Generalized Representation of Transformer Electrical Steels Working Under Heavily Saturated Conditions

Themistoklis D. Kefalas, *Member, IEEE*, George Loizos, and Antonios G. Kladas, *Member, IEEE*

***Abstract*–A generalized macroscopic representation of electrical steels used in transformer manufacturing industry is developed. The proposed representation is specifically formulated for integration in the finite element method. Usage of the specific technique enables the accurate evaluation of electromagnetic field distribution of transformer cores under heavily saturated conditions. Advantages over conventional techniques include numerical stability, numerical accuracy, and reduction of iterations of the Newton–Raphson method.**

***Index Terms*–Computer aided analysis, electromagnetic analysis, finite element methods, magnetic cores, magnetostatics, Newton–Raphson method, nonlinear magnetics, numerical analysis, power transformers, soft magnetic materials.**

I. Nomenclature

a , b	Parameters of generalized macroscopic representation of transformer electrical steel.
B_i^2	i –th squared flux density.
B_n^2	Maximum squared flux density for which ν_r is experimentally known.
d_{n-1}	Second derivative of ν_r at B_{n-1}^2.
h	Parameter equal to $B_n^2 - B_{n-1}^2$.
M	Magnetization.
ν_0	Reluctivity of vacuum.
ν_r	Relative reluctivity.
ν_{r_i}	Relative reluctivity value for B_i^2.
μ_0	Permeability of vacuum.
μ_r	Relative permeability.

II. Introduction

NOWADAYS, an ever increasing number of transformer manufacturers determine the technically and economically optimum transformer design by using advanced numerical models based on the finite element (FE) method [1], [2]. Electromagnetic field evaluation numerical techniques, present significant advantages over analytical methods as the former predict accurately the field distribution of complex, nonlinear electromagnetic devices [3]–[5]. In the case of power and distribution transformers the accurate evaluation of flux density distribution permits the calculation of key operational parameters such as no–load loss and short circuit impedance [1], [6]. Transformer manufacturers combine the aforementioned numerical techniques with optimization algorithms in order to minimize the manufacturing and operational cost of transformers [1], [7].

Recently a large number of distribution transformers are used in the renewable (wind and solar) energy market [8]. These transformers operate under heavily saturated and distorted supply voltage conditions. Conventional representation of electrical steels is not sufficient to describe transformer operation under the aforementioned conditions [9], [10]. In the present paper a generalized macroscopic representation of transformer electrical steels is developed. The advantages of the proposed formulation over the conventional one are accuracy of local field computation, numerical stability, and improvement of convergence characteristics of the Newton–Raphson iterative method. Also, the proposed representation presents advantages over contemporary approaches using a quadratic function to extrapolate the magnetization curve [11], [12]. The aforementioned approaches require the evaluation of four or three parameters whereas the proposed extrapolation method involves only two parameters.

III. Conventional Representation of Electrical Steels

If the solution of nonlinear electromagnetic problems by the FE method is based on the magnetic vector potential formulation, soft magnetic materials are expressed by the relative reluctivity versus squared flux density curve $\nu_r - B^2$ rather than the magnetization curve $B - H$, or the relative permeability versus magnetic field intensity curve $\mu_r - H$. Furthermore, nonlinearity is tackled by the Newton–Raphson iterative method.

The evaluation of ν_r for $B^2 \le B_n^2$, is carried out by cubic splines interpolation of tabulated sets of $\left(\nu_{r_i}, B_i^2\right)$, where ν_{r_i} is the relative reluctivity value for B_i^2 , B_n^2 is the maximum squared flux density for which relative reluctivity is experimentally known, and $0 \le i \le n$.

