Ant Colony Optimisation solution to distribution transformer planning problem

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Abstract: This paper proposes a stochastic optimisation method, based on Ant Colony Optimisation (ACO), for the optimal choice of transformer sizes to be installed in a distribution network. This method is properly introduced to the solution of the Optimal Transformer Sizing (OTS) problem, taking into account the constraints imposed by the load the transformer serves throughout its lifetime and the possible transformer thermal overloading. The possibility to upgrade the transformer size one or more times throughout the study period results in different sizing paths taking into account the transformer capital cost the energy loss cost during the study period.

Keywords: transformers; OTS; optimal transformer sizing; ACO; ant colony optimisation; thermal loading; energy loss cost; distribution network planning.

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1 Introduction

The objective of the Optimal Transformer Sizing (OTS) problem in a multi-year planning period is to select the transformer sizes (i.e., rated capacities) and the years of transformer installation so as to serve a distribution substation load at the minimum total cost (i.e., sum of transformer purchasing cost plus transformer energy loss cost). Oversizing a transformer can result in higher no-load losses, whereas undersizing a transformer can result in higher load losses. The proper size of a transformer depends on various economic factors as well as its losses. Owing to the large amount of installed distribution transformers in power systems, the impact of their proper sizing, also referred to as kVA sizing, is particularly crucial to their design and can result in significant economic benefits to electric utilities and other transformer users. There are several factors involved in the process of sizing a transformer, e.g., expected future growth of the load to be served, installation altitude, ambient temperature, insulation, number of phases, efficiency, losses, capital cost throughout the transformer commission time, cost of the energy and interest rate. During the solution of the transformer sizing problem, careful consideration must be given to the fact that the possible transformer capacities are discrete instead of continuous and that the option to upgrade the transformer size one

or more times throughout the study period can result in different sizing strategies that have to be considered as well.

Therefore, it is clear that the optimal choice of the transformer size cannot be straightforward, by installing a transformer of sufficient capacity to meet the load demand in the final year of the study period, a strategy usually adopted by electric utilities. On the other hand, it must be treated as a constrained minimisation problem, taking into account all possible sizing strategies. Deterministic optimisation methods may be used for the solution of this problem, such as dynamic programming (Chen and Wu, 1998) or integer programming (Jovanovic, 2003). However, the wide spectrum of transformer sizes and various load types involved in the electric utility distribution system make the transformer sizing a difficult combinatorial optimisation problem, since the space of solutions is huge. This is why stochastic optimisation methods may prove to provide more robust solutions. The use of such methods in transformer sizing is not encountered in the technical literature or is partially included in the analysis, e.g., as a tool to forecast the loads to be served (Robinson et al., 2006).

In this paper, the OTS problem is solved by means of the heuristic Ant System method using the Elitist strategy, called Elitist Ant System (EAS). EAS belongs to the family of ACO algorithms. Dorigo has proposed the first EAS in his PhD Thesis (Dorigo, 1992) and later on in Dorigo and Stützle (2004) and Dorigo et al. (1996). The EAS is a biologically inspired meta-heuristics method in which a colony of artificial ants cooperates in finding good solutions to difficult discrete optimisation problems. Cooperation is the key design component of ACO algorithms, i.e., allocation of the computational resources to a set of relatively simple agents (artificial ants) that communicate indirectly by stigmergy (Goss et al., 1989) i.e., by indirect communication mediated by the environment. In other words, a set of artificial ants cooperates in dealing with a problem by exchanging information via pheromones deposited on a graph.

In the bibliography, ACO algorithms have been applied to solve a variety of well-known combinatorial optimisation problems, such as routing problem (Stützle and Hoos, 2000), assignment problem (Maniezzo, 1999), scheduling problem (Merkle et al., 2002), fault section estimation problem (Chang et al., 1999), ship berthing problem (Tong et al., 1999) and subset (Leguizamon and Michalewicz, 1999) problems.

In power systems, ACO algorithms have been applied to solve the optimum generation scheduling problem (Yu et al., 1998; Huang, 2001), combined heat and power economic dispatch problem (Song et al., 1999), and the optimum switch relocation and network reconfiguration problems for distribution systems (Teng and Liu, 2003; Jeon et al., 2003; Vlachogiannis et al., 2005). More details on ACO implementation in the solution of other problems are described in Blum (2005).

EAS was introduced in the solution of the OTS problem in case of one (three-phase, oil-immersed, self-cooled (ONAN)) distribution transformer with constant economic factors in Amoiralis et al. (2007), whereas this paper extends the use of EAS, taking into account all details of the economic analysis, such as the inflation rate that influences the energy loss cost and the transformer investment as well as the installation and depreciation cost in a real distribution network, constituting an efficient methodology for transformer planning. The OTS problem is solved as a constrained optimisation problem, following the procedure described in Section 2. The most important constraint is the load to be served by the transformer throughout its life cycle, as well as the respective transformer thermal loading, evaluated through detailed calculation of the winding temperature variation, as described in Section 3. The objective function to be minimised

includes the transformer capital cost as well as the energy loss cost (Section 4). The EAS method (Section 5) is adapted to the considered problem and the results of its implementation (Section 6) demonstrate its suitability for the solution of OTS and the benefits from its application in comparison with simplified transformer sizing approaches.

2 Overview of the proposed method

The OTS problem consists in finding the proper capacities and technical characteristics of transformers to be installed in a distribution network so that the overall installation and energy loss cost over the study period is minimised and the peak loading condition is met (Chen and Wu, 1998). The proposed solution to the OTS problem is graphically shown in Figure 1.

Figure 1 Flow chart illustrating the main steps of the proposed method (see online version for colours)
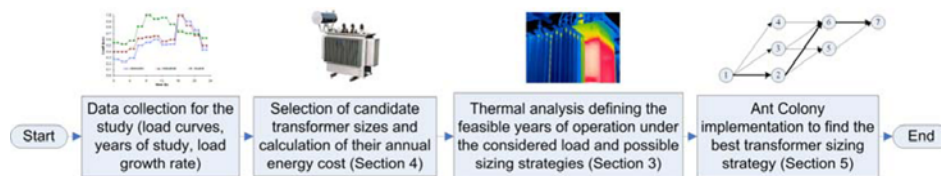
The process of the OTS is realised through the following steps, described in the diagram of Figure 1:

- Collection of data concerning the special characteristics of the load at each substation of the considered network. These data consist of the typical daily load curve and the expected load growth rate over the study period.

