Techno-economic comparative evaluation of mixed and conventional magnetic wound cores for three-phase distribution transformers

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This paper presents a comparative evaluation of conventional and mixed magnetic wound cores for three-phase distribution transformers. The authors utilize a techno-economic analysis based on an analytical approach in order to determine the variables that differentiate the mixed three-phase transformer from the conventional one. The techno-economic characteristics of two 100 kVA three-phase transformers, a conventional one manufactured of HiB electrical steel and a mixed one manufactured of conventional and HiB steels, are compared using an estimator function and laboratory tests. The authors demonstrate that the mixed three-phase transformer presents improved characteristics over the conventional one for a wide range of sales margins, material costs, magnetization levels, and geometric variables. As a consequence, mixed magnetic wound cores can be an excellent alternative for the E.U. distribution networks where it is necessary to reduce losses, greenhouse gas emissions, and energy costs.

1. Introduction

In the European Union (EU-27), according to strategies for development and diffusion of Energy Efficient Distribution Transformers (SEEDT), losses of distribution transformers are calculated at 33 TWh/year [1,2]. In addition, the reactive power losses and higher harmonics contribute a further 5TWh/year, resulting in total losses of about 38 TWh/year [1,2]. Moreover, an estimated of 5.5 million of conventional distribution transformers will be installed in the distribution grids of Europe during the next 33 years (2017–2050) [3,4]. The aforementioned represents an additional energy consumption of 72 TWh, a production of 8 Mt of CO2 emissions, and consequently a negative impact on the environment and the distribution grids.

As a result, utilities, distribution system operators, transformer manufacturers, and individual customers, are all interested in purchasing or manufacturing distribution transformers of the lowest possible total owning cost (TOC) i.e., the sum of the first cost of the transformer and the present value of its future losses, maintenance costs and so on [5]. The minimization of the TOC of a transformer, while satisfying international standards like the IEC 60076, is carried out by optimization procedures that take into account all aspects of the analysis of a transformer, like electromagnetic, thermal and so on, the capitalization of losses, availability of materials like electrical steels and winding materials etc. [6]. However, after the application of the aforementioned optimization processes there is no more room for reducing further the TOC of a transformer. As a result, other solutions and approaches must be sought.

In this paper, such an out of the box approach is used for the reduction of the TOC which is based on experimental evidence concerning the flux density non-uniformity of individual wound cores of conventional three-phase distribution transformers [7]. By using high magnetization grain-oriented electrical steel (HiB) for the wound cores operating at high flux density and conventional electrical steel for the wound cores operating at lower flux density, the overall no-load losses of the mixed three-phase transformer exhibits comparable no-load losses with a conventional distribution transformer [8], but at a lower manufacturing cost i.e., the TOC is reduced.

It is not implied that the optimum mixed three-phase distribution transformer exhibits always a lower TOC than the optimum conventional one or that the conventional optimization procedure should be abandoned all-together. Instead, it is proposed to update the existing transformer optimization procedures in order to include the mixed three-phase wound core topology.
then compare the optimum designs of the mixed and conventional three-phase wound core, and finally make the transformer selection decision based on which design presents the lowest total owning value while satisfying international standards, customer needs, and all necessary constraints.

The concept of mixed three-phase wound cores differs significantly with existing approaches appearing in research papers and patents in the technical literature. In patent [9] and in research papers [10–12] composite single-phase wound cores are developed. In Ref. [13] the aforementioned composite single phase wound cores are combined in order to manufacture the respective three-phase wound core. Also, in research papers [14–16] the combining of electrical steels is carried out on three-phase, three-legged, stack core transformers whereas in the present paper a mixed three-phase, five legged, wound core is developed that presents a significantly different topology in comparison with the stack core transformers.

The purpose of this paper is on one hand to introduce the concept of the mixed three-phase wound core and on the other hand to determine the variables and conditions that differentiate the mixed and conventional three-phase wound core designs from each other. Afterwards, it is investigated how those conditions and variables, that determine the core selection decision, favor the mixed three-phase wound core design over the conventional one.

2. Working principle of the mixed three-phase wound core distribution transformers

2.1. Brief description of conventional three-phase, five-legged distribution transformer

A large number of the installed three-phase, oil-immersed, distribution transformers are constructed of wound cores assembled about preformed windings. Ratings typically range from 50 kVA to 2 MVA. Fig. 1 shows the active part of a three-phase, five-legged transformer. The transformer core is assembled of two large, inner wound cores (cores B and C) and two smaller, outer wound cores (cores A and D). Typically, all individual wound cores are manufactured of the same grade of electrical steel.

