Models for Variable Speed Wind Turbines

Summary of simplified $dq$ models developed in PhD Thesis:

“Contribution to the analysis of variable speed wind turbines with induction generator”

by Stavros A. Papathanassiou

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1. Introduction

The technology of the variable speed wind turbines (VS WTs) comes directly from the field of the adjustable speed electrical drives. Hence, the variable speed operation of a WT can be achieved using any suitable combination of electrical generator and power electronic converters, such as squirrel cage or wound rotor induction generators and stator or rotor converter cascades, or synchronous generator with field or permanent magnet excitation and stator AC/DC/AC converter interface to the grid. Obviously, each converter-generator combination presents advantages and disadvantages regarding its cost, complexity, operating and control characteristics, dynamic performance, harmonics, output power factor regulation etc. At present, however, no scheme is clearly favoured against the others and various configurations of the electrical part are being tested and evaluated.

As in the case of the electrical drives, no general model can be introduced, which would represent with sufficient accuracy the dynamic behaviour of all VS WT schemes. In fact, each configuration requires the elaboration of an individual model, depending on the type of generator, converters and controls used, applicable only to this particular scheme (or to a family of similar schemes, with slight modifications). It is also important that the generality of the developed models strongly depends on the modelling requirements, i.e. on the time scale and nature of the phenomena to be reproduced. For instance, it would be much easier to develop general purpose models for representing the behaviour of the output power of a VS WT in relatively slow wind variations, than for calculating its stability margin or short circuit current during grid faults.

In this report dynamic models are presented for VS WT configurations which use induction generators and stator or rotor AC/DC/AC converter cascades. These are simplified dq models developed within the PhD Thesis of the author “Contribution to the analysis of variable speed wind turbines with induction generator” (NTUA, Athens, Greece, 1997). More specifically, the electrical schemes included are the following:

a) Squirrel cage induction generator connected to the grid via a dual PWM voltage-source converter cascade, as shown in figure 1(a). The switching elements shown in the figure are GTOs, although for power levels up to several hundred kWs IGBTs are also available, permitting significantly higher switching frequencies. For this scheme, the scalar and vector control alternatives of the generator-side converter are included in the report.

b) Squirrel cage induction generator and current source inverter cascade, as illustrated in figure 1(b). The machine-side converter is of the Auto-Sequential Committed Inverter (ASCI) type, whereas at the grid-side a conventional phase-controlled thyristor inverter is used.

c) The double output induction generator, known also as the static Kramer drive, shown in figure 1(c). This configuration employs a wound-rotor induction machine and a rotor converter cascade consisting of a diode rectifier and a line-commutated thyristor inverter.

A VS WT comprises the following main subsystems, which can be modelled independently:

1. The rotor aerodynamics, which determine the conversion of the wind energy to accelerating mechanical torque.
2. The drive train, i.e. the torsional subsystem of the axes, gearbox and elastic couplings, linking the turbine rotor to the electrical generator.

3. The electrical and control subsystem, consisting of the electrical generator, the power electronics converters and the associated controls.

In the following sections the modelling equations for each subsystem are presented and the main assumptions are outlined. The examined configurations are differentiated with respect only to the electrical part. In this report only stall regulated (fixed-pitch) WTs are examined. Pitch controlled machines require additional models for the blade-pitch regulation system, without any...
modification to the models of the other subsystems (with the possible exception of the WT speed controller).

2. Aerodynamic part

The conversion of the wind power to mechanical power by the wind turbine rotor can be simulated by the static relation:

\[
P_w = \frac{1}{2} \rho C_p A V_w^3
\]

where

- \(P_w\) is the rotor mechanical power (W)
- \(V_w\) the wind speed at the center of the rotor (m/s)
- \(A=\pi R^2\) the rotor surface (m\(^2\))
- \(R\) the rotor radius (m)
- \(\rho\) the air density (kg/m\(^3\)) and
- \(C_p\) the rotor aerodynamic power coefficient.

The rotor mechanical torque can be calculated from \(P_w\) by

\[
T_w = \frac{P_w}{\omega_R}
\]

where \(\omega_R\) is the rotor angular velocity, in rad/sec.

The rotor aerodynamic power coefficient, \(C_p\), is the percentage of the kinetic energy of the incident air mass that is converted to mechanical energy by the rotor, and it is expressed as a function

\[
C_p = C_p(\lambda, \beta)
\]

where \(\beta\) is the blade pitch angle and
\(\lambda\) the tip speed ratio of the blade, defined as

\[
\lambda = \frac{R \omega_R}{V_w}
\]

Using the above relations and the rotor \(C_p(\lambda)\) characteristic, the rotor aerodynamic torque and power curves can be calculated. These curves are shown in figure 2 for a stall regulated 140 kW WT. At low wind speeds, where the rotor torque and power do not exceed the rated values, the rotor speed is varied for optimal wind energy capture, i.e. for operation on the peak of the power curves (curve AB in figure 2). At higher winds, the speed is limited to its maximum value (52 rpm in figure 2) and the stall properties of the blades are used for limiting the torque and power below the design values (curve BC). Hence, the rotor speed control characteristic ABC is derived. Alternative speed control policies, such as curves ABDC or ABEC are also feasible from a static point of view. In practice, however, operation beyond the vertical line BC will result in significant over-torque and over-power situations in case of wind gusts, because the large inertia of the rotor does not permit the fast reduction of its speed, towards point C. In case of pitch controlled WTs, the above restriction may not be valid, because the pitch regulation may be used for limiting the rotor torque above the rated wind speed.
In order to reproduce the rotor aerodynamic torque harmonics (at frequencies $nP$, where $P$ the rotor speed and $n$ integer), due to the tower shadow and wind shear effects, each blade must be modelled independently. In this case, the above relations are applied for each individual blade, using the respective aerodynamic power coefficient $C_p$. For an $n$-blade rotor, the $C_p$ of each blade can be taken equal to the $1/n$ of the rotor $C_p$. The tower shadow is approximated as shown in figure 3, by considering a near sinusoidal reduction of the equivalent blade wind speed, as each blade passes in front of the tower. $\Delta V_{sh}$ is the maximum wind speed reduction and $2\theta$ the shadow angle.
The wind shear effect is represented by the well known exponential law

\[ \frac{V_w}{V_{wh}} = \left( \frac{z}{z_h} \right)^\alpha \]  

(2.5)

where \( V_w \) is the wind speed at height \( z \), \( V_{wh} \) the wind speed at the hub height \( z_h \) and \( \alpha \) the shear exponent.