- Selection of possible candidate transformer sizes according to tables of standard transformer sizes, although non-standard sizes can also be considered.

- Determination of the feasible number of years that each potential transformer can serve the load of each substation, based on thermal calculations (Section 3).

- Determination of the possible transformer sizing strategies (per substation) throughout the study period, consisting of combinations of the potential transformer capacities and the years that they may operate to serve the considered load.

- Calculation of the energy loss cost for each transformer (through the method presented in Section 4) of the network for the periods defined by the strategies of the previous step.

- Creation of the graph with all possible combinations of transformer sizes and years of replacement, for the solution of the OTS problem with the use of EAS, according to the procedure described in Section 5, for each substation of the studied distribution network. The cost of each path consists of the energy loss cost of the installed transformer S_i for the period of years of operation until the upgrade to a

larger capacity and the bid price of the new transformer S_j to be installed minus the remaining value of transformer S_i.

- Selection of the least-cost transformer sizing path per substation, with the use of EAS. This path corresponds to the OTS strategy.

3   Calculation of transformer thermal loading

The transformer thermal calculation is implemented according to the guidelines imposed by the IEEE Standard C57.91-1995 (R2002) (IEEE Std C57.91, 2002). The main equations used in these calculations are described in the following (all temperatures are expressed in °C).

3.1   Top-oil temperature calculation

The top-oil temperature rise $\Delta\Theta_{TO}$ at a time after a step load change is given by the following equation:

$$\Delta\Theta_{TO} = (\Delta\Theta_{TO,U} - \Delta\Theta_{TO,i}) \cdot (1 - e^{-1/\tau_{TO}}) + \Delta\Theta_{TO,i} \tag{1}$$

where $\Delta\Theta_{TO,i}$ and $\Delta\Theta_{TO,U}$ are the initial and ultimate top-oil rise over ambient temperature during the considered time period, whereas τ_{TO} is the oil time constant (in hours) for the considered load, deriving from the value of the time constant at the rated load $\tau_{TO,R}$, calculated through equation (2) (IEEE Std C57.91, 2002),

$$\tau_{TO} = \tau_{TO,R} \frac{\left(\dfrac{\Delta\Theta_{TO,U}}{\Delta\Theta_{TO,R}}\right) - \left(\dfrac{\Delta\Theta_{TO,i}}{\Delta\Theta_{TO,R}}\right)}{\left(\dfrac{\Delta\Theta_{TO,U}}{\Delta\Theta_{TO,R}}\right)^{\frac{1}{n}} - \left(\dfrac{\Delta\Theta_{TO,i}}{\Delta\Theta_{TO,R}}\right)^{\frac{1}{n}}} \tag{2}$$

where n is an empirical exponent equal to 0.8 for self-cooled transformers.

The constant $\tau_{TO,R}$ is a function of the weight of the core and coil assembly WCC (in kilograms), the weight of tank and fittings WT (in kilograms), the volume of oil VO (in litres), the top-oil rise over ambient temperature at rated load $\Delta\Theta_{TO,R}$ (equal to 60°C for the considered distribution transformers) and the total loss at rated load $P_{T,R}$ (in watts) according to the following equation (applicable to self-cooled transformers):

$$\tau_{TO,R} = (0.0272 \cdot WCC + 0.01814 \cdot WT + 5.034 \cdot VO) \frac{\Delta\Theta_{TO,R}}{P_{T,R}}. \tag{3}$$

A typical value of $\tau_{TO,R}$ equal to 2.5 h can be considered for distribution transformers larger than 200 kVA, whereas for smaller capacities, a value of 3.5 h may be adopted (Heunis and Herman, 2004).

The initial top-oil rise over ambient temperature $\Delta\Theta_{TO,i}$ is equal to the value $\Delta\Theta_{TO}$ calculated at the previous interval of the considered load cycle. The ultimate top-oil rise over ambient temperature is calculated with the use of the top-oil rise over ambient

temperature at rated load $\Delta\Theta_{TO,R}$ (equal to 60°C for the considered distribution transformers), the ratio K of the load at the considered interval to the rated load and the ratio R of the load loss at rated load to no-load loss:

$$\Delta\Theta_{TO,U} = \Delta\Theta_{TO,R} \cdot \left(\frac{K^2 \cdot R + 1}{R + 1} \right)^n . \tag{4}$$

For the first interval of the studied load cycle, an initial value of $\Delta\Theta_{TO,i}$ equal to zero may be arbitrarily chosen. After completing the above-mentioned calculations for all the intervals of the load cycle, the final $\Delta\Theta_{TO,U}$ of the cycle can be used as the new value of $\Delta\Theta_{TO,i}$ of the first interval and the process may be repeated to obtain stable temperature profiles.

3.2 Winding hottest spot temperature calculation

The winding hottest spot temperature rise over top-oil temperature $\Delta\Theta_H$ at a time after a step load change is given by the following equation:

$$\Delta\Theta_H = (\Delta\Theta_{H,U} - \Delta\Theta_{H,i}) \cdot \left(1 - e^{\frac{-1}{\tau_w}} \right) + \Delta\Theta_{H,i} \tag{5}$$

where $\Delta\Theta_{H,i}$ and $\Delta\Theta_{H,U}$ represent the initial and ultimate winding hottest spot temperature rise over top-oil temperature during the considered time period, respectively, and τ_w stands for the winding time constant at hot-spot location (in hours). The typical value of τ_w is less than 0.1 h (Schneider and Hoad, 1992), thus equation (3) may be simplified to the following equation:

$$\Delta\Theta_H = \Delta\Theta_{H,U} . \tag{6}$$

The ultimate winding hottest spot temperature rise over top-oil temperature is:

$$\Delta\Theta_{H,U} = \Delta\Theta_{H,R} \cdot K^{2m} , \tag{7}$$

where $\Delta\Theta_{H,R}$ is the winding hottest spot temperature rise over top-oil temperature at the rated load (a value of 15°C is suggested by the standard, when the actual test values are not available) and m is an empirical factor, equal to 0.8 for self-cooled transformers.

