

# Optimal design of single-phase shell-type distribution transformers based on a multiple design method validated by measurements

Juan C. Olivares-Galvan · Pavlos S. Georgilakis ·  
Rafael Escarela-Perez · Eduardo Campero-Littlewood

Received: 17 February 2010 / Accepted: 22 May 2011 / Published online: 7 June 2011  
© Springer-Verlag 2011

**Abstract** This paper presents a method for the design of shell-type, single-phase distribution transformers to obtain the manufacturing specifications. The method is simple, efficient and accurate. By an exhaustive analysis, it is concluded that the obtained solution is the global optimum. The following constraints are imposed: excitation current, no-load losses, total losses, impedance and efficiency. The methodology of this paper requires only six input data: transformer rating, low voltage, high voltage, connection of low-voltage coil, connection of high-voltage coil, and frequency. These data are included in the transformer nameplate. In this paper, the minimization of the following four objective functions is considered: total owning cost, mass, total losses and material cost. The consideration of these four objective functions is implemented automatically by running the optimization algorithm four times without intervention of a designer. Consequently, transformer manufacturers save design man-hours and increase capacity. A design example on a 25 kVA transformer is presented for illustration. The optimized solutions of transformer design are validated with laboratory and process measurements.

**Keywords** Conductor selection · Design · Design optimization · Distribution · Excitation current · Transformer

## List of symbols

### Constants

$\rho_{Al}$	Aluminum conductor density (kg/m <sup>3</sup> )
$\rho_{Cu}$	Copper conductor density (kg/m <sup>3</sup> )
$\rho_{core}$	Core density (kg/m <sup>3</sup> )
LF	Lamination factor (%)
B	Load loss cost rate (US\$/W)
$W_k$	Loss factor of stray losses (dimensionless)
A	No-load loss cost rate (US\$/W)
$AV_{accsa}$	Number of alternative values of aluminum conductor cross-sectional area
$AV_{ccsa}$	Number of alternative values of copper conductor cross-sectional areas
$AV_{lw}$	Number of alternative values of lamination width
$AV_{mfd}$	Number of alternative values of magnetic flux density
$AV_{lvt}$	Number of alternative values of turns of low voltage
$W_{d(Al)}$	Volumetric resistivity and material density factor for aluminum ( $\Omega$ mm <sup>4</sup> /kg)
$W_{d(Cu)}$	Volumetric resistivity and material density factor for copper ( $\Omega$ mm <sup>4</sup> /kg)

### Dependent variables

$J_{Al}$	Aluminum conductor current density (A/mm <sup>2</sup> )
$LL_{Al}$	Aluminum conductor losses (W)
$M_{Al}$	Aluminum conductor mass (kg)
$J_{Cu}$	Copper conductor current density (A/mm <sup>2</sup> )
$LL_{Cu}$	Copper conductor losses (W)
$M_{Cu}$	Copper conductor mass (kg)
$M_c$	Core mass (kg)
$E$	Core thickness (mm)

J. C. Olivares-Galvan (✉) · R. Escarela-Perez ·  
E. Campero-Littlewood  
Departamento de Energia, Universidad Autonoma  
Metropolitana-Azcapotzalco, 02200 Mexico, Mexico  
e-mail: jolivare\_1999@yahoo.com

P. S. Georgilakis  
School of Electrical and Computer Engineering, National Technical  
University of Athens (NTUA), 15780 Athens, Greece  
e-mail: pgeorg@power.ece.ntua.gr

$G$	Core window height (mm)
$F$	Core window width (mm)
$H_w, W_w$	Dimensions of the winding window (mm)
$A_{ef}$	Effective core cross-sectional area (mm <sup>2</sup> )
$\eta$	Efficiency (%)
$T_{hv}$	High-voltage thickness (mm)
LL	Load losses (W)
$T_{lv}$	Low-voltage thickness (mm)
MMC	Main materials cost (US\$)
MLT <sub>HV</sub>	Mean length of a turn of high-voltage winding (mm)
MLT <sub>LV</sub>	Mean length of a turn of low-voltage winding (mm)
NLL	No-load losses (W)
$N_{HV}$	Number of turns of high-voltage winding
% $I$	Percentage of excitation current
$A_p$	Physical core cross-sectional area (mm <sup>2</sup> )
$w_{kg}$	Specific no-load losses (W/kg)
TOC	Total owning cost (US\$)
$T_{tw}$	Total winding thickness (mm)
% $Z$	Transformer impedance (%)
VA	Volt ampere
VA <sub>kg</sub>	Volt ampere per kilogram (VA/kg)
$V_t$	Volts per turn

### Input variables

$f$	Frequency (Hz)
$V_{HV}$	High voltage (V)
$V_{LV}$	Low voltage (V)
$N_\theta$	Number of phases
$S_T$	Transformer rating (kVA)

### Solution loop variables

$S_{Al}$	Aluminum conductor cross-sectional area (mm <sup>2</sup> )
$S_{Cu}$	Copper conductor cross-sectional area (mm <sup>2</sup> )
$D$	Core width (mm)
$B$	Magnetic flux density (T)
$N_{LV}$	Number of turns of low-voltage winding
Constant values do not change, except in the cases that transformer customer requires new materials or different level of losses, the transformer manufacturing process change or there are important changes in the world economy. Values of the dependent variables are calculated by the optimization software of transformer design. Values of the input variables are chosen by the transformer designer each time that transformer designer requires a new design.	

