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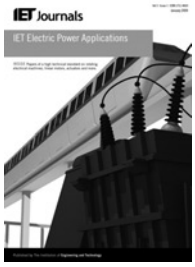

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Recursive genetic algorithm-finite element method technique for the solution of transformer manufacturing cost minimisation problem

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Abstract: The transformer manufacturing cost minimisation (TMCM), also known as transformer design optimisation, is a complex constrained mixed-integer non-linear programming problem with discontinuous objective function. This paper proposes an innovative method combining genetic algorithm (GA) and finite element method (FEM) for the solution of TMCM problem. The main contributions of the proposed method are: (a) introduction of an innovative recursive GA with a novel external elitism strategy associated with variable crossover and mutation rates resulting in an improved GA, (b) adoption of two particular finite element models of increased accuracy and high computational speed for the validation of the optimal design by computing the no-load loss and impedance and (c) combination of the innovative recursive GA with the two particular finite element models resulting in a proposed GA-FEM model that finds the global optimum, as concluded after several tests on actual transformer designs, while other existing methods provided suboptimal solutions that are 3.1–5.8% more expensive than the optimal solution.

1 Introduction

The aim of transformer design is to optimise an objective function subject to constraints imposed by international standards and transformer specification. In the bibliography of transformer design, several objective functions are optimised [1, 2]:

1. Minimisation of transformer manufacturing cost (MC) [3, 4].

2. Minimisation of total owning cost [5, 6].

3. Minimisation of transformer active part cost [7, 8].

4. Minimisation of active part mass [9].

5. Maximisation of transformer apparent power [9, 10].

Among the above-mentioned objective functions, the most commonly used functions are [1]:

1. The transformer MC, i.e. the sum of materials cost plus the labour cost. This objective function is mainly used when designing transformers for industrial and commercial users, since most of these users do not evaluate losses when they purchase transformers [11]. One of the challenges of this objective function is that the transformer MC depends on the cost of materials (copper, aluminium, steel etc.) that are stock exchange commodities with fluctuating prices on the world market.

2. The transformer total owning cost, i.e. the sum of transformer purchase cost plus the cost of transformer losses. This objective function is mainly used when designing transformers for electric utilities, since utilities usually evaluate the cost of transformer losses when they purchase transformers [11, 12]. Strategies for development and

diffusion of energy efficient distribution transformers (SEEDT) project concluded that electricity distribution companies and commercial and industrial users should use the total owning cost method to make transformer-purchasing decisions [12].

The transformer design requires knowledge of electromagnetism, magnetic circuit analysis, electric circuit analysis, loss mechanisms and heat transfer. The transformer design problem, because of its importance and complexity, has attracted the interest of many researchers [1–10]. There are two different methodologies for the solution of transformer design problem: (a) the multiple design method and (b) the mathematical programming method. The multiple design method [4, 5] is a heuristic technique that assigns many alternative values to the design variables so as to generate a large number of alternative designs and finally to select the design that satisfies all the problem constraints with the optimum value of the objective function; however, this technique is not able to find the global optimum. The geometric programming method is the most representative mathematical programming method for the solution of transformer design problem [9]; however, it has two drawbacks: (a) it requires the development of the mathematical model for each specific transformer type and configuration in advance and (b) because of the large number of coefficients in polynomial approximations, the geometric programming method is lacking flexibility and cannot be easily combined with more general transformer performance verification or cost estimation algorithms. Recently, another mathematical programming method, more specifically a parallel mixed integer programming-finite element method (MIP-FEM) technique [8], has been proposed performing better than the heuristic method [4]; however, MIP-FEM is very sensitive to the selection of the value range of design variables, so MIP-FEM often fails to find the global optimum.