### 3. Drive Train

Due to the increased compliance of the drive train of almost every wind turbine (usually achieved by “soft” axes or special elastic couplings), suitable multimass equivalents must be employed, in order to represent the low frequency torsional modes, which dominate the dynamic behaviour of the WT.

In figure 4, two typical mechanical equivalents are illustrated, consisting of rotating masses elastically coupled to each other. In figure 4(a) the three inertias correspond to the turbine rotor, the gearbox and the electrical generator. (The interconnecting axes, disk brakes etc. are incorporated in the lumped inertias of the model). The elasticity and damping elements between adjacent inertias correspond to the low and high-speed shaft elasticities and internal friction, whereas the external damping elements represent the torque losses. In order to reproduce the blade edgewise mode of oscillations, each rotor blade can be modelled by a separate inertia, elastically connected to the hub, as shown in figure 4(b) for a 3-blade rotor. In this case the inertia \( H_h \), adjacent to the gearbox, corresponds only to the hub, and not to the whole turbine rotor, as in figure 4(a).

The state equations for the drive train mechanical equivalent of figure 4(a) are the following, using the inertias’ angular positions and velocities as state variables:

\[
\frac{d}{dt} \begin{bmatrix} \theta \\ \omega \end{bmatrix} = \begin{bmatrix} I_3 \times 3 \\ -2H \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix} \begin{bmatrix} \theta \\ \omega \end{bmatrix} + \begin{bmatrix} 0 \\ 2H \end{bmatrix} T
\]

(3.1)

where

\[ \theta^T = [\theta_R, \theta_G, \theta_G] \]  

the vector of the angular positions of the rotor, gearbox and generator

\[ \omega^T = [\omega_R, \omega_G, \omega_G] \]  

the vector of the angular velocities of the rotor, gearbox and generator
Figure 4. (a) 3-mass and (b) 6-mass drive train mechanical equivalents (for a 3-blade rotor)

\[ T_T = [T_W, 0, T_G] \]
the vector of the external torques, acting on the turbine rotor (aerodynamic torque \( T_W \)) and on the generator rotor (electromagnetic torque \( T_G \)), conventionally accelerating the zero and identity 3x3 matrices, respectively

\[ [2H] = \text{diag}(2H_R, 2H_{GB}, 2H_G) \]
the diagonal 3x3 inertia matrix

\[ [C] = \begin{bmatrix} C_{HGB} & -C_{HGB} & 0 \\ -C_{HGB} & C_{HGB} + C_{GBG} & -C_{GBG} \\ 0 & -C_{GBG} & C_{GBG} \end{bmatrix} \]
the 3x3 stiffness matrix, where \( C_{HGB} \) and \( C_{GBG} \) are the hub to gearbox and gearbox to generator stiffness coefficients

\[ [D] = \begin{bmatrix} D_R + d_{HGB} & -d_{HGB} & 0 \\ -d_{HGB} & D_{GB} + d_{HGB} + d_{GBG} & -d_{GBG} \\ 0 & -d_{GBG} & D_G + d_{GBG} \end{bmatrix} \]
the 3x3 damping matrix, where \( d_{HGB} \) and \( d_{GBG} \) are the relative dampings of the elastic couplings and \( D_R, D_{GB}, D_G \) the external damping coefficients

The state equations of the 6-mass mechanical equivalent of figure 2(b) are the following:

\[ \frac{d}{dt} \begin{bmatrix} \theta \\ \omega \end{bmatrix} = -[2H]^{-1}[C]\begin{bmatrix} \theta \\ \omega \end{bmatrix} + \left[ \begin{bmatrix} 0 \\ [2H]^{-1} \end{bmatrix} T \right] \] (3.2)
where

\[ \Theta^T = [\theta_{b1}, \theta_{b2}, \theta_{b3}, \theta_h, \theta_{gb}, \theta_g] \]

the vector of the angular positions of the blades, hub, gearbox and generator

\[ \omega^T = [\omega_{b1}, \omega_{b2}, \omega_{b3}, \omega_h, \omega_{gb}, \omega_g] \]

the vector of the angular velocities of the blades, hub, gearbox and generator

\[ T^T = [T_{W1}, T_{W2}, T_{W3}, 0, 0, T_g] \]

the vector of the external torques, acting on the turbine blades (aerodynamic torques \( T_{Wi}, i=1,2,3 \)) and on the generator rotor (electromagnetic torque \( T_g \)), conventionally accelerating

\[ \mathbf{0}_{6x6} \text{ and } \mathbf{I}_{6x6} \]

the zero and identity 6x6 matrices, respectively

\[ 2H = \text{diag}(2H_B, 2H_B, 2H_B, 2H_H, 2H_{GB}, 2H_G) \]

the diagonal 6x6 inertia matrix

\[ \mathbf{C} = \begin{bmatrix}
C_{HB} & 0 & 0 & -C_{HB} & 0 & 0 \\
0 & C_{HB} & 0 & -C_{HB} & 0 & 0 \\
0 & 0 & C_{HB} & -C_{HB} & 0 & 0 \\
-C_{HB} & -C_{HB} & C_{HGB} + 3C_{HB} & -C_{HBG} & 0 & 0 \\
0 & 0 & 0 & -C_{HGB} & C_{HGB} + C_{GBG} & -C_{GBG} \\
0 & 0 & 0 & 0 & -C_{GBG} & C_{GBG}
\end{bmatrix} \]

the 6x6 stiffness matrix, where \( C_{HB}, C_{HGB} \) and \( C_{GBG} \) respectively are the blade to hub, hub to gearbox and gearbox to generator stiffness coefficients

\[ \mathbf{D} = \begin{bmatrix}
D_B + d_{HB} & 0 & 0 & -d_{HB} & 0 & 0 \\
0 & D_B + d_{HB} & 0 & -d_{HB} & 0 & 0 \\
0 & 0 & D_B + d_{HB} & -d_{HB} & 0 & 0 \\
-d_{HB} & -d_{HB} & -d_{HB} & D_B + d_{HGB} + 3d_{HB} & -d_{HBG} & 0 \\
0 & 0 & 0 & -d_{HGB} & D_B + d_{HGB} + d_{GBG} & -d_{GBG} \\
0 & 0 & 0 & 0 & -d_{GBG} & D_G + d_{GBG}
\end{bmatrix} \]

the 6x6 damping matrix, where \( d_{HB}, d_{HGB} \) and \( d_{GBG} \) are the relative dampings of the elastic couplings and \( D_B, D_H, D_{GB}, D_G \) the external damping coefficients