Finally, the winding hottest spot temperature at the considered interval is the sum of $\Delta\Theta_{TO}, \Delta\Theta_H,$ and the average ambient temperature Θ_A during the cycle:

$$\Theta_H = \Theta_A + \Delta\Theta_{TO} + \Delta\Theta_H . \tag{8}$$

3.3 Insulation aging

Although the IEEE standard specifies that operation at hottest spot temperature above 140°C results in potential risks for the transformer integrity, this value must not be considered as the maximum one during the calculations carried out for the determination of the transformer loading limits. This is because thermal aging is a cumulative process; therefore, operation above the rating should be examined in conjunction with its

consequences upon the normal life expectancy of the transformer. For the study of this paper, a maximum hot-spot temperature of 120°C has been chosen, based on the relative aging rate of the insulation in the transformer (Heunis and Herman, 2004).

3.4   Overloading capability

To determine the transformer loading limits, the calculation of the hottest spot temperature is repeated for each year of the study period, at an hourly basis, according to the daily load curve. To remain on the safe side, the peak load curve of the considered year is used in the calculations. The yearly load growth rate s is taken into account for the derivation of the per unit load K_t^k of hour t at year k of the study according to the per unit load K_t^0 of hour t at year 0:

$$K_t^k = K_t^0 \cdot (1+s)^k. \tag{9}$$

Figure 2 shows the winding hottest spot temperature variation for six distribution transformers of rated capacity 160, 250, 300, 400, 500 and 630 kVA and ratio R of the load loss at rated load to no-load loss equal to 5.61, 5.23, 5.68, 4.73, 5.44 and 7.11, respectively, serving a residential load of initial peak value equal to 230 kVA, at the 15th year of the study period, for a load growth rate of 4%. The scale of the left axis in Figure 2 corresponds to the values of the load curve K_t^k (expressed in kVA, since the per-unit values are different on the basis of the rated capacity of each transformer), illustrated as bar diagram. The scale of the right axis in Figure 2 indicates the winding hottest spot variation, in °C (corresponding to the six transformer ratings of Figure 2). The thermal calculation is based on a typical load curve of a domestic consumer, with the use of an ambient temperature equal to 40°C. As can be observed from Figure 2, the 160, 250 and 300 kVA transformers overcome the limit of 120°C so they are not suitable to serve the load at the 15th year of the study period.

Figure 2   Winding hottest spot temperature variation of six transformer (TF) ratings
(see online version for colours)
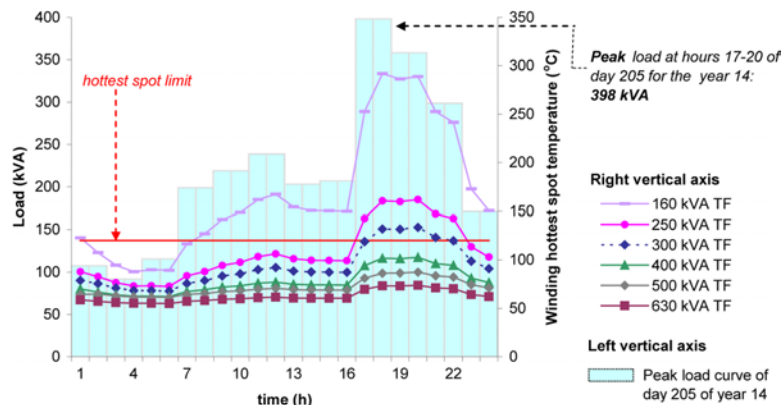
4 Calculation of transformer energy loss cost

The calculation of annual transformer energy loss cost of the potential sizing paths is realised with the use of the energy corresponding to the transformer No-Load Loss (*NLL*) E_{NLL} (in kWh) for each year of operation and the energy corresponding to the Load Loss (*LL*), E_{LL}^k (in kWh) for each k th year of operation. These energies are calculated according to equations (10) and (11), respectively:

$$E_{NLL} = NLL \cdot HPY, \tag{10}$$

$$E_{LL}^k = LL \cdot \left[l_f \cdot \frac{S_{max,0}^l}{S_{nom}^l} (1+s)^k \right]^2 \cdot HPY. \tag{11}$$

where $S_{max,0}^l$ is the initial peak load of the substation load type l (in kVA), S_{nom}^l is the nominal power of the transformer that serves load type l (in kVA), *HPY* is the number of hours per year, equal to 8760, and l_f is the load factor, i.e., the mean transformer loading over its lifetime (derived from the load curve of each consumer type served by the considered substation). The cost of total energy corresponding to the transformer *NLL* for each k th year C_{NLL}^k (in €) and the cost of energy corresponding to the transformer *LL* for each k th year C_{LL}^k (in €) are calculated as follows:

$$C_{NLL}^k = E_{NLL} \cdot CYEC^k \tag{12}$$

$$C_{LL}^k = E_{LL}^k \cdot CYEC^k \tag{13}$$

where $CYEC^k$ denotes the present value of the energy cost (in €/kWh) at the k th year. Finally, the total cost of the transformer energy loss C_L^k for the k th year is given by:

$$C_L^k = C_{NLL}^k + C_{LL}^k. \tag{14}$$

5 Elitist Ant System method

5.1 Mechanism of EAS algorithm

The EAS is an evolutionary computation optimisation method based on ants' collective problem-solving ability. This global stochastic search method is inspired by the ability of a colony of ants to identify the shortest route between the nest and a food source, without using visual cues.

