Transformer Joints FE Analysis Using Pseudo-Source Technique

Themistoklis D. Kefalas1, George Loizos1,2, and Antonios G. Kladas3

1School of Electrical and Computer Engineering, National Technical University of Athens, GR-15780 Athens, Greece
2Schneider Electric AE, GR-32011 Inofyta, Viotia, Greece

Distribution transformers losses are equal to almost 2% of the electricity generated worldwide and only in the European Union are estimated at about 33 TWh/year. Approximately 75% of the total losses are due to core losses as a result of the loading characteristics of distribution transformers. Design of the joints of magnetic cores has a profound impact on core losses and transformer efficiency. The paper introduces a finite element methodology for the analysis of transformer joints. The proposed technique consists in the application of certain boundary conditions for the excitation of the joints. The main advantages of the pseudo-source technique include minimization of the computational cost and ease of implementation. The technique is combined with a number of FE formulations and a vector hysteresis model. Two-dimensional as well as 3-D FE analysis is studied. Longitudinal and normal flux measurements were carried out for the validation of the proposed technique.

Index Terms—Finite-element methods, magnetic cores, power transformers, soft magnetic materials.

I. INTRODUCTION

Losses of the electrical grid worldwide are estimated at about 1.279 TWh/year i.e., 9.2% of global electricity generation. Almost 70% of losses take place in the distribution grid and a third of the latter in distribution transformers. This fact renders distribution transformers the second most energy intensive component of the power grid [1]. Due to the loading characteristics of distribution transformers the dominant loss component is core losses, e.g. 75% of total losses of distribution transformers in the European Union (EU-27) are due to core losses [1], [2]. As a result, there is an increasing demand on low core loss, energy efficient, distribution transformers in the US, Canada, Japan, Australia, India, and in recent years in the EU [1].

The design of the joints of wound cores has a profound impact on core losses and transformer efficiency [3]–[5]. In order to determine the optimum joint design, transformer manufacturers must evaluate accurately the flux density distribution at the joints. Nevertheless, FE analysis of wound cores requires substantial computational recourses. Typical laminated wound cores are constructed of hundreds steel sheets. The length and width of laminations ranges from 0.5 m to 1.5 m and 0.1 m to 0.6 m respectively. On the other hand, steel sheet thickness ranges from 0.5 mm to 0.23 mm and the thickness of the air-varnish composite between them is about preformed electrical winding coils. Wound cores, are formed in which overlap is gradual, flux transfer is eased, and flux normal to the sheet is minimized. This joint type is known as step-lap joint and its efficiency is determined by the overlap length and gap length, Fig. 1.

So far, researchers have used lumped circuit models [3], neural networks [4], and the application of periodic boundary conditions along with the modeling of the excitation coil [5]. However, in [3] and [4] local field distribution cannot be evaluated, and the approach used in [5] results in FE models of large mesh size, and high computational cost. For the analysis of joints a FE technique is proposed with which the required excitation is achieved by means of a pseudo-source. In this fashion the size and computational cost of the FE model is minimized.

II. MANUFACTURING OF WOUND CORE TRANSFORMERS

One-phase and three-phase, oil-immersed, distribution transformers are commonly made of wound cores assembled about preformed electrical winding coils. Wound cores, are constructed by cutting and spirally winding a flat strip of grain-oriented steel into hundreds concentric turn laminations, Fig. 1. Cutting of the electrical steel is carried out so that joints are formed in which overlap is gradual, flux transfer is eased, and flux normal to the sheet is minimized. This joint type is known as step-lap joint and its efficiency is determined by the overlap length and gap length, Fig. 1.

III. PSEUDO-SOURCE TECHNIQUE

By using properly defined boundary conditions the excitation of the step-lap joint at an arbitrary magnetization level is achieved. There is no need to model current regions, i.e., transformer windings. Only a part of a small number of steel sheets and the air-varnish composite between them are modeled. As a result, the FE model size is reduced and the computational cost is minimized. The specific technique can be implemented with any commercial FE package, e.g., ANSYS, MagNet, or FEMM.
Transformer manufacturers can use it to estimate the peak flux density distribution at transformer joints for a certain magnetization level.

