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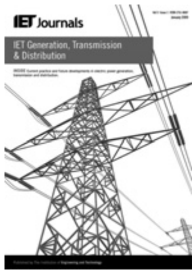

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Optimal distributed generation placement under uncertainties based on point estimate method embedded genetic algorithm

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Abstract: The scope of this study is the optimal siting and sizing of distributed generation within a power distribution network considering uncertainties. A probabilistic power flow (PPF)-embedded genetic algorithm (GA)-based approach is proposed in order to solve the optimisation problem that is modelled mathematically under a chance constrained programming framework. Point estimate method (PEM) is proposed for the solution of the involved PPF problem. The uncertainties considered include: (i) the future load growth in the power distribution system, (ii) the wind generation, (iii) the output power of photovoltaics, (iv) the fuel costs and (v) the electricity prices. Based on some candidate schemes of different distributed generation types and sizes, placed on specific candidate buses of the network, GA is applied in order to find the optimal plan. The proposed GA with embedded PEM (GA–PEM) is applied on the IEEE 33-bus network by considering several scenarios and is compared with the method of GA with embedded Monte Carlo simulation (GA–MCS). The main conclusions of this comparison are: (i) the proposed GA–PEM is seven times faster than GA–MCS, and (ii) both methods provide almost identical results.

1 Introduction

Distributed generation (DG) technologies have become more and more important in power systems [1]. Technologies that are classified as DG are categorised into renewable and fossil fuel-based sources. Renewable energy sources (RES) comprise of wind turbines, photovoltaics, biomass, geothermal, small hydro and so on. Fueled DGs are internal combustion engines, combustion turbines and fuel cells. Environmental, economic and technical factors have played an important role in DG development [2, 3]. In accordance with the Kyoto agreement on climate change, many efforts to reduce carbon emissions have taken place, and as a result the penetration of DGs in distribution systems rises [4].

DG placement significantly affects distribution network operation. Inappropriate DG placement may increase system capital and operating costs and network losses. On the other hand, optimal DG placement (ODGP) helps keep the voltage profile within the specified limits, can reduce power flows and network losses and can improve power quality and reliability of supply. The aim of the ODGP is to provide the best locations (buses) and sizes of DGs to optimise distribution network operation and planning taking into account the network operating constraints, DG operation constraints and investment constraints. The ODGP is a complex mixed integer non-linear optimisation problem, which has attracted the interest of many research efforts in the last 15 years [1].

An ordinal optimisation method is proposed in [4] for solving the ODGP. Mixed integer non-linear programming solves an ODGP model in hybrid electricity markets [5]. The optimal location of DG is determined by a sensitivity test and the optimal size of the DG is computed by a heuristic curve-fitted technique [6]. A fuzzy genetic algorithm (GA) solves a weighted multiobjective ODGP model that maximises the system loading margin and the profit of the distribution network operator [7]. Particle swarm optimisation is applied to solve an ODGP model by considering variable power load models [8].

ODGP becomes more complex considering some uncertainties that are involved, such as future load growth and the generation of non-dispatchable RES [1]. A variant of a non-dominated sorting GA in conjunction with a max–min approach solves a multiobjective ODGP that considers the uncertainties by using fuzzy numbers [9]. GA and decision theory are applied to solve an ODGP problem under uncertainty including power quality issues [10]. ODGP models with uncertainties are solved by GA in conjunction with Monte Carlo simulation (GA–MCS) in [11, 12]. An ODGP model considering the uncertainties and DG reactive capability is developed in [13]. ODGP models considering load uncertainty are solved by cuckoo optimisation algorithm and artificial neural network in [14, 15], respectively. A systematic qualitative assessment of the state of the art models and methods applied to the ODGP problem in power distribution networks together with the

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contribution of all of the reviewed ODGP works can be found in [1].

The solution of the power flow problem helps evaluate the state of the power system for a specific set of values of the input variables (generations and loads for a given network topology). In case of uncertainties in the input variables of the power system, it is desirable to assess the system output variables (bus voltages and line flows) for many load and generation conditions. It is necessary to run many times the deterministic power flow routine in order to evaluate possible system states. Many methods have been proposed for estimating the state of the power systems considering uncertainties. The most accurate method is Monte Carlo simulation (MCS), which is commonly used as benchmark method [16]. This paper proposes the point estimate method (PEM) [17] for solving the probabilistic power flow (PPF) that is involved in the ODGP under uncertainties.

This paper introduces a new technique for solving the ODGP under uncertainties, formulated as a chance constrained programming (CCP) optimisation problem, which is a type of stochastic programming. The new algorithm (GA–PEM) combines the GA and the PEM. The PEM is embedded in the GA-based developed model for evaluating each chromosome and handling the chance constraints. MCS-embedded GA (GA–MCS) has been introduced in [11, 12] for the solution of ODGP. This paper proposes the use of PEM instead of MCS, because PEM is much faster than MCS in solving each one of the many PPF problems that are required by the GA to solve the ODGP. Thus, the proposed GA–PEM method solves the ODGP problem much faster than the GA–MCS method.

The paper is organised as follows: modelling of the uncertainties that affect power flow and the state of the distribution system is given in Section 2. The PEM for PPF calculation is outlined in Section 3. In Section 4, the ODGP under uncertainties is formulated by using the mathematical model of chance constrained programming. The proposed GA–PEM method for solving the ODGP is described in Section 5. In Section 6, the proposed method is applied for solving the ODGP problem of the IEEE 33-bus distribution network and the obtained results verify the effectiveness and the validity of the proposed method. Conclusions are drawn in Section 7.

2 Modelling of the uncertainties

2.1 Output power of wind turbines

Many experiments have demonstrated that a good expression for modelling the stochastic behaviour of wind speed is the Weibull probability density function (PDF). The PDF of wind speed is given by Atwa *et al.* [18]

$$f(v) = \frac{k}{c^k} v^{k-1} \exp\left(-\left(\frac{v}{c}\right)^k\right), \quad 0 \le v < \infty \qquad (1)$$

where v is the wind speed that follows the Weibull distribution, and k and c are the shape and the scale index, respectively, of the Weibull distribution. Assuming that the wind speed PDF is known, the output power of a wind turbine can be computed as follows [18]

$$P_{\rm WT} = \begin{cases} 0, & \text{if} \quad 0 \le v \le v_{\rm ci} \\ P_{\rm WT\_n} \dfrac{(v - v_{\rm ci})}{(v_{\rm n} - v_{\rm ci})}, & \text{if} \quad v_{\rm ci} \le v \le v_{\rm n} \\ P_{\rm WT\_n}, & \text{if} \quad v_{\rm n} \le v \le v_{\rm co} \\ 0, & \text{if} \quad v_{\rm co} < v \end{cases} \qquad (2)$$

where v is the wind speed, $v_{\rm ci}$ is the cut-in wind speed, $v_{\rm co}$ is the cut-out wind speed, $v_{\rm n}$ is the nominal wind turbine speed and $P_{\rm WT\_n}$ is the nominal output power of the wind turbine.

2.2 Output power of photovoltaics

On using the historical and meteorological data for each region, it has been observed that the solar illumination intensity approximately follows the Weibull distribution [12], hence its PDF is given by

$$f(s) = \frac{k_{\rm s}}{c_{\rm s}^{k_{\rm s}}} s^{(k_{\rm s}-1)} \exp\left(-\left(s/c_{\rm s}\right)^{k_{\rm s}}\right), \quad 0 \le s < \infty \qquad (3)$$

where s is the solar illumination intensity, and k_s and c_s are the shape and the scale index, respectively, of the Weibull distribution of s.