In order to compute ν_r for $B^2 \ge B_n^2$ the conventional approach is the following. The first derivative of ν_r at B_n^2 is given by (1), where h is given by (2) and d_{n-1} is the second derivative of ν_r at B_{n-1}^2 .

$$\left.\frac{d\nu_r\left(B^2\right)}{dB^2}\right|_{B_n^2} = -\left(\frac{\nu_{r_{n-1}} - \nu_{r_n}}{h}\right) + \frac{d_{n-1}h}{6} \tag{1}$$

$$h = B_n^2 - B_{n-1}^2 \tag{2}$$

For $B^2 > B_n^2$ relative reluctivity is given by

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$$v_r(B^2) = v_{r_n} + (B^2 - B_n^2) \cdot \left. \frac{dv_r(B^2)}{dB^2} \right|_{B_n^2} \quad (3)$$

IV. Proposed Generalized Representation

For the Newton–Raphson method to convergence it is imperative to know v_r for very high values of B^2. This is why the conventional formulation of Section III is widely accepted.

Nevertheless, as $B \to \infty$ the limit of (3) yields the following equation

$$\lim_{B\to\infty} v_r(B^2) = \lim_{B\to\infty} \left(v_{r_n} + (B^2 - B_n^2) \cdot \left. \frac{dv_r(B^2)}{dB^2} \right|_{B_n^2} \right) = \infty \quad (4)$$

Equation (4) suggests that for very large excitation levels $v_r \to \infty$ or $\mu_r \to 0$. The aforementioned is of course incorrect. Considering the constitutive equation for ferromagnetic materials (5) and isolating H yields (6).

$$B = \mu_0 (M + H) \quad (5)$$

$$H = v_0 B - M \quad (6)$$

As $B \to \infty$ the value of magnetization M is negligible compared to the value of the product $v_0 B$ and the limit of reluctivity v is given by

$$\lim_{B\to\infty} v = \lim_{B\to\infty} \frac{H}{B} = \lim_{B\to\infty} \frac{v_0 B - M}{B} = \lim_{B\to\infty} \frac{v_0 B}{B} = v_0 \quad (7)$$

Equation (7) states the well known fact that the relative reluctivity v_r and relative permeability μ_r of a soft magnetic material as $B \to \infty$ tends to unity $(v_r, \mu_r \to 1)$.

From the aforementioned it follows that the conventional soft magnetic material representation produces erroneous results in cases where the excitation level is high. In the present paper a different formulation is developed. For $B^2 \le B_n^2$ the same approach is used as shown in Section III. For $B^2 \ge B_n^2$ the authors propose the following function for v_r, where a, b are parameters of the extrapolation function. The dimensions of a parameter are $[T^{-2}]$ whereas b is a dimensionless parameter.

$$v_r(B^2) = 1 - \exp[-(aB^2 + b)] \quad (8)$$

The function of (8) as well as its slope is continuous. Also, it can be seen from (9) that as $B \to \infty$, v_r tends to unity since the exponent term tends to zero. Thus, the proposed extrapolation function yields correct results and it can be integrated to the Newton–Raphson iterative technique and to the FE method.

$$\lim_{B\to\infty} v_r(B^2) = \lim_{B\to\infty} \{1 - \exp[-(aB^2 + b)]\} = 1 \quad (9)$$

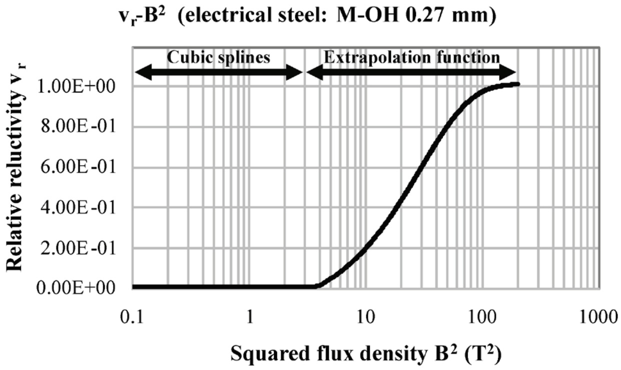


Fig. 1. Generalized representation of HiB grain-oriented steel M-OH.