Both 2-D and 3-D FE analyses are examined. In the 2-D case the magnetic vector potential (MVP) formulation is taken into consideration. In the 3-D case the MVP, edge, and magnetic scalar potential (MSP) formulations are studied.

A. 2-D FE Analysis, MVP Formulation

At the bottom edge of the model an arbitrary vector potential $A_{Z1}$ is applied, Fig. 2. At the top edge a potential $A_{Z2}$ is applied, where $B_p$ is the magnetization level, $n$ is the number of steel sheets, $c_{eff}$. $L_{th}$ are the lamination stacking factor and thickness. At the other two edges a natural boundary condition occurs. Peak flux density distribution is obtained, after the solution of the nonlinear magnetostatic problem, using the postprocessor of the FE code.

B. 3-D FE Analysis, MVP Formulation

The FE model is designed in the $X - Y$ plane and extruded along the $Z$ axis, Fig. 3. Two blocks defined as volumes of air are added at the sides of the model in order to investigate the impact of end-effects. At the bottom and top facet of the model the same boundary conditions used in the 2-D case are applied. At the side facets a flux parallel condition is applied by constraining $A_X$, $A_Y$ to zero. At the front and back facet a flux normal condition is imposed by constraining $A_X$ to zero.

C. 3-D FE Analysis, Edge Formulation

An edge based approach is not effective, as it does not permit to impose directly boundary conditions concerning vector potential components at the bottom and top facet of the FE model.

D. 3-D FE Analysis, MSP Formulation

At the back facet of the model an arbitrary scalar potential $\Phi_1$ is applied Fig. 4. At the front facet a scalar potential $\Phi_2$ reduced by $\Phi_2$ is applied. At the other four facets a flux parallel natural condition occurs. The disadvantage of the MSP formulation is that the value of $\Phi_2$ that corresponds to a certain magnetization level is not known a priori as in the case of the MVP formulation. Three different scalar potential formulations are examined, the reduced scalar potential (RSP), the difference scalar potential (DSP), and the general scalar potential (GSP).

1) DSP and GSP Formulations: The scalar potential $\Phi$ is computed by solving (1) where $\mu$ is the permeability tensor and $H_g$ is a guess field that is evaluated by using a two-step (DSP) or a three-step (GSP) scheme

$$\nabla \cdot (\mu \cdot (H_g + \nabla \Phi)) = 0. \quad (1)$$

The specific formulations are used when there are current regions present in the 3-D FE model. Therefore, neither is suitable for implementation in the pseudo-source technique.

2) RSP Formulation: The RSP formulation can be integrated in the pseudo-source technique since it is used for 3-D FE models that do not contain current regions. $H_g$ is replaced by the Biot-Savart field. As a one-step procedure is used for the evaluation of $H_g$, the computational effort of the RSP formulation is significantly lower than that of the DSP and GSP formulations.

IV. VECTOR HYSTERESIS MODEL

Complex loss mechanisms take place at step-lap joints. Alternate losses, rotational losses, and the highly anisotropic nature of electrical steels must be taken into consideration for the exact computation of the local flux density time evolution. Only the peak flux density distribution can be evaluated by implementing the pseudo-source technique with a commercial FE code, and transformer manufacturers can determine losses using experimental specific core loss curves expressed as a function of peak flux density. Thus, it follows that a suitable vector hysteresis model must be adopted.

The hysteresis model is based on the inverse, vector Jiles-Atherton model. It can be integrated in the 2-D MVP based FE analysis, where $B$ in each element can be directly evaluated by the curl of $A$. In contrast with the scalar model, in the vector model the differential of magnetization $dM$ is a function of vector variables, and the five parameters of the original Jiles-Atherton model are replaced by five tensor quantities, the saturation magnetization tensor $M_s$, tensor $a$ representing inter-domain coupling, tensor $k$ which is a parameter of the anhysteretic magnetization, tensor $\alpha$ related to the pinning of do-
main walls, and tensor \( \mathbf{c} \) associated with the reversible component \( \mathbf{M}_{\text{rev}} \).