The operation mode of EAS algorithm is as follows: the artificial ants of the colony move, concurrently and asynchronously, through adjacent states of a problem, which can be represented in the form of a weighted graph. This movement is made according to a transition rule, called *random proportional* rule, through a stochastic mechanism. When ant k is in node i and has so far constructed the partial solution s^p, the probability of going to node j is given by:

$$p_{ij}^k = \begin{cases} \dfrac{\tau_{ij}^\alpha + n_{ij}^\beta}{\sum_{c_{il} \in N(s^P)} \tau_{il}^\alpha + n_{il}^\beta}, & \text{if } c_{ij} \in N(s^p) \\ 0, & \text{otherwise} \end{cases} \qquad (15)$$

where $N(s^p)$ is the set of feasible nodes when being in node i, i.e., edges (i, l) where l is the node not yet visited by the ant k. The parameters α and β control the relative importance of the pheromone vs. the heuristic information value η_{ij}, given by:

$$n_{ij} = \frac{1}{d_{ij}} \qquad (16)$$

where d_{ij} is the weight of each edge. Regarding parameters α and β, their role is as follows: if $\alpha = 0$, those nodes with better heuristic preference have a higher probability of being selected. However, if $\beta = 0$, only the pheromone trails are considered to guide the constructive process, which can cause a quick stagnation, i.e., a situation where the pheromone trails associated with some transitions are significantly higher than the remainder, thus making the ants build the same solutions. Hence, there is a need to establish a proper balance between the importance of heuristic and pheromone trail information.

Individual ants contribute their own knowledge to other ants in the colony by depositing pheromones, which act as chemical 'markers' along the paths they traverse. Through indirect communication with other ants via foraging behaviour, a colony of ants can establish the shortest path between the nest and the food source over time with a positive feedback loop known as stigmergy. As individual ants traverse a path, pheromones are deposited along the trail, altering the overall pheromone density. More trips can be made along shorter paths and the resulting increase in pheromone density attracts other ants to these paths. The main characteristic of the EAS technique is that (at each iteration) the pheromone values are updated by all the k ants that have built a solution in the iteration itself. The pheromone τ_{ij}, associated with the edge joining nodes i and j, is updated as follows (Dorigo and Stützle, 2004; Bonabeau et al., 1999):

$$\tau_{ij} = (1 - \rho) \cdot \tau_{ij} + \sum_{m=1}^{k} \Delta\tau_{ij}^k + \varepsilon \cdot \tau_{ij}^{\text{elite}} \qquad (17)$$

where $\rho \in (0, 1)$ is the evaporation rate, k is the number of ants, ε is the number of elitist ants, and $\Delta\tau_{ij}^k$ is the quantity of pheromone laid on edge (i, j) by ant k, calculated as follows:

$$\Delta\tau_{ij}^k = \begin{cases} \dfrac{Q}{L_k}, & \text{if ant } k \text{ used edge } (i, j) \text{ in its tour} \\ 0, & \text{otherwise} \end{cases} \qquad (18)$$

where Q is a constant for pheromone update and L_k is the length (or the weight of the edge) of the tour constructed by ant k. Furthermore, shorter paths will tend to have higher pheromone densities than longer paths since pheromone density decreases over time owing to evaporation (Dorigo and Stützle, 2004; Dorigo et al., 1996). This shortest path represents the global optimal solution and all the possible paths represent the feasible region of the problem.

5.2 OTS implementation using the EAS algorithm

In this work, our interest lies in finding the optimum choice of distribution transformers capacity sizing, so as to meet the load demand for all the years of the study period. To achieve so, a graph shown in Figure 3 is constructed, representing the sizing paths.

**Figure 3** The directed graph used for the OTS problem
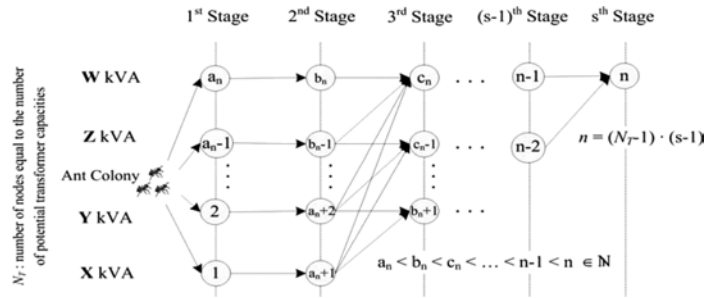
The graph has *s* stages and each stage indicates a time period (in years) the limits of which are defined by the need to replace one of the considered transformer sizes owing to violation of its thermal loading limits. Therefore, stage *s* has one node less in comparison with stage $(s - 1)$, stage $(s - 2)$ has one node less in comparison with stage $(s - 1)$, etc. The first stage indicates the beginning of the study, comprising number of nodes equal to the number of potential transformer capacities N_T, whereas *s* represents the end of the study period (consisting of the largest necessary rated capacity able to serve the load at the final year of the study). Symbols *X*, *Y*, *Z*, *W* refer to the different rated powers $(X < Y < Z < W)$. It is important to point out that it is meaningless to connect for example node 1 to node $(c - 1)$ because it corresponds to installation of a transformer with lower rated power than the one already in operation (only upgrade of the sizes can be considered). Furthermore, the arcs between the nodes are directed from the previous stage to the next one (backward movement is not allowed) since each stage represents a forward step in the time of the study. Nodes 1 to a_n (Figure 3) are designated as the source nodes corresponding to each potential transformer size and node *n* is designated as the destination node (Figure 3). The use of multiple source nodes enables the examination of more potential solutions of the problem. The numbering of *n* nodes of the graph of Figure 3 is realised upwards for each stage, i.e., the bottom node of the 1st stage is node 1, whereas the last node of the 1st stage is a_n (where a_n corresponds to the number of nodes of the 1st stage). Accordingly, the 2nd stage begins with node $a_n + 1$ whereas its last node is numbered b_n (where b_n is the number of nodes of the 3rd stage), 3rd stage begins with node $b_n + 1$ and ends at node c_n (where c_n is the number of nodes of the 3rd stage), etc., concluding to the *s* th stage, which comprises only node *n*. The objective of the colony agents is to find the least-cost path between nodes that belong to first stage and node *n* that belongs to *s* th stage. Three quantities are associated with each arc: the arc weight, the pheromone strength and the agent learning parameter. On the basis of these characteristics, the weight of each arc is calculated.