## 1 Introduction

In order to compete successfully in a global economy, transformer manufacturers need to design software capable of producing optimal designs in a very short time [1]. Traditionally, the transformer design problem has been surrounded by much art. The first transformer design by computer was performed in 1955 [2]. After 1955, more research in transformer design using computers was presented by [3–7]. Several design procedures for low-frequency and high-frequency transformers have appeared in the literature after 1970 [8,9]. Rubaii [10] describes single-phase core type and shell-phase transformer design software that is part of an electric machinery class, each student optimizes the design by observing the effect of the parameter variations on the transformer performance, and the design is limited to the specifications for the core and coils of the transformer. His software has 21 input parameters and the values of the independent variables are chosen by the students. Jewell [11] makes a functional proposal with students in electrical engineering in which the students design, build, test, model and analyze a 10 VA toroidal iron core transformer. The program has applications in both the classroom and in the industry, and it is useful for designing the following types of transformers: 1–1,000 kVA rating, 20–500 Hz fundamental operating frequency, single-phase shell-type transformer or three-phase core-type transformers. Reference [12] deals with the teaching of design of three-phase or single-phase dry type transformers, based on a computer program, where the user optimizes its design based on trial and error. Reference [13] is a study on the relationship of weight and volume against frequency for high-frequency transformers. Poloujadoff and Findlay [14] present the variation in the price of the transformer as a function of the primary turns, which is approximately a hyperbolic function. They also presented cost curves of the transformer against magnetic flux density and against current density. Andersen [15] presented an optimizing routine, called Monica, which is based on Monte Carlo simulations. Basically, his routine uses random numbers to generate feasible designs from which the lowest cost design is chosen. Breslin et al. [16] presented a web-based transformer design system where users can create new optimized transformer designs and collaborate on previous designs through a shared information space which allows for collaboration among users where designs can be shared and analyzed. In [17], simulated annealing technique was proposed for obtaining the optimal design of a three-phase power transformer. Other researchers considered the use of genetic algorithm techniques in transformer design [18,19]. Singh and Saxena [20] presented an optimum design of distribution transformer using aluminum conductor. Jabr [21] show that transformer design optimization problem can be formulated in geometric programming format. This method guarantees that the obtained solution is the global optimum.

**Table 1** Software capabilities

Accurate	The output is correct or sufficiently precise for transformer designer for the case of single-phase transformer
Complete	Everything needed for the software is included; the user needs to supply only six input data
Efficient	Computing resources are not wasted (time, memory)
Easy to use	Only six input data are required: transformer rating, low voltage, high voltage, connection of low-voltage coil, connection of low-voltage coil, and frequency
Expandable	New features and functions can be added, for example the three-phase transformer
Measurable	The software performance (time) is measured (Fig. 3)
Self contained	The software performs all the necessary functions itself, e.g., initializes variables, checks inputs, etc

**Table 2** Effort required on various activities of software development

Activity	%
Specification (The general functional requirements of the system were analyzed and transformed into a concrete set of specifications for the software. The input and output were precisely defined)	20
Design (Algorithms were identified for implementation. Flowcharts were written)	10
Coding (The software code was written and preliminary debugging was done)	20
Testing (The program was tested to see if it performs according to the specifications and design)	25
Integration (The various programs were put together to build the whole system)	10
Documentation (This activity is part of each of the other activities. Some documentation is for the team that is building the software and some is for the software users)	15

Hernandez et al. [22] presented an intelligent design assistant that consists in a knowledge-based system that design of distribution transformers. The presented system is formed by a user interface developed in Visual C++, a knowledge base implemented in CLIPS and an inference engine that processes the knowledge in a forward manner. They validated the intelligent design assistant designing a 1,500 kVA, 13.2/0.22 kV distribution transformer.

This paper proposes a design methodology capable of minimizing the objective function while ensuring the fulfillment of constraints for a single-phase transformer manufactured with electrical steel with 3% silicon [23]. Four objective functions are considered: total owning cost, mass, total losses and cost of materials. The optimal solution for each one of the four objective functions is obtained automatically by running the optimization algorithm four times. In addition to the construction limitations, the following five constraints are considered: limits on excitation current, no-load losses, total losses, impedance and efficiency. The high-voltage winding of this paper is manufactured with copper and the low-voltage winding with aluminum due to validation since experimental results were available with this configuration. Today, many distribution transformer manufacturers are moving toward the manufacturing of aluminum transformers. There are companies that manufacture distribution transformers with aluminum, e.g. Cooper Power Systems (<http://www.cooperpower.com/library/pdf/99028.pdf>, accessed Nov 2009).