2.2. Experimental findings on flux density non-uniformity in conventional three-phase transformers

Experiments carried out on the conventional five-legged, wound transformer core show that the flux density of the individual cores is not the same [7]. More specifically the flux density of the outer cores is lower than the inner wound cores. Only after a certain magnetization level the flux density of the outer wound cores reaches that of the inner wound cores due to the magnetic saturation of the electrical steel. The aforementioned is valid regardless of the grade of the electrical steel. Fig. 2 shows the peak flux density as obtained experimentally in Ref. [7], of the wound cores of a 100 kVA three-phase transformer core manufactured 100% with HiB steel for two calculated magnetization levels, 1.2T and 1.6T. Fig. 2 shows that the flux density of the outer wound cores A and D is significantly lower than the inner wound cores B and C.

The magnetization level of the three-phase transformer is calculated using a standard method applied in the transformer industry. The specific method considers the connection of the excitation winding and a balanced, three-phase, sinusoidal, voltage source. As a result, and by applying Faraday’s law, the magnetization level of the three-phase transformer is given by Eq. (1) in the case of a delta connection. \( N_i \) is the number of winding turns of the excitation winding, \( c_f \) is the core stacking factor (0.965), \( E_0 \) is the thickness of the wound cores shown in Fig. 2, \( D \) is the axial length of the core, \( f \) is the frequency, and \( V_{rms} \) is the rms value of the line-to-line voltage.

\[
B = \frac{\sqrt{2}}{4 \cdot \pi} \cdot \frac{V_{rms}}{c_f \cdot E_0 \cdot D \cdot f \cdot N_i}
\] (1)

On the other hand, the peak magnetic flux density of individual wound cores shown in Fig. 2, is obtained experimentally by using search coils and an experimental setup based on a PC equipped with a data-acquisition PCI card of National Instruments (NI6143), current probes based on the Hall Effect, and active differential voltage probes. The captured current and voltage waveforms are post-processed, using virtual instruments created with LabVIEW software, in order to compute iron losses, values of local peak flux density, and the harmonic content of the flux density waveforms [7,8].

Even though, the peak flux density of the outer wound cores is lower than the inner cores, the induced voltage waveform in all of the secondary windings has the same amplitude and it is offset from the others by 120°. This is due to the high harmonic content of the flux density waveforms [17,18] of the outer and inner wound cores caused by the unsymmetrical magnetic circuit topology of the three-phase, wound core, transformer which was first proposed in Ref. [19]. Detailed experimental and theoretical studies of the five-legged three-phase wound core transformer are given respectively in Refs. [7,20].
2.3. Concept of the mixed three-phase wound cores

The working principle of the mixed three-phase wound cores is based on the aforementioned experimental findings concerning the local flux density distribution non-uniformity of the inner and outer wound cores. It is also based on the fact that high magnetization grain oriented electrical steel presents improved losses as well as improved magnetization characteristics over conventional grain oriented cold rolled steels but at a higher unit cost [21].

It would therefore be logical to place the HiB steel grade at the two inner cores where the flux density is high and the conventional grade at the outer cores where the flux density is lower, as an attempt to concentrate the magnetic flux to the low loss HiB steel and away from the standard loss electrical steel. As a result, the mixed three-phase transformer core exhibits comparable core losses with a transformer core constructed of the high cost, high permeability electrical steel up to a certain magnetization level and at a lower manufacturing cost [8]. This is due to the aforementioned flux density non-uniformity of the three-phase wound core and the fact that core loss is a non-linear function of the flux density.

As a consequence, by using conventional steel for the outer cores, and HiB steel for the inner cores, transformer manufacturers may reach an optimum trade-off between the manufacturing cost and the cost of future no-load losses and effectively reduce the TOC of the three-phase distribution transformer. The topology of the proposed mixed three-phase wound core is shown in Fig. 3.

Another advantage of the proposed concept is that the transformer manufacturers will not have to make any kind of investment on specialized machinery, any changes of their existing production line, or alterations of the annealing process of the wound cores in order to achieve the aforementioned economic benefits. This is due to the same manufacturing processes of both the conventional and the mixed three-phase wound cores since the mechanical properties of different grades of electrical steels are similar. The only difference between the mixed three-phase transformer and the conventional one is that the outer cores of the mixed three-phase transformer are constructed of a standard electrical steel and the inner cores are constructed of HiB steel.