The relationship between the output power of a photovoltaic and the illumination intensity is [12]

$$P_s = \begin{cases} P_{s\_{\rm n}} \dfrac{s}{s_{\rm n}}, & 0 \le s \le s_{\rm n} \\ P_{s\_{\rm n}}, & s_{\rm n} \le s \end{cases} \qquad (4)$$

where s is the illumination intensity, $s_{\rm n}$ is the nominal illumination intensity of the photovoltaic panel and $P_{s\_{\rm n}}$ is the nominal output power of the photovoltaic panel.

2.3 Uncertain load growth

Owing to the sustainable development of technology and industry, electricity demand has increased. By using statistical studies and the historical data, it has been found that the load growth of bus i at year t, $\Delta P_{Li}(t)$, follows the normal distribution with mean $\mu_i(t)$ and standard deviation $\sigma_i(t)$ [12].

2.4 Uncertain fuel prices

The operating cost of fueled DGs mainly consists of fuel price cost. The fuel price is dependent on the laws of supply and demand of fuel, affected by numerous unforeseen geopolitical factors such as weather, political and military crises, availability of refining units, subsidies or taxation and therefore it cannot be predicted accurately. Generally, it has been observed that the price of fuel tends to follow the Geometric Brownian Motion (GBM) described by the following formula [19]

$$p_f(t) = p_f(t-1) \exp\left[\left(\mu_f - \frac{1}{2}\sigma_f^2\right)t + \sigma_f W(t)\right] \qquad (5)$$

where $p_f(t)$ is the price in year t; $p_f(t-1)$ is the price in year $t-1$; μ_f and σ_f are the mean value and standard deviation of price in year t; and the variable $W(t)$ is the Brownian motion and $W(t) \sim N(0, t)$.

Hence, the notation $p_f(t) \sim {\rm GBM}(p_f(t-1), \mu_f, \sigma_f)$ means that the variable $p_f(t)$ follows the GBM in year t.

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2.5 Uncertain electricity prices

It is supposed that electricity prices, such as the on-grid price C^L, also follows the GBM in year t, which means that $C^L(t) \sim \mathrm{GBM}(C^L(t-1), \mu_L, \sigma_L)$ [18].

3 PEM for solving the PPF problem

The PEM is applied in order to calculate the statistical moments of a random variable that is a function of several random variables. It was first developed by Rosenblueth in 1975 [20] and since then, many methods that improve the original Rosenblueth's method have been presented. The PPF model efficiently assesses the uncertainties the stochastic variables involve in the power flow calculation. Hong's PEM [21] is adopted in this paper for the solution of the PPF problem.

Let us assume that the function F is the set of non-linear power flow equations that relate the input and the output variables; $\boldsymbol{Z}$ is the vector of stochastic output variables and p_i is the ith random input variable; then, the set $\boldsymbol{Z}$ of random output variables can be expressed as follows

$$\boldsymbol{Z}(l, k) = F\left(p_1, p_2, \ldots, p_l, \ldots, p_m\right) \tag{6}$$

PEM concentrates all the statistical information provided by the first central moments of the stochastic input variables and computes K points for each variable, named concentrations. The kth concentration $(p_{l,k}, w_{l,k})$ of a random variable p_l can be defined as a pair of a location $p_{l,k}$ and a weight $w_{l,k}$. The location is the kth value of the variable p_l at which function F is evaluated and the weight $w_{l,k}$ is a weighting factor that accounts for the relative importance of this evaluation in the random output variable [17].

By using Hong's PEM, the function F has to be evaluated only K times for each random variable p_l by maintaining the mean value μ of all the other random variables $m-1$, that is, if $\boldsymbol{Z}(l, k)$ is the set of random output variables of the lth variable for the kth concentration, then $\boldsymbol{Z}(l, k)$ is computed as follows: $\boldsymbol{Z}(l, k) = F(\mu_{p1}, \mu_{p2}, \ldots, p_{l,k}, \ldots, \mu_{pm})$. The total number of simulations depends on the number of points K that will be selected, and the number of random input variables m of the power system. Therefore the total amount of power flow computations is equal to $k \times m$. In this paper, 2PEM $(2m+1)$ is used with $2m+1$ simulations that give very accurate results running only for several times and it is used for solving the PPF [17].

The location $p_{l,k}$ is given by

$$p_{l,k} = \mu_{p_l} + \xi_{l,k}\sigma_{l,k} \tag{7}$$

where μ_{p_l} is the mean value of variable p_l, $\sigma_{l,k}$ is the standard deviation of variable p_l and $\xi_{l,k}$ is the standard location.

The standard location $\xi_{l,k}$ and the weight $w_{l,k}$ are calculated by solving the non-linear system of the following equations

$$\sum_{k=1}^{K} w_{l,k} = \frac{1}{m} \tag{8a}$$

$$\sum_{k=1}^{K} w_{l,k}\left(\xi_{l,k}\right)^j = \lambda_{l,j} \tag{8b}$$

where $\lambda_{l,j}$ is the jth standard central moment of p_l random variable, given by the following formulae

$$\lambda_{l,k} = \frac{M_j(p_l)}{\sigma_{p_l}^j} \tag{9}$$

$$M_j(p_l) = \int_{-\infty}^{+\infty} \left(p_l - \mu_{p_l}\right)^j f_{p_l} dp_l \tag{10}$$

Thus, by considering the scheme of 2PEM with the $2m+1$ simulations $(K=3)$ the standard location $\xi_{l,k}$ and the weight $w_{l,k}$ are computed by (11) and (12), respectively

$$\xi_{l,k} = \frac{\lambda_{l,3}}{2} + (-1)^{3-k}\sqrt{\lambda_{l,4} - \frac{3}{4}\left(\lambda_{l,3}\right)^2}, \quad \text{for} \quad k = 1, 2$$
$$\xi_{l,3} = 0 \tag{11}$$

$$w_{l,k} = \frac{(-1)^{3-k}}{\xi_{l,k}\left(\xi_{l,1} - \xi_{l,2}\right)}, \quad \text{for} \quad k = 1, 2$$
$$w_{l,3} = \frac{1}{m} - \frac{1}{\lambda_{l,4} - \left(\lambda_{l,3}\right)^2} \tag{12}$$

More specifically, using as data the probability distribution of the random variables that are input to the power system, first, the locations and the weights are computed and next a deterministic load flow is executed for every point-concentration as follows

$$\boldsymbol{Z}(l, k) = F\left(\mu_{p1}, \mu_{p2}, \ldots, p_{l,k}, \ldots, \mu_{pm}\right) \tag{13}$$

where $\boldsymbol{Z}(l, k)$ is the set of random output variables of concentration k of variable p_l. The output variable $\boldsymbol{Z}(l, k)$ refers to: (i) the active power flow (P_{ij}) and reactive power flow (Q_{ij}) of the branch $i-j$ of the network, (ii) the voltage magnitude (V) and the voltage angle (δ) of the buses, (iii) the total power losses (P_{loss}) and (iv) the active power injections (P_i) and reactive power injections (Q_i). $F(.)$ stands for the set of non-linear equations of deterministic power flow that relate the input variables with the output variables.

