Journal of Materials Processing Technology 181 (2007) 281–285

Journal of Materials Processing Technology

www.elsevier.com/locate/jmatprotec

# A decision tree method for the selection of winding material in power transformers

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**Abstract**

In this paper, a decision tree method is proposed for the selection of the winding material in power transformers. The proposed method is very fast and effective, since it divides by two the effort of the transformer design engineer. The accuracy of the proposed method is 94%, which makes it very efficient for industrial use.


*Keywords:* Power transformer; Winding material; Transformer design; Transformer manufacturing; Optimum transformer; Transformer manufacturing cost; Decision trees

## 1. Introduction

The variation in the cost of the materials used in the transformer manufacturing has direct impact in the design of the optimum transformer, i.e. the transformer that meets the specification with the minimum manufacturing cost. The material of the transformer windings can be copper (CU) or aluminum (AL). Since CU and AL are stock exchange commodities, their prices can significantly change from time to time. In addition, CU and AL conductors have different technical characteristics. Consequently, in some transformer designs it is more economical to use CU windings instead of AL windings and vice versa. However, this has to be checked in every transformer design, which means that for each design, there is a need to optimize twice the transformer (once with CU and once with AL windings) and afterwards to select the most economical design. In this paper, a decision tree (DT) method is proposed for the selection of the winding material. The proposed method is very fast and effective, since it divides by two the effort of the transformer design engineer. The accuracy of the proposed method is 94%, which makes it very efficient for industrial use.

The paper is organized as follows: Section 2 briefly presents the procedure to find the optimum transformer. Section 3 presents an overview of the decision tree methodology. Sections 4 and 5 are focused on the application of the decision tree methodology for the selection of winding material in power transformers. Section 4 describes the creation of the learning and test sets and Section 5 presents the results. Section 6 concludes the paper.

## 2. Optimum transformer

The power transformers considered in this paper are three-phase, wound core and their magnetic circuit is of shell type. In the industrial environment considered, the optimum transformer is calculated with the help of a suitable computer program, which uses 134 input parameters in order to make the transformer design as parametric as possible [1]. The computer program allows many variations in design variables. These variations permit the investigation of enough candidate solutions. For each one of the candidate solutions, it is checked if all the specifications (limits) are satisfied, and if they are satisfied, the cost is estimated and the solution is characterized as acceptable. On the other hand, the candidate solutions that violate the specification are characterized as non-acceptable solutions. Finally, among the acceptable solutions, the transformer with the minimum manufacturing cost is selected, which is the optimum transformer. Some of these 134 input parameters have very strong impact on the determination of the optimum transformer, e.g. the input parameters such as the unit cost (in $/kg) of the magnetic material, the unit cost of the winding material and the type of the winding material (CU or AL).

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0924-0136/$ – see front matter 
doi:10.1016/j.jmatprotec.2006.03.036

## 3. Overview of decision trees

The decision tree methodology [2–4] is a non-parametric learning technique able to produce classifiers about a given problem in order to reduce information for new, unobserved cases. The DT is a tree structured upside down, built on the basis of a Learning Set (LS). The LS comprises a number of preclassified states defined by a list of candidate attributes.

In order to detect if a node is terminal, i.e. "sufficiently" class pure, the classification entropy of the node with a minimum preset value $H_{\min}$ is compared. If it is lower than $H_{\min}$, then the node is sufficiently class-pure and it is not further split. Such nodes are labeled LEAVES. Otherwise, a suitable test is sought to divide the node, by applying the optimal splitting rule. In the case that no test can be found with a statistically significant information gain, the node is declared a DEADEND and it is not split.

DTs are tested using test sets (TS), comprising a number of similar, preclassified, but independent transformer designs. The class (CU or AL) of each of these transformer designs is compared to the class of the terminal node finally led to by applying the tests of the various non-terminal nodes. This comparison provides the DT classification error rate.

Each node possesses a subset of transformer designs with the following characteristics:

$E_{\mathrm{n}}$: the transformer designs subset of node n of the DT.
$N$: size (number of transformer designs) of $E_{\mathrm{n}}$.
$n_{\mathrm{CU}}$: number of transformer designs with CU windings in $E_{\mathrm{n}}$.
$n_{\mathrm{AL}}$: number of transformer designs with AL windings in $E_{\mathrm{n}}$.

The relative frequencies of transformer designs with CU and AL windings for node n will be

$$f_{\mathrm{CU}} = \frac{n_{\mathrm{CU}}}{n_{\mathrm{CU}} + n_{\mathrm{AL}}} = \frac{n_{\mathrm{CU}}}{N} \quad \text{and} \quad f_{\mathrm{AL}} = \frac{n_{\mathrm{AL}}}{n_{\mathrm{CU}} + n_{\mathrm{AL}}} = \frac{n_{\mathrm{AL}}}{N} \tag{1}$$

The entropy of $E_{\mathrm{n}}$ with respect to the class partition of its elements, is defined as

$$H_{\mathrm{c}}(E_{\mathrm{n}}) = -(f_{\mathrm{CU}} \log f_{\mathrm{CU}} + f_{\mathrm{AL}} \log f_{\mathrm{AL}}) \tag{2}$$

A test $T$ is defined at node n as

$$T : A_i \leq t \tag{3}$$

where $A_i$ is the value of attribute $i$ of a particular transformer design and $t$ is a threshold value.

