# Global Transformer Optimization Method Using Evolutionary Design and Numerical Field Computation

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**This paper addresses the complex optimum transformer design problem, which is formulated as a mixed-integer nonlinear programming problem, by introducing an integrated design optimization method based on evolutionary algorithms and numerical electromagnetic and thermal field computations. The main contributions of this research are: i) introduction of a new overall transformer optimization method, minimizing either the overall transformer materials cost or the overall transformer materials and operating cost, ii) expansion of the solution space by innovative techniques that define the variation of crucial design variables such as the conductors' cross-section, ensuring global optimum transformer designs, and iii) incorporation of numerical field computation in order to validate the feasibility of the optimum designs. The proposed method is compared with a heuristic optimization method of the transformer manufacturing industry and the results demonstrate the robustness and the superiority of this new approach.**

***Index Terms*—Design method, finite element method (FEM), mixed integer nonlinear programming, optimization methods, transformer.**

## I. Introduction

THE difficulty in achieving the optimum balance between the transformer cost and performance is becoming even more complicated nowadays, as the main materials used in transformer manufacturing (copper or aluminum for transformer windings, steel for magnetic circuit) are variable stock exchange commodities and their prices are modified on a daily basis. Techniques that include mathematical models employing analytical formulas, based on design constants and approximations for the calculation of the transformer parameters are often the base of the design process adopted by transformer manufacturers [1].

The overall transformer manufacturing cost minimization is scarcely addressed in the technical literature. On the other hand, the main approaches deal with the minimization of specific transformer cost components, such as the cost of magnetic material [2], [3], or the active part cost [4].

This paper introduces the application of a mixed integer nonlinear programming (MINLP) in conjunction with the branch and bound (BB) [5] technique to the overall transformer design optimization, developing a novel implementation of MINLP linked to finite element method (FEM). The novelties of the proposed method can be categorized as follows: i) the deterministic MINLP technique is successfully applied to the overall cost minimization of transformer active part and mechanical part, ii) crucial design variables such as the conductors' cross-section are added to the optimization algorithm and the solution space is effectively enlarged and traversed through innovative implementation techniques, ensuring global optimum transformer designs, and iii) both magnetic and thermal FEM are employed for the overall design validation. The proposed method finds the global optimum transformer design by minimizing either the overall transformer materials cost (i.e., the transformer active part cost plus transformer mechanical part cost) or the overall transformer materials and operating cost taking into account proper loss evaluation factors, while simultaneously satisfying all the constraints imposed by international standards and transformer user needs, instead of focusing on the optimization of only one parameter of transformer performance (e.g., no-load losses or short-circuit impedance). Using the proposed technique, a user-friendly software package is developed that combines transformer design with analysis, optimization and visualization tools, useful for both design optimization and educational purposes. The method is applied for the design of distribution transformers of several ratings and loss categories and the results are compared with a heuristic transformer design optimization method (which is already used by the transformer industry), resulting to significant cost savings.

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## II. Proposed Methodology

### A. Mixed Integer Nonlinear Programming Method in Combination With Branch and Bound Technique

Recently, the area of MINLP [5] has experienced tremendous growth and a flourish of research activity. In the transformer design optimization area, MINLP techniques are very suitable and effective due to the fact that the design variables can assume not only continuous values but also integer values (e.g., number of winding turns). In this context, this paper proposes a BB optimization algorithm tailored to a MINLP formulation, completing previous research [4]. MINLP refers to mathematical programming with continuous and discrete variables and nonlinearities in the objective function and constraints. A general MINLP can be written as

$$\begin{aligned} & \min f(x,y) \\ \text{subject to} \quad & H(x,y)=0 \text{ and } G(x,y)\le 0 \\ & x\in\mathbb{R}^n \text{ and } y\in\mathbb{Z}^m \end{aligned}$$

where $x$ is a vector of $n$ continuous variables and $y$ is a vector of $m$ integer variables ($\mathbb{R}$ denotes the real numbers and $\mathbb{Z}$ denotes the integers). The function $f$ is a scalar valued objective function, while the vector functions $H$ and $G$ express linear or nonlinear constraints.