The vector $\boldsymbol{Z}(l, k)$ is used to evaluate the first j moments of the random output variables of the power system as follows

$$E(Z) = \sum_{k=1}^{K}\sum_{l=1}^{m} w_{l,k} Z(l, k) \tag{14}$$

$$E\left(Z^j\right) = \sum_{k=1}^{K}\sum_{l=1}^{m} w_{l,k} (Z(l, k))^j \tag{15}$$

where $E(Z)$ is the expected value and $E(Z^j)$ is the jth moment of output of the random output variable Z, respectively. For $j=2$, the standard deviation of Z is evaluated.

Therefore the algorithm for solving PPF using Hong's PEM is shown in Fig. 1.

4 Formulation of the ODGP problem under uncertainties

The design variables (unknowns) of the ODGP problem are the following: (i) the buses at which the DGs will be installed, (ii) the installed capacity of each DG unit and (iii) the type of each DG (fueled DG, microturbine, wind turbine, photovoltaic, biomass unit etc.) to be installed.

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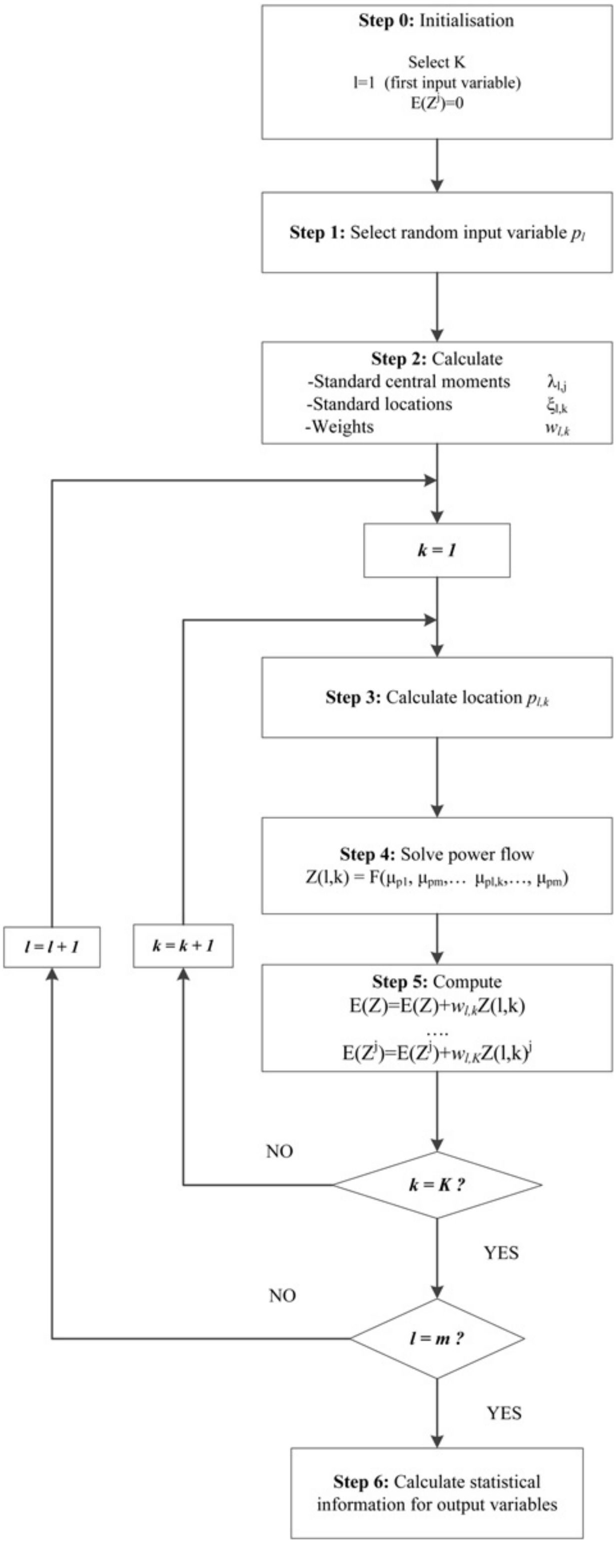

Fig. 1 *Flowchart of PEM for the PPF problem*

The placement of the DG units, and especially the RES placement, is affected by many factors such as wind speed, solar irradiation, environmental factors, geographical topography, political factors and so on. For example, wind turbines cannot be installed near residential areas, either because of the reactions of the residents, either because of legislation or even because of interference from environmental organisations.

The type of the DG units to be installed depends directly on both the installation costs and the operating costs. Owing to rising fuel prices, fueled DGs, despite the low investment costs, become more expensive to operate, unlike RES, which have higher investment costs but virtually no operating costs.

In this work, the types of DG to be studied are: (i) wind turbines, (ii) photovoltaics and (iii) fueled DGs. The uncertainties that affect the state of the distribution network have been modelled in Section 2. The ODGP will be solved for the case of the peak load. Given the complexity of the problem, some scenarios will be used, through which the optimal solution will be selected.

4.1 Objective function

In ODGP, the main purpose is to minimise or maximise an objective function, choosing the suitable one depending on the problem [1]. In this paper, costs associated with the installation of DGs in a distribution system are the investment cost, operating cost, maintenance cost, capacity adequacy cost and network loss cost and thus the objective function is the minimisation of the total costs and is described by the following formula in compact form [12]

$$\min f = b_1 C^{\mathrm{I}} + b_2 C^{\mathrm{M}} + b_3 C^{\mathrm{O}} + b_4 C^{\mathrm{Lt}} + b_5 C^{\mathrm{A}} \qquad (16)$$

or equivalently by the following formula in detailed form [12]

$$\begin{aligned}\min f = {} & b_1 \sum_{k=1}^{N_{\mathrm{type}}} \sum_{i \in N_{\mathrm{DG}k}} \left(C^{\mathrm{I}}_{\mathrm{DG}k} P^{N}_{\mathrm{DG}ki}\right) \\ & + b_2 \sum_{k=1}^{N_{\mathrm{type}}} \sum_{i \in N_{\mathrm{DG}k}} \left(C^{\mathrm{M}}_{\mathrm{DG}k} T_{\mathrm{DG}ki} P^{N}_{\mathrm{DG}ki}\right) \\ & + b_3 \sum_{k=1}^{N_{\mathrm{type}}} \sum_{i \in N_{\mathrm{DG}k}} \left(C^{\mathrm{O}}_{\mathrm{DG}k} T_{\mathrm{DG}ki} P^{N}_{\mathrm{DG}ki}\right) + b_4 C^{L} W_{\mathrm{loss}} \\ & + b_5 C^{L} \sum_{k=1}^{N_{\mathrm{type}}} \sum_{i \in N_{\mathrm{DG}k}} T_{\mathrm{DG}ki}\left(P^{N}_{\mathrm{DG}ki} - P_{\mathrm{DG}ki}\right)\end{aligned} \qquad (17)$$

where $b_1 + b_2 + b_3 + b_4 + b_5 = 1$; b_1, b_2, b_3, b_4 and b_5 are the weighting coefficients; C^{I}, C^{M}, C^{O} and C^{A} are the costs ($) for DGs investment, maintenance, operation and the capacity adequacy cost ($), respectively; C^{Lt} is the loss cost ($) of the distribution network; C^{L} is the electricity price ($/kWh); W_{loss} is the energy loss (kWh) of the distribution network; $C^{\mathrm{I}}_{\mathrm{DG}k}$, $C^{\mathrm{M}}_{\mathrm{DG}k}$ and $C^{\mathrm{O}}_{\mathrm{DG}k}$ are the per-unit investment, maintenance and operation cost, respectively, of the kth type of DG; $P^{\mathrm{N}}_{\mathrm{DG}ki}$ is the installed capacity of the kth type of DG at bus i; $P_{\mathrm{DG}ki}$ is the active power output of the kth type of DG at bus i; N_{type} is the number of different DG types; $N_{\mathrm{DG}k}$ is the set of candidate buses for installing DG of type k; and $T_{\mathrm{DG}ki}$ is the equivalent generation hours of the kth type of DG at bus i.