In order to complete the proposed macroscopic material presentation there are two conditions that must be met.

1. The continuity of the first derivative of v_r at B_n^2.
2. The continuity of v_r at B_n^2.

The relative reluctivity at B_n^2 and the first derivative of v_r at B_n^2 are given by (10) and (11) respectively.

$$v_r(B_n^2) = 1 - \exp[-(aB_n^2 + b)] \quad (10)$$

$$\left. \frac{dv_r(B^2)}{dB^2} \right|_{B_n^2} = a \exp[-(aB_n^2 + b)] \quad (11)$$

Parameter a is evaluated by substituting (10) to (11) and rewriting, where $dv_r(B^2)/dB^2|_{B_n^2}$ is given by (1) and $v_r(B_n^2)$ is given by (3) by replacing B^2 with B_n^2.

$$a = \left. \frac{dv_r(B^2)}{dB^2} \right|_{B_n^2} / [1 - v_r(B_n^2)] \quad (12)$$

Substituting (10) to (12) and rewriting yields

$$\left. \frac{dv_r(B^2)}{dB^2} \right|_{B_n^2} / a = \exp[-(aB_n^2 + b)] \quad (13)$$

Parameter b is evaluated by taking the natural logarithm of both sides of (13) and rewriting, where $dv_r(B^2)/dB^2|_{B_n^2}$, a are given by (1) and (12) respectively.

$$b = -aB_n^2 - \ln\left(\left. \frac{dv_r(B^2)}{dB^2} \right|_{B_n^2} / a \right) \quad (14)$$

An application of the proposed generalized core material representation, for the commonly used in transformer industry, HiB grain–oriented steel M–OH 0.27 mm is shown in Fig. 1. For $B^2 \le 4.04T^2$ the $v_r - B^2$ curve is represented by cubic splines interpolation. For $B^2 \ge 4.04T^2$ the $v_r - B^2$ curve is represented by the proposed function given by (8). The a parameter is equal to $3.4234799 \cdot 10^{-2}\ T^{-2}$ and the b parameter is equal to –0.1320405.

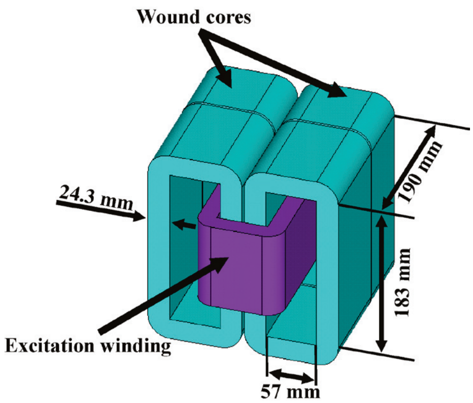


Fig. 2. One-phase, shell type, wound core transformer.

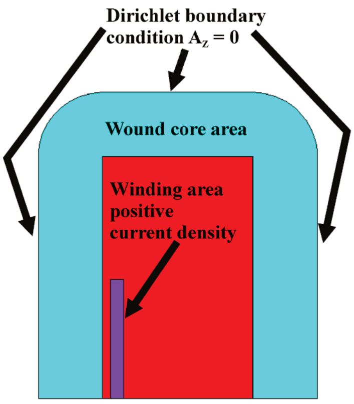


Fig. 3. FE model of one-phase, shell type, wound core transformer.

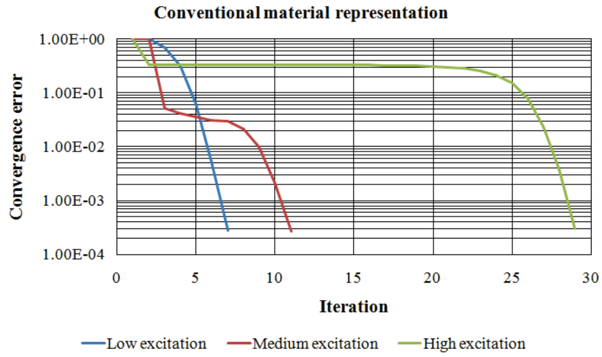


Fig. 4. Newton-Raphson convergence for conventional core material representation.