BB algorithms for MINLP [6] constitute a well-known approach for solving combinatorial optimization problems to optimality. Essentially, BB techniques use an implicit enumeration scheme for exploring the search space in an "intelligent" way. This is done by partitioning the search space and producing upper and lower bounds of the solutions attainable in each partition. Thus, the search performed by the algorithm can be represented as a tree that is traversed in a certain way. The most efficient (in terms of the number of iterations required to find the optimum and prove its optimality) is to use a depth-first traversal.

The proposed recursive BB algorithm solves continuous optimization problems, while constraining some variables into sets of standard values, which may consist of discrete or integer values. The associated discrete programming problem is recursively divided into two sub-problems, by fixing the discrete variables to the closest above and below standard values. The search starts by solving a nonlinear programming (NLP) relaxation, and using the solution as the lower bound of the problem. If the solutions of the discrete variables are all equal to the values defined at the standard discrete set, then the optimum solution is reached and the search is stopped. Otherwise, the search branches on the first discrete variable that has non-standard solution. The closest discrete values above and below the current solution are identified. If both above and below values exist, the NLP with the fixed above values becomes the first sub-problem. The first discrete variable with non-standard solution is identified. Subsequently, a new equality constraint to fix this variable to the above value is added to the original constraints, and the NLP sub-problem subject to the updated constraints is solved. If the NLP sub-problem converges, and yields the superior solution over the existing lower bound, then this solution becomes the new lower bound. The branching continues recursively to the next discrete value with non-standard solution. Otherwise, the node is fathomed. If this happens, the algorithm backtracks to the ascendant node, and then resumes branching at the sub-problem associated with below values.

In this paper, the sequential quadratic programming (SQP) method is proposed for solving transformer cost minimization NLP sub-problems with BB [6], which enforces early detection and termination of infeasible or inferior NLP solutions. The SQP implementation consists of three main stages: 1) at each major iteration a positive definite quasi-Newton approximation of the Hessian of the Lagrangian function is calculated using the Broyden-Fletcher-Goldfarb-Shanno (BFGS) method, 2) at each major iteration of the SQP method, a quadratic programming problem is solved, and 3) the line search algorithm is a safeguarded cubic polynomial method, which requires fewer function evaluations but more gradient evaluations.

## B. Finite Element Method

The MINLP solution is validated by a convenient FEM technique. Magnetic FEM is used for transformer performance parameters (no-load losses and short-circuit impedance) calculation [3], [7], while thermal FEM is used for the calculation

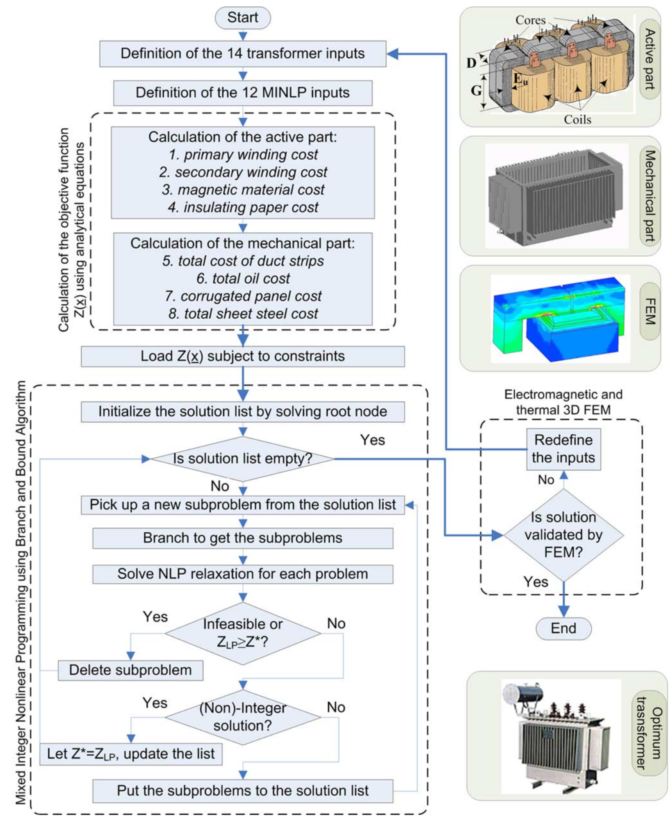


Fig. 1. Flowchart of the proposed technique.