4.2 Constraints modelling

4.2.1 Deterministic equality constraints: The power flow equations (18) and (19) are used for computing the output variables of the distribution system, such as the power flow of each branch, the voltage magnitude and angle per bus, the total power losses and so on. The

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Newton–Raphson method is applied to solve the power flow problem for each state of random input variables

$$P_{\mathrm{DG}i} - P_{Li} - V_i^2 G_{kk} - V_i \sum_{k \in A(i)} V_k \left(G_{ik} \cos \delta_{ik} + B_{ik} \sin \delta_{ik} \right) = 0 \tag{18}$$

$$Q_{\mathrm{DG}i} - Q_{Li} + V_i^2 B_{kk} - V_i \sum_{k \in A(i)} V_k \left(G_{ik} \sin \delta_{ik} - B_{ik} \sin \delta_{ik} \right) = 0 \tag{19}$$

where $P_{\mathrm{DG}i}$ and $Q_{\mathrm{DG}i}$ are the real and reactive power produced at bus i, P_{Li} and Q_{Li} are the real and reactive power consumed at bus i, V_i is the voltage magnitude at bus i, δ_{ik} is the voltage angle between bus i and bus k, $Y_{ik} = G_{ik} + jB_{ik}$ is the element of the bus admittance matrix that refers to the line between buses i and k and $A(i)$ corresponds to the set of buses connected to bus i.

4.2.2 Deterministic inequality constraints: Deterministic inequality constraints are strict and cannot be violated. They have direct relationship with technical specifications of the power system and are commonly formed by the network designers and engineers for the best possible quality of voltage and power supplied. These include the upper limit of real and reactive output power produced by DG units ($P_{\mathrm{DG}i\max}$, $Q_{\mathrm{DG}i\max}$), the permitted total capacity of DGs installed in the distribution system and the lower limit of RES penetration for the carbon emissions reduction and for empowering the penetration of RES in distribution system as proportion of the total DG penetration. More specifically, the following deterministic inequality constraints have to be met

$$P_{\mathrm{DG}i} \le P_{\mathrm{DG}i\max}, \quad i = 1, 2, \ldots, N_{\mathrm{DG}} \tag{20}$$

$$Q_{\mathrm{DG}i} \le Q_{\mathrm{DG}i\max}, \quad i = 1, 2, \ldots, N_{\mathrm{DG}} \tag{21}$$

$$\sum_{i=1}^{N_{\mathrm{DG}}} P_{\mathrm{DG}i}^{N} \le \mathrm{DG\,pen} \sum_{i=1}^{N_{\mathrm{B}}} P_{Li} \tag{22}$$

$$\sum_{i=1}^{N_{\mathrm{RES}}} P_{\mathrm{DG}i}^{N} \ge \mathrm{RES\,pen} \sum_{i=1}^{N_{\mathrm{DG}}} P_{\mathrm{DG}i}^{N} \tag{23}$$

where N_{DG} is the number of installed DGs, N_{B} is the number of buses of the distribution system, N_{RES} is the number of installed RES, DGpen is the maximum penetration of DGs and RESpen is the minimum penetration of RES in distribution system as a fraction of the total installed DGs capacity.

4.2.3 Chance constraints: Chance constraints are not crucial limitations and it is possible to be violated a few times under a confidence level a. The following chance constraints are considered [12, 22]

$$\Pr\left\{ S_{ij} \le S_{ij\max} \right\} \ge a, \quad i, j = 1, 2, \ldots, N_{\mathrm{b}} \tag{24}$$

$$\Pr\left\{ V_{\min} \le V_i \le V_{\max} \right\} \ge a, \quad i, j = 1, 2, \ldots, N_{\mathrm{B}} - 1 \tag{25}$$

where S_{ij} and $S_{ij\max}$ are the power flow (MVA) and the maximum permissible power flow in branch $i-j$, respectively; N_b is the number of branches of the distribution system and $N_{\mathrm{B}} - 1$ is the number of distribution system buses except the slack bus, which has predefined voltage magnitude V_1 and angle $\delta_1 = 0^{\mathrm{o}}$.

4.3 Mathematical model

Owing to the uncertainties included, the ODGP problem has to be formulated with a mathematical model of stochastic programming. CCP is a method of stochastic programming [23]; its constraints and its objective function contain stochastic variables. The developed model of the CCP-based optimal DG placement has the following form

$$\begin{cases} \min\limits_{X} \{ f(\boldsymbol{X}, \boldsymbol{\xi}) \} \\ \text{subject to: } \Pr\left\{ f(\boldsymbol{X}, \boldsymbol{\xi}) \le \bar{f} \right\} \ge b \\ \Pr\{ g(\boldsymbol{X}, \boldsymbol{\xi}) \le 0 \} \ge a \\ \boldsymbol{G} = \boldsymbol{0} \\ \boldsymbol{H}_{\min} \le \boldsymbol{H} \le \boldsymbol{H}_{\max} \end{cases} \tag{26}$$

where $\boldsymbol{X}$ is the vector of the design variables, ξ is the set of stochastic variables, $f(\boldsymbol{X}, \xi)$ is the objective function, $\bar{f}$ is the optimal value of the objective function satisfying the confidence level b, $g(\boldsymbol{X}, \xi)$ presents the inequalities (chance constraints) described by (24) and (25); $\boldsymbol{G} = \boldsymbol{0}$ and $\boldsymbol{H}_{\min} \le \boldsymbol{H} \le \boldsymbol{H}_{\max}$ correspond to the deterministic equality and inequality constraints, respectively; Pr{ev} denotes the probability of the event ev.

5 Proposed solution for the ODGP problem

The traditional method for solving a CCP-based optimisation problem is to convert chance constraints into deterministic constraints according to the given confidence level. A GA with embedded PEM (GA–PEM) is introduced for the solution of the CCP described in (26), that is, for the solution of the ODGP under uncertainties. More specifically, the GA searches the best solution among a number of possible solutions, whereas the PEM is proposed for the solution of the PPF problems, which are necessary for the evaluation of each chromosome of the GA. The flowchart of the proposed method is shown in Fig. 2. Complete explanation of the method is presented in Sections 5.1–5.5.

The GA has been well introduced and analysed in many power system problems [24–26]. Certain features of the GA, such as the structure of the chromosome, the coding of the design variables, the creation of the initial population, the decoding of the encoded chromosome, the handling of constraints and the evaluation of fitness function, will be discussed in the following.