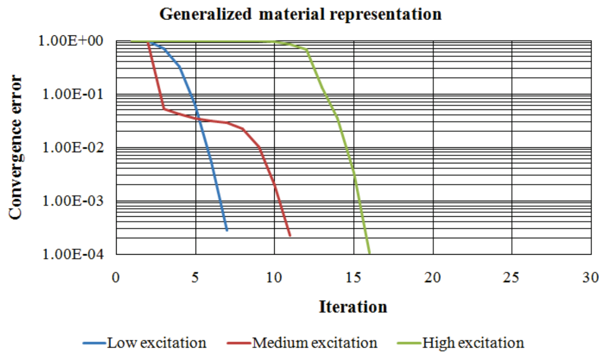


Fig. 5. Newton-Raphson convergence for generalized core material representation.

V. Integration to FE Method

The generalized macroscopic representation of Section IV can be integrated to 2D or 3D FE method based on the magnetic vector potential formulation and the Newton–Raphson method.

The exact same procedure used for the integration of the conventional representation of electrical steels to FE method, is used for the generalized representation. The proposed representation of electrical steels is achieved by using the composite function described in Section IV and shown in Fig. 1. The function as well as its first derivative is continuous for $0 \le B^2 \le B_n^2$ and for $B^2 \ge B_n^2$ and the aforementioned two conditions are essential for the Newton–Raphson iterative method to converge. The magnetic vector potential formulation, the Newton–Raphson method, and the integration of the conventional material model to the FE method are well known and full details may be sought in [9], [10].

The two parameters of the generalized macroscopic representation, a and b, are evaluated only once for each electrical steel and there is no need to reevaluate them at every iteration step of the Newton–Raphson method.

A 2D FE nonlinear package based on the magnetic vector potential and Newton–Raphson method has been developed by the authors. The specific FE package consists of a pre–processing code, a first order triangle mesh generator, the nonlinear FE solver, and a graphics post–processor. Both material representations were integrated to the nonlinear solver of the FE package.

VI. Comparison of Conventional and Proposed Representation

For the comparison of the conventional and generalized material representation, the one–phase, shell type, wound core transformer was used. It is shown in Fig. 2 and it is constructed of two or four wound cores assembled about a preformed winding as described in [5], [13], [14].

The preprocessor code was used to set up the FE model of the one–phase shell type transformer. Due to symmetry only one quarter of the transformer was modeled. The model is shown in Fig. 3 and is comprised by the wound core area surrounding a winding area of positive current density. The nonlinear properties of the conventional grain–oriented electrical steel M4 0.27 mm were assigned to the wound core area. For the grain–oriented steel M4 0.27 mm, the a parameter is equal to $1.6012561 \cdot 10^{-2}$ T^{-2} and the b parameter is equal to $-5.7915371 \cdot 10^{-2}$.

Magnetostatic analyses were carried out for a low, a medium, and a high level of excitation by applying a current density of $1.19 \cdot 10^{4}$ A/m^2, $7.46 \cdot 10^{4}$ A/m^2, and $2.15 \cdot 10^{9}$ A/m^2 to the winding area. Figs. 4, 5 show respectively the convergence characteristics of the Newton–Raphson method when the conventional and generalized material representation is used. Figs. 6, 7 show the flux density distribution of the one–phase transformer for conventional and generalized material representation respectively.