Due to its simplicity, some transformer manufacturers have successfully used the principle of the proposed multiple design optimization method in the manufacturing of distribution transformers, but to our knowledge this methodology has not been widely studied yet [24], most probably because the efficiency and accuracy of the transformer design optimization is a strong differentiation point among transformer manufactures that are competing in a global marketplace. The proposed technique can be extended to other types of transformers and other electromagnetic devices. The advantage of the proposed methodology over others is that its implementation in software is very economical. The proposed transformer design methodology and software has a number of different capabilities that are somewhat independent (see Table 1). Table 2 gives the total activities for this software. There is a universal agreement that software costs are high. Programmer productivity is usually measured in lines per man-month, which we abbreviate lines per month. The results show a wide variation: productivity ranged from 5 to 5,000 lines per month. The cost of the proposed software was US\$1 per line considering Mexican salaries. In extreme cases, costs can reach \$1,000 per line [25]. This is especially important for many of the small transformer manufacturing companies, which have used spreadsheets to satisfy their own design needs. These companies could save substantial design man-hours, increase their design capacity and minimize transformer material cost with the minimum intervention of a designer.

## 2 Main design formulas for the distribution transformer

The proposed transformer design methodology presented here covers the following: single phase transformer, shell-type core construction, wound-type core construction, low-high-low windings, rating: 5–167 kVA, primary voltage: 13.2–33 kV, secondary voltage: 240/120 V.

The most important formulas for the design are presented below. Transformers can be manufactured with the results of the program. The authors are working in the development of commercial software for transformer design based on this methodology.

The following assumptions for the design example (Sect. 6) are made: the magnetic flux density is constant throughout the core. The low-voltage coil is wound closest to the core (low-high-low winding). The number of turns is rounded to the nearest integer during the optimization process. Material waste costs are ignored. High-voltage winding is manufactured with copper round conductor, while low-voltage winding is manufactured with aluminum laminated conductor. Standardization is applicable in many transformer design variables, e.g. in core dimensions (core width, core window width and height). Minimum efficiencies used in this work are for power factor equal to one. Operation frequency is 60 Hz. Lamination factor or space factor is 97.6% for M3 lamination. Transformer voltage class is 15 kV. Margin is 1 cm (for 95 kV BIL). Primary voltage is 13,200 Grd Y/7,620 V, secondary voltage is 240/120 V. Main high-low-voltage barrier is 5 mm.

### 2.1 Core cross-sectional area

Assuming a frequency of 60 Hz, the effective area of the core is expressed as follows [26]:

$$A_{\text{ef}} = \frac{225,078.215 \cdot (V_{\text{LV}}/N_{\text{LV}})}{B \cdot f} = \frac{225,078.215 \cdot (V_{\text{HV}}/N_{\text{HV}})}{B \cdot f} \quad (1)$$

where  $V_t = V_{\text{LV}}/N_{\text{LV}} = V_{\text{HV}}/N_{\text{HV}}$  are volts per turn, which are equal for primary and secondary windings. In [27], it is presented a good approximation of volts per turn for single-phase transformers and it is calculated with  $V_t = 81.1988 S_T^{-0.4527}$ .

### 2.2 Core thickness

After selecting the lamination width, the core thickness  $E$  is calculated from [28]:

$$E = \frac{A_{\text{ef}}}{2 \cdot D \cdot \text{LF}} \quad (2)$$

Note that the factor 1/2 appears in (2) because the assembly of every winding of shell-type transformer requires two cores (see Fig. 1a.)

### 2.3 Core window

The dimensions of the core window,  $F$  (width) and  $G$  (height), are derived from coil height and thickness, respectively [28]. The transformer cores are manufactured with steel with 3% silicon. Interested readers in amorphous cores can consult [29].

### 2.4 Core mass

The core mass is derived from [30]:

$$M_C = (2 \cdot E(F + G) + \Pi \cdot E^2) \cdot \text{LF} \cdot D \cdot \rho_{\text{core}} \cdot 10^{-6} \quad (3)$$

The dimensions of the core are shown in Fig. 1b.

### 2.5 No-load losses

The no-load losses depend on the magnetic flux density at which the transformer operates as well as the physical characteristics of the magnetic material [31]. The specific no-load losses at 60 Hz for magnetic material with 0.23 mm lamination are obtained from [32]:

$$w_{\text{kg}} = -43.34266 + 16.92744 \cdot B - 2.62944 \cdot B^2 + 0.20421 \cdot B^3 - 0.00791 \cdot B^4 + 0.00012 \cdot B^5 \quad (4)$$

The no-load losses are obtained from [28] as:

$$\text{NLL} = M_C \cdot w_{\text{kg}} \quad (5)$$

### 2.6 Excitation current

Volt ampere per kilogram at 60 Hz for 0.23 mm core lamination are obtained from [32]:

$$\text{VA}_{\text{kg}} = -0.54004 + 0.52255 \cdot B - 0.16564 \cdot B^2 + 0.02506 \cdot B^3 - 0.00169 \cdot B^4 + 0.0000426 \cdot B^5 \quad (6)$$

The excitation current is determined using:

$$\% I = \frac{\text{VA}}{10 \cdot S_T} \quad (7)$$

Interested readers in calculation of waveforms of excitation current are referred to [33].