6   Results and discussion

The proposed method is applied for the optimal choice of the transformer sizes of a practical distribution network. Figure 4 illustrates the single line diagram of the examined distribution network, corresponding to a typical 20 kV network encountered in rural areas of Greece. A 150/20 kV power transformer supplies a distribution feeder comprising mostly overhead lines, but also underground cable segments. Sixteen of the 23 nodes in the network of Figure 4, symbolised as $N_1 - N_{23}$, correspond to distribution substations, whereas the remaining nodes correspond to intersection points. The type of loads as well as their initial peak value (at the year 0 of the study period) are indicated on the diagram (Figure 4). Three consumer types are encountered in the nodes of Figure 4, namely domestic, tourist and industrial, with daily loading profiles that correspond to load factor l_f equal to 0.55, 0.75 and 0.61, respectively. The load of each node is served by a distribution substation, comprising a 20/0.4 kV distribution transformer, as illustrated in the diagram.

Figure 4   Single line diagram of the examined distribution network
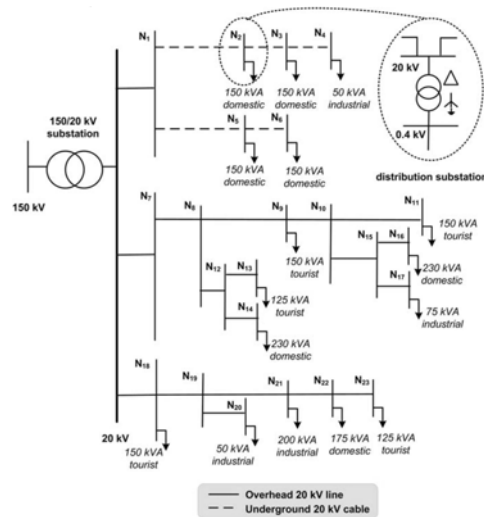
Depending on the load type and peak value, the substations of Figure 4 may be divided into the following seven types, designating the number of transformer sizing strategies that must be defined during the solution of the OTS problem:

1   *Type 1* (nodes N_2, N_3, N_5 and N_6): Domestic load of initial peak value equal to 150 kVA (at the year 1 of the study) with a growth rate equal to 4% for 29 years, resulting in final peak value of 467.80 kVA (at the 30th year of the study).

2   *Type 2* (nodes N_9, N_{11} and N_{18}): Tourist load of initial peak value equal to 150 kVA with a growth rate equal to 4.5% for 29 years, resulting in final peak value of 537.61 kVA (at the 30th year of the study).

3   *Type 3* (nodes N_{13} and N_{23}): Tourist load of initial peak value equal to 125 kVA with a growth rate equal to 4.5% for 29 years, resulting in final peak value of 448.00 kVA (at the 30th year of the study).

4   *Type 4* (node N_{22}): Domestic load of initial peak value equal to 175 kVA
     with a growth rate equal to 4% for 29 years, resulting in final peak value
     of 545.76 kVA (at the 30th year of the study).

5   *Type 5* (node N_{21}): Industrial load of initial peak value equal to 200 kVA
     with a growth rate equal to 4% for 29 years, resulting in final peak value
     of 623.73 kVA (at the 30th year of the study).

6   *Type 6* (nodes N_{14} and N_{16}): Domestic load of initial peak value equal to 230 kVA
     with a growth rate equal to 4% for 29 years, resulting in final peak value of
     717.29 kVA (at the 30th year of the study).

7   *Type 7* (nodes N_4 and N_{20}): Industrial load of initial peak value equal to 50 kVA
     with a growth rate equal to 2.7% for 29 years, resulting in final peak value
     of 108.27 kVA (at the 30th year of the study).

8   *Type 8* (node N_{17}): Industrial load of initial peak value equal to 75 kVA
     with a growth rate equal to 2.7% for 29 years, resulting in final peak value
     of 162.41 kVA (at the 30th year of the study).

In the case of Type 1–6 substations, six transformer ratings are considered, namely 160, 250, 300, 400, 500 and 630 kVA. In the case of Type 7–8 substations, owing to the smaller initial peak value and growth rate of the considered load, intermediate capacities (between 160 and 250 kVA) were included in the study, resulting in a range of five ratings consisting of 160, 190, 220, 250 and 300 kVA. The ratings of 400, 500 and 630 kVA were not included in the range for this type of substation, since their capacities are significantly larger than the served load. Table 1 lists the main technical characteristics and bid price of the considered transformers. Initially, thermal calculation study was carried out to find the exact periods where each transformer can meet the load expectations of the above six types of substations. The calculations described in Section 3 and illustrated in Figure 2 were repeated for the six transformers and each year of the study, resulting in the time periods of Table 1 (indicated as years of thermal durability, i.e., years that the transformer is able to withstand the respective thermal loading). According to Table 1, transformers up to 500 kVA are suitable to serve the loads of Type 1–4 substations, since their thermal durability covers the study period (therefore, the thermal durability of 630 kVA transformers is not examined in these cases). In the case of Type 5–6 substations, transformers of 160 kVA cannot be used since the initial peak load results in excessive thermal overloading, and only the ratings 250–630 kVA can be considered. In the case of Type 7–8 substations, all considered ratings are suitable to serve the load up to the final year of the study. The periods derived in Table 1 were used to define the stages of the graph of Figure 3. To define the weight of each arc in the graph of Figure 3, the energy loss cost calculation of each transformer for the studied period was based on the annual energy loss cost calculation described in Section 4. The energy loss cost $C_L^{i \rightarrow j}$ corresponding to the transition from node i (p th year of the study period) to j (q th year of the study period) is computed by:

$$C_L^{i \rightarrow j} = \sum_{k=p+1}^{q} C_L^k. \tag{19}$$

Table 1   Technical parameters and thermal withstand of the transformers used
in the solution of the OTS problem