For a low excitation level the convergence characteristics of the Newton–Raphson method as well as the resulting flux density distribution are identical for both core material

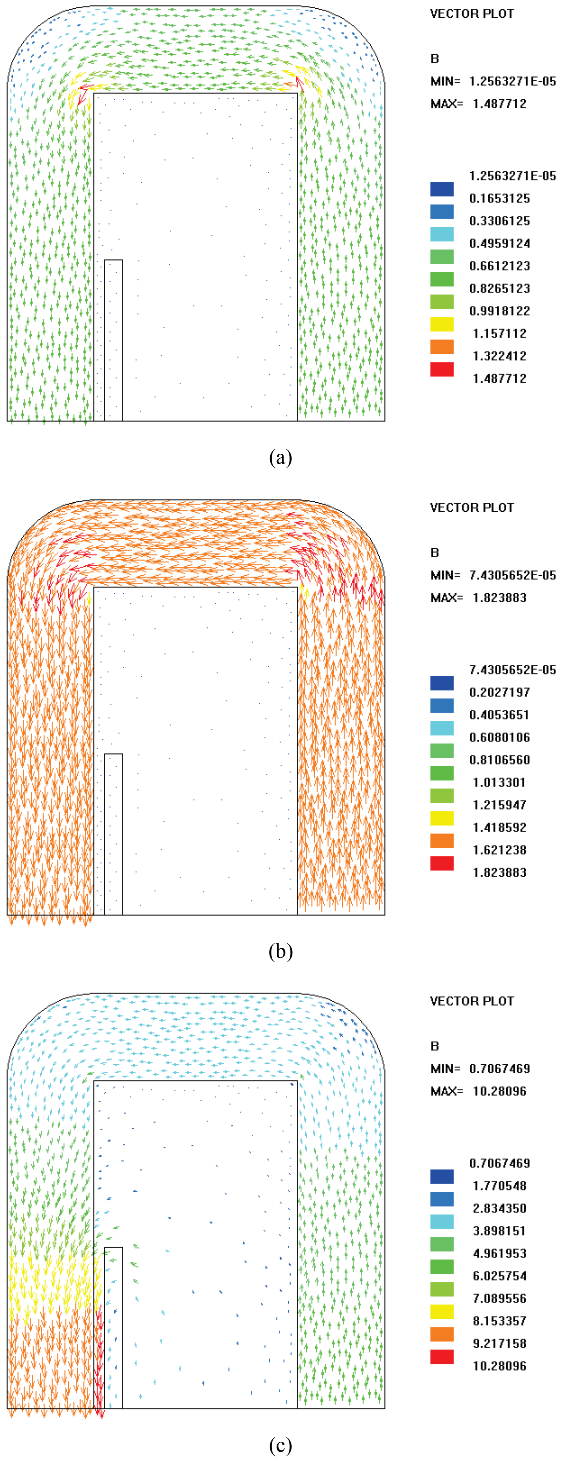


Fig. 6. Flux density vector plot of one-phase transformer using conventional core material representation, (a) $1.19{\cdot}10^4$ A/m^2, (b) $7.46{\cdot}10^4$ A/m^2, (c) $2.15{\cdot}10^9$ A/m^2.