### 2.7 Winding conductor mass and load losses

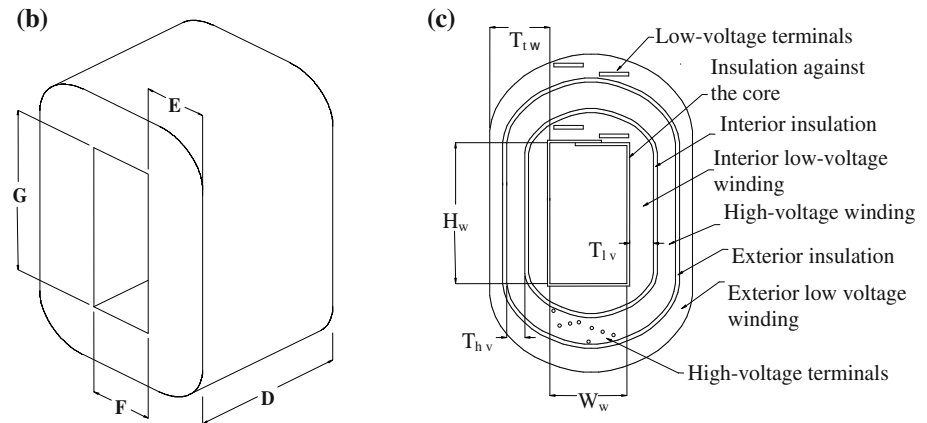
The conductor mass and load losses are derived from:

$$M_{\text{Al}} = \text{MLT}_{\text{LV}} \cdot N_{\text{LV}} \cdot N_{\theta} \cdot S_{\text{Al}} \cdot \rho_{\text{Al}} \cdot 10^{-6} \quad (8a)$$

$$M_{\text{Cu}} = \text{MLT}_{\text{HV}} \cdot N_{\text{LV}} \cdot N_{\theta} \cdot S_{\text{Al}} \cdot \rho_{\text{Cu}} \cdot 10^{-6} \quad (8b)$$

**Fig. 1** Active element.

**a** Single-phase shell-type transformer picture taken during manufacturing; the cores are assembled; **b** core dimensions, **c** low-high-low winding dimensions. High-voltage conductor is made of copper wire and low-voltage conductor is made of sheets of aluminum



$$LL_{Al} = J_{Al}^2 \cdot M_{Al} \cdot W_{d(Al)} \cdot W_k \quad (9a)$$

$$LL_{Cu} = J_{Cu}^2 \cdot M_{Cu} \cdot W_{d(Cu)} \cdot W_k \quad (9b)$$

$$LL = LL_{Al} + LL_{Cu} \quad (9c)$$

The dimensions of the low-high-low winding are shown in Fig. 1c. The interested reader on low-high-low windings of single-phase transformers is referred to [34].

## 2.8 Efficiency

The efficiency is computed for full-load at unity power factor. The efficiency is computed by:

$$\eta = \frac{S_T}{LL + NLL + S_T} 100\% \quad (10a)$$

## 2.9 Total owning cost

The total owning cost takes into account not only the initial transformer cost, but also the cost to operate and maintain the transformer over its life. The TOC is given by:

$$TOC = MMC + A \cdot NLL + B \cdot LL \quad (10b)$$

From [35],  $A = 8.16$  US\$/W (no-load loss cost rate) and  $B = 4.02$  US\$/W (load loss cost rate). MMC is the cost of transformer main materials, i.e., core cost, winding

cost, mineral oil cost, etc. The authors have made a sensitivity analysis fluctuating coefficients  $A$  and  $B$  for selection of core lamination thickness in distribution transformers. If TOC is minimized, 79% of the analyzed transformers have a lower TOC when designed with M3 lamination and 21% when designed with M2 lamination [36].

## 3 Performance constraints

Performance constraints are used to determine the feasibility design region where optimal design parameters of the transformer can be determined. Following are the constraints that are included in the design optimization method.

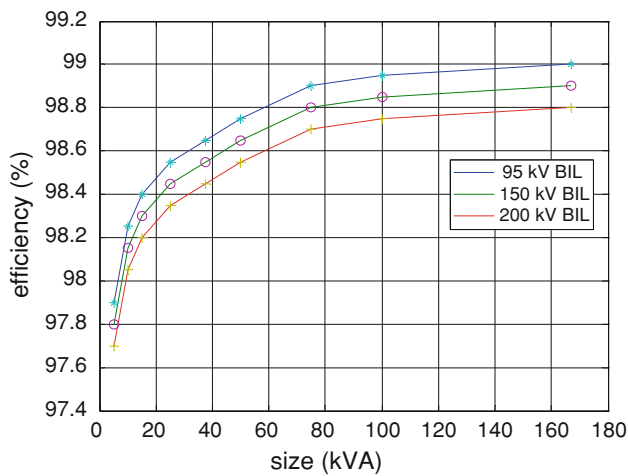
### 3.1 Efficiency

The minimum efficiencies versus transformer rating and insulation class for single-phase transformers can be seen in Fig. 2, part of which has been taken from the Mexican standard [37].

### 3.2 Excitation current

According to [37], the excitation current should not exceed 1.5% in all single-phase and three-phase transformers with capacity greater than 45 kVA. For three-phase transformers





**Fig. 2** Minimum efficiencies required by the Mexican standard for single-phase transformer [37]

**Table 3** Maximum no-load losses (W) and maximum total losses (W) for single-phase transformers [37]

kVA	Basic insulation level					
	95 kV		150 kV		200 kV	
	No load losses	Total losses	No load losses	Total losses	No load losses	Total losses
5	30	107	38	112	63	118
10	47	178	57	188	83	199
15	62	244	75	259	115	275
25	86	368	100	394	145	419
37.5	114	513	130	552	185	590
50	138	633	160	684	210	736

up to 45 kVA, the excitation current should not be larger than 2.0%.

