Taxonomy of PMU Placement Methodologies

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***Abstract*—Utilization of phasor measurement units (PMUs) in the monitoring, protection and control of power systems has become increasingly important in recent years. The aim of the optimal PMU placement (OPP) problem is to provide the minimal PMU installations to ensure full observability of the power system. Several methods, based on mathematical and heuristic algorithms, have been suggested for the OPP problem. This paper presents a thorough description of the state of the art of the optimization methods applied to the OPP problem, analyzing and classifying current and future research trends in this field.**

***Index Terms*—Heuristic algorithms, mathematical algorithms, observability, optimal PMU placement (OPP), optimization, phasor measurement units (PMUs).**

I. INTRODUCTION

RESTRUCTURING of the electric power industry has transformed state estimation (SE) from an important application into a critical one. It is a key function in modern energy management systems (EMS), by providing a complete, consistent, and accurate database as an input to all other online applications in the control center. Conventional state estimators use a set of measurements, consisting of bus voltages, real and reactive power flows and injections, in order to estimate bus voltage phasors in the system. Until recently, these measurements were obtained only through the supervisory control and data acquisition (SCADA) system, which gathers the real-time measurements from the remote terminal units (RTUs) installed in substations.

With the advent of global positioning system (GPS), the measurement set can be enlarged to include time synchronized phasor measurements provided by phasor measurement units (PMUs). A PMU installed at a bus is able to measure the voltage phasor of the installed bus and the current phasors of some or all the branches incident to the bus, assuming that the PMU has sufficient number of channels. As the availability of PMUs at substations is increased, the performance of different essential functions concerning the monitoring, protection, and control of interconnected power systems is improved [1]–[4].

With the growing number of PMUs planned for installation in the near future, both utilities and research institutions are looking for the best solutions to their placement. The solution methodologies for the optimal PMU placement (OPP) problem can be classified into two categories: mathematical and heuristic algorithms.

This paper proposes a taxonomy of OPP methods, offering a unifying description of a relatively large number of works devoted to the subject. It significantly enhances and updates a preliminary survey on the topic [5]. This review serves as a guide to aid researchers and power system engineers on the available PMU placement methodologies.

The paper is organized as follows. Section II provides the problem formulation with its extensions and generalizations. Sections III and IV outline the published mathematical and heuristic methods, respectively. Sections V and VI present the contribution and the implementation details of the reviewed works, respectively. Section VII suggests possible future work and Section VIII concludes the paper.

II. FORMULATION OF OPTIMAL PMU PLACEMENT PROBLEM

Phasor measurement units are devices offering advanced protection, analysis and control in power systems using satellite technology. They can provide measurements of the voltage phasors of the installed buses and the current phasors of some or all the lines connected to those buses.

The typical OPP problem concerns about the determination of the minimum number of PMUs, n_{PMU}, and the optimal location set, $S(n_{\mathrm{PMU}})$, of the n_{PMU} PMUs, ensuring that the entire power system remains a single observable island. This model can be generalized to include additional constraints or contingencies as follows:

$$\begin{aligned}&\min_{n_{\mathrm{PMU}}}\left\{\min_{S(n_{\mathrm{PMU}})} G\left(n_{\mathrm{PMU}}, S(n_{\mathrm{PMU}})\right)\right\}\\ &\text{s.t.} \quad f\left(n_{\mathrm{PMU}}, S(n_{\mathrm{PMU}})\right)=1\end{aligned} \tag{1}$$

where $G(n_{\mathrm{PMU}}, S(n_{\mathrm{PMU}}))$ is the number of unobservable buses, and $f(n_{\mathrm{PMU}}, S(n_{\mathrm{PMU}}))$ is a multi-objective evaluation logical function given by

$$\begin{aligned}f\left(n_{\mathrm{PMU}}, S(n_{\mathrm{PMU}})\right)=&\,O_{bs}\left(n_{\mathrm{PMU}}, S(n_{\mathrm{PMU}})\right)\\ &+O_{bs}\left(n_{\mathrm{PMU}}, S(n_{\mathrm{PMU}})\right)\bullet C_{on}\left(n_{\mathrm{PMU}}, S(n_{\mathrm{PMU}})\right)\end{aligned} \tag{2}$$

where $O_{bs}(.)$ is the observability evaluation logical function, $C_{on}(.)$ is the constraints evaluation logical function, and $+$ and $\bullet$ denote the logical OR and AND operators, respectively. The OPP problem is NP-hard and does not have a unique solution. Depending upon the starting point, the optimization scheme may yield different sets of optimal solutions with the same minimum number of PMUs. Different formulations of the PMU