On the other hand, the alteration of the magnetic properties of the three-phase wound transformer core results in changes of the parameters of the equivalent electric circuit of the transformer [22,23]. Thus, the no-load current and the series impedance of the mixed three-phase wound core transformer are increased in comparison with the conventional one. As the magnetization level increases beyond a critical value, the increase in no-load losses of the mixed three-phase transformer becomes unacceptable [8]. The specific critical induction value as well as the flux density distribution of the wound cores is also affected by the leakage fluxes and the corresponding air gap between adjacent wound cores [20,24].

3. Techno-economic analysis of conventional and mixed three-phase transformers

In this section a systematic techno-economic analysis of the conventional and the mixed three-phase transformers is presented in order to determine the conditions that favor the mixed over the conventional three-phase transformer designs.

The specific analysis is based on an analytical approach where a complex optimization problem, like the transformer design optimization problem, is broken into its component parts in order to determine only the variables and conditions that differentiate the manufacturing and future operational costs of the conventional and mixed, three-phase wound core designs. The presented analytical approach differs from statistical approaches making use of a large number of data of manufactured distribution transformers [21]. The results obtained from the proposed analytical approach are verified by a systematic and detailed experimental case study in Section 5 i.e., the experimental results are not the basis of the analysis but instead are used in order to verify the analytical approach developed in the present paper.
3.1. Comparison of conventional and mixed magnetic wound cores

In order to determine the differences between the mixed and the conventional three-phase transformer designs, the core topologies are compared by constraining all their geometrical variables i.e., identical three-phase wound cores as shown in Figs. 1 (b) and 3 (b) for the conventional and the mixed wound cores. It follows then, that the total volume of the three-phase wound cores are identical and considering that the density of all the grades of electrical steel $d_{ms}$ (kg/m$^3$) is the same, it follows that the total mass of the three-phase wound cores $M_{CRGO}$ is the same. What changes is the mass of the HiB electrical steel $M_{HiB}$, and the mass of the standard electrical steel $M_{SM}$. In the case of the conventional three-phase wound core, the two inner cores and the two outer cores are manufactured of the HiB steel. As a result, it follows that:

$$M_{HiB_{con}} = M_{CRGO} = 2 \cdot (M_{Inner} + M_{Outer}) \quad M_{SM_{con}} = 0 \quad (2)$$

In the case of the mixed three-phase wound core, the two inner cores are manufactured of the HiB steel and the two outer cores are manufactured of the conventional steel. As a result, it follows that:

$$M_{HiB_{mix}} = 2 \cdot M_{Inner} \quad M_{SM_{mix}} = 2 \cdot M_{Outer} \quad M_{HiB_{mix}} + M_{SM_{mix}} = M_{CRGO} \quad (3)$$

3.2. Magnetic wound cores geometrical variables and derivation of masses

An approximate method for evaluating the mass of the three-phase wound core follows next. It can be easily verified by the reader and it is valid for both the conventional and the mixed three-phase magnetic wound cores.

The variable $x_3$ is the thickness of the wound core, $x_4$ and $x_5$ are the window width and height, and $x_6$ is the axial length of the core. The optimum values of the specific variables are determined using the stochastic optimization algorithm simulated annealing with restarts [5]. For the optimization problem under consideration, the specific algorithm presents improved characteristics over other non-deterministic optimization algorithms such as the genetic algorithms, or other optimization approaches such as heuristic optimization algorithms [25].

The volume of the outer (small) core is the sum of the sub-volumes $v_3$ to $v_5$, as can be seen in Fig. 4, and is given in Eq. (4). The volume of each sub-volume is given in Eq. (5).

$$v_{outer} = 4 \cdot v_3 + 2 \cdot v_4 + 2 \cdot v_5 \quad (4)$$

$$v_3 = \pi \cdot x_3^2 \cdot x_6 / 4 \quad v_4 = x_3 \cdot x_4 \cdot x_6 \quad v_5 = x_3 \cdot x_5 \cdot x_6 \quad (5)$$

By replacing Eq. (5) into Eq. (4) yields the volume of the outer core:

$$v_{outer} = \pi \cdot x_3^2 \cdot x_6 + 2 \cdot x_3 \cdot x_4 \cdot x_6 + 2 \cdot x_3 \cdot x_5 \cdot x_6 \quad (6)$$