By applying the test $T$ to all transformer designs (TD) of node n, $E_{\mathrm{n}}$ is split into two subsets $E_{\mathrm{n1}}$ and $E_{\mathrm{n2}}$

$$E_{\mathrm{n1}} = \{\mathrm{TD} \in E_{\mathrm{n}} : A_i \leq t\} \quad \text{and} \quad E_{\mathrm{n2}} = \{\mathrm{TD} \in E_{\mathrm{n}} : A_i > t\} \tag{4}$$

If $n_i$ is the number of transformer designs in $E_{\mathrm{n}i}$, $i = 1, 2$, the corresponding frequencies are given by

$$f_1 = \frac{n_1}{n_1 + n_2} = \frac{n_1}{N} \quad \text{and} \quad f_2 = \frac{n_2}{n_1 + n_2} = \frac{n_2}{N} \tag{5}$$

The entropy of $E_{\mathrm{n}}$ with respect to the partition induced by $T$ is

$$H_T(E_{\mathrm{n}}) = -(f_1 \log f_1 + f_2 \log f_2) \tag{6}$$

where $H_T(E_{\mathrm{n}})$ is a measure of the uncertainty of the outcome of test $T$.

The mean conditional entropy of $E_{\mathrm{n}}$, given the outcome of test, corresponds to the residual entropy after the application of $T$ and is defined as

$$H_{\mathrm{c}}(E_{\mathrm{n}}|T) = f_1 H_{\mathrm{c}}(E_{\mathrm{n1}}) + f_2 H_{\mathrm{c}}(E_{\mathrm{n2}}) \tag{7}$$

The information gained from the application of test $T$ is expressed by the achieved reduction of the learning subset entropy

$$I(E_{\mathrm{n}}; T) = H_{\mathrm{c}}(E_{\mathrm{n}}) - H_{\mathrm{c}}(E_{\mathrm{n}}|T) \tag{8}$$

A more objective (less biased) estimator of the merit of test $T$ is provided by the normalized information gain, defined as

$$C(E_{\mathrm{n}}; T) = \frac{2I(E_{\mathrm{n}}; T)}{H_{\mathrm{c}}(E_{\mathrm{n}}) + H_T(E_{\mathrm{n}})} \in [0, 1] \tag{9}$$

Under the hypothesis of no correlation between the test $T$ and the class partition in the Universe $U$ of the transformer designs (resp. $U_{\mathrm{n}}$), that is for zero actual increase in information, the random variable $NI(E_{\mathrm{n}};T)$, which is an estimator of the total actual information gain, is $X^2$-distributed with one degree of freedom and its expected value is positive and inversely proportional to the size of the subset $E_{\mathrm{n}}$

$$NI(E_n; T) \sim X^2(1) \tag{10}$$

If $\alpha$ is the risk level of not detecting situations of only apparent information gain and $X_{\mathrm{cr}}$ the value such that $P(X > X_{\mathrm{cr}}) = \alpha$, where $X$ a random variable following an $X^2$ distribution with one degree of freedom, then the following statistical test can be formulated: "the node splitting test $T$ is rejected as uncorrelated with the class partition if $Q_1 = NI(E_{\mathrm{n}};T) < X_{\mathrm{cr}}$", where $N$ is the number of learning states in $E_{\mathrm{n}}$.

## 4. Creation of the learning and test sets

For the creation of the learning and test sets, six power ratings (250, 400, 630, 800, 1000 and 1600 kVA) are considered. For each transformer, nine categories of losses are taken into account, namely AA′, AB′, AC′, BA′, BB′, BC′, CA′, CB′, CC′, according to Ref. [5]. Seven different unit costs (in \$/kg) are considered for the CU and the AL winding. Based on the above, $6 \times 9 \times 7 = 378$ transformer design optimizations with CU winding (CU designs) and 378 transformer design optimizations with AL winding (AL designs) are realized. For each design, either the CU design or the AL design is the final optimum design (with the least manufacturing cost). In total, $6 \times 9 \times 7^2 = 2646$ final optimum designs are collected and stored into databases. The databases are composed of sets of final optimum designs (FOD) and each FOD is composed of a collection of input/output pairs. The input pairs or *attributes* are the parameters affecting the selection of winding material. Attributes have been selected

Table 1
Attributes

| Symbol | Attribute name |
|---|---|
| $I_1$ | CU unit cost ($/kg) |
| $I_2$ | AL unit cost ($/kg) |
| $I_3$ | $I_1/I_2$ |
| $I_4$ | Magnetic material unit cost ($/kg) |
| $I_5$ | $I_4/I_1$ |
| $I_6$ | $I_4/I_2$ |
| $I_7$ | Guaranteed no-load losses (W) |
| $I_8$ | Guaranteed load losses (W) |
| $I_9$ | $I_7/I_8$ |
| $I_{10}$ | Rated power (kVA) |
| $I_{11}$ | Guaranteed short-circuit voltage (%) |
| $I_{12}$ | $I_7/I_{10}$ |
| $I_{13}$ | $I_8/I_{10}$ |

based on extensive research and transformer designers' experience. The list of 13 attributes, initially selected, is shown in Table 1. The output pairs comprise the type of winding (CU or AL) that corresponds to each FOD. The learning set is composed of 1350 sets of FODs and the test set has 1296 independent sets of FODs.