4. Induction Generator

The induction generator is simulated by the standard 4\textsuperscript{th} order dq model, the equations of which are the following, expressed in the arbitrary reference frame:

\[
\begin{align*}
u_{sd} &= -r_s i_{sd} - \omega \Psi_{sq} + p \Psi_{sd} \\
u_{sq} &= -r_s i_{sq} + \omega \Psi_{sd} + p \Psi_{sq} \\
u_{rd} &= 0 =r_r i_{rd} - (\omega - \omega_r) \Psi_{rq} + p \Psi_{rd} \\
u_{rq} &= 0 =r_r i_{rq} + (\omega - \omega_r) \Psi_{rd} + p \Psi_{rq}
\end{align*}
\]

(4.1)

where the flux linkages \( \Psi_{sd}, \Psi_{sq}, \Psi_{rd}, \Psi_{rq} \) are given by

\[
\begin{align*}
\Psi_{sd} &= -X_s i_{sd} + X_m i_{rd} \\
\Psi_{sq} &= -X_s i_{sq} + X_m i_{rq} \\
\Psi_{rd} &= -X_m i_{sd} + X_r i_{rd} \\
\Psi_{rq} &= -X_m i_{sq} + X_r i_{rq}
\end{align*}
\]

(4.2)

where \( \omega_0 = 2\pi 50 \text{ rad/sec} \) the base electrical angular frequency

\( \omega \) the arbitrary dq frame angular frequency (p.u.)
\[ \omega_r \text{ the generator speed (p.u.)} \]
\[ u_{sd}, u_{sq} \text{ the stator voltage } d \text{ and } q \text{ components (p.u.)} \]
\[ i_{sd}, i_{sq}, i_{rd}, i_{rq} \text{ the stator and rotor } d \text{ and } q \text{ windings currents (p.u.)} \]
\[ r_s, r_r \text{ the stator and rotor windings resistance} \]
\[ X_s, X_r \text{ the stator and rotor windings reactance} \]
\[ X_m \text{ the magnetizing reactance} \]
\[ p \equiv (1/\omega_0)(d/dt) \]

Substituting (4.2) in (4.1) and solving for the derivatives of the currents, the state equations of the machine with the currents as state variables are derived:

\[
\begin{bmatrix}
  \frac{d}{dt} i_{sd} \\
  \frac{d}{dt} i_{sq} \\
  \frac{d}{dt} i_{rq}
\end{bmatrix} =
\begin{bmatrix}
  -r_s X_s & (\alpha D + \omega_0 X_m^2) & -r_s X_s & -\omega_0 X_m \\
  -r_s X_s & 0 & \alpha D X_m & 0 \\
  -v_0 X_m & -r_s X_m & 0 & (\alpha D - \omega_0 X_m)(\alpha D - \omega_0 X_m)
\end{bmatrix}
\begin{bmatrix}
  i_{sd} \\
  i_{sq} \\
  i_{rq}
\end{bmatrix} +
\begin{bmatrix}
  -v_s \\
  0 \\
  0
\end{bmatrix}
\]

where \( D = X_s X_r - X_m^2 \).

The generator electromagnetic torque \( T_e \) is given by

\[
T_e = \mathbf{u}_{rd} i_{rq} - \mathbf{u}_{rq} i_{rd} = X_m (i_{sd} i_{rd} - i_{sq} i_{rq})
\]

where all quantities are expressed in p.u. and generator convention is assumed for \( T_e \).

5. Voltage Source Converter Scheme

5.1 Network Side Converter and DC link

![Figure 5. Voltage source converters simplified diagram.](image1)

![Figure 6. Fundamental frequency equivalent circuit and respective vector diagram for the network-side inverter output filters.](image2)
In figure 5 the simplified diagram of the voltage source converters is shown. The fundamental frequency equivalent circuit and vector diagram of the converter output filters are illustrated in figure 6. \( V_i, \omega_i \) are the inverter output voltage magnitude and frequency and \( V_b, \omega_b \) the bus voltage magnitude and frequency. \( R_f, X_f \) are the filters resistance and reactance.

\[
\begin{align*}
V_{bd} & \equiv V_i - R_f I_d + \omega_b X_f V_i I_q + \frac{X_f}{\omega_0} \frac{dI_d}{dt} - \omega_0 (R_f I_d + \omega_b X_f V_i I_q + V_i I_d - V_b) / X_f \\
V_{bq} & \equiv 0 = V_i - R_f I_q - \omega_b X_f V_i I_d + \frac{X_f}{\omega_0} \frac{dI_q}{dt} - \omega_0 (R_f I_q - \omega_b X_f V_i I_d + V_i I_q) / X_f
\end{align*}
\]

where \( \omega_0 = 2\pi \cdot 50 \text{ rad/sec} \) is the base electrical angular velocity, \( \omega_b \) the bus voltage vector angular velocity (in p.u.), \( I_d, I_q \) the d and q components of the inverter output current, \( \vec{V} \) (in p.u.), \( R_f, X_f \) the filters resistance and reactance, \( V_b \) the bus voltage magnitude, \( V_{id}, V_{iq} \) the d and q components of the inverter voltage \( \vec{V}_i \), which are given by:

\[
\begin{align*}
V_{id} &= V_i \cos \delta_i \\
V_{iq} &= V_i \sin \delta_i \\
\frac{d\delta_i}{dt} &= \omega_0 (\omega_i - \omega_b)
\end{align*}
\]

The dc voltage differential equation is the following (see figure 5)

\[
C \frac{dV_{dc}}{dt} = I_c = I_{dc1} - I_{dc2}
\]

where the currents \( I_{dc1} \) and \( I_{dc2} \) are calculated from the active power relations in the following.

From the dc-side input power relation, expressed in p.u., \( I_{dc1} \) is derived:

\[
P_{dc1} = \frac{2}{3} V_{dc} I_{dc1} \Rightarrow I_{dc1} = \frac{3 P_{dc1}}{2 V_{dc}}
\]

Ignoring the power losses of the output converter, the active power balance yields:

\[
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\]
Depending on the pulse width modulation (PWM) technique used, the inverter fundamental ac voltage $V_i$ is related with the dc voltage $V_{dc}$ and the control system reference voltage $V_{ref}$, with a relation of the form:

$$V_i = k_v V_{ref} V_{dc}$$  \hspace{1cm} (5.7)

The proportionality constant $k_v$ depends on the operating mode of the inverter and can be assumed to be constant when the inverter operates in the linear region (i.e. below the output voltage saturation point).