5.1 Chromosome structure

Encoding of potential possible solutions is a basic tool for the efficient application of the GA. The accurate calculation of the objective function depends on the installed capacity and the allocation of DG units. Therefore every potential solution (chromosome) has to be represented with a three-part vector that has as many parts as the types of DGs to be installed in the distribution system

$$\boldsymbol{X} = \left(\boldsymbol{X}^{\mathbf{WDG}}, \boldsymbol{X}^{\mathbf{SDG}}, \boldsymbol{X}^{\mathbf{FDG}} \right) \tag{27}$$

where $\boldsymbol{X}^{\mathbf{WDG}}$ is an L_{W} dimension vector corresponding to

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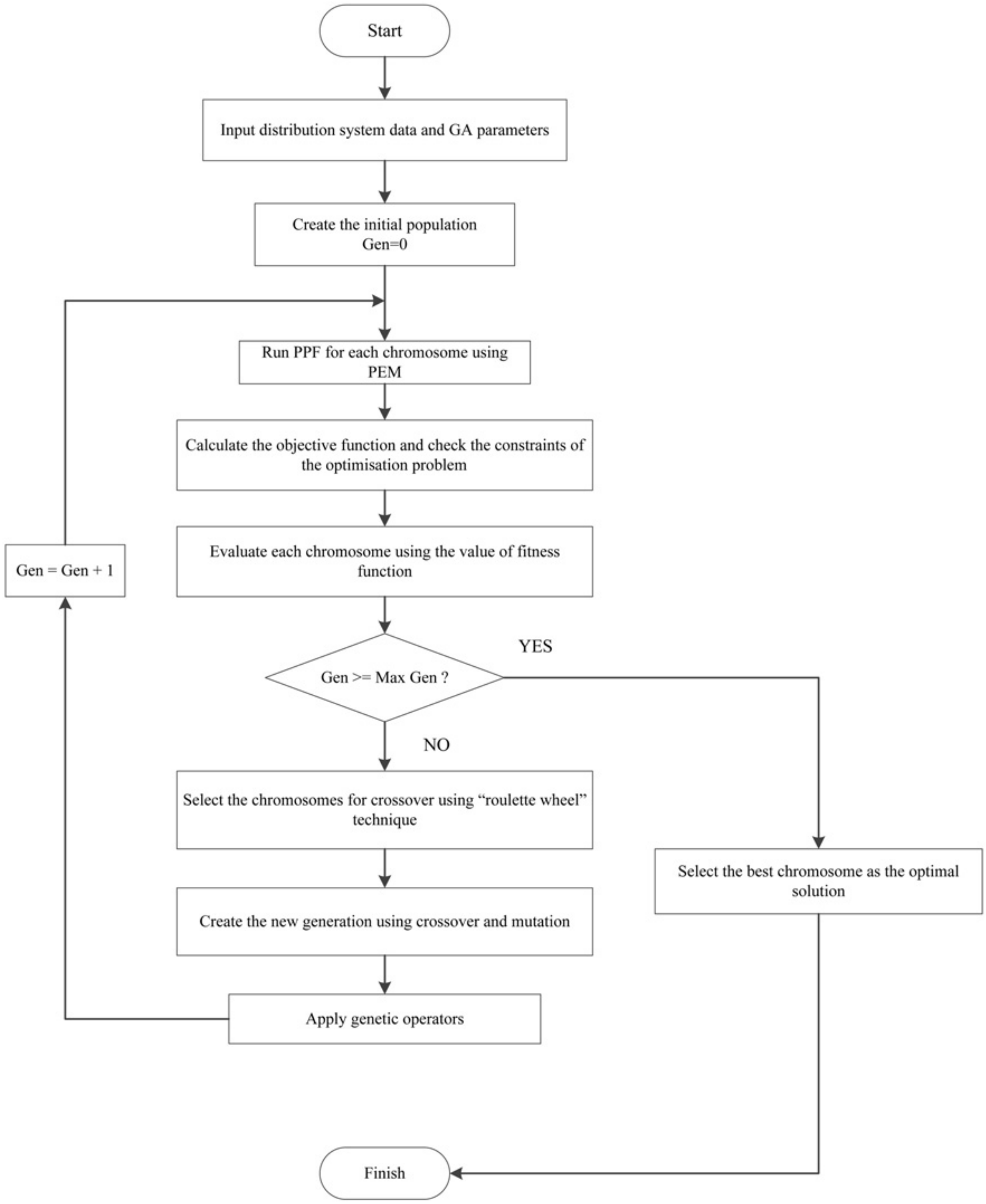

Fig. 2 *Flowchart of the proposed GA–PEM method*

the installed capacity P_i^{WDG} of wind turbines, in each of the candidate buses for wind turbine installation; L_{W} is the number of candidate buses for wind turbine installation; $\boldsymbol{X}^{\mathbf{SDG}}$ is an L_S dimension vector corresponding to the installed capacity P_i^{SDG} of photovoltaics, in each of the candidate buses for installation of photovoltaics; L_S is the number of candidate buses for installation of photovoltaics; $\boldsymbol{X}^{\mathbf{FDG}}$ is an L_{F} dimension vector corresponding to the installed capacity P_i^{FDG} of fueled DGs, in each of the candidate buses for installation of fueled DGs; L_{F} is the number of candidate buses for installation of fuelled DGs. Consequently, the dimension L of the chromosome is equal to $L_W + L_S + L_{\mathrm{F}}$.

Each element of vector $\boldsymbol{X}$ (each gene of the chromosome in the GA) is represented by an integer, selected through a set of integer values, that is,

$$\boldsymbol{X} = \begin{cases} 0, & \text{if} \quad \text{there is no DG} \\ 1 \text{ or } 2 \text{ or } \cdots \text{ or } \mathrm{N_C}, & \text{if} \quad \text{there is DG} \end{cases} \quad (28)$$

where $X = 0$ corresponds to the absence of DG, whereas $X = \{1 \text{ or } 2 \text{ or } \ldots \text{ or } N_{\mathrm{C}}\}$ corresponds to the first, or the second, or …., or the N_{C}th candidate DG capacity [24].

For instance, considering the first candidate bus for installing wind turbines and assuming that this is the bus number 4 of the distribution system, having five possible scenarios of installed capacities (e.g. 20, 40, 60, 80 and 100 kW, such as those of Table 5), the value $X = 1$

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corresponds to 20 kW installed capacity, whereas the value $X = 5$ corresponds to 100 kW installed capacity.

5.2 Initial population

The GA starts with the creation of the initial random population of chromosomes (the initial population of possible solutions of the problem), creating a table with dimensions $N_{pop} \times N_{par}$ with zero elements, where N_{pop} is the number of chromosomes and N_{par} is the number of genes of each chromosome.

Step 1: for each possible solution, an integer number is randomly selected between 1 and N_{par}.
Step 2: an h dimension vector $\boldsymbol{H}$ is randomly selected with integer elements between 1 and N_{par}. For example, let us suppose that $N_{par} = 8$ and that $h = 5$ is randomly chosen, then the vector $\boldsymbol{H}$ is filled with integer numbers in the interval [1, …, 8], for example, $\boldsymbol{H} = \{1, 4, 8, 3, 6\}$ where $\boldsymbol{H}$ represents the 1st, 4th, 8th, 3rd and the 6th gene.
Step 3: finally, in an iterative process, for each element of $\boldsymbol{H}$ a random number is generated between 1 and N_C, that is, some of the candidate scenarios are placed randomly in the gene of $\boldsymbol{H}$. For example, let us suppose that $H_1 = 1$, so the first gene of the chromosome will randomly pick a value between 1 and N_C; if, for example, $H_2 = 4$, then the fourth gene of the chromosome will randomly pick a value between 1 and N_C and so on.

In fact, the initial population includes zeros (no DG placement) and random sizing of capacity installed in each candidate bus. In this way, faster convergence to a good solution can be achieved and an initial population with random penetration rates of DG units is created.

