₛₑ (kg)</td>
<td>120.0</td>
<td>0</td>
<td>57.9</td>
</tr>
<tr>
<td>Mᵢₑ (kg)</td>
<td>0</td>
<td>133.8</td>
<td>59.2</td>
</tr>
<tr>
<td>Mₑₗ₁ (kg)</td>
<td>35.6</td>
<td>31.8</td>
<td>34.9</td>
</tr>
<tr>
<td>g(x) (S)</td>
<td>3.706</td>
<td>3.687</td>
<td>3.616</td>
</tr>
<tr>
<td>Δg (%)</td>
<td>0</td>
<td>0.51</td>
<td>2.43</td>
</tr>
<tr>
<td>SAR evaluations</td>
<td>3,057</td>
<td>3,208</td>
<td>4,526</td>
</tr>
</tbody>
</table>

Aₜₚₑₑ = 8 S/W, P = 0.5 pu, Bₜₚₑₑ = 2 S/W, SM = 0.5
Cₑₑ = 7.46 $, Cₑₑₑ = 3.73 $, Cₑₑₑₚₑₑ = 3.357 $, EL = 0.1 $/kWh
eₑₑ = 0.965, dₑₑₑ = 8,930 kg/m³, dₑₑₑₑ = 7,650 kg/m³, ρ = 1.7·10⁸ Ω·m

f = 50 Hz, J = 3·10⁴ A/m², Pᵢₑₑ = 560 W, Pₛₑₑ = 130 W

9. CURRENT & FUTURE DEVELOPMENTS

Probably conventional transformers will continue to be the dominant component for transmitting and distributing electrical energy for a long time despite recent development of radically new transformer topologies and the advent of superconducting transformers. This is due to the robustness, reliability, and efficiency of the conventional transformer. Nevertheless, present energy and transformer materials costs are driving utilities and transformer manufacturers to reduce losses and manufacturing costs. A simple and effective way to achieve this, without revolutionizing the topology of the transformer and magnetic materials, is the composite magnetic core technique with which the manufacturer can achieve an optimum tradeoff between manufacturing and operating cost.

Even though patents related to composite magnetic cores can be traced back to at least 1929, the specific technique remains a challenging and innovative design approach and it has not been used extensively due to inherent difficulties in determining the optimum configuration of the composite magnetic core design variables. The present paper introduces a novel method that reduces the sum of first cost and PV of future losses of wound core distribution transformers. It consists in the evaluation of specific design variables of multiple grade laminations wound cores by minimizing properly defined objective functions. The proposed methodology can be used after the design optimization of the transformer or it can be directly integrated to a typical optimization procedure. The minimization of the objective functions is based on a formulation that combines FEM anisotropy models of low computational cost with stochastic optimization algorithms.

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CONFLICT OF INTEREST

I would like to declare that there are no other Conflicts of Interest or potential Conflicts of Interest. Furthermore, no patents which are in various stages of legal litigation have been cited.

REFERENCES