The control system of the output converter is shown in block diagram form in figure 8, where only the blocks relevant to the «normal» operation of the system have been retained. The control system consists of two distinct regulation loops. The first regulates the dc voltage level to the reference value $V_{dc,ref}$, using a PID controller (and an additional derivative controller, activated in case of large $V_{dc}$ excursions). Output of the loop is the inverter frequency (actually, its reference value $\omega_{ref}$), permitting the regulation of the power angle $\delta$ between the inverter and bus voltage vectors (see figure 6). The reactive output current control loop, shown in figure 8(b), regulates the magnitude of the inverter ac voltage, in order to control the reactive output power and hence the power factor of the WT, which may take even leading values (i.e. the WT may produce reactive power, for instance in order to provide voltage support to weak grids).
The outputs $V_{\text{ref}}$ and $\omega_{\text{ref}}$ of the control system loops determine the voltage magnitude and frequency of the output converter, as shown in figure 9. The time constant of the delay blocks depends on the switching frequency of the converter and typically is of the order of a few msec or less. The voltage magnitude also depends on the dc voltage level, according to relation (5.7).

![Figure 9. Voltage source converter model.](image)

5.2 Generator-Side Converter

5.2.1 Scalar Control

![Figure 10. Scalar control system of the generator-side converter. (a) Speed and generator frequency control. (b) Generator voltage control.](image)
In figure 10 the block diagram of the scalar control system is shown for the generator-side converter. The generator frequency control subsystem of figure 10(a) basically consists of the PI speed controller. Its output, the slip frequency \( \omega_{sl} \), is added to the rotor speed, to determine the stator frequency (actually the reference value \( \omega_{ref} \), to the converter control circuits). The speed controller gain is assumed to vary between a minimum and a maximum value, depending on the slip frequency (output of the PI block). In addition, it comprises two derivative-type blocks (with 1st order stabilizing terms), the role of which are to provide damping to torque and dc voltage oscillations. Their inputs are respectively the real component of the generator current and the measured dc voltage. For regulating the generator voltage the constant V/f control principle is employed, as shown in figure 10(b). The term proportional to the dc voltage deviation from its nominal value compensates the dependence of the converter ac voltage on the dc voltage levels (relation (5.7)). The stabilizer block accepts as input the reactive component of the generator current in order to damp oscillations of the current at low torque and speed values.

For the generator-side converter, the block diagram of figure 9 is also valid, relating the control system commands \( V_{ref} \) and \( \omega_{ref} \) with the actual voltage and frequency, \( V_s \) and \( \omega_s \), applied to the machine. The induction generator is modelled using the 4th order dq model of Section 4.

5.2.2 Vector Control

The vector control system of the generator converter comprises the two subsystems illustrated in figure 11, next page. The subsystem of figure 11(a) calculates the angular position \( \theta_v \) of the rotor flux reference frame, \((dq)_v\), and the torque and flux components of the stator current, \( i_v^r \) and \( i_v^q \), using the measured stator phase currents and rotor position. The \((dq)_v\) reference frame, in which the machine vector control equations are expressed, is the frame aligned with the rotor flux vector, as shown in figure 12, next page, where the rotor reference frame \((dq)_r\) and the stationary frame \((dq)_s\) are also shown.

In the block diagram of figure 11(a) the stator current components expressed in the stationary reference frame \((dq)_s\), are first calculated from

\[
\begin{bmatrix}
i_{sd}^s \\
i_{sq}^s
\end{bmatrix} = \frac{2}{3} \begin{bmatrix}
1 & -0.5 \\
\sqrt{3}/2 & -\sqrt{3}/2
\end{bmatrix} \begin{bmatrix}
i_a \\
i_b \\
i_c
\end{bmatrix}
\]

and subsequently transformed to the rotor reference frame:

\[
\begin{bmatrix}
i_{sd}^r \\
i_{sq}^r
\end{bmatrix} = \begin{bmatrix}
\cos \theta_r & \sin \theta_r \\
-\sin \theta_r & \cos \theta_r
\end{bmatrix} \begin{bmatrix}
i_{sd}^v \\
i_{sq}^v
\end{bmatrix}
\]

where \( \theta_r \) is the angular position of the rotor. The rotor flux components and the angle of the rotor flux vector \( \theta_v \) are then using relations:

\[
T_r \frac{d\Psi_{rd}^r}{dt} + \Psi_{rd}^r = -X_m i_{sd}^r
\]

\[
\Psi_{rd}^r = -\frac{X_m}{1 + sT_r} i_{sd}^r
\]

\[
T_r \frac{d\Psi_{rq}^r}{dt} + \Psi_{rq}^r = -X_m i_{sq}^r
\]

\[
\Psi_{rq}^r = -\frac{X_m}{1 + sT_r} i_{sq}^r
\]

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Figure 11. Block diagram of the vector control system. (a) Induction machine vector model. (b) Calculation of the torque and flux currents and regulation of the ac voltage and frequency.

Figure 12. Orthogonal reference frames and relative angles. (dq)$_s$: stationary frame, (dq)$_r$: rotor frame, (dq)$_\psi$: rotor flux frame (used for vector control).
and

\[ \theta_v = \theta_r + \tan^{-1} \left( \frac{\Psi_{\text{req}}}{\Psi_{\text{ref}}} \right) \]  

(5.11)

where \( T_r \) is the rotor time constant, defined as

\[ T_r = \frac{1}{\omega_0} \frac{X_L}{r_r} \]  

(5.12)

The torque and flux components of the stator current, \( i_{s\theta} \) and \( i_{s\phi} \) respectively, are also calculated from the \((dq)\) components using:

\[ \begin{bmatrix} i_{s\phi} \\ i_{s\theta} \end{bmatrix} = \begin{bmatrix} \cos \theta_v & \sin \theta_v \\ -\sin \theta_v & \cos \theta_v \end{bmatrix} \begin{bmatrix} i_{s\phi} \\ i_{s\theta} \end{bmatrix} \]  

(5.13)

