Advanced Computational Tools for Wound Core Distribution Transformer No–Load Analysis

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Abstract — The paper deals with the accurate representation of laminated wound cores with a low computational cost using 2D and 3D finite element (FE) method. An inverse anisotropy model is developed in order to model laminated wound cores. The inverse anisotropy model was integrated to the 2D and 3D FE method. A comparison between 2D and 3D FE techniques was carried out. FE techniques were validated by experimental analysis. In the case of no–load operation of wound core transformers both 2D and 3D FE techniques yields the same results. Computed and experimental no–load losses agree within 2% to 6%. The originality of the paper consists in the development of an inverse anisotropy model, specifically formulated for laminated wound cores, and in the effective representation of electrical steels using a composite single–valued function. By using the aforementioned techniques the FE computational cost is minimised, the numerical stability and convergence of the Newton–Raphson iterative method are improved, and the 3D FE analysis of wound cores is rendered practical.

Keywords — Electromagnetic analysis, Finite element methods, Magnetic cores, Power transformers

I. INTRODUCTION

Nowadays an ever increasing number of researchers and transformer manufacturers evaluate key operational parameters of transformers by using advanced numerical models, Cranganu–Cretu [1], Hernández [2], Soto [3], Kefalas [4], and Kefalas [5]. The no–load loss analysis of wound core transformers is based on the exact evaluation of the flux density distribution of wound cores. In order to compute accurately the flux density distribution of laminated wound cores, each individual steel sheet and the air–varnish composite between successive sheets must be modelled. This approach was followed recently for the finite element (FE) analysis of toroidal cores consisting of a small number of steel sheets, Zurek [6].

Nevertheless, FE analysis of wound cores requires substantial computational recourses. Typical laminated wound cores are constructed of hundreds steel sheets. The length and width of laminations ranges from 0.5 m to 1.5 m and 0.1 m to 0.6 m respectively. On the other hand, steel sheet thickness ranges from 0.5 mm to 0.23 mm and the thickness of the air–varnish composite between two successive steel sheets ranges from 10 μm to 8 μm. Consequently, the detailed modelling of a typical wound core requires a mesh size of the order $10^8$ to $10^9$ elements in the 2D FE case, and $10^{11}$ to $10^{12}$ elements in the 3D FE case, Kefalas [7].

In the present paper the aforementioned problem is addressed by replacing the laminated wound core by a bulk material that accurately models the complicated laminated core structure, Kefalas [7]. The properties of the bulk material are represented by an inverse anisotropy model that can be integrated to magnetic vector potential (MVP) based 2D and 3D FE formulations. The advantage of the inverse anisotropy model presented in this paper is its robustness and computational efficiency since it is specifically formulated for wound core transformers. In contrast, anisotropy models appearing in the literature are developed for stack core transformers, Sande [8], and are not suitable for wound core transformers since wound core and stack core topologies differ significantly.

Accurate modelling of the electrical steel nonlinearity under all excitation conditions, including heavily saturated conditions, is taken into consideration by using a composite single–valued function. And in this case the authors formulated the aforementioned composite function so that it can be integrated directly to MVP based 2D and 3D FE formulations and to the Newton–Raphson iterative technique.
II. LAMINATED WOUND CORE REPRESENTATION

An accurate representation of the wound core is achieved by considering the iron–laminated material as homogeneous and anisotropic media at the level of finite elements. An elliptic anisotropy model is best suited for the wound core transformer in contrast with the stack core transformer, Sande [8]. The anisotropy model developed in the present paper is an inverse one i.e., in contrast with the anisotropy model presented by Kefalas in [7], it is based on the assumption that the flux density $B$ has an elliptic trajectory for the modulus of magnetic field intensity $H$ constant. As a result, it can be integrated to MVP based 2D and 3D FE formulations where $B$ in each element of the FE mesh can be directly evaluated by the curl of the magnetic vector potential $A$.

A simplified graphical interpretation of this assumption is given in Figs. 1, 2. Fig. 1 depicts $B–H$ characteristics of grain–oriented steel for various angles to the rolling direction. For constant magnetic field intensity the flux density tends to decrease as the angle to the rolling direction increases. By projecting the values of the flux density for different directions of the field to the $B_p–B_q$ plane, where $p$ and $q$ is the direction tangential and normal to the rolling direction respectively, a curve is formed as shown in Fig 2.