The volume of the inner (large) core can be derived in an analogous manner and from Fig. 4 it can be easily seen that the volume of the inner core is equal to Eq. (7). This is due to the width of the window of the inner cores being twice the width of the window of the outer core.

$$v_{inner} = v_{outer} + 2 \cdot v_4 \quad (7)$$

By replacing Eqs. (5) and (6) into Eq. (7) yields the volume of the inner core:

$$v_{inner} = \pi \cdot x_3^2 \cdot x_6 + 4 \cdot x_3 \cdot x_4 \cdot x_6 + 2 \cdot x_3 \cdot x_5 \cdot x_6 \quad (8)$$

The five-legged wound transformer core is assembled of two outer and two inner wound cores. Thus, the total volume of the three-phase wound core is given by:

$$V_{CRGO} = 2 \cdot v_{outer} + 2 \cdot v_{inner} \quad (9)$$

By replacing Eqs. (6) and (8) into Eq. (9) it yields the following expression for the total volume of the three-phase wound core:

$$V_{CRGO} = 4 \cdot \pi \cdot x_3^2 \cdot x_6 + 12 \cdot x_3 \cdot x_4 \cdot x_6 + 8 \cdot x_3 \cdot x_5 \cdot x_6 \quad (10)$$

As can be seen from the expressions for the volume of the cores Eqs. (6), (8) and (10), not all geometrical variables are equally important. The most significant is the thickness of the core $x_3$ followed by the axial length of the core $x_6$. The aforementioned variables are present in all three terms and furthermore for the first term the thickness of the core $x_3$ is raised in the second power. The variables $x_4$ and $x_5$ representing the width and the height of the window of the core are less important and determine mostly the volume of the winding material. Finally, the mass of the wound cores is given by Eq. (11) where $d_{ms}$ is the density of the electrical steel (kg/m$^3$) and $C_{sf}$ is the core stacking factor.

$$M_{CRGO} = C_{sf} \cdot d_{ms} \cdot V_{CRGO} \quad M_{Outer} = C_{sf} \cdot d_{ms} \cdot v_{outer} \quad M_{Inner} = C_{sf} \cdot d_{ms} \cdot v_{inner} \quad (11)$$

3.3. Derivation of the objective function used for the transformer analysis

Both transformer designs are subject to the same constraints imposed by the IEC 60076 standard. As a consequence, the TOC function is reduced to the following objective function representing the manufacturing and operational costs of the active part of the five-legged, wound core transformer, where $C_{CRGO}$ is the unit cost of the electrical steel ($$/kg$$), $M_{CRGO}$ is the mass of the electrical steel (kg), $C_{Cu}$ is the unit cost of the winding material ($$/kg$$), $M_{Cu}$ is the
mass of the winding material ($\text{kg}$), and $SM$ is the sales margin. The $A_{\text{factor}}$ and $B_{\text{factor}}$ ($\text{$/W$}$) are the present value of 1 W of no-load loss and load loss respectively over the life of the transformer, whereas $P_{\text{IL}}$ and $P_{\text{LL}}$ are the no-load loss and load loss (W).

$$f(x) = (C_{\text{CRGO}} \cdot M_{\text{CRGO}} + C_{\text{Cu}} \cdot M_{\text{Cu}})/SM + A_{\text{factor}} \cdot P_{\text{IL}} + B_{\text{factor}} \cdot P_{\text{LL}}$$

(12)

The objective function can be simplified further due to the fact that the comparison is carried out on transformer cores of exactly the same geometrical parameters. In this manner both transformer designs have windings of the same dimensions and material, and consequently of the same mass and winding losses. Therefore, in the next paragraphs it is shown that the respective terms can be omitted. In the generalized case where the transformer core is manufactured of different grades of grain-oriented electrical steels, the term $C_{\text{CRGO}} \cdot M_{\text{CRGO}}$ is partitioned into two terms as in the following equation, where $C_{\text{HiB}}$ and $C_{\text{SM}}$ is the unit cost of the HiB and standard electrical steels ($\text{$/kg$}$).

$$f(x) = (C_{\text{HiB}} \cdot M_{\text{HiB}} + C_{\text{SM}} \cdot M_{\text{SM}} + C_{\text{Cu}} \cdot M_{\text{Cu}})/SM + A_{\text{factor}} \cdot P_{\text{IL}} + B_{\text{factor}} \cdot P_{\text{LL}}$$

(13)