5.3 Decoding and chromosome evaluation

Each chromosome is decoded using a decoding procedure. This procedure takes as argument three tables, one per each type of DG, and a coded chromosome (genes) and returns a decoded chromosome (phenotypes). Each table contains the candidate scenarios (potential sizes of DG for each candidate bus). Therefore the zeros and integers 1, 2, …, N_C are translated into DG absence and DG installed capacity, respectively.

For the evaluation of each chromosome, a PEM is solved in order to obtain all the statistical information that is necessary for calculating the objective function and controlling the satisfaction of the constraints. If a constraint is violated, a penalty is given in the objective function value, as described in Section 5.4.

5.4 Handling of constraints and calculation of fitness function

In the GA, in order to handle the violation of constraints while seeking the best solution, a penalty function is applied. Penalty function measures the degree the objective function will be charged [27]. Penalties are incorporated into fitness function, which is the evaluation function of the chromosome

$$F_{\text{fitness}} = \bar{f}(\boldsymbol{X}, \boldsymbol{\xi}) + \sum_{i=1}^{N_{\text{constra ints}}} \text{penalty}_i \qquad (29)$$

where $\bar{f}(\boldsymbol{X}, \boldsymbol{\xi})$ is the value of the objective function as it is calculated using PEM; $N_{\text{constra int}\,s}$ is the set of constraints; penalty_i is the penalty because of ith constraint violation. The penalty is calculated by the following formula

$$\text{penalty}_i = C_i d_i^n \qquad (30)$$

where d_i is the distance from the upper or the lower limit, in the case the constraint is violated; C_i is the coefficient of violation equals to W_1 near the limits and equals to W_2 far from the limits, with $W_1 \ll W_2$ and n usually equals to two [28].

5.5 Next generations and GA termination

After evaluating the initial population, the best chromosomes are selected as prospective parents, the pairs are selected and the genetic operators are applied (crossover, mutation, special genetic operators [26]), for the creation of the new population. Comparing the population of the new generation with the one of the previous generation, the best N_{pop} chromosomes are

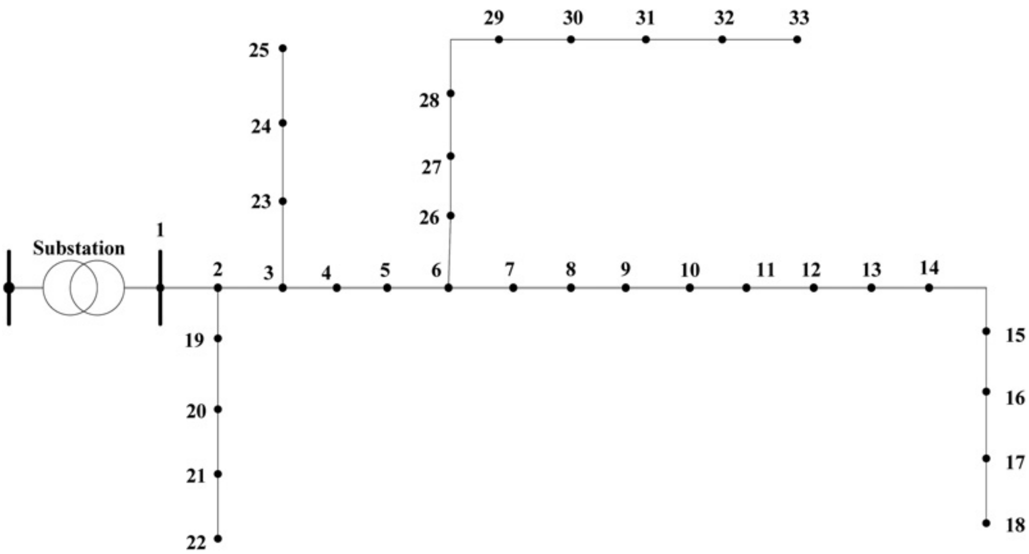

Fig. 3 *IEEE 33-bus radial distribution system*

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Table 1 Data for the IEEE 33-bus distribution system

Branch	Sending bus	Receiving bus	Resistance, R, Ω	Reactance, X, Ω	Real power load at receiving bus, MW	Reactive power load at receiving bus, MVAr
1	1	2	0.0922	0.0477	0.1	0.06
2	2	3	0.493	0.2511	0.09	0.04
3	3	4	0.366	0.1864	0.12	0.08
4	4	5	0.3811	0.1941	0.06	0.03
5	5	6	0.819	0.707	0.06	0.02
6	6	7	0.1872	0.6188	0.2	0.1
7	7	8	1.7114	1.2351	0.2	0.1
8	8	9	1.03	0.74	0.06	0.02
9	9	10	1.04	0.74	0.06	0.02
10	10	11	0.1966	0.065	0.045	0.03
11	11	12	0.3744	0.1238	0.06	0.035
12	12	13	1.468	1.155	0.06	0.035
13	13	14	0.5416	0.7129	0.12	0.08
14	14	15	0.591	0.526	0.06	0.01
15	15	16	0.7463	0.545	0.06	0.02
16	16	17	1.289	1.721	0.06	0.02
17	17	18	0.732	0.574	0.09	0.04
18	2	19	0.164	0.1565	0.09	0.04
19	19	20	1.5042	1.3554	0.09	0.04
20	20	21	0.4095	0.4784	0.09	0.04
21	21	22	0.7089	0.9373	0.09	0.04
22	3	23	0.4512	0.3083	0.09	0.05
23	23	24	0.898	0.7091	0.42	0.2
24	24	25	0.896	0.7011	0.42	0.2
25	6	26	0.203	0.1034	0.06	0.025
26	26	27	0.2842	0.1447	0.06	0.025
27	27	28	1.059	0.9337	0.06	0.02
28	28	29	0.8042	0.7006	0.12	0.07
29	29	30	0.5075	0.2585	0.2	0.6
30	30	31	0.9744	0.963	0.15	0.07
31	31	32	0.3105	0.3619	0.21	0.1
32	32	33	0.341	0.5302	0.06	0.04

Table 2 Average value (μ) and standard deviation (σ) of load growth for the IEEE 33-bus distribution system

Bus	μ (kW)	σ (kW)
1	0	0
2	0.0035	0.0013
3	0.00315	0.0018
4	0.0042	0.0018
5	0.0021	0.0012
6	0.0021	0.00085
7	0.007	0.0031
8	0.007	0.00327
9	0.0021	0.00096
10	0.0021	0.00125
11	0.001575	0.00061
12	0.0021	0.0012
13	0.0021	0.00082
14	0.0042	0.0025
15	0.0021	0.00071
16	0.0021	0.00073
17	0.0021	0.00113
18	0.00315	0.0011
19	0.00315	0.0015
20	0.00315	0.0014
21	0.00315	0.00123
22	0.00315	0.00138
23	0.00315	0.00149
24	0.0147	0.0046
25	0.0147	0.0078
26	0.0021	0.0012
27	0.0021	0.00084
28	0.0021	0.00112
29	0.0042	0.00218
30	0.007	0.0037
31	0.00525	0.0022
32	0.00735	0.0036
33	0.0021	0.00084

selected as the new generation. The algorithm terminates when it has exceeded the maximum number of generations or when a better solution than the current best solution cannot be found for a certain number of generations.

6 Results and discussion

The IEEE 33-bus radial distribution system is used for demonstrating the proposed method. The GA–PEM and GA–MCS algorithms were developed in Matlab 7.12 and the computer program was executed in a PC having the following specifications: processor Intel Core 2 Duo 2.00 GHz, 3 GB RAM, running under MS Windows 7 Pro version 2009.