This paper proposes a new power transformer design methodology based on a novel recursive genetic algorithm-finite element method (GA-FEM) technique. The proposed method successfully combines the optimisation capabilities of an improved GA (Section 2.3) as well as the accuracy and the computational speed of two particular finite element models (Section 2.2) that are adopted for the validation of the optimal design by computing the no-load loss (NLL) and impedance. The five main contributions and features of the proposed improved GA of Section 2.3 are: (a) introduction of an innovative recursive GA with a novel external elitism strategy assuring that the solution at a current GA run is better than or at least the same as the solution at the previous GA run, (b) incorporation of an internal elitism strategy assuring the copy of the best solution to the next GA generation, (c) incorporation of the optimal solution provided by MIP-FEM method [8] into the initial population of the initial GA run, which in combination with the external and internal elitism strategies assures that the proposed GA-FEM will converge to a better or at least the same solution with the MIP-FEM method, (d) adoption of variable crossover and mutation rates resulting in improved GA search and (e) optimal configuration for the parameters of the improved GA. In this paper, the minimisation of transformer MC has been considered as transformer design objective; however, the proposed recursive GA-FEM method can be also applied for all other transformer design objective functions, e.g. the minimisation of transformer total owning cost. Application results (Section 3) confirm that the proposed GA-FEM technique finds the global optimum solution to transformer design problem in very short time, while two other methods find suboptimal solutions.

2 Proposed GA-FEM methodology

2.1 Problem formulation

The objective of transformer manufacturing cost minimisation (TMCM) problem, also called transformer design optimisation problem, is to design the transformer so as to minimise the transformer MC, i.e. the sum of materials cost plus labour cost, subject to constraints imposed by international standards and transformer user needs. These constraints are:

1. *Induced voltage constraint*: it expresses the relation between the induced voltage in the secondary winding and the magnetic induction.

2. *Turns ratio constraint*: the turns ratio is equal to the voltage ratio.

3. *NLL constraint*: the designed NLL must be smaller than a maximum NLL.

4. *Load loss (LL) constraint*: the designed LL is required to be smaller than a maximum LL.

5. *Total loss (i.e. NLL plus LL) constraint*: the designed total loss must be smaller than a maximum total loss.

6. *Impedance constraint*: the designed impedance must be between a minimum and a maximum impedance.

7. *Magnetic induction constraint*: the designed magnetic induction is required to be smaller than a saturation magnetic induction.

8. *Heat transfer constraint*: the total heat produced by the transformer total loss (i.e. NLL plus LL) must be smaller than the total heat that can be carried away by the combined effects of conduction, convection and radiation.

9. *Temperature rise constraint*: the transformer temperature rise (because of NLL and LL) must be smaller than a maximum temperature rise.

10. *Efficiency constraint*: the transformer efficiency is required to be greater than a minimum efficiency.

11. *No-load current constraint*: the transformer no-load current is required to be smaller than a maximum no-load current.

12. *Voltage regulation constraint*: the transformer voltage regulation is required to be smaller than a maximum voltage regulation.

13. *Thickness of layer insulation constraint*: the thickness of layer insulation must withstand the induced voltage test and the impulse voltage test. More specifically: (a) the induced voltage must be smaller than a maximum induced voltage that the insulation can withstand and (b) the impulse voltage must be smaller than a maximum impulse voltage that the insulation can withstand.

14. *Tank dimensions constraints*: (a) the tank length must be smaller than a maximum tank length, (b) the tank width must be smaller than a maximum tank width and (c) the tank height must be smaller than a maximum tank height.

The TMCM is a complex constrained mixed-integer non-linear programming problem. The TMCM problem is further complicated by the fact that the objective (i.e. the MC) function is discontinuous [5].

2.2 Finite element models

The FEM is a powerful tool for the analysis and design of power transformers. In particular for the TMCM problem of wound core type transformers, it is proposed to use two FE models, the first to compute the transformer NLL and the second to evaluate the transformer impedance. In particular, a permeability tensor FE model is adopted for the computation of the NLL, since this model accurately represents the core material and the geometry of wound cores [13]. Moreover, an efficient FE model with detailed representation of winding geometry and cooling ducts is adopted for impedance evaluation [14]. Both FE models are based on a particular magnetic scalar potential formulation [15], which is advantageous in terms of computational speed in comparison to FEM based on magnetic vector potential, as there is only one unknown at each node of the FE mesh. The accuracy and the computational speed are the main advantages of the above two FE models that make them ideal for the solution of the TMCM problem.