### 3.3 No-load losses and total losses

Table 3 shows the maximum no-load and total loss constraints for distribution transformers.

### 3.4 Impedance

Table 4 shows the impedance constraints for single-phase and three-phase distribution transformers. The impedance depends on the insulation class and the transformer rating.

## 4 Transformer design optimization methodology

The transformer design is an optimization problem of the form: Minimize an objective function, subject to a set of con-

**Table 4** Impedance constraints for single-phase transformer from 5 to 167 kVA [37]

Insulation class (kV)	Impedance (%)
1.2–25	1.5–3.00
25	1.50–3.25
34.5	1.50–3.50

straints: maximum limits on magnetizing current, the no-load losses, total losses, max and min limits for the impedance and a minimum limit for the efficiency. In Sect. 5, we have shown the actual values of the constraints.

The objective function can be one of the following: (1) transformer total owning cost, (2) transformer material cost (conductor cost + core cost + oil cost + insulation cost + tank cost), (3) transformer mass (conductors mass + core mass + oil mass + insulation mass + tank mass); or (4) transformer total losses (no-load losses + load losses). The optimization problem for a specific 25 kVA transformer design example, for all the objective functions, is formulated as:

min TOC

subject to  $\%I < 1.5\%$ ,  $NLL < 86 \text{ W}$ ,

$NLL + LL < 368 \text{ W}$ ,  $1.5 < \%Z < 3.0$ ,

$\eta \geq 98.55\%$  (11)

min Transformer material cost

subject to  $\%I < 1.5\%$ ,  $NLL < 8 \text{ W}$ ,

$NLL + LL < 368 \text{ W}$ ,  $1.5 < \%Z < 3.0$ ,

$\eta \geq 98.55\%$  (12)

min Transformer mass

subject to  $\%I < 1.5\%$ ,  $NLL < 86 \text{ W}$ ,

$NLL + LL < 368 \text{ W}$ ,  $1.5 < \%Z < 3.0$ ,

$\eta \geq 98.55\%$  (13)

min Transformer total losses

subject to  $\%I < 1.5\%$ ,  $NLL < 86 \text{ W}$ ,

$NLL + LL < 368 \text{ W}$ ,  $1.5 < \%Z < 3.0$ ,

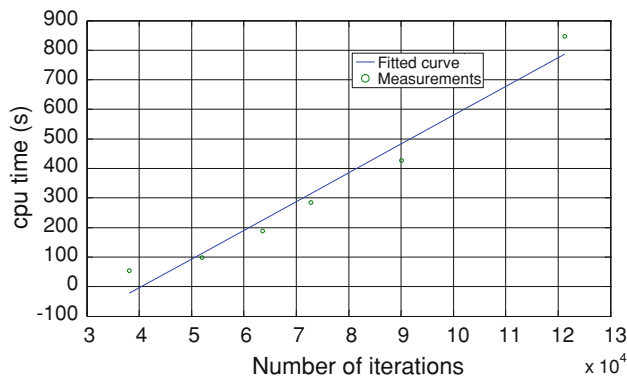
$\eta \geq 98.55\%$  (14)

The process of finding the optimum transformer using the proposed methodology was implemented in MATLAB, which uses the following input data: transformer rating, low voltage, high voltage, connection of low-voltage coil, connection of high-voltage coil and frequency.

The computer program allows the variation of all the design parameters (low-voltage turn number, low-voltage cross-section area, high-voltage conductor, lamination width,

**Table 5** Ranges of solution loop variables

Variable	Initial value	Final value	Alternative values
Low-voltage turns	$81.1988S_T^{-0.4527} - 5$	$81.1988S_T^{-0.4527} + 5$	11
Low-voltage conductor cross-sectional area (mm <sup>2</sup> )	34.29	452.12	7
High-voltage conductor cross-sectional area (mm <sup>2</sup> )	6 AWG	15 AWG	10
Lamination width (mm)	152.4	203.2	3
Magnetic flux density (T)	1.5	1.7	20

**Fig. 3** Computational cost versus number of designs calculated or iterations. (The CPU time =  $0.0097 \times \text{number of designs} - 392.2013$ , i.e., a function of degree 1 that fits the measurements in a least squares sense)

magnetic-flux density) for the ranges shown in Table 5, which permit the investigation of all feasible solutions. For each one of the feasible solutions, the algorithm checks if all the constraints are satisfied, and if they are satisfied the solution is called acceptable. The feasible solutions that violate the constraints are called non-acceptable solutions. From the acceptable solutions, the transformer with the minimum value for the objective function is the optimum transformer.

The number of evaluated designs is calculated using combinational analysis. Based on the last column of Table 5, we have that the number of designs is  $11 \cdot 7 \cdot 10 \cdot 3 \cdot 20 = 46,200$ . The computing time for a transformer design using MATLAB software depends on the number of designs. The tests were made on a computer Intel Celeron D CPU 430@1.80 GHz, 960 MB RAM. In Fig. 3, the computational cost is plotted versus the number of designs. It can be seen that the computational cost is approximately linear with the number of evaluated designs. This is also valid for different transformer ratings.