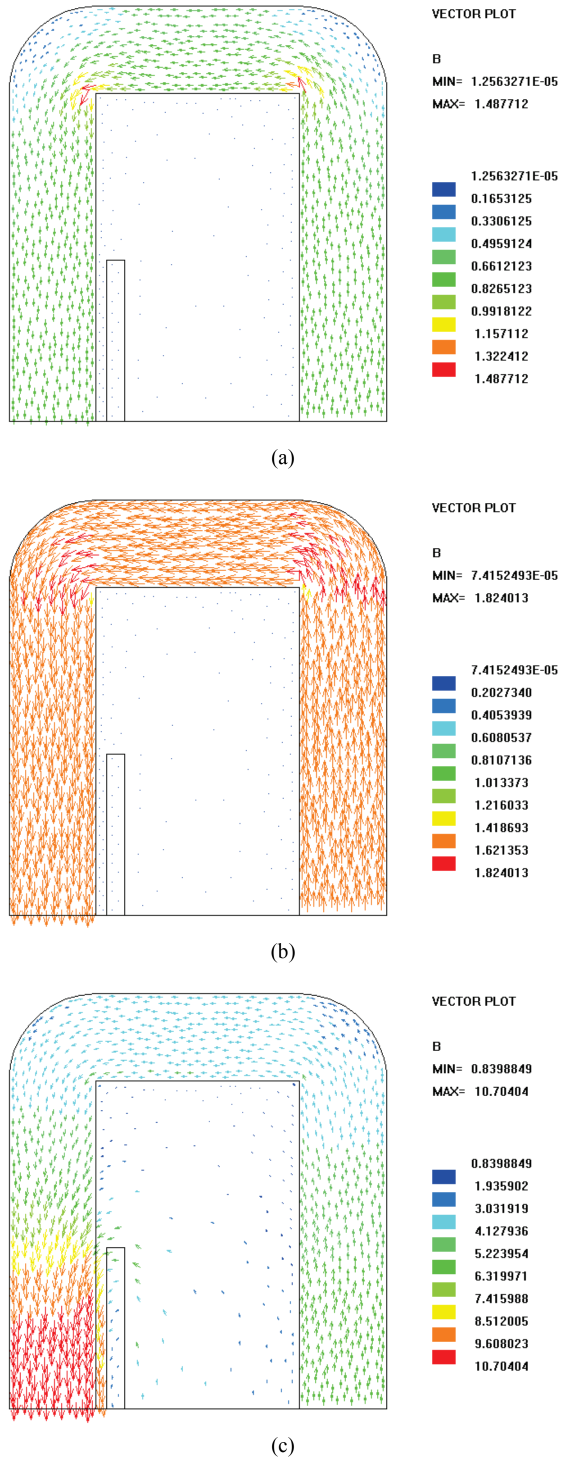


Fig. 7. Flux density vector plot of one-phase transformer using proposed generalized core material representation, (a) $1.19{\cdot}10^4$ A/m^2, (b) $7.46{\cdot}10^4$ A/m^2, (c) $2.15{\cdot}10^9$ A/m^2.

representations as can be seen from Figs. 4, 5 and Figs 6(a), 7(a) respectively. This is due to the fact that for low excitation levels the $v_r - B^2$ curve is represented by cubic splines interpolation in both cases.

For a medium excitation level the convergence error of the Newton–Raphson method is improved, $2.71{\cdot}10^{-4}$ for the conventional and $2.22\cdot10^{-4}$ for the proposed generalized representation. Also Figs. 6(b), 7(b) show that there is a difference in the flux density distribution.

For a high excitation level the Newton–Raphson method needs 29 iterations in the case of the conventional representation and only 16 when the generalized macroscopic representation is adopted, i.e. the computational effort is almost doubled when the conventional representation is used. Fig. 6(c) shows that the flux density magnitude of the core opposite the winding is lower than that of air between the core and the winding, i.e. the core material has a relative permeability lower than unity. This is the main drawback of the conventional core material representation as explained in Section IV. The aforementioned problem is solved when the proposed generalized representation is used, where Fig. 7(c) depicts the correct flux density distribution and clearly shows that the flux density in the core material is larger than that of air.

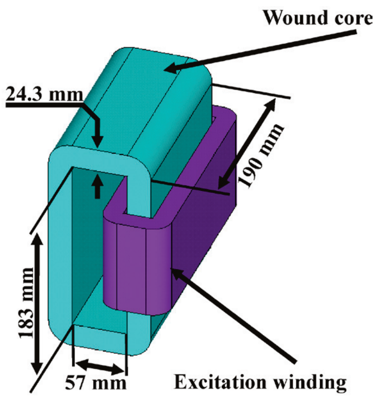


Fig. 8. One-phase, core type, wound core transformer.

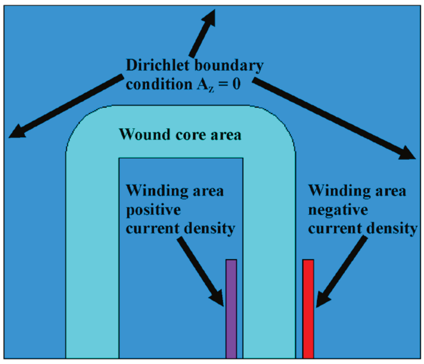


Fig. 9. FE model of one-phase, core type, wound core transformer.