The specific model incorporates two significant improvements over conventional vector Jiles-Atherton models. The anhysteretic component of magnetization \( \mathbf{M}_{\text{an}} \), is expressed as a vector rather than a scalar quantity, resulting in this manner to improved representation of the magnetization. Also, the differential of the anhysteretic component \( d\mathbf{M}_{\text{an}} \), is expressed as a function of the effective field \( \mathbf{H}_e \) using (2) thus improving numerical stability [6], [7]

\[
d\mathbf{M}_{\text{an}} = \left[ \frac{d\mathbf{M}_{\text{an}}}{d\mathbf{H}_e} \right] = \xi \cdot d\mathbf{H}_e,
\]

(2)

According to [6], [7] \( d\mathcal{M} \) is evaluated by (3) if \( \mathbf{x}_f \cdot d\mathbf{H}_e > 0 \) and by (4) if \( \mathbf{x}_f \cdot d\mathbf{H}_e \leq 0 \), where \( \xi \) is given by (2) and \( \mathbf{x}_f \) is given by (5)

\[
d\mathcal{M} = \frac{1}{\mu_0} [1 + \mathbf{x}_f \cdot \mathbf{x}_f]^{-1} \mathbf{x}_f (1 - \alpha) + c \xi (1 - \alpha)^{-1} \\
\cdot \mathbf{x}_f \cdot \mathbf{x}_f \cdot \mathbf{x}_f + c \xi / \mu_0
\]

(3)

\[
d\mathcal{M} = \frac{1}{\mu_0} [1 + c \xi (1 - \alpha)^{-1}] \cdot [c \xi / \mu_0
\]

(4)

\[
\mathbf{x}_f = k^{-1} (\mathbf{M}_{\text{an}} - \mathbf{M}).
\]

(5)

The advantages of the specific hysteresis model over the Preisach model [8], [9] are the low computational cost, ease of integration to the FE analysis, and small number of required model parameters. Also, it presents superior accuracy over energy-based time-harmonic models [10] and viscous-type vector hysteresis techniques [11]. Moreover, it enables eddy-current effects consideration in step-lap joint regions indirectly, by providing different characteristics along the lamination rolling direction (longitudinal flux) and transverse to the lamination rolling direction (normal flux). This constitutes an alternative possibility for considering eddy currents in step-lap joints presenting reduced computational requirements when compared to direct eddy-current term consideration through diffusion equation [12], [13]. This hysteresis model was integrated to a time-stepping 2-D MVP model developed by the authors [2]. The nonlinear problem was solved using the Newton-Raphson iterative technique.

V. RESULTS AND DISCUSSION

A. Experimental Setup

A 15-turn excitation coil was supplied with a sinusoidal voltage waveform from a programmable ac power supply in order to magnetize a wound core constructed of 90 steel sheets of HiB grain-oriented electrical steel with \( L_{\text{th}} = 0.27 \text{ mm} \), \( c_{\text{sf}} = 0.965 \), and tangential and transverse to the rolling direction parameters shown in Table I. The core window height, width, and length were 183 mm, 57 mm, and 190 mm. Longitudinal and normal flux measurements were carried out by inserting 10 two turns search coils (17A to 17G and 18A to 18C) at the 17th and 18th sheet, Fig. 5. The interlaminar gap has been appropriately enlarged with respect to typical core values in order to enable the above mentioned search coil insertion for experimental validation purposes. The voltages induced in the search coils were captured by directly connecting them into the inputs of a National Instruments NI6143 DAQ card. A current probe based on the Hall effect was used for capturing the no-load current. Analysis of the captured data was carried out using LabVIEW software [14].