of winding and core hottest spot temperature. If validation fails, i.e., the deviation of the calculated losses and short-circuit impedance value from the prescribed values exceeds the permissible tolerance, or the winding hottest spot temperature overcomes the respective limit, the MINLP process is repeated (Fig. 1). Validation through numerical field computation enhances the accuracy of the proposed method and eliminates the possibility of infeasible optimum designs. The magnetic and thermal analyses are based on a 3D model developed to provide accurate solutions within limited execution times [7], suitable for an optimization algorithm.

## C. Problem Formulation

This section introduces the mathematical formulation of the proposed method. This technique is integrated in Matlab environment, using suitable graphical user interface (GUI).

The proposed method is shown in the flowchart of Fig. 1. The 14 transformer inputs (Fig. 1) concern design parameters, such as rated power, vector group, voltages, etc., while the 12 MINLP inputs (Fig. 1) comprise the upper/lower bounds and the initial value of the design vector.

A MINLP for optimizing the transformer design is based on the minimization of the overall transformer cost function

$$\min Z(\underline{x}) = \min_{\underline{x}} \sum_{j=1}^{8} c_j f_j(\underline{x}) \tag{1}$$

where $c_j$ and $f_j$ are the unit cost (euro/kg) and the weight (kg) of each component $j$ (active and mechanical part, Fig. 1), and $\underline{x}$ is the vector of the four design variables, i.e., the number of low voltage turns, the magnetic induction magnitude $(B)$, the width of core leg $(D)$ and the core window height $(G)$ (Fig. 1).

The minimization of the objective function is subject to

$$\mathrm{DNLL} + \mathrm{DLL} - 1.10 \cdot (\mathrm{GNLL} + \mathrm{GLL}) < 0 \quad (2)$$

$$\mathrm{DNLL} - 1.15 \cdot \mathrm{GNLL} < 0 \quad (3)$$

$$\mathrm{DLL} - 1.15 \cdot \mathrm{GLL} < 0 \quad (4)$$

$$0.90 \cdot \mathrm{GU} < \mathrm{DU} < 1.10 \cdot \mathrm{GU} \quad (5)$$

$$\mathrm{DLL} + \mathrm{DNLL} < H_c \quad (6)$$

$$0.5 \cdot D - 2 \cdot E_u \leq 0 \quad (7)$$

$$2 \cdot E_u - 0.9 \cdot D \leq 0 \quad (8)$$

$$D - G \leq 0 \quad (9)$$

$$lb_j \leq x_j \leq ub_j, \quad j = 1, 2, \ldots, n \quad (10)$$

$$x_j \geq 0, \quad j = 1, 2, \ldots, n \quad (11)$$

where DNLL denotes the designed no-load loss (W), DLL the designed load loss (W), DU the designed short-circuit impedance (%), GNLL the guaranteed no-load loss (W), GLL the guaranteed load loss (W), GU the guaranteed short-circuit impedance (%), $H_c$ is the heat dissipated (by convection) through the transformer cooling system (W), while $D, G, E_u$ are the geometric characteristics of the active part (Fig. 1), and $\underline{\mathrm{lb}}$ and $\underline{\mathrm{ub}}$ are $n \times 1$ matrices of lower and upper bounds on $\underline{x}$. The coefficients appearing in (2)–(5) are based on the tolerances specified by IEC 60076-1, while the respective coefficients in (6)–(9) are based on the transformer manufacturer specifications.

Upon user selection, the transformer loss cost can also be integrated into (1) enabling to seek for the optimum design based on the total owning cost (TOC), i.e., the transformer purchasing cost plus the transformer operating cost

$$\min_{x_j} \mathrm{TOC} = \min_{x_j} \left( \frac{\left[ \left( \sum_{j=1}^{8} c_j \cdot x_j \right) + \mathrm{CRM} + \mathrm{LC} \right]}{(1 - M)} + A \cdot \mathrm{DNLL} + B \cdot \mathrm{DLL} \right) \quad (12)$$

where CRM denotes the cost of the transformer remaining materials (euro), LC denotes the labor cost (euro), $M$ denotes the transformer sales margin (%), $A$ denotes the equivalent no-load loss cost rate (euro/W), and $B$ denotes the equivalent load loss cost rate (euro/W). The strong point of the proposed software is that the designer can define the loss evaluation factors ($A$ and $B$) using 1) the IEEE standard method [9], 2) the simple yet effective industrial method of [10], or 3) his own admission, utilizing the friendly and easy-to-use GUI. The fractional part of (12) is called transformer bid price (BP).