Transformer technical parameters				Transformer thermal withstand (yr) per type of substation load						
Size (kVA)	Bid price (€)	NLL (kW)	LL (kW)	Type 1	Type 2	Type 3	Type 4	Type 5	Type 7	Type 8
160	5275	0.454	2.544	5	4	8	3	0	30	30
190	5801	0.537	2.920	–	–	–	–	–	30	30
220	6327	0.619	3.296	–	–	–	–	–	30	30
250	6853	0.702	3.672	16	14	18	14	10	30	30
300	6932	0.738	4.186	21	18	23	19	15	30	30
400	9203	0.991	4.684	28	25	28	26	22	–	–
500	10,296	1.061	5.771	30	30	30	30	28	–	–
630	12,197	1.094	7.774	–	–	–	–	30	–	–

When the transition from node i to j corresponds to transformer size upgrade from S_i to S_j, the installation cost $I_{i \to j}$ derives from:

$$I_{i \to j} = BP_{S_j}^p - R_{S_i}^p, \tag{20}$$

$$R_{S_i}^p = BP_{S_i} \cdot \left[\frac{(1+r)^N - (1+r)^m}{(1+r)^N - 1} \right], \tag{21}$$

where $BP_{S_j}^p$ is the present value of the bid price of the transformer to be installed at the p th year of the study period, $R_{S_i}^p$ is the remaining value of the uninstalled transformer, BP_{S_i} is the transformer S_i bid price, r is the inflation rate, N are the years of the transformer lifetime and m are the years that the transformer was under service.

Table 2 lists the cost of four arcs of Figure 5, calculated according to Sections 4 and 6.

Figure 5   The directed graph used by the proposed ACO method for the Type 5 substation load (see online version for colours)
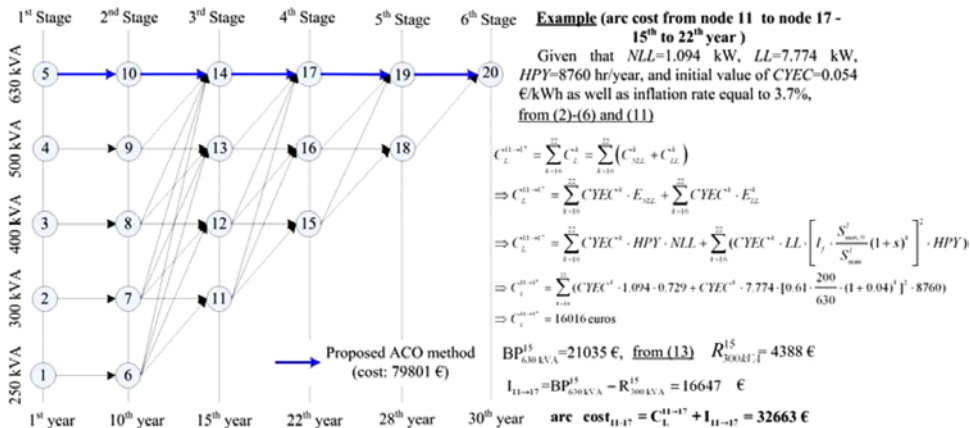
Table 2 Cost of indicative arcs in the graph of Figure 5 (substation Type 5)

Arc	Cost of the arc	Value (€)	Arc	Cost of the arc	Value (€)
$2 \rightarrow 7$	$C_L^{2 \rightarrow 7} + BP_{300}^{\text{1st-year}}$	17,972	$17 \rightarrow 19$	$C_L^{17 \rightarrow 19}$	23,601
$11 \rightarrow 17$	$C_L^{11 \rightarrow 17} + I_{11 \rightarrow 17}$	32,663	$19 \rightarrow 20$	$C_L^{19 \rightarrow 20}$	11,185
$1 \rightarrow 6$	$C_L^{1 \rightarrow 6} + BP_{250}^{\text{1st-year}}$	19,389	$12 \rightarrow 15$	$C_L^{12 \rightarrow 15}$	19,673
$6 \rightarrow 11$	$C_L^{6 \rightarrow 11} + I_{6 \rightarrow 11}$	14,816	$15 \rightarrow 19$	$C_L^{15 \rightarrow 19} + I_{15 \rightarrow 19}$	47,230
$11 \rightarrow 15$	$C_L^{11 \rightarrow 15} + I_{11 \rightarrow 15}$	31,156	$5 \rightarrow 10$	$C_L^{5 \rightarrow 10} + BP_{630}^{\text{1st-year}}$	21,738
$15 \rightarrow 18$	$C_L^{15 \rightarrow 18} + I_{15 \rightarrow 18}$	45,575	$10 \rightarrow 14$	$C_L^{10 \rightarrow 14}$	72,618
$18 \rightarrow 20$	$C_L^{18 \rightarrow 20} + I_{18 \rightarrow 20}$	43,831	$14 \rightarrow 17$	$C_L^{14 \rightarrow 17}$	16,016