2.3 Introduction and configuration of an improved recursive GA

GAs are powerful optimisation methods inspired by natural genetics and biological evolution. Their main advantages are: (a) GAs explore several areas of the search space simultaneously, reducing the probability of being trapped in local optima and (b) GAs do not require any prior knowledge, space limitations or special properties of the function to be optimised, such as smoothness, convexity, unimodality or existence of derivatives [16].

This paper introduces an improved GA for the solution of the TMCM problem. This section presents the contributions, features and optimal parameter settings of the improved GA.

Since the GA is a stochastic optimisation method, in general, it converges to different solution each time the GA is executed. That is why this paper proposes to implement a novel *recursive GA* approach, i.e. to run N times the GA and to introduce an *external elitism strategy* that copies the best solution found at the end of each GA run to the initial population of the next GA run. This innovative external elitism strategy assures that after the completion of each GA run, a solution is provided that is better than or at least the same as the solution of the previous GA run. As will be shown in Section 3, after 7–10 GA runs, the global optimum is reached for the TMCM problem.

An *internal elitism strategy* is also adopted, i.e. the best solution of every generation is copied to the next generation so that the possibility of its destruction through a genetic operator is eliminated.

The initial population of candidate solutions is created randomly. However, in the initial population of the initial GA run, the worst solution (i.e. the one with the maximum MC) is substituted by the solution that is computed by the MIP-FEM method proposed in [8]. The incorporation of the MIP-FEM solution into the initial population of the initial GA run in combination with the external and internal elitism strategies assures that the proposed method will converge to a better or at least the same solution with MIP-FEM method.

To improve the GA search by assuring a good exploration at the beginning of evolution, and more and more exploitation capability while optimisation goes on, *variable crossover and mutation rates* were tested. After enough experimentation, it was found that the best results were obtained with the following variable crossover and mutation probabilities

$$P_{\mathrm{ck}} = 0.35 + 0.45\left[\frac{k-1}{N_{\mathrm{g}}-1}\right] \tag{1}$$

$$P_{\mathrm{mk}} = 0.055 - 0.045\left[\frac{k-1}{N_{\mathrm{g}}-1}\right] \tag{2}$$

where P_{ck} is the crossover probability at generation k, P_{mk} is the mutation probability at generation k, and N_{g} is the number of generations.

The first column of Table 1 presents the seven-design variables that have been used for the solution of the TMCM problem by the proposed GA. In Table 1 and throughout this paper, LV stands for low voltage and HV stands for high voltage. The fifth column of Table 1 shows that the first five design variables are of integer type, while the rest two design variables are of real type. The fourth column of Table 1 shows

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Table 1 Determination of the number of bits of GA chromosome

Design variable	Symbol	Unit	Possible values	Type	Bits
number of LV turns	x_1	—	$8 \leq x_1 \leq 1000$	integer	10
magnetic material type	x_2	—	$1 \leq x_2 \leq 12$	integer	4
magnetic induction	x_3	G	$10\,000 \leq x_3 \leq 18\,500$	integer	15
width of core leg	x_4	mm	$80 \leq x_4 \leq 500$	integer	9
Core window height	x_5	mm	$80 \leq x_5 \leq 500$	integer	9
LV current density	x_6	A/mm^2	$1.5 \leq x_6 \leq 5.5$	real	7
HV current density	x_7	A/mm^2	$1.5 \leq x_7 \leq 5.5$	real	7
		number of bits of GA chromosome			61

the range of possible values that each design variable can take. This range of possible values has been determined from a large database of actual transformer designs with the following main characteristics: three-phase, oil-immersed, wound core distribution transformers from 25 kVA up to 2000 kVA, with voltages up to 36 V. Binary coding is used for chromosome representation. The last column of Table 1 presents the number of bits used for each design variable. As can be seen from the last row of Table 1, the GA chromosome has 61 bits.

After trial and error, it was found that a population size of 40 chromosomes and a number of 30 generations provide very good results for TMCM.