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TABLE I
DIFFERENT FORMULATIONS OF THE OPP PROBLEM

Formulation characteristics	Published works
Effect of zero-injection buses	[6], [9], [10], [11], [12], [13], [14], [16], [17], [19], [20], [23], [26], [27], [30], [31], [32], [33], [34] and [37]
Effect of Conventional Measurements	[6], [7], [8], [11], [12], [15], [16], [19], [22], [23], [33], [38] and [43]
Single or Multiple PMU Loss Contingency	[10], [13], [15], [16], [21], [22], [32], [40] and [41]
Single Branch Outage Contingency	[9], [10], [13], [15], [21], [22], [24], [32] [39], [40] and [41]
Contingency of Single Line Outage or Single PMU Loss	[10], [15], [21], [22], [32], [36] and [40]
Effect of PMU Channel Capacity	[10], [17] and [18]

placement problem with additional constraints have been presented in the literature, as can be seen in Table I.

III. MATHEMATICAL ALGORITHMS

A. Integer Programming (IP)

Integer programming (IP) is a mathematical programming method of solving an optimization problem having integer design variables, while the objective function and the constraints are linear, nonlinear, or quadratic, thus leading to integer linear programming (ILP), integer nonlinear programming (INLP) and integer quadratic programming (IQP) algorithms, respectively [44].

The objective of method [6] is to accomplish the task of PMU placement at strategic buses making the entire system observable. The existence of constraints considering PMU measurements and injections that may be zero injections or measured injections as well as PMU measurements and conventional injections and flows, is also discussed. In [7], a feasible numerical method is presented for PMU placement that transforms existing critical measurements into redundant ones. The main goal is to improve bad data detection and identification capability of the state estimator for a given system by taking advantage of the PMU technology.

An IP formulation of the OPP problem is proposed in [8]. This proposal improves the bad data processing capability of the state estimator, assuming that any bad data appearing on a single measurement will be detectable. Depending on the measurement configuration and the system topology, the critical measurements are transformed into redundant. The problem is extended to incorporate conventional measurements as candidates for placement and can be used to determine optimal locations when a desired level of local redundancy is considered in the system. The description of a simple modified ILP based optimal placement method, ensuring complete topological observability of the system under intact and critical contingency cases, is presented in [9]. A voltage stability based contingency ranking method is carried out to screen few critical contingencies, which have been considered in the optimal PMU placement.

Different additional contingency conditions, i.e., line outages and loss of measurements, are considered separately or simultaneously for solving the OPP problem in [10]. Additionally, communication constraints of power networks are considered as measurement limitations and are included in the model in order to restrict the number of PMUs. In [11], the optimal redundant PMU placement formulation for full and incomplete observability, considering situations with and without zero injection measurements, shows that a linear OPP problem can be modelled and solved by ILP. A similar formulation, considering or not the conventional power flow and injection measurements, is proposed in [12].

The procedure for multistaging of PMU placement in [13], ascertains that final placement obtained by phasing is identical to the one obtained without imposing phasing constraints, develops a linear model for zero injection constraints, and proposes the bus observability index (BOI) and the system observability redundancy index (SORI) to rank these multiple placement solutions. A hybrid two-stage PMU placement technique is proposed in [14]. The first stage utilizes an ILP based approach to ensure topological observability, whereas the second stage determines the numerical observability, by looking at whether the measurement Jacobian matrix, which relates current and voltage phasor measurements to bus voltage states in the linear model, is of full rank.

The description of an optimal placement strategy for fully observable network using only "branch" PMUs is presented in [15]. The study takes also into account the PMU failures and network contingencies that involve topology changes as well as bad data detectability of PMU measurements. A unified approach, considering the impacts of both existing conventional measurements and the possibility of single or multiple PMU loss into the decision strategy of the OPP problem, is presented in [16]. The problem is formulated as a binary integer linear programming (BILP) problem.

A BILP formulation is proposed in [17], for optimal PMU placement where channel limits of PMUs are taken into account. The method also accounts for any existing zero injection measurements. An optimal solution to the PMU placement problem for network observability, given a specified number of available channels for the candidate PMUs, is also studied in [18].