One of the crucial design variables during the transformer design optimization is the calculation of the conductors' cross-section. The conductors' cross-section derives from the current density of the high voltage (HV) and low voltage (LV) winding, which consist crucial design parameters, dependent on the transformer rating and loss category. In the proposed method, three new approaches are proposed with the aim of

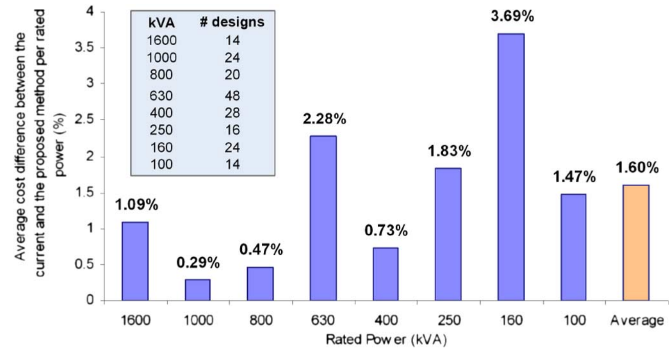


Fig. 2. Average cost difference, i.e., the average difference between the costs of the optimum transformer designs produced by the proposed method versus the current method employed in the manufacturing industry, for each kVA category considered in the study.

successfully defining the values of the HV and LV winding current density (in A/mm$^2$), denoted as $\mathrm{WCD_{HV}}$ and $\mathrm{WCD_{LV}}$, respectively. At the first approach, the transformer designer can define directly the value of the $\mathrm{WCD_{HV}}$ and $\mathrm{WCD_{LV}}$. The main drawback of this approach is that the transformer designer should be quite experienced in order to correctly set this value and direct the method to the optimal solution. At the second approach, an interval with a set of discrete $c_{\mathrm{LV}}$ and $c_{\mathrm{HV}}$ values for the LV and HV winding, respectively, can be defined. In this case, the proposed method will calculate $c_{\mathrm{LV}} \cdot c_{\mathrm{HV}}$ optimum transformer designs, and finally will keep the best optimum transformer design among them. Although this approach is time-consuming, it assures a global optimum design. At the third approach, the designer can increase the vector of the four design variables into six. In particular, the correct definition of the current density value is under the rules (supervision) of the MINLP optimization method. In this way, the transformer designer defines the initial, the upper and the lower value of the $\mathrm{WCD_{HV}}$ and $\mathrm{WCD_{LV}}$ and the proposed method finds an optimum transformer design, designating the values of the six variables of the design vector $\underline{x}$.

## III. Results and Discussion

The robustness of the proposed method is presented in comparison with that of current method [1] that is already applied in a transformer manufacturing industry. The proposed method minimizes the overall transformer cost (1), subject to the constraints (2)–(11) by seeking the optimum settings of the four design variables, namely, the core constructional parameters D and G shown in Fig. 1 (continuous variables), the magnetic induction (continuous variable), and the number of turns (integer variable). Two more design variables can be optionally added: $\mathrm{WCD_{HV}}$ and $\mathrm{WCD_{LV}}$ (continuous variables).

The proposed method has been applied in a wide spectrum of actual transformers, of different voltage ratings and loss categories. In particular, 188 optimum transformer designs were created and compared with the current method [1]. Fig. 2 depicts the results. It should be noted that experiments were carried out using constant $\mathrm{WCD_{HV}}$ and $\mathrm{WCD_{LV}}$ values (1st approach for the current density determination, described in Section II-C) because the current heuristic technique [1] could not support the other two approaches.