Among the four different selection schemes tested, i.e. roulette wheel, tournament, deterministic sampling and stochastic remainder sampling [16], the tournament selection scheme produced the best results and convergence for TMCM.

2.4 Overview of proposed method

The flowchart of the proposed optimisation model for the solution of TMCM problem, shown in Fig. 1, is composed of two submodels:

1. MIP-FEM submodel: initially, MIP-FEM deterministic optimisation method [8] is used to solve the TMCM problem. Let S_0 be the solution provided by that method.

2. Recursive GA-FEM submodel (N GA-FEM runs): After the execution of MIP-FEM submodel, N runs of the proposed recursive GA-FEM submodel are executed. Each run of GA-FEM submodel requires two internal runs:

(a) *GA run*: The recursive GA-based optimisation model, described in Section 2.3, is executed to solve the TMCM problem. The solution S_0 provided by the MIP-FEM submodel is included in the initial population of the initial GA run. In all the other GA runs, the best solution S_i

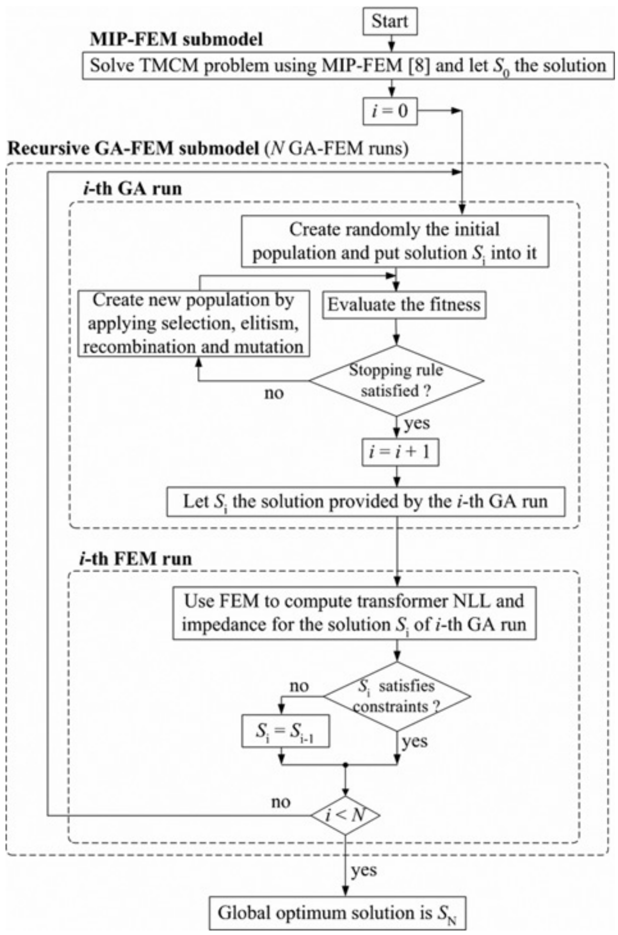

Figure 1 *Flowchart of the proposed method for TMCM problem*

provided by the previous GA-FEM run is included at the initial population of the next GA run. This approach assures that the solution S_i is better than or at least the same as the solution S_{i-1} (see Section 2.3).

(b) *FEM run*: The two FE models of Section 2.2 are used for the computation of transformer NLL and impedance (unlike the analytical formulas used in the GA run) in order to provide more accurate results and better convergence to the optimal solution.

3 Results and discussion

3.1 Application of proposed method to 1600 kVA transformer design

The proposed GA-FEM method has been used for the solution of the TMCM problem of an actual 1600 kVA transformer design with the following main specifications: rated frequency 50 Hz, rated HV 20 kV, rated LV 0.4 kV, prescribed NLL 1700 W, prescribed LL 20 000 W and prescribed impedance 6%. The NLL, LL and impedance tolerances are according to IEC 60076-1 international standard, i.e. the maximum NLL is 1955 W, the maximum LL is 23 000 W, the maximum total loss is 23 870 W, the minimum impedance is 5.4% and the maximum impedance is 6.6%. Table 2 compares the results of the proposed method with a heuristic [4] and a MIP-FEM method [8]. As can be seen from Table 2, the three techniques converged to three different solutions. In particular, the proposed recursive GA-FEM method, after seven GA-FEM runs that are implemented into 3.42 min, provides the best result, since it converges to the global minimum MC of $23 271.