A BILP model for simultaneous placement of PMUs and traditional power flow measurements is formulated in [19], by introducing auxiliary variables and constraints. The mixed-integer programming is used to incorporate the stochastic nature of components and their outage probabilities whereas an efficient linearization technique is proposed to convert the nonlinear function representing the probability of observability into a set of linear expressions [20]. The approach considers the PMU placement in a staged time span.

A multiobjective IQP formulation of the OPP problem that minimizes the number of PMUs, ensuring the system observability for normal operating conditions as well as for single branch outages or single PMU outages, and maximizes the measurement redundancy at the buses, is presented in [21]. Method [22] is a similar IQP placement process that also considers the existence of conventional power flow and injection measurements and other special requirements, such as the specific redundancy levels at certain buses, and the installation of PMUs in certain critical or preferred buses.

B. Exhaustive Search

Exhaustive search is a general optimization technique that systematically enumerates all possible candidates for the solution and selects the candidate that satisfies the constraints at the optimum value of the objective function. Its main advantage is that it guarantees the finding of the global optimum. However, it is not suitable for large-scale systems with huge search space.

An exhaustive binary search method is implemented in [23] to solve the OPP problem considering single branch outages with or without the existence of zero injections. In case of multiple solutions, an algorithm is proposed to select the most preferable set based on measurement redundancy.

IV. HEURISTIC ALGORITHMS

A. Genetic Algorithm (GA)

A genetic algorithm (GA) is an optimization method based on the conjecture of natural selection and genetics. GA operates on a population of individuals, known as "chromosomes", which are potential solutions to a given problem and are combined to breed new individuals [45].

A combinatorial method of a graph theoretic procedure that estimates individual optimal solutions of objectives and a simple nondominated sorting GA (NSGA) that finds the best tradeoffs between competing objectives are proposed in [24]. An adaptive clonal algorithm (CLONALG) is used in [25] to find the globally and the approximately optimal solutions, ensuring the system observability.

B. Tabu Search (TS)

Tabu Search (TS) is a gradient-descent optimization method with memory [45].

In [26], the OPP is solved by the TS algorithm and a fast observability analysis method based on augmented incidence matrix that only manipulates integer numbers and can fast, conveniently and quantitatively assess the network observability as far as PMU placement scheme is concerned. Comparisons between TS and other methods are presented in [27].

C. Simulated Annealing (SA)

Simulated annealing (SA) is a generic heuristic optimization method inspired from annealing in metallurgy [45].

The pioneering proposal [28] on OPP problem solution, using a modified bisecting search to fix the number of PMUs and a simulated-annealing-based method, looks for a placement set that leads to an observable network for a fixed number of PMUs. In [27], three modifications concerning the unobservable buses, the cooling schedule and the selection rules for PMUs according to the previous solutions, are made.

A graph theoretic approach for placing PMUs based on incomplete observability and SA to solve the pragmatic communication-constrained PMU placement problem, is presented in [29]. A sensitivity constrained method, by placing PMUs on buses with higher sensitivities and considering the optimal PMU placement and the dynamic data of power system for complete observability, is proposed in [30]. Comparisons between SA and other methods are reported in [12].

D. Differential Evolution (DE)

Differential evolution (DE) is a heuristic optimization method that uses the randomly sampled pair differences of objective vectors to guide the mutation operation instead of using probability distribution functions [45].

The algorithm proposed in [31] is an organic integration of Pareto non-dominated sorting and differential evolution algorithm.

E. Particle Swarm Optimization (PSO)

Particle swarm optimization (PSO) is a population-based optimization method in which each potential solution (called a particle) is assigned a randomized velocity and then flows through the problem hyperspace. PSO has been found to be extremely effective in solving a wide range of engineering optimization problems [45].

The binary PSO proposal in [32] satisfies the constraints of PMU loss or branch outage effect and its results are compared with those of [6], [10], [22], [26], [28], and [29]. A similar methodology, taking into account any available data from existing conventional measurements, the number and location of zero injection buses, the number and location of installed PMUs, and the system topology, is proposed in [33].

F. Immune Algorithm (IA)

The immune algorithm (IA) is a search strategy based on genetic algorithm principles and inspired by protection mechanisms of living organisms against bacteria and viruses.

In [34], the OPP problem solution is determined using an immunity genetic algorithm (IGA) that utilizes some characteristics and knowledge of the problem for restraining the degenerative phenomena during evolution and improving its efficiency.