The minimization of the total owning cost function, and the objective function under consideration, is a constrained multi-objective optimization problem [26], subject to a number of constraints according to the IEC 60076-1 standard and related technical international standards, that include the following:

- Short circuit impedance inequality constraint $|U_k| < 1.1U_{k}^{\text{spec}}$, where $U_{k}^{\text{spec}}$ is the specified short circuit impedance (V).
- No-load loss inequality constraint $P_{\text{NLL}} < 1.15P_{\text{NLL}}^{\text{spec}}$, where $P_{\text{NLL}}^{\text{spec}}$ is the specified no-load loss (W).
- Load-loss inequality constraint $P_{\text{LL}} < 1.15P_{\text{LL}}^{\text{spec}}$, where $P_{\text{LL}}^{\text{spec}}$ is the specified load loss (W).
- Apparent power constraint.
- Induced voltage constraint.
- Temperature rise constraint.
- No-load current constraint.

The short circuit impedance is evaluated by 3D finite element (FE) analysis [5] and the no-load losses by a 2D nonlinear FE analysis [1]. In the case of the no-load losses evaluation, accuracy is depended on the suitable modeling of complex mechanisms such as the hysteresis phenomena [27].

3.4. Objective function of the conventional three-phase transformer

In the case of the conventional three-phase transformer, the inner and outer cores are manufactured of HiB steel and therefore the total masses of the HiB and conventional magnetization grain-oriented steels are given by Eqs. (14) and (15).

$$M_{\text{HiBconv}} = 2 \cdot (M_{\text{inner}} + M_{\text{outer}})$$

(14)

$$M_{\text{SMconv}} = 0$$

(15)

As a result, by substituting Eqs. (14), (15) into the objective function of Eq. (13) yields the objective function $f(x)^{\text{conv}}$ of the conventional three-phase transformer.

$$f(x)^{\text{conv}} = [2 \cdot C_{\text{HiB}} \cdot (M_{\text{inner}} + M_{\text{outer}}) + C_{\text{Cu}} \cdot M_{\text{Cu}}]/SM + A_{\text{factor}} \cdot P_{\text{ILconv}} + B_{\text{factor}} \cdot P_{\text{LL}}$$

(16)

3.5. Objective function of the mixed three-phase wound core

In the case of the mixed three-phase wound core, the two inner cores are manufactured of the HiB steel whereas the two outer cores are manufactured of the conventional electrical steel.

$$M_{\text{HiBmixed}} = 2 \cdot M_{\text{inner}}$$

(17)

$$M_{\text{SMmixed}} = 2 \cdot M_{\text{outer}}$$

(18)

By replacing Eqs. (17), (18) into Eq. (13) yields the objective function of the mixed three-phase, five-legged wound core transformer $f(x)^{\text{mixed}}$.
\[ f(x)_{\text{mixed}} = [2 \cdot (C_{\text{HM}} \cdot M_{\text{inner}} + C_{\text{SM}} \cdot M_{\text{outer}}) + C_{\text{Cu}} \cdot M_{\text{Cu}}] / (SM + A_{\text{factor}} \cdot P_{\text{ILmixed}} + B_{\text{factor}} \cdot P_{\text{LL}}) \]

4. Variables and conditions that favor the mixed three-phase transformer over the conventional transformer

4.1. Comparison of the objective functions

The comparison of the sum of manufacturing cost and cost of future losses of the conventional and mixed transformer designs is carried out by an estimator function that includes only the variables that differentiate the two designs. The estimator function is derived by subtracting the objective functions (16) and (19) and by considering that they concern transformers of identical geometrical parameters and of the same winding material i.e., copper material. As a consequence the terms \( B_{\text{factor}} \cdot P_{\text{LL}} \) are cancelled.

\[ f(x)_{\text{conv}} - f(x)_{\text{mixed}} = \frac{2 \cdot (C_{\text{HM}} \cdot (M_{\text{inner}} + M_{\text{outer}}) - C_{\text{HM}} \cdot M_{\text{inner}} - C_{\text{SM}} \cdot M_{\text{outer}})}{SM + A_{\text{factor}} \cdot (P_{\text{ILconv}} - P_{\text{ILmixed}})} \]