Table 2 Comparison of proposed GA-FEM method with two existing transformer design methods for a 1600 kVA transformer design

Parameter	Heuristic	MIP-FEM	GA-FEM
number of LV turns	10	10	11
magnetic material type	1 (i.e. HiB)	2 (i.e. M4)	1 (HiB)
magnetic induction, G	16 012	16 991	18 000
width of core leg, mm	290	322	325
Core window height, mm	338	322	354
LV current density, A/mm^2	4.3	4.6	4.3
HV current density, A/mm^2	4.0	3.8	4.6
NLL, W	1581	1952	1791
LL, W	19 035	18 767	21 151
Total loss, W	20 616	20 719	22 942
impedance, %	5.89	6.41	6.20
MC, $	24 814	24 446	23 271
number of algorithm runs	1	1	7
Total execution time, min	0.45	0.79	3.42

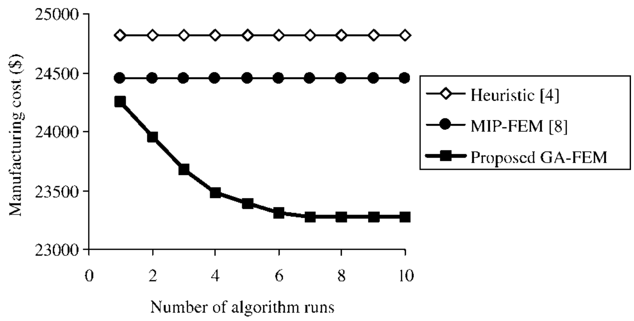

Figure 2 *Comparative results for a 1600 kVA transformer design*

Fig. 2 compares the minimum MC computed by the above three techniques for the solution of the 1600 kVA TMCM problem. Since the heuristic and the MIP-FEM are both deterministic optimisation techniques, they always converge to the same minimum MC, i.e. $24 814 for the heuristic and $24 446 for the MIP-FEM. On the other hand, the proposed recursive GA-FEM, because of its special design presented in Section 2.3, manages to progressively reduce the MC, as the number of GA-FEM algorithm runs increases. In particular, after seven GA-FEM runs, the global minimum MC is achieved, which is 4.8% cheaper than the MC computed by a MIP-FEM method [8] and 6.2% cheaper than the MC computed by a heuristic method [4]. As can be seen from Fig. 2, after the seventh GA-FEM run, the MC is not further decreased, which means that seven GA-FEM runs are enough to obtain the global optimum solution to TMCM problem.

Table 3 Comparison of average manufacturing cost saving of proposed GA-FEM against heuristic [4] and MIP-FEM [8]

Rated power, kVA	Number of designs	Cost saving of proposed against heuristic	Cost saving of proposed against MIP-FEM
100	25	5.3	1.8
160	25	4.9	2.6
250	25	8.0	4.4
400	25	6.5	3.0
630	25	6.0	2.8
800	25	5.9	2.0
1000	25	3.7	3.2
1600	25	6.3	4.9
average		5.8	3.1

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3.2 Generalisation of results

The proposed GA-FEM method has been tested on 200 actual transformer designs, of eight power ratings and various loss categories and voltage ratings. As can be seen from Table 3, the proposed GA-FEM method finds the global optimum solution that is, on average (a) 5.8% cheaper than the solution of a heuristic technique [4] and (b) 3.1% cheaper than the solution of a MIP-FEM method [8].

4 Conclusion

This paper has proposed an innovative recursive GA-FEM method for the solution of the complex constrained mixed-integer non-linear TMCM problem. When tested on 200 actual transformer designs, the proposed GA-FEM technique converged to the global optimum, thus GA-FEM provides significant MC savings ranging from 3.1% to 5.8%, in comparison with two deterministic optimisation methods that converged to local optimum solutions. The proposed recursive GA approach can be also very useful for the solution of other optimisation problems in electric machines and power systems.

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