G. Iterated Local Search (ILS)

Iterated local search (ILS) is a global optimization technique that explores a sequence of solutions created as perturbations of the current best solution, the result of which is refined using an embedded heuristic.

The algorithm of [35] solves the OPP problem assuming that a PMU placed at one node is capable of measuring all current phasors leaving the node. The proposed method suggests an initial PMU distribution which makes the network observable and then ILS is used to minimize the size of the PMU configuration needed to observe the network.

H. Spanning Tree Search

Spanning tree search (STS) algorithm dynamically determines the best path from source to destination avoiding bridge loops that can cause misinterpret results.

In [29], the spanning trees of the power system graph are used to find the optimal locations of PMUs based on the concept of depth of unobservability. The objective is to reduce the number of PMUs compared to the number required for complete observability, limiting the distance between unobserved and observed buses and guaranteeing a near uniform distribution of PMUs around the network. A spanning tree search of multiple solutions with a minimum set of PMUs, by means of direct and indirect voltage and current measurements is proposed in [36].

I. Greedy Algorithm

The greedy algorithm is an optimization methodology that follows the problem solving heuristic of making the locally optimal choice at each stage with the hope of finding the global optimum.

In [37], a virtual data elimination pre-processing method and a matrix reduction algorithm have been introduced to reduce the size of the placement model and the computational effort for the determination of the optimal placement set. A low complexity method that facilitates the placement of secure PMUs to defend against data injection attacks and handling different types of PMU measurements, is developed in [38]. In [39], the goal is to maximize the overall sensor response while minimizing the correlation among sensor outputs so as to minimize the redundant information provided by multiple sensors for effective assessment of the dynamic performance of the power system. The bus voltage magnitude, and the angle and frequency coherency indices estimated by statistical sampling of power system response signals, are used as sensor responses.

J. Recursive Security N Algorithm

The recursive security N algorithm is a spanning tree search of multiple solutions, with a different starting point.

In [36], recursive and single shot security N approaches of PMU placement, with the aim of linear static state estimation, are presented ensuring the network observability. Recursive and single shot security $N-1$ algorithms considering both line losses and PMU outages are also presented.

K. Decision Tree

A decision tree is a tree in which each branch node represents a choice between a number of alternatives and each leaf node represents a classification or decision.

A PMU placement technique is suggested in [40], to determine the critical locations of new PMUs, increasing the reliability of classification for fast voltage security monitoring.

L. Practical Heuristic Algorithms

A PMU placement algorithm under single measurement loss and any single-branch outage is suggested in [41]. The numerical observability is performed by using the minimum condition number of the normalized measurement matrix as the criteria in conjunction with the sequential elimination. A sequential addition is used to select the redundancy measurements under the contingency. The binary integer programming is also applied to select the optimal redundant measurements and a heuristic approach is also used to minimize the number of PMU placement sites. A minimal PMU placement strategy such that the fault location observability can be achieved, considering the installation cost of PMUs, is proposed in [42]. This scheme can accurately and quickly determine a fault point on the power grid whereas the prefault transmission network models are not necessary.

The PMU location algorithm in [43] determines the slack bus and the PMU installed at the slack bus in each subsystem of a distributed state estimator utilizing synchronized phasor measurements. The direct combination (DC) method suggested in [27] avoids the time consuming feature of random search, by using a simple but very effective heuristic rule.

V. CONTRIBUTION OF THE REVIEWED OPP METHODS

Table II describes chronologically the main contribution by all the published OPP works reviewed in this paper.

VI. IMPLEMENTATION DETAILS

The most frequently simulated systems are the IEEE 14-bus and the IEEE 57-bus systems. The systems having the largest number of buses are the Polish and Entergy Corporation systems with 2746 and 2285 buses, respectively. The majority of the proposed OPP methodologies have been implemented in MATLAB. A significant number of MATLAB-based developments make use of TOMLAB optimization toolbox. Other OPP implementations are based on the IBM ILOG, CPLEX, and GAMS.