From Eq. (20) and by regrouping the terms under the sales margin \( SM \) and \( A_{\text{factor}} \) it follows that:

\[ f(x)_{\text{conv}} - f(x)_{\text{mixed}} = \frac{2 \cdot (C_{\text{HM}} \cdot M_{\text{inner}} + M_{\text{outer}}) - C_{\text{HM}} \cdot M_{\text{inner}} - C_{\text{SM}} \cdot M_{\text{outer}})}{SM + A_{\text{factor}} \cdot (P_{\text{ILconv}} - P_{\text{ILmixed}})} \]

In Eq. (21) the term \( C_{\text{HM}} \cdot M_{\text{inner}} \) is cancelled and it yields:

\[ f(x)_{\text{conv}} - f(x)_{\text{mixed}} = \frac{2 \cdot (C_{\text{HM}} \cdot M_{\text{outer}} - C_{\text{SM}} \cdot M_{\text{outer}})}{SM + A_{\text{factor}} \cdot (P_{\text{ILconv}} - P_{\text{ILmixed}})} \]

Finally by regrouping Eq. (22) the difference of the two objective functions of the two transformer designs is simplified into Eq. (23).

\[ f(x)_{\text{conv}} - f(x)_{\text{mixed}} = \frac{2 \cdot M_{\text{outer}}}{SM} \cdot \left( C_{\text{HM}} - C_{\text{SM}} \right) + A_{\text{factor}} \cdot \left( P_{\text{ILconv}} - P_{\text{ILmixed}} \right) \]

The difference of the objective functions can be categorized as follows:

1st case \( f(x)_{\text{conv}} - f(x)_{\text{mixed}} > 0 \)
2nd case \( f(x)_{\text{conv}} - f(x)_{\text{mixed}} = 0 \)
3rd case \( f(x)_{\text{conv}} - f(x)_{\text{mixed}} < 0 \)

In the first case the mixed three-phase transformer has a lower TOC. In the second case the two designs have exactly the same owning cost and in the third case the conventional transformer design present a lower TOC.

From the first case and Eq. (23) it follows that:

\[ \frac{2 \cdot M_{\text{outer}}}{SM} \cdot (C_{\text{HM}} - C_{\text{SM}}) + A_{\text{factor}} \cdot (P_{\text{ILconv}} - P_{\text{ILmixed}}) > 0 \]

By moving the term representing the cost of future no-load losses to the right hand side of the equation it yields,

\[ \frac{2 \cdot M_{\text{outer}}}{SM} \cdot (C_{\text{HM}} - C_{\text{SM}}) > A_{\text{factor}} \cdot (P_{\text{ILmixed}} - P_{\text{ILconv}}) \]

and by dividing with the right hand term it follows that:

\[ \frac{2 \cdot M_{\text{outer}}}{A_{\text{factor}} \cdot SM} \cdot \frac{C_{\text{HM}} - C_{\text{SM}}}{P_{\text{ILmixed}} - P_{\text{ILconv}}} > 1 \]

By taking the logarithm of the left hand side of Eq. (26) we define the dimensionless estimator function \( E_{\text{function}} \). The logarithm is applied in the left hand side of Eq. (26) for practical and graphical representation purposes.

\[ E_{\text{function}} = \log_{10} \left( \frac{2 \cdot M_{\text{outer}}}{A_{\text{factor}} \cdot SM} \cdot \frac{C_{\text{HM}} - C_{\text{SM}}}{P_{\text{ILmixed}} - P_{\text{ILconv}}} \right) \]

Also Eq. (27) can be written as follows where \( \Delta C_{\text{CRGO}} \) is the difference in cost of the Hb and conventional steel, and \( \Delta P_{\text{IL}} \) is the difference in iron loss of the mixed and conventional three-phase wound cores where \( P_{\text{IL}} = f(B) \) is a function of flux density.

\[ E_{\text{function}} = \log_{10} \left( \frac{2 \cdot M_{\text{outer}}}{A_{\text{factor}} \cdot SM} \cdot \frac{\Delta C_{\text{CRGO}}}{\Delta P_{\text{IL}}} \right) \]

4.2. Decision for the selection of mixed or conventional three-phase wound cores based on an estimator function

The estimator function of Eq. (28) provides an insight on the selection of the transformer design that presents the minimum sum of manufacturing cost and cost of future losses. If the value of the estimator function is positive this means that \( f(x)_{\text{conv}} - f(x)_{\text{mixed}} > 0 \) i.e., the sum of manufacturing cost and cost of future losses of the mixed three-phase transformer is lower than the conventional one.