Figures 5–7 illustrate the graph used by the proposed ACO method for the solution of the OTS problem for Type 5, Type 7 and Type 8 substation load of Figure 4, respectively, using the transformers of Table 1. The graphs of Figures 6 and 7 comprise more nodes than the one of Figure 5, since all candidate transformer sizes can serve the load up to the final year of the study, therefore resulting in the presence of five nodes (i.e., five capacities) at each stage, an overall of 30 nodes. This configuration is consistent to the general graph of Figure 3, where, for the purpose of generality, the lack of several nodes in the ultimate stages is indicated. Moreover, since, according to Table 1, the years to serve the load are equal for all transformer capacities (i.e., 30 years), the thermal durability could not be used as a criterion for the derivation of the stages of Figures 6 and 7. For this purpose, the whole period of the study was divided to five equal periods of six years, to derive the same number of stages as the ones in the graphs corresponding to the other types of substations. We tested several values for each parameter, i.e., $\alpha \in \{0, 0.5, 1, 2, 5\}$, $\beta \in \{0, 0.5, 1, 2, 5\}$, $\rho \in \{0.1, 0.3, 0.5, 0.7, 1\}$. Table 3 includes the data of the optimal sizing strategies for each type of substation of Figure 4 and Table 1. The optimal solutions of Table 3 were obtained using $k = 20$, $\alpha = 2$, $\beta = 0.5$, $\rho = 0.5$, $Q = 2.7$, max iterations = 2000. According to Figure 5, the optimal sizing path yielded by the proposed method corresponds to installation of the largest rated capacity that can serve the expected load at the end of the study period. The same optimal path is selected for Types 1–4 and 6 of the substation loads of the considered network, corresponding to the costs listed in Table 3. This is because transformers with rated power significantly larger than the served load operate under low load current, thus consuming less annual energy losses (and consequently having less annual energy loss cost), in comparison with transformers of rated power close to the served load. In the case of Figure 6, the optimal sizing path corresponds to the installation of the rated capacity larger than the maximum peak load at the end of the study (equal to 108.27 kVA for this type of substation), i.e., 160 kVA, instead of the maximum available capacity that can serve the expected load at the end of the study period (i.e., 300 kVA). The same conclusion applies for Type 8 substation, where the installation of the 190 kVA

capacity from the first year of the study is selected, i.e., the capacity that is larger than the maximum peak load at the end of the study (equal to 162.41 kVA for this type of substation).

Figure 6   The directed graph used by the proposed ACO method for the Type 7 substation load (see online version for colours)
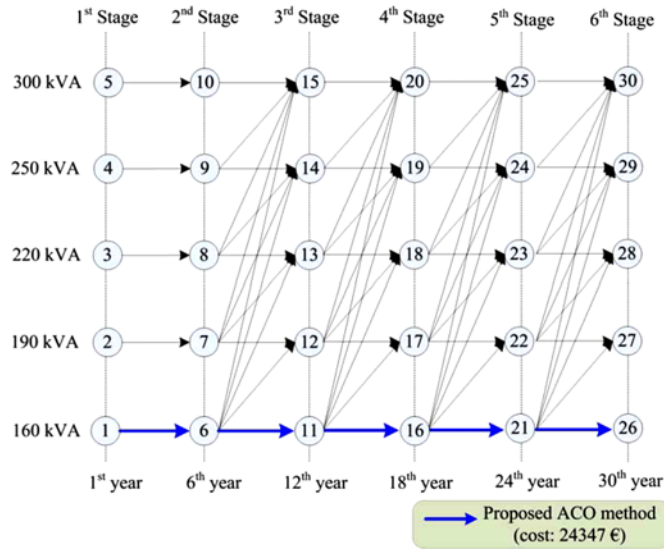

Figure 7   The directed graph used by the proposed ACO method for the Type 8 substation load (see online version for colours)
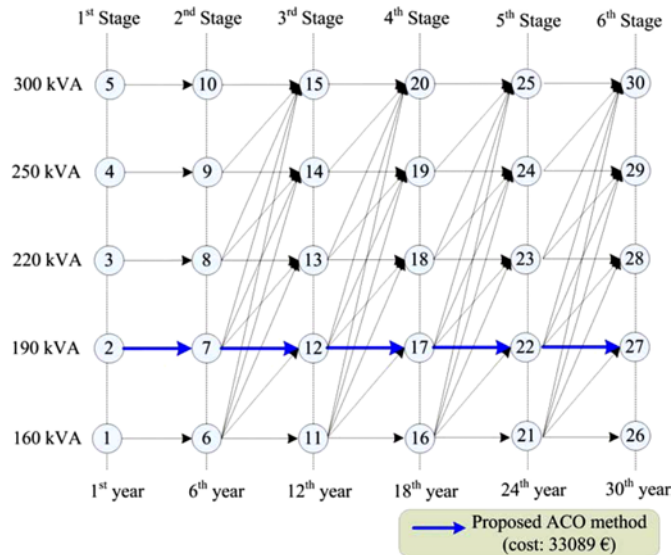
Table 3   Results of the proposed method for the OTS problem of Figure 4

Transformer cost	Substation load types							
	Type 1	*Type 2*	*Type 3*	*Type 4*	*Type 5*	*Type 6*	*Type 7*	*Type 8*
Installation cost (€)	10677	10296	10677	10677	12648	12648	5470	6015
Energy loss cost (€)	48529	61195	66372	56025	67153	70047	18877	26863
Total cost (€)	59206	71491	77049	66702	79801	82695	24347	32878
Number of substations	4	3	2	1	1	2	2	1
Total installation cost (€)	42708	30888	21354	10677	12648	25296	10940	6015
Total energy loss cost (€)	194116	183585	132744	56025	67153	140094	37754	26863
Overall cost (€)	236824	214473	154098	66702	79801	165390	48694	32878

According to the results of Table 3, the energy loss cost is the main component that influences the total cost difference between the proposed and the conventional sizing method, compared with the installation cost. The importance of transformer energy losses can be observed through the curves of Figure 8, indicating the variation of the energy loss cost of the considered transformer capacities for the Type 1 substation load throughout the study period (corresponding to load factor equal to 0.55, for domestic load). Figure 9 illustrates the variation of the energy loss cost of the considered transformer capacities for the Type 7 substation load throughout the study period (corresponding to load factor equal to 0.61, for industrial load), where the difference in the energy loss cost between 160 kVA (the optimal transformer) and 300 kVA (the largest available capacity) is verifying the selection of the optimal sizing path. Finally, Figure 10 shows the variation of the energy loss cost of the considered transformer capacities for Type 8 substation load throughout the study period (corresponding to load factor equal to 0.61, for industrial load), which has different behaviour in comparison with Figures 8 and 9. In this case, it is clear that for the first 15 years of the study, the 160 kVA transformer is the one with the best (less) annual energy cost, while for the next 15 years of the study it becomes the worst transformer, in terms of annual energy cost. Under these circumstances, neither the larger nor the smaller transformer capacity through the study period can serve successfully the considered load (75 kVA). As a result of the above and based on the optimisation algorithm, i.e., the proposed ACO method, the transformer rating of 190 kVA is selected as the optimum one.