The main part of the vector control system, illustrated in figure 11(b), comprises the PI speed controller and the blocks where the \( d \) and \( q \) reference currents and the stator voltage magnitude and frequency are calculated. The gain of the PI speed controller depends on the level of its output, the torque reference \( T_{\text{ref}} \), changing from \( K_{s,\text{min}} \) to \( K_{s,\text{max}} \), with a time constant \( T_{fK_s} \), when \( T_{\text{ref}} \) exceeds \( T_{\text{max}} \). The generator reference flux, \( \Psi_{\text{ref}} \), is determined by the external signal \( \Psi_{\text{ext}} \), taking into account the dc voltage levels and whether the machine has entered the field weakening region. The following relations are used for this purpose:

\[ \Psi_{\text{ref}} = \begin{cases} \Psi_{\text{ext}} & \omega_{\text{stat}} \leq \omega_{FW} \\ \Psi_{\text{ext}} \frac{\omega_{FW}}{\omega_{\text{stat}}} & \omega_{\text{stat}} > \omega_{FW} \end{cases} \]  

(5.14)

\[ \omega_{FW} = \frac{V_{dc}}{\Psi_{\text{ext}}} \omega_{FW,\text{nom}} \]  

(5.15)

where \( \omega_{\text{stat}} \) is the static term of the stator frequency, defined in the following and \( \omega_{FW,\text{nom}} \) the nominal field weakening frequency.

The “Torque Current” block in figure 11(b) calculates the reference value \( i_{q,\text{ref}} \) of the stator current torque component:

\[ i_{q,\text{ref}} = \frac{\Psi_{r,\text{nom}}}{\Psi_{\text{ref}}} \frac{T_{\text{ref}}}{T_{e,\text{nom}}} \]  

(5.16)

where the subscript “nom” denotes machine nominal quantities. Similarly, the «Flux Current» block performs the flux component reference value calculation

\[ i_{d,\text{ref}} = \frac{\Psi_{\text{ref}}}{X_m} \]  

(5.17)

using an approximation of the magnetizing curve for determining the magnetizing reactance \( X_m \).

If the machine-side converter were current controlled, the remaining blocks in diagram 11(b) would not be required. In the case considered here, however, where only the output voltage and frequency of the converter are directly controlled, these variables are respectively used for regulating the torque and flux components of the stator current at their reference values. Due to the limited bandwidth of the PI regulators used, the two loops present a certain coupling,
particularly at higher frequencies, and therefore the considered control system deviates from the strict vector control principle.

As shown in figure 11(b), the reference value $\omega_{\text{ref}}$ of the converter output frequency is determined by the sum

$$\omega_{\text{ref}} = \omega_{\text{stat}} + \omega_{\text{sl,dyn}}$$  \hspace{1cm} (5.18)

where $\omega_{\text{sl,dyn}}$ is the “dynamic” slip frequency term, output of the PI controller, whose purpose is to eliminate the error in the torque current regulation. The “static” frequency is given by the sum

$$\omega_{\text{stat}} = \omega_{\text{sl,stat}} + \omega_r$$  \hspace{1cm} (5.19)

where $\omega_r$ is the rotor speed and $\omega_{\text{sl,stat}}$ the “static” slip frequency term, given by

$$\omega_{\text{sl,stat}} = -\frac{\omega_0}{T_r} \frac{i_{\text{sq}}^v}{I_{\text{ar}}} = \frac{\omega_0 X_m}{T_r} \frac{i_{\text{sq}}^v}{\Psi_r}$$  \hspace{1cm} (5.20)

Similarly, the converter voltage reference is determined by the sum of a static and a dynamic term:

$$V_{\text{ref}} = V_{\text{stat}} + V_{\text{dyn}}$$  \hspace{1cm} (5.21)

The static voltage regulator utilizes the following relations:

$$u_d = -r_s i_{d,\text{ref}}$$

$$u_q = -r_s i_{q,\text{ref}} + \frac{\omega_{\text{stat}}}{\omega_{\text{FW}}} \omega_{\text{FW}} \omega_{\text{FW,nom}} V_{\text{ref}} \frac{1}{V_{\text{dc}}}$$  \hspace{1cm} (5.22)

$$u_{\text{stat}} = \sqrt{u_d^2 + u_q^2}$$  \hspace{1cm} (5.23)

which provide a crude approximation of the induction generator stator voltage equations:

$$u_{sd}^v = -r_s i_{sd}^v + \omega_r X_{sd}^v - \frac{1}{\omega_0} X' \frac{di_{sd}^v}{dt} + \frac{1}{\omega_0} X_r \frac{d\Psi_r}{dt}$$

$$u_{sq}^v = -r_s i_{sq}^v - \omega_r X_{sq}^v + \omega_r X_{sq} X_{sd} + \frac{1}{\omega_0} X'_r \frac{di_{sq}^v}{dt}$$  \hspace{1cm} (5.24)

where $X' = X_s - \frac{X_m^2}{X_r}$ is the transient reactance of the machine. The dynamic voltage term, output of the respective PI regulator, is required for eliminating then stator voltage error.

5.2.3 Speed Reference Determination

From the rotor aerodynamic torque and power characteristics, such as the ones shown in figure 2, the rotor speed control characteristic is determined, which is curve ABC for the diagrams of figure 2, as explained in Section 2. Using this characteristic and a suitable wind speed measurement, the rotor speed reference, input to the converters control system, may be determined, as shown in figure 13. The first order low-pass filter acts as a moving average of the measured wind speed, filtering out the high frequency fluctuations, which the turbine is unable to follow due to the high rotor inertia.
An alternative speed control strategy, which does not require a wind speed measurement (which is generally unreliable), is based on the rotor speed measurement and the subsequent determination of the generator torque reference, using the rotor speed control characteristic (curve ABC in figure 2), as shown in figure 14. In this case, the speed control loop of the generator-side converter control system, figure 11(b), is redundant, since the torque reference is directly calculated. Apparently, application of this strategy requires that the WT control system utilizes a torque reference signal, as is the case with the vector control system.

6. Current Source Converter Scheme

In figure 16 the simplified diagram of the current-source inverter scheme is shown. The equations of the system are derived in the following, with all quantities are expressed in p.u.