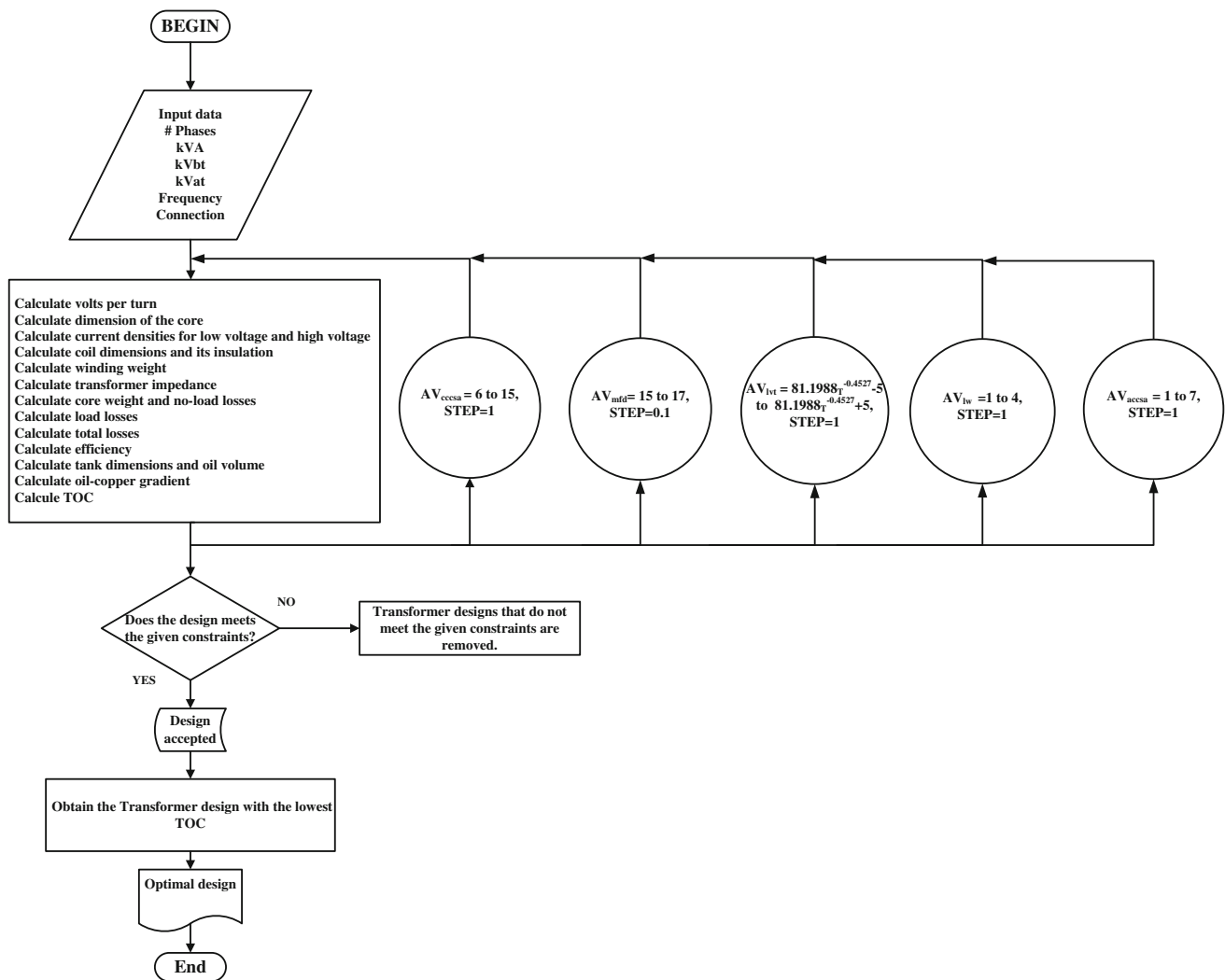
Figure 4 shows the flow chart for optimizing TOC, where  $N_{MFD}$  is the number of possibilities for the magnetic flux

density,  $N_{CG}$  is the number of alternatives for high-voltage conductors,  $N_{LVT}$  is the number of choices for low-voltage turns,  $N_{LW}$  is the number of options for laminations width and  $N_{LVA}$  is the number of substitutions for low-voltage conductors. Other objective functions can substitute TOC in line 19 of Fig. 4.

### 5 Design example: 25 kVA transformer

Table 6 shows the three solutions with the lowest TOC and the three designs with the highest TOC of a total of 955 feasible solutions. It is observed that the total owning cost of the optimum transformer is US\$ 2,267.01. The optimum solution is 28% less expensive than the most expensive one. The proposed methodology was used to design a 25 kVA–13,200 Grd Y/7620 V–240/120 V transformer at 60 Hz. The following design constraints were imposed:  $\%I < 1.5\%$ ,  $NLL < 86 \text{ W}$ ,  $NLL + LL < 368 \text{ W}$ ,  $1.5 < \%Z < 3.0$ ,  $\eta \geq 98.55\%$ . With this information, the transformer design software produces the optimum transformers for different objective functions as shown in Table 7. The minimum TOC transformer is shown in the second column. The results of minimum mass design are given in the third column of Table 7. The transformer with the same input data was designed to minimize total loss and results are shown in column 4. Finally, column 5 of Table 7 presents the optimization of transformer material cost. The problem of minimizing losses is equivalent to the problem of maximizing efficiency. In Table 8, it is shown that there is a small difference between the laboratory and process measurements and design values obtained with this methodology for the 25 kVA transformer using as objective function the TOC.