VII. Application of the Generalized Macroscopic Representation

In the present section the generalized macroscopic representation is applied to the one–phase, core type wound core transformer. FE simulations were carried out for different magnetization levels. Also, two grain–oriented electrical steels were used as core materials, the conventional M4 0.27 mm and the HiB M–OH 0.27 mm. Fig. 8 shows the one–phase, core type, wound core transformer which is constructed of a single wound core assembled about a preformed winding [14].

The preprocessor code was used to setup the FE model of the one–phase core type transformer shown in Fig. 9. One half of the transformer was modeled due to symmetry. The model was placed inside a square defined as air. To three edges of the square a Dirichlet boundary condition was assigned whereas to the bottom edge no boundary condition was assigned.

Fig. 10 shows the flux density distribution vector plot of the one–phase core type transformer constructed of the HiB electrical steel M–OH 0.27 mm for three different magnetization levels, 1.5 T, 1.7 T, and 2.0 T. Fig. 11 shows the flux density distribution of the transformer constructed of the conventional electrical steel M4 0.27 mm and for the same magnetization levels.

Due to the different magnetization characteristics of the two electrical steels, there is a notable difference in the flux density distribution for the same magnetization level as shown in Figs. 10, 11. Also, the maximum local flux density is considerably higher than the mean magnetization level in all cases.