TABLE I
DESIGN OPTIMIZATION RESULTS FOR THE 400 KVA TRANSFORMER USING DIFFERENT CURRENT DENSITY DEFINITION METHODS

| Characteristics of the optimum transformer design | Current Density Definition Method (Section II.C) | | |
|---|---|---|---|
| | Constant LV and HV current density | Discrete values of LV and HV current density | Addition of LV and HV current density to the design vector |
| Low voltage turns | 18 | 18 | 19 |
| D (mm) | 239 | 219 | 230 |
| G (mm) | 248 | 237 | 261 |
| B (Gauss) | 18000 | 18000 | 18000 |
| $WCD_{LV}$ (A/mm$^2$) | 3 | 3.4 | 3.4 |
| $WCD_{HV}$ (A/mm$^2$) | 3 | 3.6 | 3.3 |
| DNLL (W) | 859 | 841 | 818 |
| DLL (W) | 4288 | 4945 | 4890 |
| Cost (€) | 4203 | 3866 | 3954 |
| Cost saving versus method [1] (%) | 1.46 | 9.36 | 7.29 |

TABLE II
OPTIMIZATION RESULTS USING AS OBJECTIVE FUNCTION (1) AND (12)

| Symbol | Equation (1) | Equation (12) |
|---|---|---|
| $\sum_{j=1}^{8} c_j f_j(\underline{x})$ (€) | 4203 | 4428 |
| BP | 9684 | 10006 |
| DNLL (W) | 859 | 719 |
| DLL (W) | 4288 | 4613 |
| A (€/W) | 8.31 | 8.31 |
| B (€/W) | 2.49 | 2.49 |
| TOC (€) | 27500 | 27467 |

As part of the results, the application of the method to a transformer of 400 kVA is reported (GLL $=$ 4600 W and GNLL $=$ 750 W, the rated primary and secondary voltages are 20/0.4 kV, the vector group is Dyn11, the frequency is 50 Hz, and $\mathrm{WCD_{HV}}$ and $\mathrm{WCD_{LV}}$ are both equal to 3 A/mm$^2$), yielding optimum design with an average cost saving of 1.46% (Table I) in comparison with the existing method [1]. Moreover, the second current density determination method is used, defining variation of the $\mathrm{WCD_{LV}}$ at discrete values of the [3 3.6] interval with step $0.2\,(c_{\mathrm{LV}} = 4)$ and variation of the $\mathrm{WCD_{HV}}$ at discrete values of the [3 3.6] interval with step $0.15\,(c_{\mathrm{HV}} = 5)$. In this case, the proposed method detects $4 \cdot 5 = 20$ optimum transformer designs and selects the most cost effective among them, corresponding to an optimum cost of 3866 euro (Table I), i.e., 9.36% cheaper than the optimum transformer of the current method [1]. Finally, the third current density determination method of Section II-C is applied by adding the $\mathrm{WCD_{HV}}$ and $\mathrm{WCD_{LV}}$ to the design vector with upper and lower bounds corresponding to the same intervals as the ones defined at the second approach, resulting in an optimum transformer cost of 3954 euro (Table I), i.e., 7.29% cheaper than the optimum transformer of the current method [1]. Detailed results for each current density approach are shown in Table I.

Finally, a comparison of the optimization results incorporating the transformer operating cost, using (12), has been conducted, for the same case study of the 400 kVA transformer. Table II shows the results of the proposed method using as objective function the (1) and (12). Although the use of (12) leads to an optimum transformer where the costs of the eight materials are euro $=225$ more expensive than the transformer yielded by the use of (1), its respective TOC is slightly cheaper. This difference relies on the transformer loss cost, rendering the optimization of (12) a compromise between manufacturing and operating cost.

## IV. CONCLUSION

The proposed method is very effective because of its robustness, its high execution speed and its ability to effectively search the large solution space. The validity of this method is illustrated by its application to a wide spectrum of actual transformers, of different power ratings and losses, resulting to optimum designs with an average cost saving of 1.60% in comparison with the existing heuristic method used by a transformer manufacturer. This technique has proven to be reasonably efficient on transformer design optimization problem. The development of user-friendly software based on this method provides significant improvements in the design process of the manufacturing industry.


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