The IEEE 33-bus distribution system is shown in Fig. 3 and its data are given in Table 1. Bus 1 is the slack bus and as a result DG units cannot be connected there. The other buses

Table 3 Investment, maintenance and operating costs of DGs and energy loss cost of the distribution system

Cost component	DG type		
	Wind DG	Photovoltaics DG	Fueled DG
investment cost C^I, \$/kW	1800	2000	850
maintenance cost C^M, \$/kWh	0.05	0.03	0.02
operation cost C^O, \$/kWh	0	0	GBM (0.03, 0.02, 0.01)
energy loss cost C^L, \$/kWh	GBM (0.08, 0.09, 0.02)		

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Table 4 Technical specifications of DGs

DG type	Technical specification
wind turbines	v_{ci} = 4 m/s v_{co} = 25 m/s v_n = 15 m/s Poper factor = 0.9 lagging
photovoltaics	s_n = 1000 W/m^2 power factor = 1.0
fueled DGs	stable power power factor = 0.9 lagging

Table 5 Candidate schemes for the type, allocation and sizing of DGs

Bus	Installed capacity, kW					DG type
4	20	40	60	80	100	1, 2
7	40	80	120	160	200	1, 2, 3
8	40	80	120	160	200	1, 2
14	20	40	60	80	100	1, 2
18	20	40	60	80	100	1, 2, 3
24	100	200	300	400	500	1, 2, 3
25	100	200	300	400	500	1, 2, 3
30	40	80	120	160	200	1, 2
32	40	80	120	160	200	1, 2, 3

Note for DG type*:* 1 – wind DG, 2 – photovoltaics DG and 3 – fueled DG

are PQ buses. Voltage in slack bus is supposed to be $V_1 = 1.02$ pu, the base power is 1 MVA and the base voltage is 12.66 kV. The total load of the distribution system is 3.715 MW and 2.3 MVAr.

Table 6 Parameters of the GA

population size	50
rate of population for mating	30%
crossover probability	0.9
mutation probability	0.2
special genetic operators probability	0.2
elitism	1 chromosome
maximum generation number	200
number of consecutive generations that a better chromosome has not been found	25

Table 7 Definition of scenarios

Scenario	Wind speed parameters	Solar radiation parameters	Weights of the objective function
1	$k_v = 2.1$, $c_v = 7.5$	$k_s = 1.4$, $c_s = 5.5$	$b_1 = 0.1$, $b_2 = 0.11$, $b_3 = 0.34$, $b_4 = 0.34$, $b_5 = 0.11$
2	$k_v = 1.8$, $c_v = 6$	$k_s = 1.8$, $c_s = 6.5$	$b_1 = 0.1$, $b_2 = 0.11$, $b_3 = 0.34$, $b_4 = 0.34$, $b_5 = 0.11$
3	$k_v = 2.1$, $c_v = 7.5$	$k_s = 1.4$, $c_s = 5.5$	$b_1 = 0.34$, $b_2 = 0.11$, $b_3 = 0.34$, $b_4 = 0.11$, $b_5 = 0.10$

The load is expected to increase in the next year (year after the reference year) as is shown in Table 2. The costs of DG are as shown in Table 3. The technical characteristics of DGs are shown in Table 4. The network constraints are as follows: the voltage magnitude cannot exceed ±6% of the nominal grid voltage and power flow on the lines of the network should not exceed 4 MVA. These restrictions are

Table 8 Optimal DG placement by GA–PEM for the IEEE 33-bus distribution system for the Scenarios 1 and 2 of Table 7 that correspond to different wind speed and solar radiation parameters

DG type	Bus	Installed capacity (kW)		
		Before DG placement	After DG placement – Scenario 1	After DG placement – Scenario 2
wind DG	14	—	60	0
	18	—	40	40
	32	—	80	0
photovoltaics DG	14	—	60	60
	18	—	0	80
	25	—	0	100
	30	—	0	120
	32	—	160	40
fueled DG	7	—	160	80
	18	—	80	100
	25	—	200	400
	32	—	160	40
energy losses, MWh		1765.0	1199.4	1277.4
network loss ratio, %		5.14	3.33	3.54
DG penetration, %		0	26.01	27.03
RES penetration, %		0	10.40	10.92
RES/DG		0	0.40	0.40
probability of chance constraint Pr{$S_{ij} \leq S_{ij\max}$} to be satisfied		—	0.901	0.90
probability of chance constraint Pr{$V_{\min} \leq V \leq V_{\max}$} to be satisfied		—	0.93	0.94
investment cost C^{I}, $		—	1 274 000	1 399 000
maintenance cost C^{M}, $		—	215 290	241 906
operating cost C^{O}, $		—	162 839	168 372
energy loss cost C^{Lt}, $		—	107 642	114 569
capacity adequacy cost C^{A}, $		—	70 862	79 974
objective value *f*, $		—	250 840	271 507

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not strict and must be met with a probability greater than 0.9, which means that these are chance constraints. Penetration of DG should not exceed 50% of the total load and the percentage of renewable energy must be at least 40% of total DG in order to achieve the target set by the network operator for carbon emissions reduction and energy saving.

The candidate schemes of DG installation for the 33-bus distribution system are shown in Table 5, where the candidate buses are presented together with the candidate sizes and type of DG that may be installed per bus. The parameters of GA used are shown in Table 6. The confidence level for the chance constraints is $a = 0.9$.

The three scenarios of Table 7 have been designed in order to investigate the effect of uncertain parameters (shape and scale parameters of the Weibull distribution of wind speed and solar radiation) as well as the impact of the weights of the objective function on the results. It should be noted that Scenario 3 uses the optimal weights of the objective function (17) computed by an analytic hierarchy process in [12].

The proposed GA–PEM algorithm was executed 5–7 times and it gave practically the same optimal solution for each execution. The GA converged in the optimal solution after 45–55 generations. Thus, it can be concluded that running five times the proposed GA–PEM, good results can be obtained. More specifically, the application of the proposed GA–PEM provides the optimal DG placement results shown in Table 8 for the case of the IEEE 33-bus distribution system for two different scenarios of values of uncertain parameters (Scenarios 1 and 2 of Table 7). It is concluded that the DGs are placed in the areas where voltage drop seems to be more appreciable and out of the limits. Despite the random load growth in the period under examination, the energy losses reduce from 1765 MWh (before DG placement) to 1199 MWh (Scenario 1) and 1277 MWh (Scenario 2), respectively. Chance constraints and the ratio RES/DG converge close to the specified limitations. The total RES penetration is equal to 10.40% (Scenario 1) and 10.92% (Scenario 2), respectively, of total load of the examined distribution system. The optimal solution of Table 8 satisfies all the constraints and minimises the total cost. Table 8 shows that in Scenario 2, the penetration of photovoltaics increases, whereas the installation of wind generation decreases. This is because of the lower level of wind speed and the higher level of solar illumination of Scenario 2 in comparison to Scenario 1 (Table 7).