If the estimator function is zero this means that the TOC of the two designs is the same. However, in this case the mixed three-phase transformer presents lower manufacturing cost over the conventional one and increased cost of losses in comparison with the conventional one.

In the case where the value of the estimator function is negative \( f(x)_{\text{conv}} - f(x)_{\text{mixed}} < 0 \), this means that the conditions and constraints favor the conventional design.

The aforementioned are depicted graphically in Fig. 5 where the part of the graph of the estimator function that lies in the right area “I” corresponds to values of the variable that favor the mixed three-phase transformer and the part that lies in the left area “II” corresponds to values of the respective variable that favor the conventional design.

The estimator is a function of five variables. There are two variables in the numerator and three variables in the denominator. The variables in the numerator are the mass of the outer core \( M_{\text{outer}} \), and the difference in cost of the two grades of steel \( \Delta C_{\text{CRGO}} \). The three variables in the denominator are the \( A_{\text{factor}} \), the sales margin, and the difference in iron loss of the mixed and conventional three-phase transformers.

If the estimator function is positive this means that it favors the mixed transformer core design over the conventional one. Consequently, large values in the numerator and small values in the denominator of Eq. (28) favor the mixed transformer core.
Large values of the mass of the outer core means that the mass of the three-phase transformer core is also large i.e., three-phase transformers of high rating power. This implies that the mixed three-phase transformer design is going to be more successful if applied to large transformers of high ratings. In the case where the difference of the cost of the HiB and standard grain-oriented steels is large this favors the mixed three-phase design as well. The aforementioned depends on the grades of electrical steel, commodity price, magnitude of order, currencies, and so on. If the difference of the cost of the two grades of steel is large this means that there is going to be a large difference between the magnetization and loss characteristics of the two grades of steel.

5. Case study of 100 kVA three-phase wound core distribution transformers

In order to examine the effect of the various variables on the first cost and operational cost of the mixed and conventional three-phase transformers a case study is investigated concerning 100 kVA three-phase distribution transformers. For that purpose, a couple of three-phase, five-limbed, transformer cores are analyzed and tested with the exact same geometrical variables, a conventional manufactured of 100% of the HiB steel M-0H 0.27 mm, and a mixed transformer core manufactured of the conventional electrical steel M4 0.27 mm and the M-0H steel. Fig. 6 shows a comparison of the magnetization and loss characteristics of the aforementioned electrical steels, where $P_{SCL}$ is the specific core losses (W/kg). Table 1 shows the data and variables of the mixed and conventional three-phase transformer cores under study.

The experimental setup used for the no-load loss evaluation of the conventional and mixed wound core is shown in Fig. 7 [28]. It includes a programmable power supply MX30, a National Instruments data acquisition device, a noise rejecting BNC connector block, active differential voltage probes, and current probes based on the Hall Effect. Three twenty turns excitation coils in delta connection are supplied with a three-phase, 50 Hz, sinusoidal voltage waveform, from the power supply in order to magnetize the conventional and mixed three-phase wound cores to the same induction levels.

Four virtual instruments are created with the use of LabVIEW software [28]. The first two are used for capturing the voltage and magnetizing current waveforms. The last two are used for manipulating the acquired voltage and current data in order to determine the no-load losses. A comparison of the respective experimental no-

![Table 1](image)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Conventional wound core</th>
<th>Mixed wound core</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$ (mm)</td>
<td>24.3</td>
<td>M-0H 0.27 mm</td>
</tr>
<tr>
<td>$x_4$ (mm)</td>
<td>57</td>
<td>M-0H 0.27 mm &amp; M4 0.27 mm</td>
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<td>$x_5$ (mm)</td>
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<td></td>
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<td>$x_6$ (mm)</td>
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<tr>
<td>$M_{total}$ (kg)</td>
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</tr>
<tr>
<td>$M_{loss}$ (kg)</td>
<td>80.9</td>
<td></td>
</tr>
<tr>
<td>Electrical steel grade</td>
<td>M-0H 0.27 mm</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 6. Characteristics of conventional and HiB electrical steels. (a) $B$ versus $H$. (b) $P_{SCL}$ versus $B$.

Fig. 7. Experimental setup for three-phase transformers cores.
load losses for the conventional and mixed wound cores are shown in Table 2 for magnetization levels ranging from 1.0 T to 1.74 T.