The relation between the fundamental frequency component of the generator current, $I_s$, and the dc current $I_d$, is the following:

$$I_s = \frac{2\sqrt{3}}{\pi} I_d$$  \hspace{1cm} (6.1)
For the simulation of the induction generator, a \(dq\) reference frame is selected with its \(d\) axis aligned with the fundamental stator current vector. Then, the stator current \(d\) and \(q\) components are given by

\[
i_{sd} \equiv I_s = \frac{2\sqrt{3}}{\pi} I_d
\]

\[
i_{sq} \equiv 0
\] (6.2)

Ignoring the converter losses, the active power balance yields the following relation for the dc voltage \(V_{d1}\):

\[
\frac{2}{3} V_{d1} I_d = u_{sd} i_{sd} + u_{sq} i_{sq} \Rightarrow V_{d1} = \frac{3\sqrt{3}}{\pi} u_{sd}
\] (6.3)

The output converter dc voltage, \(V_{d2}\), is given by the well known relation:

\[
V_{d2} = -\frac{3\sqrt{3}}{\pi} V_b \cos \alpha
\] (6.4)

where \(V_b\) is the bus voltage magnitude and \(\alpha\) the inverter firing angle. The negative sign is due to the selection of the \(V_{d2}\) reference polarity. If a cosine firing angle controller is used for the inverter, then the above equation is written as:

\[
V_{d2} = -\frac{3\sqrt{3}}{\pi} V_c
\] (6.5)

where \(V_c\) is the reference voltage of the cosine controller, given in p.u. by:

\[
V_c = V_b \cos \alpha
\] (6.6)

For the fundamental output current \(I_b\), a relation similar to (6.1) holds:

\[
I_b = \frac{2\sqrt{3}}{\pi} I_d
\] (6.7)

The dc link RL filter differential equation is the following

\[
V_{d1} - V_{d2} = R_d I_d + L_d \frac{dI_d}{dt}
\] (6.8)

where \(R_d\) and \(L_d\) are the choke resistance and inductance and all quantities are in absolute values (\(V\), \(A\), \(Ω\) and \(H\)). Expressing (6.8) in p.u., eliminating \(V_{d1}\) and \(V_{d2}\) by (6.3) and (6.5), substituting \(i_{sd}\) for the dc current \(I_d\) from (6.2), it is derived:

\[
u_{sd} + V_c = R_d' i_{sd} + X_d' \frac{di_{sd}}{\omega_0 dt}
\] (6.9)

where the \(R_d'\) and \(X_d'\) are the dc filter resistance and reactance, referred to the stator of the induction machine:

\[
R_d' = \frac{\pi^2}{18} \frac{R_d (Ω)}{Z_B}
\]

\[
X_d' = \frac{\pi^2}{18} \frac{\omega_0 L_d (H)}{Z_B}
\] (6.10)

where \(Z_B\) is the base resistance and \(\omega_0\) the base electrical angular velocity.
The induction machine is simulated using the 4th order dq model of Section 4. Expressing its equations (4.3) in the stator current reference frame, where $i_{sq}=0$, the state equations are reduced to three, since $i_{sq}$ is no longer a state variable and the respective equation is redundant. Eliminating subsequently $u_{sd}$ from the other three equations, using (6.9), the 3rd order model of the system is obtained, with the currents $i_{sd}$, $i_{rd}$ and $i_{rq}$ as state variables:

$$
\frac{di_{sd}}{dt} = \begin{bmatrix}
\frac{RX_{r}}{D_1} & \frac{r_{r}X_{m}}{D_1} & \frac{\omega_{r}X_{m}X_{r}}{D_1} \\
\frac{R_{r}X_{m}}{D_1} & -\omega_{r} - \frac{r_{r}X_{r}}{D_1} & -\frac{r_{r}X_{m}X_{r}}{D_1} \\
(\omega_{r} - \omega_{s}) & -\frac{X_{r}}{X_{r}} & -(\omega_{r} - \omega_{s})
\end{bmatrix} \begin{bmatrix}
i_{sd} \\
i_{rd} \\
i_{rq}
\end{bmatrix} + i_{rd} \begin{bmatrix}
\frac{X_{r}}{D_1} \\
\frac{X_{r}}{D_1} \\
0
\end{bmatrix}
$$

(6.11)

where $D_1 = X_1X_r - X_m^2$, $R_1 = r_s + R_d'$ and $X_1 = X_s + X'_d$.

The stator voltages $u_{sd}$ and $u_{sq}$ are respectively calculated from (6.9) and the second of (4.3), using (6.11) for the elimination of the derivative terms:

$$
u_{sd} = \left(\frac{R_d' - R_s X_r X'_d}{D_1}\right)i_{sd} - \frac{r_s X_m X'_d}{D_1}i_{rd} - \omega_r X_m X'_d i_{rd} + \left(\frac{X_r X'_d}{D_1} - 1\right)V_c
$$

(6.12)

$$
u_{sq} = -\frac{1}{X_r} \left(D \omega_s - X_m^2 \omega_r\right) i_{sd} + \omega_r X_m i_{rd} - r_r \frac{X_m}{X_r} i_{rq}
$$

(6.13)

The generator electromagnetic torque is given by (4.4), substituting $i_{sq}=0$:

$$T_e = -X_m i_{sd} i_{rq}
$$

(6.14)

The control system of the current source converters scheme is illustrated in figure 17, in the next page. It is essentially a V/f control scheme, similar to the scalar controller of the voltage source converter, shown in figure 10. The stator voltage of the induction generator is determined by the stator frequency and the V/f control characteristic. The inner current and voltage regulation loops stabilize the system (which presents static and dynamic instability in open loop) and προσδίδουν to the generator “voltage-fed” characteristics. The stator frequency $\omega_s$ is determined by the output $\omega_{sl}$ of the speed controller (the “slip-frequency”), to which the rotor speed $\omega_r$ or the respective reference $\omega_{r,ref}$ is added. Using the speed feedback $\omega_r$, the slip control principle is implemented, which is much more suitable and presents significantly improved dynamic characteristics than using the speed reference $\omega_{r,ref}$. In the latter case, the stator frequency is essentially determined directly by the output of the speed controller, since the $\omega_{r,ref}$ input is an externally defined and slowly varying quantity. The torque stabilizer is similar in principle to the equivalent block of the voltage source converter scalar controller, shown in figure 10, and accepts as input an estimation of the electromagnetic torque:

$$\hat{T}_e = \frac{2V_d I_d}{3\omega_r}
$$

(6.15)

The speed reference, $\omega_{r,ref}$, is found from the measured wind speed, as discussed in Section 5.2.3. 
Figure 17. Control system of the current source converter scheme.