VII. FUTURE RESEARCH

Important factors to investigate and further explore can be:

1) *Advanced hybrid optimization methods*. There exist many possibilities of hybridizing individual heuristics with each other and/or with other optimization methods. A large number of publications has proven the great success and benefits of such hybrids in many different combinatorial optimization problems. Moreover, particularly promising possibilities combining heuristics with ILP techniques are also suggested for future OPP research.
2) *Further investigation of optimal solutions*. The OPP problem does not have a unique solution. Different solutions providing the same minimum number of PMUs at different locations may be obtained based on the observability criterion. Methods that will provide the best choice among these optimal solutions, taking into consideration different criteria, have to be developed.
3) *Multi-objective optimization strategy*. Generalization of the OPP problem considering multiple objectives including installation cost, redundancy, performance, and other planning constraints, may be considered to generate more practical results.
4) *Additional constraints*. Influence of PMU malfunction, communication failures, measurement redundancy and uncertainty, number of measurement channels, metering accuracy including frequency, voltage, current and phase angle range limitations, environmental conditions (e.g., temperature, humidity) and existence of other metering devices, are some constraints that can be taken into account.
5) *Decision variables*. A significant number of the proposed methods minimize an objective function containing integer or binary placement decision variables. The OPP problem

TABLE II
CONTRIBUTION OF THE REVIEWED OPP WORKS

Year	Authors	Reference	Contribution
1993	T. L. Baldwin, L. Mili, M. B. Boisen Jr., and R. Adapa	[28]	A dual search algorithm which uses both a modified bisecting search and a simulated-annealing-based method. The procedure is accelerated by an initial PMU placement provided by a graph-theoretic procedure.
2001	K.-S. Cho, J.-R. Shin, and S. Ho Hyun	[27]	A modified simulated annealing (MSA) method in which the current state can be considered to speed up the convergence as well as a direct combination (DC) and a tabu search (TS) method considering the same simple heuristic rule in order to reduce the search space and guarantee the entire system observability.
2001	R. F. Nuqui, A. G. Phadke, R. P. Schulz, and N. Bhatt	[40]	Decision trees (DT) are used to identify the critical locations of additional PMUs improving the accuracy of quick voltage security classification in systems that PMUs pre-exist.
2002	I. Kamwa and R. Grondin	[39]	The OPP problem is motivated by wide-area monitoring and control of large disturbances affecting the normal operation of the interconnected system. The optimal solution is based on a suitable electrical distance measure, such as the statistical average of the entropy or coherency representing the signatures of the candidate bus set, which has the property of maximizing the overall PMU responses while minimizing their time-space cross correlation.
2002	G. B. Denegri, M. Invernizzi, and F. Milano	[36]	This work suggests four different deterministic PMU placement methods. The first two methods are based on a recursive and a single shot security N algorithm, respectively. Both of them try to achieve a faster solution. The next two methods explore the OPP solution with an N-1 security criterion, considering a single PMU loss or a single line outage. All techniques attempt to determine multiple solution at a time.
2003	B. Milosevic and M. Begovic	[24]	Incorporation of two competing objectives: minimization of the number of PMUs and maximization of the measurement redundancy. A nondominated sorting genetic-based algorithm (NSGA) is used to provide an entire Pareto-optimal front instead of a single point solution, and could be implemented in multiobjective optimization problems in which a prohibitively large enumerative search space is required.
2004	B. Xu and A. Abur	[6]	An integer programming formulation of OPP problem with or without the existence of conventional measurements allowing the easier analysis of network observability.