B. Methodology

Using the experimental setup, the authors were able to evaluate the peak value and waveform of the flux density as well as local hysteresis loops at 10 positions (17A to 17G and 18A to 18C). 2-D and 3-D models of the step-lap joint as shown in Fig. 5 were built using the commercial package ANSYS in order to compare computed and experimental peak flux density. Also, a model was built using the 2-D FE code developed by the authors and the vector hysteresis model of Section IV in order to compare computed and experimental flux density waveforms and local hysteresis loops.

C. 2-D and 3-D FE Analysis Using Commercial FE Package

Magnetostatic analyses for \( B_p \) ranging from 1.1 T to 1.75 T were carried out using the 2-D MVP, 3-D MVP, and 3-D MSP formulations. A typical desktop PC was used for the FE analysis. Fig. 6 shows a detail of the computed 2-D flux lines and peak flux density distribution of the 3-D and 2-D MVP models, illustrating the inhomogeneous local field distribution around gaps 2 and 3 of the FE model. Table II shows a comparison between computed and experimental peak flux density at four different positions (18A, 17D–17F) and for \( B_p = 1.3 \text{ T} \). The difference between computed and experimental results ranges from 5% to 10%. Results obtained by all formulations agree within 0.3% to 1%. Table III shows that the computational effort of the 3-D analysis is a multiple of that of the 2-D analysis. From the aforementioned it follows that the 3-D analysis is not practical and the end-effects are insignificant. This is due to the inherent 2-D symmetry of the step-lap joint problem.

D. 2-D FE Analysis Using the Vector Hysteresis Model

Time-stepping analyses were carried out using the dynamic vector hysteresis model of Section IV. In this case the excitation

### Table I

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<tr>
<th>Parameters of the HiB Electrical Steel</th>
<th>Tangential to the rolling</th>
<th>Transverse to the rolling</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_a (\text{A/m}) )</td>
<td>1.987 \times 10^6</td>
<td>1.249 \times 10^6</td>
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<tr>
<td>( \alpha )</td>
<td>9.124 \times 10^{-4}</td>
<td>6.736 \times 10^{-4}</td>
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<tr>
<td>( \sigma (\text{A/m}) )</td>
<td>39.34</td>
<td>32.68</td>
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<tr>
<td>( k (\text{A/m}) )</td>
<td>43.45</td>
<td>36.91</td>
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<tr>
<td>( c )</td>
<td>0.123</td>
<td>0.381</td>
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**Fig. 5.** Step-lap joint and search coils arrangement (not to scale).
is achieved by applying at the top edge of the 2-D FE model a sinusoidal vector potential waveform. Fig. 7 shows a comparison of computed and experimental flux density waveform at positions 17A, 17B for $B_p = 1.7 \, \text{T}$. In all cases the flux density waveform is highly distorted. This is due to the deviation of the flux from the lamination rolling direction and the anisotropic behavior of the electrical steel, represented by the implemented hysteresis model. Also, the computed peak flux density is overestimated. Fig. 8 shows local $B$-$H$ loops at positions 17C, 17E for $B_p = 1.4 \, \text{T}$. It is clearly shown that the computed losses are also overestimated.

VI. CONCLUSION

The proposed pseudo-source technique presents significant advantages over previous methods used for joints analysis. The paper shows that it can be easily implemented using a commercial code or a FE code integrating a state of the art vector hysteresis model. It is expected to offer great services to transformer manufacturers in determining the optimum joint configuration. Finally, the paper is expected to contribute to the widespread adoption of the FE method by the transformer industry the use of which is hindered by the excessive computational effort required to model laminated cores.

TABLE II

<table>
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<th>$B_p = 1.3 , \text{T}$</th>
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<tr>
<td>2D MVP</td>
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<tr>
<td>0.569554</td>
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TABLE III

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<th>Comparison of Computational Cost of FE Formulations</th>
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<td>FE formulation</td>
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<tr>
<td>2D MVP</td>
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<td>3D MVP</td>
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<td>3D RSP</td>
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REFERENCES