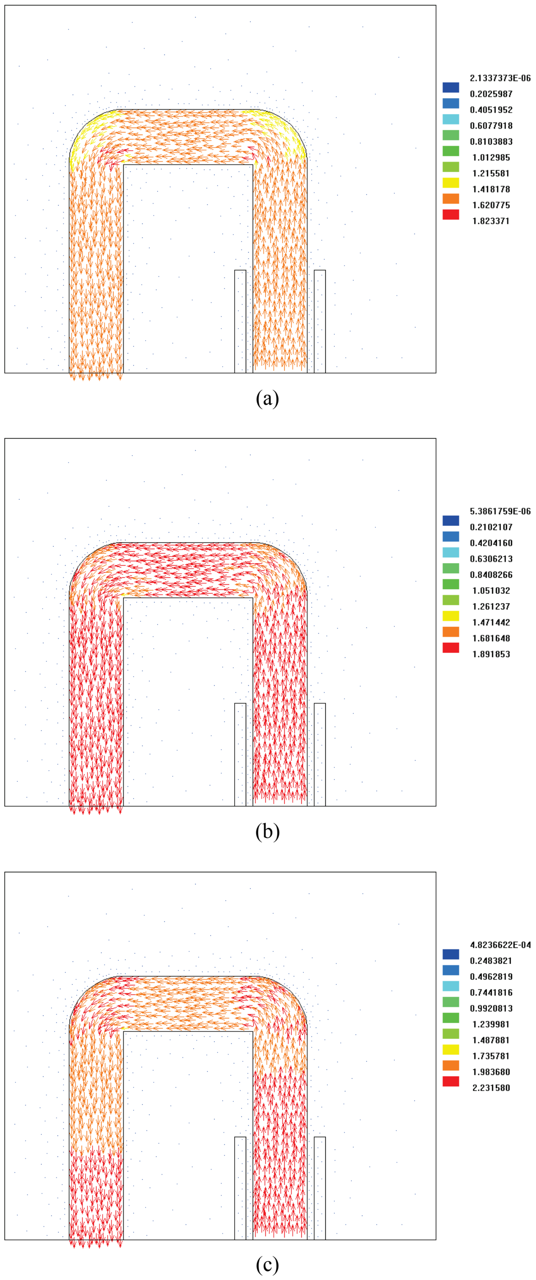


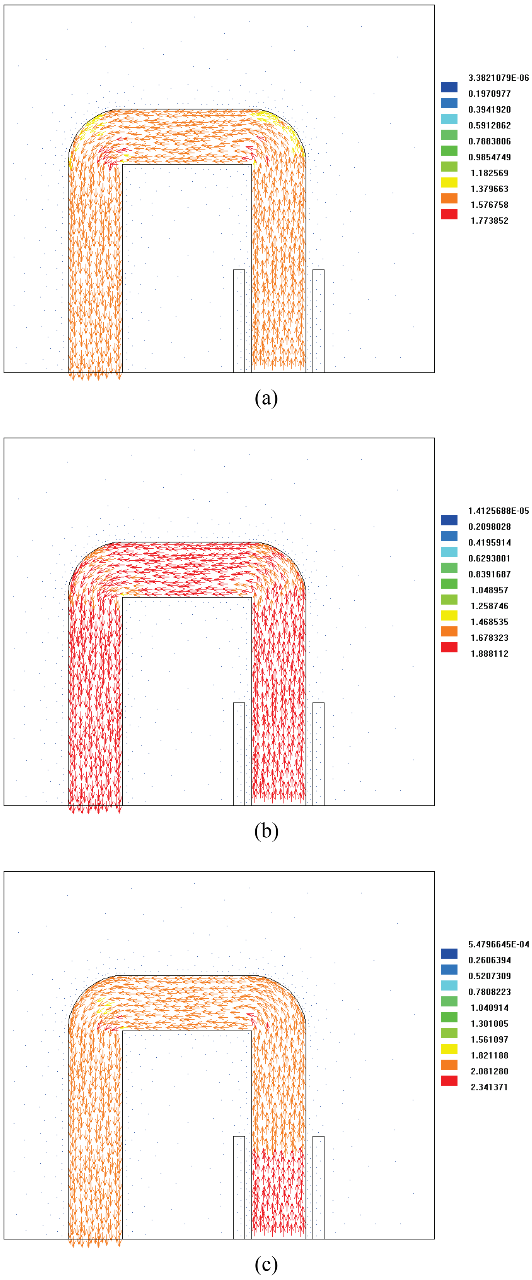
Fig. 10. Flux density vector plot of a wound core constructed of M-OH 0.27 mm, (a) 1.5 T, (b) 1.7 T, (c) 2.0 T.

VIII. Conclusion

The proposed generalized macroscopic representation predicts accurately the magnetization characteristics of electrical steels regardless of the excitation level. It yields the same results for low excitation levels as the conventional representation, but for medium and high excitation levels it yields improved results and reduces significantly the computational effort.

Since the generalized macroscopic representation is based on a continuous composite function with a continuous slope, it can be integrated directly to the Newton–Raphson technique and the FE method.

It is proven to be extremely efficient for the flux density distribution evaluation of distribution transformers used in the renewable energy market which operate under heavily saturated and distorted supply voltage conditions, e.g. under a PWM voltage waveform. The proposed generalized macroscopic core material representation is expected to aid



(a)

(b)

(c)

Fig. 11. Flux density vector plot of a wound core constructed of M4 0.27 mm, (a) 1.5 T, (b) 1.7 T, (c) 2.0 T.

transformer manufacturing industry in identifying transformer designs of minimum manufacturing and operational cost for use in the renewable energy market.

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X. Biographies

Themistoklis D. Kefalas (M'09) was born in Greece in 1977. He received the Electrical Engineering Educator degree from the School of Pedagogical and Technological Education, Athens, Greece, in 1999 and the Diploma and the Ph.D. degree in electrical engineering from the National Technical University of Athens, Athens, Greece, in 2005 and 2008, respectively.

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George Loizos was born in Greece in 1972. He received BEng and MSc in Electronics engineering in 1994 and 1995, a second MSc in Electrical Engineering and a Ph.D. in 1998 and 2002 respectively, all degrees from Cardiff University, Wales, UK.

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