7. Double Output Induction Generator Scheme

The simplified diagram of the double output induction generator scheme is shown in figure 18. The dq model of the wound rotor induction machine is given by the same equations, presented in Section 4, where the rotor voltages $u_{rd}$ and $u_{rq}$ are now non-zero. Hence, equations (4.3), expressed in the arbitrary $dq$ reference frame, now become:

$$\begin{align*}
\frac{di_{sd}}{dt} &= \frac{\alpha_0}{D} \left[ -r_r X_r (aD+\omega_r X_m^2) -r_r X_m \omega_r X_m \right] + \frac{\alpha_0}{D} \left[ -X_r 0 X_m 0 \right] \left[ i_{rd} \right] \\
\frac{di_{sq}}{dt} &= \frac{\alpha_0}{D} \left[ -aD \omega_r X_m \right] -r_r X_r \omega_r X_m -r_r X_m \left( aD-\omega_r X_r \right) \left[ i_{rd} \right] \\
\frac{di_{rd}}{dt} &= \frac{\alpha_0}{D} \left[ -r_r X_m \omega_r X_m \right] -r_r X_m \left( aD-\omega_r X_r \right) -r_r X_r \left[ i_{rd} \right] \\
\frac{di_{rq}}{dt} &= \frac{\alpha_0}{D} \left[ -r_r X_m \omega_r X_m \right] -r_r X_m \left( aD-\omega_r X_r \right) -r_r X_r \left[ i_{rd} \right]
\end{align*}$$

(7.1)
where $D = X_s X_r - X_m^2$ and $\omega_0$ the base electrical angular velocity.

The inputs $u_{sd}$ and $u_{sq}$ of the model are directly available from the known stator voltage. The voltages $u_{rd}$ and $u_{rq}$ of the rotor, on the other hand, will be calculated from the converters and dc filter equations. The following relations hold for the diode rectifier and the thyristor inverter in absolute values (V and A):

\[
U_{d1} = \frac{3\sqrt{3}}{\pi} U_r
\]

\[
I_r = \frac{2\sqrt{3}}{\pi} I_d
\]

\[
U_{d2} = -\frac{3\sqrt{3}}{\pi} U_i \cos \alpha = -\frac{1}{n_T} \cdot \frac{3\sqrt{3}}{\pi} U_i \cos \alpha
\]

\[
I_i = \frac{2\sqrt{3}}{\pi} I_d
\]

where $U_{d1}, U_{d2}$ are the rectifier and inverter dc voltages (average values)

$I_d$ the dc current (average value)

$U_r, I_r$ the fundamental rotor voltage and current (peak phase values)

$U_i, I_i$ the fundamental inverter output voltage and current (peak phase values)

$\alpha$ the inverter firing angle

$U_i$ the bus voltage (peak phase values) and

$n_T = U_s/U_i$ the rotor transformer ratio

The dc link RL filter differential equation is the following

\[
U_{d1} - U_{d2} = R_d I_d + L_d \frac{dI_d}{dt}
\]

where $R_d$ and $L_d$ are the choke resistance and inductance and all quantities are in absolute values (V, A, Ω και Η). Expressing (7.6) in p.u., eliminating $U_{d1}$ and $U_{d2}$ by (7.2) and (7.4) and substituting $I_r$ for the dc current $I_d$ from (7.3), it is derived:

\[
U_r + U_c = R'_d I_r + X'_d \frac{dI_r}{\omega_0 dt}
\]

where $U_c$ is the p.u. control voltage of the inverter cosine firing angle controller

\[
U_c = \frac{n_u}{n_T} U_s \cos \alpha
\]

where the $R'_d$ and $X'_d$ are the dc filter resistance and reactance, referred to the stator of the induction machine:

\[
R'_d = n_M^2 \frac{\pi^2}{18} R_d(\Omega), \quad X'_d = n_M^2 \frac{\pi^2}{18} \frac{\omega_0 L_d(\Omega)}{Z_B^s}
\]

$Z_B^s$ is the stator base resistance and $n_M$ the equivalent stator/rotor turn ratio of the induction machine.

The rotor voltage and current, $U_r$ and $I_r$, are related with the respective $d$ and $q$ components by:
Ignoring the harmonics and the commutation phenomena of the diode rectifier, its reactive power consumption is zero and therefore the rotor voltage and current are displaced by 180\(^{0}\) as shown in figure 19 (the rotor current conventionally enters the rotor terminals). Hence:

\[
\begin{align*}
\bar{U}_r &= \frac{\bar{I}_r}{I_r} U_r \Leftrightarrow \left(u_{rd} = \frac{i_{rd}}{I_r} U_r, u_{rq} = \frac{i_{rq}}{I_r} U_r \right) \\
(7.12)
\end{align*}
\]

Derivating (7.11) it is obtained:

\[
\frac{dI_r}{dt} = \frac{i_{rd} (di_{rd}/dt) + i_{rq} (di_{rq}/dt)}{I_r} = \frac{\text{Re}\{\frac{d\bar{I}_r}{dt}\bar{I}_r^*\}}{I_r} \\
(7.13)
\]

where the complex representation \(\bar{F}\) of a \(dq\) quantity \(f\) (voltage, current or flux) and its derivatives is defined as

\[
\bar{F} = f_d + j f_q \quad \text{and} \quad \frac{d\bar{F}}{dt} = \frac{df_d}{dt} + j \frac{df_q}{dt} \\
(7.14)
\]

and its complex conjugate is denoted by “\(^{*}\)”. Substituting in (7.13) the derivatives of the rotor currents from (7.1), it is obtained:

\[
\frac{dI_r}{dt} = -\frac{\omega_d}{D_{Ir}} \left[X_m P_1 + r_x X_s I_r^2 - X_s (u_{rd} i_{rd} + u_{rq} i_{rq}) \right] = -\frac{\omega_d}{D_{Ir}} \left(X_m P_1 + r_x X_s I_r^2 - X_s U_r I_r \right) \\
(7.15)
\]

where the quantity \(P_1\) is given by

\[
P_1 = \text{Re}\{[\bar{U}_r + (r_x + j \omega_x X_s) \bar{I}_r] \bar{I}_r^*}\} \\
(7.16)
\]

Combining (7.15) with (7.7) and solving for the rotor voltage \(U_r\), the following relation is derived

\[
U_r = \frac{(r_x X_s X_d' - DR_s') I_r + DU_r + X_m X_s' P_1 / I_r}{D + X_s X_d'} \\
(7.17)
\]