The small differences in no-load losses of the mixed three-phase wound core transformer and the conventional one shown in column 2 and column 3 of Table 2, provide experimental evidence of the validity of the mixed three-phase transformer concept. As a result, even though the mixed three-phase transformer has a reduced manufacturing cost in comparison with the conventional one, there is only a marginal increase in no-load losses. However, there is a critical magnetization level where beyond that the increase in losses becomes unacceptable [8].

In the following paragraphs the variation of the estimator function versus a number of its independent variables is shown. The temperature rise is not included as it is taken into consideration in the transformer design optimization problem as a constraint function [29].

Fig. 8 shows the estimator function versus the magnetization level of the transformer for various values of the $A_{factor}$. The SM is set to 0.5 i.e., 100% sales margin and the difference in cost of HiB and conventional electrical steels $\Delta C_{CRGO}$ is set to 1.0 $$/kg. As the magnetization level increases the value of the estimator function is reduced. This means that low magnetization levels favor designs of the mixed three-phase transformer and high magnetization levels favor the designs of conventional transformers.

The estimator function is zero for a "critical" magnetization level where the designs of the mixed and the conventional three-phase transformers have the same TOC. This means that any gain from reduced manufacturing cost i.e., lower cost of electrical steel, is compensated from an increased operational cost i.e., higher cost of no-load losses, and vice-versa. As the $A_{factor}$ increases the graph of the estimator function is shifted to the left i.e., to the side of the conventional three-phase transformer. Also as the $A_{factor}$ increases the “critical” magnetization level decreases. From the aforementioned it follows that very high values of the $A_{factor}$ favor designs of the conventional three-phase transformer, as in this case mixed three-phase transformer designs are superior to the conventional one, only for very low magnetization levels.

Table 2

<table>
<thead>
<tr>
<th>$B_p$ (T)</th>
<th>$P_{loss}$ (W)</th>
<th>$P_{loss expulsion}$ (W)</th>
<th>$\Delta P_{loss}$ (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>35.8</td>
<td>36.88</td>
<td>1.08</td>
</tr>
<tr>
<td>1.2</td>
<td>50.92</td>
<td>52.28</td>
<td>1.36</td>
</tr>
<tr>
<td>1.4</td>
<td>70.16</td>
<td>71.96</td>
<td>1.80</td>
</tr>
<tr>
<td>1.5</td>
<td>82.28</td>
<td>85.12</td>
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<tr>
<td>1.6</td>
<td>96.4</td>
<td>101.7</td>
<td>5.30</td>
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<td>105.3</td>
<td>113.5</td>
<td>8.20</td>
</tr>
<tr>
<td>1.7</td>
<td>115.6</td>
<td>126.4</td>
<td>10.80</td>
</tr>
<tr>
<td>1.72</td>
<td>121.1</td>
<td>134.2</td>
<td>13.10</td>
</tr>
<tr>
<td>1.74</td>
<td>126.2</td>
<td>141.6</td>
<td>15.40</td>
</tr>
</tbody>
</table>

Fig. 9. Estimator function versus magnetization level for five different sales margins (SM).

Fig. 10. Estimator function versus the $A_{factor}$ for five magnetization levels.

6. Conclusion

The mixed three-phase transformer design cannot be considered as superior or inferior to the conventional three-phase transformer but instead it depends on the value of specific variables if the mixed three-phase transformer is techno-economically
a better choice in comparison with the three-phase conventional transformer. Those variables are five and more specifically, the mass of the outer wound core $M_{outer}$, the sales margin $SM$, the difference in unit cost of the HiB and conventional electrical steels $\Delta C_{CRGO}$, the present value of 1 W of no-load loss $A_{factor}$, and the difference in no-load losses of the mixed and the conventional three-phase transformers $\Delta NL$.

As a consequence, the mixed three-phase transformer design is favored over the conventional one for high transformer ratings, high sales margins, and high values of $\Delta C_{CRGO}$. Also, the mixed three-phase transformer design is favored over the conventional one for low values of $A_{factor}$ i.e., for a short transformer lifespan, and for small differences of mixed and conventional no-load losses i.e., for reduced magnetization levels. Careful consideration of the aforementioned findings from, utilities, DSOs, and transformer manufacturers, is going to provide valuable services in determining the most techno-economically efficient distribution transformer for usage in the distribution electricity network. Subsequently this is going to lead to the reduction of losses, costs, and greenhouse gas emissions, considering the large number of distribution transformers that are going to be installed in the European distribution network in the next thirty years.

References


Fig. 11. Estimator function versus the difference in cost of steels grades for five magnetization levels.