In the case of laminated wound cores, this curve can be approximated by an ellipse and this approximation leads to an error that rarely exceeds a few percentage points. This is due to the fact, that the reluctivity of the wound core along direction $q$ i.e., normal to the rolling direction of the electrical steel, is three to four orders of magnitude larger than the reluctivity along direction $p$ i.e., tangential to the rolling direction, as shown in Fig. 3. Therefore, if $v_p$ is the magnetic reluctivity tangential to the electrical steel rolling direction, $v_q$ is the reluctivity normal to the rolling direction, and $r$ is the ratio of the ellipse $p$ semi–axes to the ellipse $q$ semi–axes, shown in Fig 4, then
Fig. 4 illustrates an ellipse in the $B_p - B_q$ plane for an arbitrary value of magnetic field intensity, where $B(0^\circ)$, $B(90^\circ)$ are the respective values of the flux density tangential and normal to the rolling direction. The equation of the ellipse of Fig. 4 is given by (2) and the ratio of the ellipse semi–axes is given also by (3).

$$\frac{B_p^2}{B(0^\circ)} + \frac{B_q^2}{B(90^\circ)} = 1$$  \hspace{1cm} (2)

$$r = \frac{B(0^\circ)}{B(90^\circ)}$$  \hspace{1cm} (3)

Substituting (3) into (2) yields

$$B_p^2 + r^2 B_q^2 = B(0^\circ)^2$$  \hspace{1cm} (4)

For an arbitrary angle $\theta$ to the rolling direction and from Fig. 4, it follows that the two components $B_{p\theta}$, $B_{q\theta}$ of the flux density are given by

$$B_{p\theta} = B \cos \theta, \quad B_{q\theta} = B \sin \theta$$  \hspace{1cm} (5)

The point $(B_{p\theta}, B_{q\theta})$ satisfies also the equation of the ellipse. So by substituting (5) into (4) it follows that

$$B(\theta) = B(0^\circ)\sqrt{1 + (r^2 - 1)\sin^2 \theta}$$  \hspace{1cm} (6)

In order to determine the $B – H$ characteristics of the homogeneous anisotropic media of the laminated wound core, one can express from magnetic circuit concepts the magnetic reluctivity tangential and normal to the rolling direction by (7) and (8) respectively

$$v_p = v_0 v_r \left[ c_{p\theta} + (1 - c_{p\theta}) v_r \right]$$  \hspace{1cm} (7)

$$v_q = c_{q\theta} v_0 v_r + (1 - c_{q\theta}) v_0$$  \hspace{1cm} (8)

where $v_0$ is the reluctivity of air, $v_r$ is the relative reluctivity of the electrical steel obtained by the normal magnetisation curve of the electrical steel and $c_{p\theta}$ is the stacking factor of the wound core. The reluctivity $v_s$ orthogonal to the $p – q$ plane, shown in Fig. 3, is taken to be equal to the reluctivity along the rolling direction of the electrical steel $v_p$.

Even though the aforementioned assumption is not true it does not affect the flux density distribution evaluation of wound cores due to the axial symmetry of the FE problem. Finally, due to the geometry of the wound core, shown in Fig. 3, the reluctivity tensor must be rotated to a different coordinate system depending on the location of the area or volume of the 2D and 3D FE model respectively.

### III. Electrical Steel Representation Under Heavily Saturated Conditions

If the solution of nonlinear electromagnetic problems by the FE method is based on the MVP formulation, soft magnetic materials are expressed by the relative reluctivity versus squared flux density curve $v_r - B^2$ and nonlinearity is tackled by the Newton–Raphson iterative method.
The evaluation of $v_r$ for $B^2 \leq B^2_n$, is carried out by cubic splines interpolation of tabulated sets of $(v_r, B^2)$, where $v_r$ is the relative reluctivity value for $B^2$, $B^2_n$ is the maximum squared flux density for which relative reluctivity is experimentally known, and $0 \leq i \leq n$.

In order to compute $v_r$ for $B^2 \geq B^2_n$, the conventional approach is to use a linear or a quadratic function to extrapolate the magnetisation curve Fujiwara [9]. In the case of the linear extrapolation function as $B \to \infty$, $v_r \to \infty$ or $\mu_r \to 0$. The aforementioned is of course incorrect. Considering the constitutive equation for ferromagnetic materials it is easily verified that the relative reluctivity $v_r$ of a soft magnetic material as $B \to \infty$ tends to unity ($v_r \to 1$).

As a result, it follows that the conventional soft magnetic material representation produces erroneous results in cases where the excitation level is high. In the present paper an alternative extrapolation function is proposed for $B^2 \geq B^2_n$ (9), where $a$, $b$ are parameters of the extrapolation function. The dimensions of $a$ parameter are $[T^{-2}]$ whereas $b$ is a dimensionless parameter.

$$v_r(B^2) = 1 - \exp[-(ab^2 + b)]$$

The function of (9) as well as its slope is continuous. Also, it can be seen from (9) that as $B \to \infty$, $v_r$ tends to unity since the exponent term tends to zero. Thus, the proposed extrapolation function yields correct results and it can be integrated to the Newton–Raphson iterative technique and to the FE method. The values of the two parameters are obtained by satisfying the following two conditions.

1. The first derivative of function $v_r$ at $B^2_n$ must be continuous.
2. Function $v_r$ at $B^2_n$ must be continuous.