The generator electromagnetic torque is given by the same relation (4.4), as for the conventional induction machine (in p.u. and using generator convention):

\[
T_e = \Psi_{rd} i_{rq} - \Psi_{rq} i_{rd} = X_m (i_{rd} i_{rd} - i_{rd} i_{rd}) \\
(7.18)
\]

The state equations (7.1), along with the algebraic relations (7.10), (7.11), (7.12), (7.16), (7.17) and (7.18), constitute the 4\(^{th}\) order \(dq\) dynamic model of the system, expressed in the arbitrary reference frame.
The block diagram of the DOIG control system is shown in figure 20. It comprises an inner PI current control loop, which acts essentially as a torque controller, due to the direct proportion of the electromagnetic torque to the dc current. The external loop is the speed controller, which may be absent if a torque control scheme is implemented, as discussed in Section 5.2.3. The gain of the PI speed controller is variable, depending on its output. The sign of the speed error, input of the PI speed controller, depends on whether the system operates in motor or generator mode, i.e. in sub- or super-synchronous speeds. A derivative torque stabilizer block is also implemented, which increases the damping of low and middle frequency torque oscillations.

Figure 20. Control system block diagram of the double output induction generator scheme.
APPENDIX
The per unit system

Mechanical System

If $S_B$ is the base power (VA), $\omega_0$ the base electrical angular velocity (rad/sec) and $P$ the number of poles of the generator, then the base values at the high speed side (generator-side) of the drive train are defined as follows:

$$
\omega_B = \frac{\omega_0}{P/2}
$$

the base mechanical speed, in mechanical rad/sec

$$
T_B = \frac{S_B}{\omega_B}
$$

the base torque, in Nm

$$
\theta_B = \omega_B
$$

the base mechanical angle, in mechanical rad/sec

$$
J_B = \frac{S_B}{0.5\omega_B^2} = \frac{T_B}{0.5\omega_B}
$$

the base inertia, in Nm/(rad/sec)

$$
C_B = \frac{T_B}{\omega_B} = \frac{S_B}{\omega_B^2}
$$

the base stiffness coefficient, in Nm/(rad/sec)

$$
D_B = d_B = \frac{T_B}{\omega_B} = \frac{S_B}{\omega_B^2}
$$

the base damping coefficient, in Nm/(rad/sec)

The low speed side (rotor-side) base quantities are calculated from the above quantities using the gearbox ratio $n$ as follows:

$$
\omega_B' = n \omega_B'' \\
J_B' = n^2 J_B'' \\
\theta_B' = n \theta_B'' \\
D_B' = n^2 D_B'' \\
C_B' = n^2 C_B''
$$

where primed and double-primed respectively are the low and high speed side base quantities.

ABC Base Values

The stator abc rms base current is found from the base power $S_B$ and the stator abc base voltage $V_{Babc}^s$ (rms phase value) as

$$
I_{Babc}^s = \frac{S_B}{3V_{Babc}^s}
$$

The instantaneous base values are always $\sqrt{2}$ times the respective rms ones. The base resistance is defined as

$$
Z_{Babc}^s = \frac{V_{Babc}^s}{I_{Babc}^s} = \frac{3(V_{Babc}^s)^2}{S_B}
$$

The rotor winding base values for the slip-ring induction machine are defined using the respective stator quantities and the effective stator-rotor turns ratio $n_M$:

$$
V_{Babc}^r = \frac{V_{Babc}^s}{n_M} \\
I_{Babc}^r = n_M I_{Babc}^s
$$

wherefrom the base resistance is given by
For the double output inverter generator, the base quantities at the \textit{ac} side of the output inverter (secondary side of the recovery transformer) are defined using the stator quantities and the recovery transformer turns ratio, \( n_T \):

\[
V_{\text{Babc}}^{r} = \frac{V_{\text{Babc}}^{s}}{n_T}, \quad I_{\text{Babc}}^{r} = n_T I_{\text{Babc}}^{s},
\]

\[
Z_{\text{Babc}}^{r} = \frac{Z_{\text{Babc}}^{s}}{n_T}, \quad Z_{\text{Babc}}^{i} = \frac{Z_{\text{Babc}}^{i}}{n_T}.
\]

**DQ Base Values**

The \( dq \) system base voltage and current are taken equal to the respective \( abc \) instantaneous base values:

\[
V_{\text{Bdq}} = \sqrt{2}V_{\text{Babc}}, \quad I_{\text{Bdq}} = \sqrt{2}I_{\text{Babc}}
\]

Using these definitions, the base power \( S_B \) is given by

\[
S_B = \frac{3}{2} V_{\text{Bdq}} I_{\text{Bdq}}
\]

whereas the base resistance is equal to the respective \( abc \) value

\[
Z_{\text{Bdq}} = \frac{V_{\text{Bdq}}}{I_{\text{Bdq}}} = Z_{\text{Babc}}
\]

**DC Base Values**

The \( dc \) side base values are taken equal to the \( abc \) instantaneous base values of the winding where the converter is connected. Hence, for a stator converter cascade it holds:

\[
V_{\text{Bdc}} = \sqrt{2}V_{\text{Babc}}^{s}, \quad I_{\text{Bdc}} = \sqrt{2}I_{\text{Babc}}^{s},
\]

\[
Z_{\text{Bdc}} = \frac{V_{\text{Bdc}}}{I_{\text{Bdc}}} = Z_{\text{Babc}}^{s}
\]

while for the rotor converter cascade:

\[
V_{\text{Bdc}} = \sqrt{2}V_{\text{Babc}}^{r} = \sqrt{2} \frac{V_{\text{Babc}}^{s}}{n_M},
\]

\[
I_{\text{Bdc}} = \sqrt{2}I_{\text{Babc}}^{r} = \sqrt{2}n_M I_{\text{Babc}}^{s},
\]

\[
Z_{\text{Bdc}} = Z_{\text{Babc}}^{r} = \frac{1}{n_M} Z_{\text{Babc}}^{s}
\]

The base power is then given by

\[
S_B = \frac{3}{2} V_{\text{Bdc}} I_{\text{Bdc}}
\]

Hence, the \( dc \) side active power is given in p.u. by a relation of the following type:

\[
P = \frac{2}{3} V_{\text{dc}} I_{\text{dc}} \quad \text{p.u.}
\]