2005	R. F. Nuqui and A. G. Phadke	[29]	The concept of "depth-of-unobservability" is introduced. A graph theoretic placement technique guarantees a near uniform distribution of PMUs around the network, making use of spanning trees of a power system graph and limiting the distance between unobserved and observed buses. A simulated annealing (SA) placement algorithm dealing with the optimal location for new communication facilities is also proposed.
2005	J. Chen and A. Abur	[7]	It proposes an integer programming algorithm that significantly improves the bad data detection capability adding a minimum number of PMUs at strategic locations. All critical measurements are transformed into redundant.
2005	H.-S. Zhao, Y. Li, Z.-Q. Mi, and L. Yu	[30]	The concept of sensitivity constrained optimal PMU placement for complete observability is introduced. An initial guess is made by using an observability topology analysis method and placing PMUs on the buses with more incidence branches and higher node sensitivities in unobservable region. The final placement is based on a simulated annealing solution ensuring the entire system observability.
2006	J. Chen and A. Abur	[8]	The OPP problem is formulated as an integer programming (IP) problem eliminating measurement criticality in the entire system and extended to also incorporate conventional measurements as candidates for placement. The same formulation is used when a desired level of local redundancy is considered in the system.
2006	J. Peng, Y. Sun, and H. F. Wang	[26]	This work proposes a global optimization tabu search (TS) algorithm providing optimal solutions, for full network observability and enough redundancy, with high accuracy and less computational effort. A priority list based on heuristic rule is embedded to accelerate optimization.
2006	K.-P. Lien, C.-W. Liu, C.-S. Yu, and J.-A. Jiang	[42]	A PMU placement strategy considering the fault-location observability of transmission network and the installation cost of PMUs. The fault location observability can be achieved by the so-called "one-bus spaced deployment strategy".
2006	X. Bian and J. Qiu	[25]	This work proposes a new objective function, in which the domain of feasible solution is extended to make the search more convenient. The optimal solution is obtained by maximization of the objective function using the adaptive clonal algorithm (CLONALG) in order to accelerate the optimizing process and prevent the search from locally optimal traps.
2007	W. Jiang, V. Vittal, and G. T. Heydt	[43]	The goal of the proposed PMU placement algorithm is to place PMUs in each subsystem of a distributed state estimator to reduce the variances of the SE errors and at the same time increase the local redundancy.
2007	C. Rakpenthai, S. Premrudeepreechacharn, S. Uatrongjit, and N. R. Watson	[41]	A placement method based on the minimum condition number of the normalized measurement matrix considering the full system observability and the contingency conditions of single measurement loss and single-branch outage. The positions of these measurements are rearranged by a heuristic algorithm in order to minimize the number of PMU placement sites.
2007	S. Chakrabarti, D. Eliades, E. Kyriakides, and M. Albu	[21]	An integer quadratic programming (IQP) approach to solve the OPP problem making the system observable under normal operating conditions, as well as for single branch or single PMU outages. This formulation minimizes the number of PMUs and maximizes the measurement redundancy.
2008	S. Chakrabarti and E. Kyriakides	[23]	A binary exhaustive search algorithm ensuring the network observability with or without conventional measurements and for single branch outages. The benchmarking of optimal solutions and a strategy to select the best among competing solutions are introduced.
2008	D. Dua, S. Dambhare, R. K. Gajbhiye, and S. A. Soman	[13]	A multistage integer linear programming (ILP) placement algorithm in a given time horizon with or without zero injections and single PMU loss. Zero injections are modelled as linear constraints. Two indices, bus observability index (BOI) and system observability redundancy index (SORI), are introduced to further rank the multiple solutions.