On the basis of the aforementioned case studies and taking into consideration Figures 5–7, we conclude that the key to deal with the OTS problem is to select an ideal transformer size at the first year of the study period (30 years), which will remain constant throughout this period. In particular, it is not worth upgrading the transformer size during the study because the present value of the bid price (of the new transformer to be installed) increases dramatically the cost of the upgrade arc value (i.e., the arc that represents transformer size upgrading, namely arcs that direct from smaller to higher transformer sizes), while at same time the value of the energy cost remains relatively smaller. The present remaining value of the uninstalled transformer is usually not high enough to compensate this cost difference. This observation can also be confirmed from Figures 8–10. To be more precise, it is clear that there are transformer sizes able to serve

the considered load with less annual energy losses and consequently less annual energy loss cost in comparison with the remaining transformer designs during the period of the 30 years (e.g., in the case of Figure 10, the 160 kVA rating corresponds to less energy loss than the selected 190 kVA rating, thus in terms of less energy consumption, the selection to upgrade from 160 kVA to 190 kVA would be the optimal one), however the economic surplus owing to the additional transformer purchase overcomes the need to upgrade the transformer during the study. In a nutshell, the initial transformer choice is a key issue in the OTS problem, based on the load properties and the economic forecast.

Figure 8   Total annual energy loss cost curves corresponding to the five transformer ratings serving load of Type 1 substation (see online version for colours)
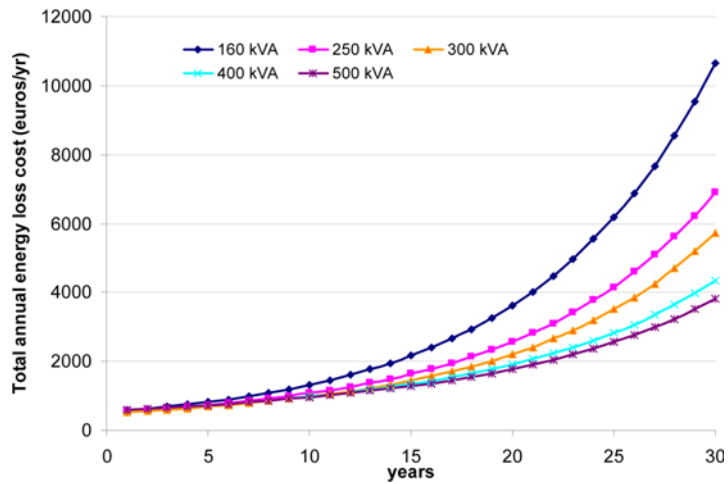

Figure 9   Total annual energy loss cost curves corresponding to the five transformer ratings serving load of Type 7 substation (see online version for colours)
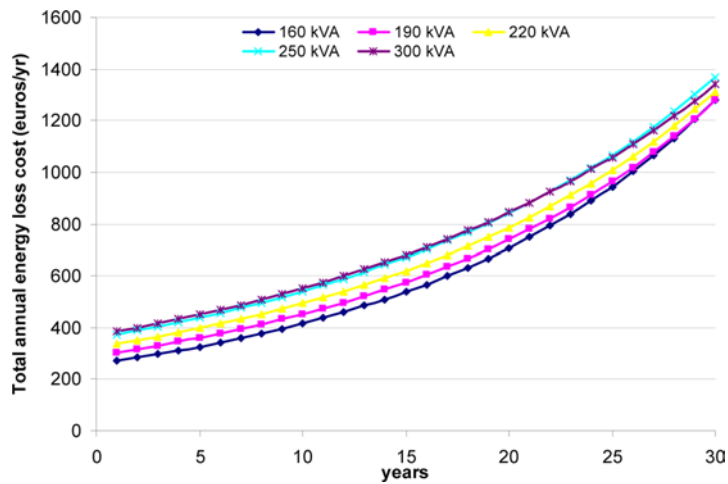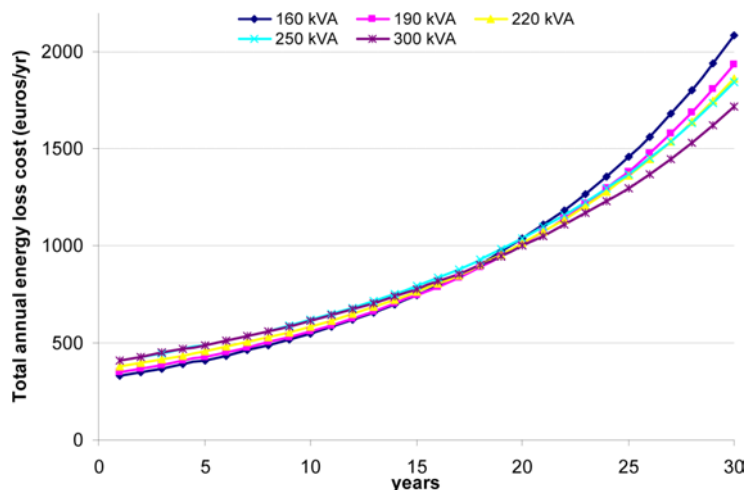
Figure 10 Total annual energy loss cost curves corresponding to the five transformer ratings serving load of Type 8 substation (see online version for colours)

7 Conclusions

In this paper, an EAS algorithm is proposed for the solution of the OTS planning problem by minimising the overall transformer cost (i.e., the sum of the transformer purchasing cost plus the transformer energy loss cost) over the planning period, while satisfying all the problem constraints (i.e., the load to be served and the transformer thermal loading limit). The method is applied for the selection of the optimal size of the distribution transformers in a real network, comprising 16 distribution substations to serve a load over a period of 30 years. The application results show that the proposed EAS algorithm is very efficient because it always converges to the global optimum solution of the OTS problem. According to these results, the initial transformer choice is the key issue in the OTS problem, based on the load properties and the economic forecast.

Acknowledgements

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