An advantage of analyzing all possible designs is that running the optimization algorithm multiple times the designer gets the answer for all objective functions without intervention of a designer. Only sorting is needed once the solution is obtained.



**Fig. 4** Simplified flow diagram for transformer optimization using TOC as an objective function

**Table 6** Acceptable solutions sorted by transformer total owning cost

Number	TOC (US\$)	Total material cost (US\$)	Core mass (kg)	Conductor mass (kg)	Core losses (W)	Total losses (W)	$B$ (T)	LV turns	Efficiency (%)
1	2,267.01	668.08	84.83	40.76	85.21	309.41	1.51	19	98.78
2	2,275.7	590.15	83.59	34.49	83.93	332.17	1.51	19	98.69
3	2,277.5	668.31	83.02	41.41	84.84	312.34	1.52	19	98.77
...	...	...	...	...	...	...	...	...	...
953	3,115.56	1,303.68	82.27	93.21	82.63	364.97	1.51	21	98.56
954	3,127.35	1,303.69	82.27	93.21	84.08	366.42	1.52	21	98.56
955	3,139.5	1,303.68	82.27	93.21	82.27	364.61	1.53	21	98.56

## 6 Conclusions

The development and application of an optimal design procedure for shell-type distribution transformers has been

presented. The proposed methodology has many advantages for transformer manufacturers, since they can save design man-hours and they can increase their design capacity. The advantage of this methodology over others is that its



**Table 7** 25 kVA–13,200 Grd Y/7,620 V–240/120 V transformer design at 60 Hz

Parameter	Objective function optimized			
	Minimum TOC	Minimum transformer mass	Minimum total losses	Minimum transformer material cost
No-load losses (W)	85.21	83.06	85.98	83.79
Core mass (kg)	84.83	68.6	87.08	79.14
Core cost (US\$)	192.80	155.89	197.9	179.87
Load losses (W)	224.2	279.27	176.1	282.17
Conductor mass (kg)	40.76	38.81	95.39	29.93
Winding cost (US\$)	414.06	384.53	1,013.94	288.98
LV turns	19	21	22	20
HV turns	1,266	1,399	1,466	1,333
$B$ (T)	1.51	1.61	1.5	1.54
Efficiency (%)	98.78	98.63	98.96	98.58
TOC (US\$)	2,267.01	2,397.9	2,699.44	2,380.05
Transformer mass (kg)	221.50	172.11	267.95	184.89
Total losses (W)	309.41	362.33	262.08	365.96
Transformer material cost (US\$)	668.08	594.89	1,287.66	529.34

**Table 8** Relative errors between measurements and calculations for 25 kVA transformer using as objective function the TOC

Variables	$\frac{ \text{Design value} - \text{measured value} }{\text{measured value}} \cdot 100\%$
Weight of high-voltage conductor	1.1
Weight of low-voltage conductor	1.05
Core weight	0.2
No-load losses	3.4
Load losses	11.4
Percentage of excitation current	13.2
Percentage of impedance	0.3
Efficiency	0.5

codification is very easy. Therefore, it is ideal for use in small transformer manufacturing companies that could implement it in few weeks. Inexperienced engineers can successfully use this software.

The approach of this paper consists in the methodological analysis of all possibilities. Therefore, avoiding the common trial-and-error based on knowledge and experience of transformer designer. In this paper, we have optimized the transformer design using four different objective functions: (1) minimum total owning cost, (2) minimum mass, (3) minimum total losses and (4) minimum transformer material cost. The software was tested by designing a 25 kVA–13,200 Grd Y/7,620 V, 240/120 V, oil-filled, single-phase distribution transformer resulting in a small error with respect to laboratory and process measurements.

## References

- Georgilakis PS (2009) Spotlight on modern transformer design. Springer, London
- Williams SB, Abetti PA, Magnusson EF (1955) How digital computers aid transformer designer. Gen Electr Rev 58:24–25
- Williams SB, Abetti PA, Magnusson EF (1956) Application of digital computers to transformer design. AIEE Trans Part III 75:728–735
- Sharpley W, Oldfield JV (1958) The digital computer applied to the design of large power transformers. Proc IEE Part A 105:112–125
- Williams SB, Abetti PA, Mason HJ (1959) Complete design of power transformers with a large size digital computer. AIEE Trans Part III 77:1282–1291
- Wu HH, Adams R (1970) Transformer design using time-sharing computer. IEEE Trans Magn 6:67
- Andersen OW (1967) Optimum design of electrical machines. IEEE Trans Power Apparatus Syst 86:707–711
- Odessee PH (1974) Transformer design by computer. IEEE Trans Manuf Technol 3:1–17
- Judd FF, Kressler DR (1977) Design optimization of small low-frequency power transformers. IEEE Trans Magn 13:1058–1069
- Rubaai A (1994) Computer aided instruction of power transformer design in the undergraduate power engineering class. IEEE Trans Power Syst 9:1174–1181
- Jewell WT (1990) Transformer design in the undergraduate power engineering laboratory. IEEE Trans Power Syst 5:499–505
- Grady WM, Chan R, Samotyj MJ, Ferraro RJ, Bierschenck JL (1992) A PC-based program for teaching the design and analysis of dry-type transformers. IEEE Trans Power Syst 7:709–717
- Gu WJ, Liu R (1993) A study of volume and weight vs. frequency for high-frequency transformers. In: Proceedings of the IEEE Power Electronics Specialists Conference, pp 1123–1129
- Poloujadoff M, Findlay RD (1986) A procedure for illustrating the effect variations of parameters on optimal transformers design. IEEE Trans Power Syst 1:202–206