Since the proposed representation of electrical steels is based on a continuous single–valued composite function with a continuous slope, it can be integrated directly to the Newton–Raphson iterative technique and the FE method. Also, the proposed representation presents advantages over contemporary approaches using a quadratic function to extrapolate the magnetisation curve Fujiwara [9]. The aforementioned approaches require the evaluation of four or three parameters whereas the proposed extrapolation method involves only two parameters.

IV. FE ANALYSIS AND EXPERIMENTAL VERIFICATION

The inverse elliptic anisotropy model was integrated to the MVP based 2D and 3D nonlinear FE method. A conventional procedure was used since the inverse elliptic anisotropy model is specifically formulated for the MVP based FE formulation, Silvester [10]. Also, the proposed composite single–valued function of Section III was integrated to the MVP based 2D and 3D FE method as well as the Newton–Raphson method using the exact same procedure used for the integration of the conventional representation of electrical steels, Silvester [10]. The two parameters of the proposed extrapolation function, $a$ and $b$, are evaluated only once for each electrical steel and there is no need to re–evaluate them at every iteration step of the Newton–Raphson iterative method.

A 2D and 3D FE nonlinear package based on the magnetic vector potential and Newton–Raphson method has been developed by the authors. The specific FE package consists of a pre–processing code, a first order triangle and tetrahedral mesh generator, the nonlinear 2D and 3D FE solvers, and a graphics post–processor.

Figs. 5 to 8 show the 2D and 3D peak flux density vector plot that corresponds to magnetisation levels from 0.3 T to 1.75 T of a wound core constructed of the HiB grain–oriented electrical steel M–OH 0.27 mm. The thickness of the core is 51.3 mm whereas the core window height, width, and length are 183 mm, 57 mm, and 190 mm. The simulated magnetisation of the wound core to an arbitrary level is achieved by using magnetostatic FE analysis and a systematic iterative procedure based on the bisection technique. Typically 20
iterations are enough in order to determine the current density which produces the desired magnetisation level with an error of the order of $10^{-4}$.

The evaluation of the peak flux density distribution with the FE method is used in conjunction with the experimentally determined local specific core loss $SCL$, for the evaluation of the wound core no–load loss. The authors developed two post–processing algorithms for the evaluation of the no–load loss in the 2D and 3D case respectively. In both cases the no–load loss of each element that belongs to the core domain i.e., triangle in the 2D case and tetrahedral in the 3D case, is computed based on the local specific core losses that correspond to the peak flux density of the element, the core stacking factor $s_{cf}$, the electrical steel density $msd$, and the volume $V$ of each element, where in the 2D case the volume is equal to the product of the area of each element with the length of the core.

The experimental setup used for the no–load loss evaluation of wound cores consists of a 20–turn excitation coil which is supplied with a sinusoidal voltage waveform from a programmable ac power supply, in order to magnetise the wound core. No–load current and voltage are captured using a current probe based on the Hall effect, an active high–voltage differential probe, and a National Instruments NI6143 data acquisition card. Analysis of the captured data was carried out using LabVIEW software, Loizos [11], Loizos [12]. Table I summarises the computed and measured no–load loss of the tested wound core for different working induction ratings. Calculated and measured no–load losses agree within 2% to 6%.

<table>
<thead>
<tr>
<th>Magnetization level (T)</th>
<th>Computed 2D no–load loss (W)</th>
<th>Computed 3D no–load loss (W)</th>
<th>Measured no–load loss (W)</th>
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<tr>
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<td>18.7891</td>
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<tr>
<td>1.72</td>
<td>46.53759</td>
<td>46.85404</td>
<td>49.43109859</td>
</tr>
</tbody>
</table>

Figure 5. Detail of 2D flux density vector plot (B = 0.3 T)

Figure 6. Detail of 2D flux density vector plot (B = 0.6 T)

Figure 7. Detail of 2D flux density vector plot (B = 1.3 T)

Figure 8. 3D flux density vector plot (B = 1.75 T)
V. CONCLUSION

By using the inverse elliptic anisotropy model in order to model the laminated wound core there is no need to model hundreds electrical steel sheets and the air–varnish composite between them. The resulting FE model is very simple and produces accurate results. The mesh size in the 2D case is reduced from $10^8$ to $10^6$ elements to, $10^6$ to $10^4$ elements and consequently the computational cost is minimised. Also, the aforementioned anisotropy model makes practical the 3D FE analysis of laminated wound cores by reducing the mesh size from $10^{11}$ to $10^8$ elements.

Even though, the computational effort of the 3D analysis is a multiple of that of the 2D analysis, the results obtained by the 2D and 3D FE analysis agree within 0.3% to 0.7%. This is due to the inherent 2D symmetry of wound cores. As a result, it follows that in the case of no–load analysis of wound core transformers the 3D FE analysis is not necessary.

REFERENCES