may be formulated, by using continuous placement decision variables and relaxing the constraints [46].

6) *Graph based algorithms*. Alternative formulations of the OPP problem, based on the vertex covering or dominating set problems in graphs [47]–[49], will be interesting future research directions.

7) *Large-scale networks*. The majority of the proposed methods are tested with small systems. The design and im-

TABLE II
(CONTINUED)

Year	Authors	Reference	Contribution
2008	B. Gou	[11]	A generalized integer linear programming formulation for optimal PMU placement under different cases of redundant PMU placement, complete and incomplete observability, considering the situations with and without zero injection measurements.
2008	B. Gou	[12]	An ILP optimal PMU placement algorithm with and without conventional power flow and injection measurements.
2008	M. Zhou, V. A. Centeno, A. G. Phadke, Y. Hu, D. Novosel, and H. A. R. Volskis	[37]	A virtual data elimination preprocessing method and a matrix reduction algorithm are introduced to reduce the scale of a placement study. Greedy algorithm is employed to find the optimal set choosing to install a PMU at the bus that covers the largest number of uncovered buses.
2009	S. Chakrabarti, E. Kyriakides, and D. G. Eliades	[22]	An IQP placement algorithm considering the full network observability with or without loss of a single transmission line or PMU as well as conventional measurements existence and maximizing the measurement redundancy at all buses.
2009	N. H. Abbasy and H. M. Ismail	[16]	A unified binary ILP approach considering the impacts of both existing conventional measurements and the possibility of single or multiple PMU loss, while preserving the system observability and lowest system metering economy. The connectivity matrix is built only once based on the original network topology.
2009	F. Aminifar, C. Lucas, A. Khodaei, and M. Fotuhi-Firuzabad	[34]	This work proposes an immune genetic algorithm (IGA). Three effective vaccines are abstracted using the rules associated with topological observability analysis, while their injection into the individuals of generations reveals acceleration in the convergence process. The effect of preventing from familial reproduction is also modelled and a novel rule in the observability assessment is suggested.
2009	R. Sodhi, S.C. Srivastava, and S.N. Singh	[9]	A modified ILP approach to incorporate a voltage stability based contingency ranking method as well as a graph theoretic approach to modify the constraints under contingencies is proposed.
2009	M. Korkali and A. Abur	[17]	A binary ILP formulation taking into account PMU channel limitations and the existence of zero injections.
2010	F. Aminifar, A. Khodaei, M. Fotuhi-Firuzabad, and M. Shahidehpour	[10]	Line outage or PMU loss contingency conditions with or without the existence of zero injections are considered separately or simultaneously in the proposed model making it more flexible. Communication constraints are also considered as measurement limitations and included in the model. The results are compared with other formulation in literature.
2010	R. Emami and A. Abur	[15]	A “branch” PMU placement method for complete observability taking into account PMU failures and network contingencies as well as an additional placement method ensuring bad data detectability are proposed.
2010	C. Peng, H. Sun, and J. Guoa	[31]	A non-dominated sorting and differential evolution (NSDE) approach to improve the performance of the multi-objective OPP problem considering the number of PMUs and an N-1 reliability test under the precondition of network observability.
2010	M. Hurtgen and J.-C. Maun	[35]	A heuristic PMU placement method is utilized in two phases. The initial PMU configuration that makes the network observable is obtained using the PageRank placement algorithm and modified to minimize its size using an Iterated Local Search algorithm. An advantage of this method is its easy understanding and implementation.
2010	R. Sodhi, S. C. Srivastava, and S. N. Singh	[14]	A two-stage method ensures the numerical observability along with the topological observability: The first determines the minimum number of PMUs that makes the system topologically observable by an ILP algorithm, while the second ensures, if the Jacobian matrix which relates current and voltage phasor measurements to bus voltage states is not of full rank, the numerical observability using a sequential elimination algorithm.
2010	M. Korkali and A. Abur	[18]	A mixed ILP algorithm for network observability formulation considering a specified channel capacity for the candidate PMUs.
2010	R. Kavasseri and S. K. Srinivasan	[19]	A mixed ILP algorithm that minimizes the objective function which incorporates the cost of PMUs and traditional line flow measurements and takes into account or not the zero-injection buses.
2011	M. Hajian, A. M. Ranjbar, T. Amraee, and B. Mozafari	[32]	A modified discrete binary version of particle swarm (BPSO) ensuring the topological observability and considering the contingency conditions of PMU loss or branch outage is suggested. A new rule based on observability analysis of zero-injection buses is also proposed.
2011	F. Aminifar, M. Fotuhi-Firuzabad, M. Shahidehpour, and A. Khodaei	[20]	This work considers the expansion of generation facilities and transmission networks which influences the topological observability along with the multistage PMU placement which incorporates the stochastic nature of components and their outage probabilities. The expression of observability index probability is nonlinear and converted into linear by an effective linearization technique.
2011	T. T. Kim, and H. V. Poor	[38]	A fast greedy algorithm that strategically places secure PMUs at key buses in the network to defend against data injection attacks.
2011	A. Ahmadi, Y. Alinejad-Beromi, and M. Moradi	[33]	A binary particle swarm optimization (BPSO) that tries to minimize the number of PMUs needed for complete observability and maximize the measurement redundancy, with or without the existence of conventional measurements.

plementation of robust optimization algorithms applicable to large-scale networks should be investigated.

VIII. CONCLUSIONS

This paper presents a thorough description of the state of the art of the optimization methods applied to OPP problem, analyzing and classifying current and future research trends in this field. The solution methodologies for the OPP problem are classified into two major categories: mathematical algorithms and heuristic algorithms. The most frequently used techniques for the solution of the OPP problem are the *IP* and the *SA* algorithm. The IEEE bus systems (14-bus to 300-bus system) are used as benchmark systems in the majority of the research works. The biggest system, for which the solution of the OPP has been reported so far, is the Polish power system that contains 2746 buses and 3514 lines. The majority of the proposed OPP methodologies have been implemented in MATLAB. Future research areas include advanced hybrid optimization methods, further investigation of optimal solutions, modification of the OPP problem formulation, and multi-objective placement strategies.

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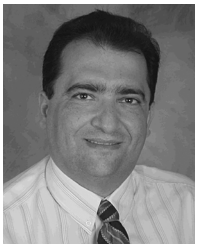

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