15. Andersen OW (1991) Optimized design of electric power equipment. *IEEE Comput Appl Power* V4:11–15
16. Breslin JG, Hurley WG (2003) A web-based system for transformer design. In: KES 2003, LNAI vol 2773, Springer, Berlin, pp 715–721
17. Padma S, Bhuvaneswari R, Subramanian S (2006) Optimal design of power transformer using simulated annealing techniques. In: *Proceedings of IEEE International Conference on Industrial Technology*, vol 2, Mumbai, India, pp 1015–1019
18. Hui L, Li H, Bei H, Shunchang Y (2001) Application research based on improved genetic algorithm for optimum design of power transformers. In: *Proceedings of the International Conference on Electrical Machines and Systems*, vol 1, pp 242–245
19. Nims JW, Smith RE, El-Keib AA (1996) Application of a genetic algorithm to power transformer design. *Electr Power Compon Syst* 24:669–680
20. Singh B, Saxena RB (1987) Optimum design of small distribution transformer using aluminum conductors. *Electr Power Compon Syst* 12:271–280
21. Jabr RA (2005) Application of geometric programming to transformer design. *IEEE Trans Magn* 41:4261–4269
22. Hernandez C, Arjona MA (2008) An intelligent assistant for designing distribution transformers. *Expert Syst Appl* 34:1931–1937
23. Knyazev NM (1997) Steels and alloys for high-precision technology. *Metal Sci Heat Treat* 39:257–261
24. Georgilakis PS, Tsili MA, Souflaris AT (2007) A heuristic solution to the transformer manufacturing cost optimization problem. *J Mater Process Technol* 181:260–266
25. Rice JR (1983) Numerical methods, software, and analysis, international student edition. McGraw-Hill, Singapore
26. Lur'e AI (2008) Transformer connection under no-load and short-circuit events. *Russ Electr Eng* 79:57–70
27. Say MG (1958) The performance and design of alternating current machines, 3rd edn. Pitman Paperbacks, Great Britain
28. Olivares JC, Kulkarni SV, Cañedo J, Escarela-Perez R, Driesen J, Moreno P (2003) Impact of the joints design parameters of the wound core in distribution transformer losses. *Acta Press* 23:151–157
29. Schulz R, Alexandrov N, Tdtreault J, Simoneau R, Roberge R (1995) Development and application of amorphous core-distribution transformers in Québec. *J Mater Eng Perform* 4:430–434
30. Cogent Power Inc. (2001) Catalog of design and manufacture of transformer cores, Canada
31. Thompson JE (1967) A review of some interdisciplinary work on grain-oriented silicon-iron and its use in large power transformers. *J Mater Sci* 2:395–403
32. AT1 Allegheny Ludlum (1996) Technical data silectron, grain-oriented silicon steel, USA, p 14
33. Theocharis AD, Milias-Argitis J, Zacharias Th (2008) Single-phase transformer model including magnetic hysteresis and eddy currents. *Electr Eng* 90:229–241
34. Bean RL, Chackan N, Moore HR Jr, Wentz EC (1959) Transformer for the electric power industry, Westinghouse Electric Corporation. McGraw-Hill Book Company, New York
35. Arce-Salas JJ (2004) Factors of evaluation for transformers. Comisión Federal de Electricidad, Subdirección de Distribución, Circular 31.312.02. [http://desarrollo.azc.uam.mx/curso/tesis/Factores\\_de\\_evaluacion.pdf](http://desarrollo.azc.uam.mx/curso/tesis/Factores_de_evaluacion.pdf). Accessed Dec 2009
36. Olivares-Galvan JC, Georgilakis PS, Campero-Littlewood E, Escarela-Perez R (2010) Core lamination selection for distribution transformers based on sensitivity analysis. *Electr Eng* (submitted). [http://desarrollo.azc.uam.mx/curso/tesis/Lamination\\_paper\\_EPCS\\_1\\_Dec\\_2009\\_revised\\_paper.pdf](http://desarrollo.azc.uam.mx/curso/tesis/Lamination_paper_EPCS_1_Dec_2009_revised_paper.pdf). Accessed Feb 2010
37. Norma Mexicana ANCE, NMX-J-116 ANCE (2005) Transformadores de distribución tipo poste y tipo subestación, Especificaciones