Fig. 4 shows the evolution of the best chromosome per generation of the GA for Scenario 1. It can be observed that

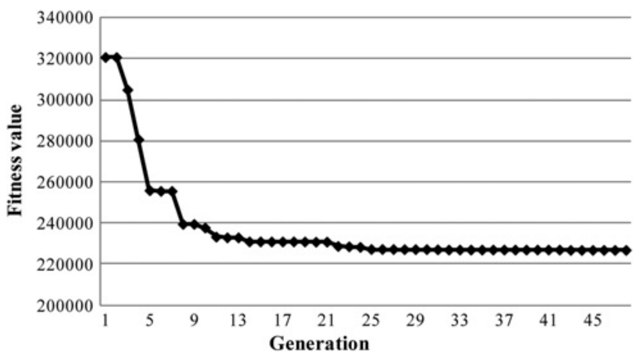

Fig. 4 *Fitness value evolution of the best chromosome per generation of the GA for Scenario 1*

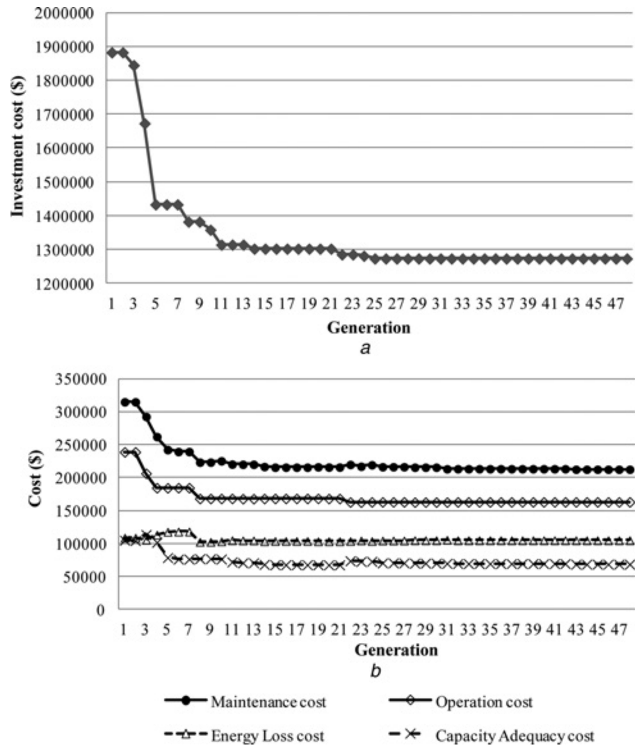

Fig. 5 *Evolution of individual costs of the best chromosome per generation of the GA for Scenario 1*

a Investment cost
b Maintenance, operation, energy loss and capacity adequacy cost

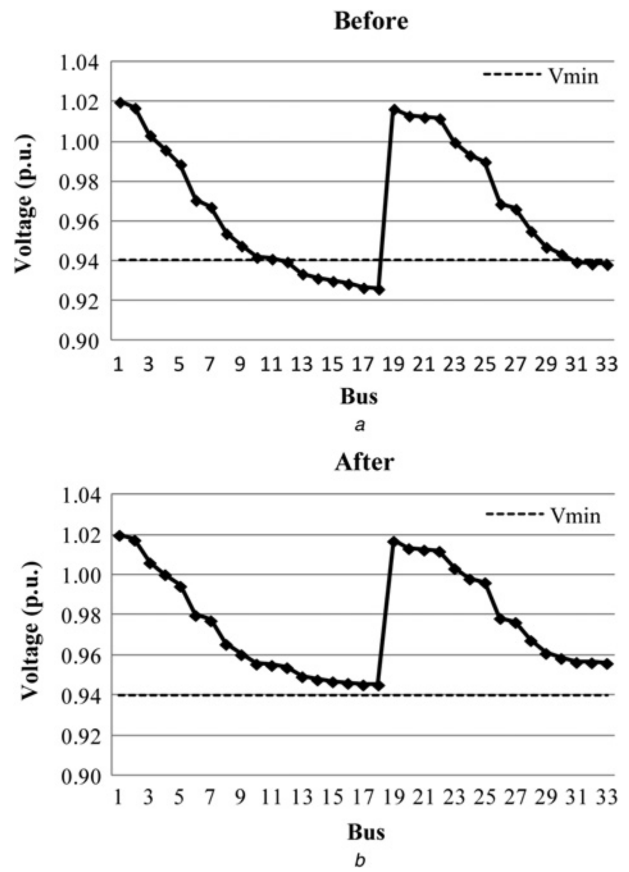

Fig. 6 *Voltage variations at each node of the 33-bus distribution system before and after DG placement for Scenario 1*

a Before DG placement
b After DG placement

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Table 9 Comparison of the optimum solution found by GA–PEM and GA–MCS for the IEEE 33-bus distribution system for the Scenarios 1 and 3 of Table 7 that correspond to different weights of the objective function

	Scenario 1		Scenario 3	
	GA–PEM	GA–MCS	GA–PEM	GA–MCS
energy losses, MWh	1199.4	1173.8	1175.6	1149.9
network loss ratio, %	3.33	3.37	3.26	3.30
DG penetration, %	26.01	26.01	27.04	27.04
RES penetration, %	10.40	10.40	10.92	10.92
RES/DG	0.40	0.40	0.40	0.40
probability of chance constraint $\Pr\{S_{ij} \le S_{ij\max}\}$ to be satisfied	0.901	0.906	0.909	0.96
probability of chance constraint $\Pr\{V_{\min} \le V \le V_{\max}\}$ to be satisfied	0.93	0.99	0.97	0.99
investment cost C^I, $	1 274 000	1 274 000	1 335 000	1 335 000
maintenance cost C^M, $	215 290	212 659	224 850	223 150
operating cost C^O, $	162 839	163 132	168 232	168 221
energy loss cost C^L, $	107 642	104 964	105 508	102 957
capacity adequacy cost C^A, $	70 862	68 634	75 607	74 213
objective value f, $	250 840	249 495	554 999	554 388
number of generations	48	51	51	54
total time elapsed, min	77.15	569.31	81.63	604.83

the optimal solution of the ODGP problem has been found after 48 generations. Fitness value corresponds to the objective function value, as it is minimised, and these two values are equal when all the constraints are satisfied, that is, when there is no penalty cost. Fig. 5 illustrates the evolution of individual costs of DGs in GA procedure for Scenario 1. After DG placement, although the load increases at each bus of the distribution system, an improvement in the voltage profile is observed, as can be seen in Fig. 6.

Table 9 compares the optimum solution found by the proposed GA–PEM with that provided by the GA–MCS of [11, 12] for Scenarios 1 and 3 of Table 7. It can be observed from Table 9 that both methods provide practically the same results. However, the proposed GA–PEM is seven times faster than the GA–MCS. More specifically, in case of Scenario 1, the GA–PEM converged to the best solution after 77.15 minutes, whereas the optimum solution of the GA–MCS was found after 569.31 minutes. The much faster execution of the proposed GA–PEM is due to the fact that PEM is much faster than MCS in the solution of the PPF problem, which has to be solved many times evaluating each chromosome in each generation. Table 9 also shows the impact of the weights of the objective function on the results. More specifically, in Scenario 3, the value of the objective function is increased in comparison to Scenario 1, mainly because of the increased value of the weight of the investment cost.

7 Conclusion

In this paper, chance constrained programming was introduced, as a stochastic programming model and a PEM-embedded GA-based approach was proposed as a new methodology for solving the ODGP problem considering the uncertainties of load growth, wind power production, protovoltaics production and the volatile future fuel prices and electricity prices. The new method was demonstrated on the IEEE 33-bus radial distribution system and compared with the GA–MCS method. It was found that the two methods provide practically the same results, however, the proposed GA–PEM is seven times faster than the GA–MCS because of the fact that PEM solves much faster than MCS the many PPF problems that are evaluated by the GA.

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