## 5. Results and discussion

### 5.1. Decision tree

Fig. 1 shows the decision tree for the selection of the winding material. The decision tree is automatically built on the basis of the learning set of 1350 FODs with the 13 attributes list of Table 1. The notation used for the DT nodes is explained in Fig. 2.

Each terminal node of the decision tree of Fig. 1 produces one decision rule, on the basis of its CU index, i.e. the ratio of CU designs over the FODs of that node. For example, from terminal node 21 the following rule is derived: if $I_3 > 0.7616$ and $I_5 \leq 0.4452$ and $I_3 > 0.9564$ and $I_7 > 860$, then choose AL, since the CU index of node 21 is 0.0. Table 2 presents the 13 decision rules of the 13 terminal nodes of the decision tree of Fig. 1.

Table 3
Transformer attribute values

| Attribute | Value | Unit |
|---|---|---|
| $I_1$ | 4.81 | $/kg |
| $I_2$ | 5.13 | $/kg |
| $I_3$ | 0.94 | – |
| $I_4$ | 1.78 | $/kg |
| $I_5$ | 0.37 | – |
| $I_6$ | 0.35 | – |
| $I_7$ | 610 | W |
| $I_8$ | 6000 | W |
| $I_9$ | 0.10 | – |
| $I_{10}$ | 400 | kVA |
| $I_{11}$ | 4 | % |
| $I_{12}$ | 1.53 | W/kVA |
| $I_{13}$ | 15 | W/kVA |

Among the 13 candidate attributes, the decision tree automatically selected the following five most important attributes:

$$I_3 = \frac{\text{CU unit cost}}{\text{AL unit cost}}$$

$$I_5 = \frac{\text{magnetic material unit cost}}{\text{CU unit cost}}$$

$$I_7 = \text{guaranteed no load losses [W]}$$

$$I_8 = \text{Guaranteed load losses [W]}$$

$$I_{13} = \frac{\text{guaranteed load losses}}{\text{rated power}}\left[\frac{\text{W}}{\text{kVA}}\right]$$

The selection of the above five attributes is reasonable and expected, since they are all related to the selection of winding material (CU or AL) in power transformers.

### 5.2. Selection of winding material

Let us suppose that the decision tree of Fig. 1 has to select the winding material of a transformer that the attribute values

Table 2
Decision tree rules

| Number | Node | Rule |
|---|---|---|
| 1 | 4 | If $I_3 \leq 0.7616$ and $I_5 \leq 0.3760$ then AL |
| 2 | 7 | If $I_3 \leq 0.7616$ and $I_5 > 0.3760$ and $I_8 > 17240$ then AL |
| 3 | 8 | If $I_3 \leq 0.7616$ and $I_5 > 0.3760$ and $I_8 \leq 17240$ and $I_3 \leq 0.6399$ then CU |
| 4 | 9 | If $I_3 \leq 0.7616$ and $I_5 > 0.3760$ and $I_8 \leq 17240$ and $I_3 > 0.6399$ then CU |
| 5 | 15 | If $0.7616 < I_3 \leq 0.9564$ and $I_5 \leq 0.4452$ and $I_7 > 1121$ then AL |
| 6 | 17 | If $0.7616 < I_3 \leq 0.9564$ and $I_5 \leq 0.4452$ and $I_7 \leq 1121$ and $I_{13} > 13.3377$ then AL |
| 7 | 18 | If $0.7616 < I_3 \leq 0.9564$ and $I_5 \leq 0.4452$ and $I_7 \leq 1121$ and $I_{13} \leq 13.3377$ and $I_9 \leq 0.1600$ then CU |
| 8 | 19 | If $0.7616 < I_3 \leq 0.9564$ and $I_5 \leq 0.4452$ and $I_7 \leq 1121$ and $I_{13} \leq 13.3377$ and $I_9 > 0.1600$ then AL |
| 9 | 21 | If $I_3 > 0.9564$ and $I_5 \leq 0.4452$ and $I_7 > 860$ then AL |
| 10 | 22 | If $I_3 > 0.9564$ and $I_5 \leq 0.4452$ and $I_7 \leq 773$ then AL |
| 11 | 23 | If $I_3 > 0.9564$ and $I_5 \leq 0.4452$ and $773 < I_7 \leq 860$ then AL |
| 12 | 24 | If $I_3 > 0.7616$ and $I_5 > 0.4452$ and $I_7 \leq 1121$ then CU |
| 13 | 25 | If $I_3 > 0.7616$ and $I_5 > 0.4452$ and $I_7 > 1121$ then AL |

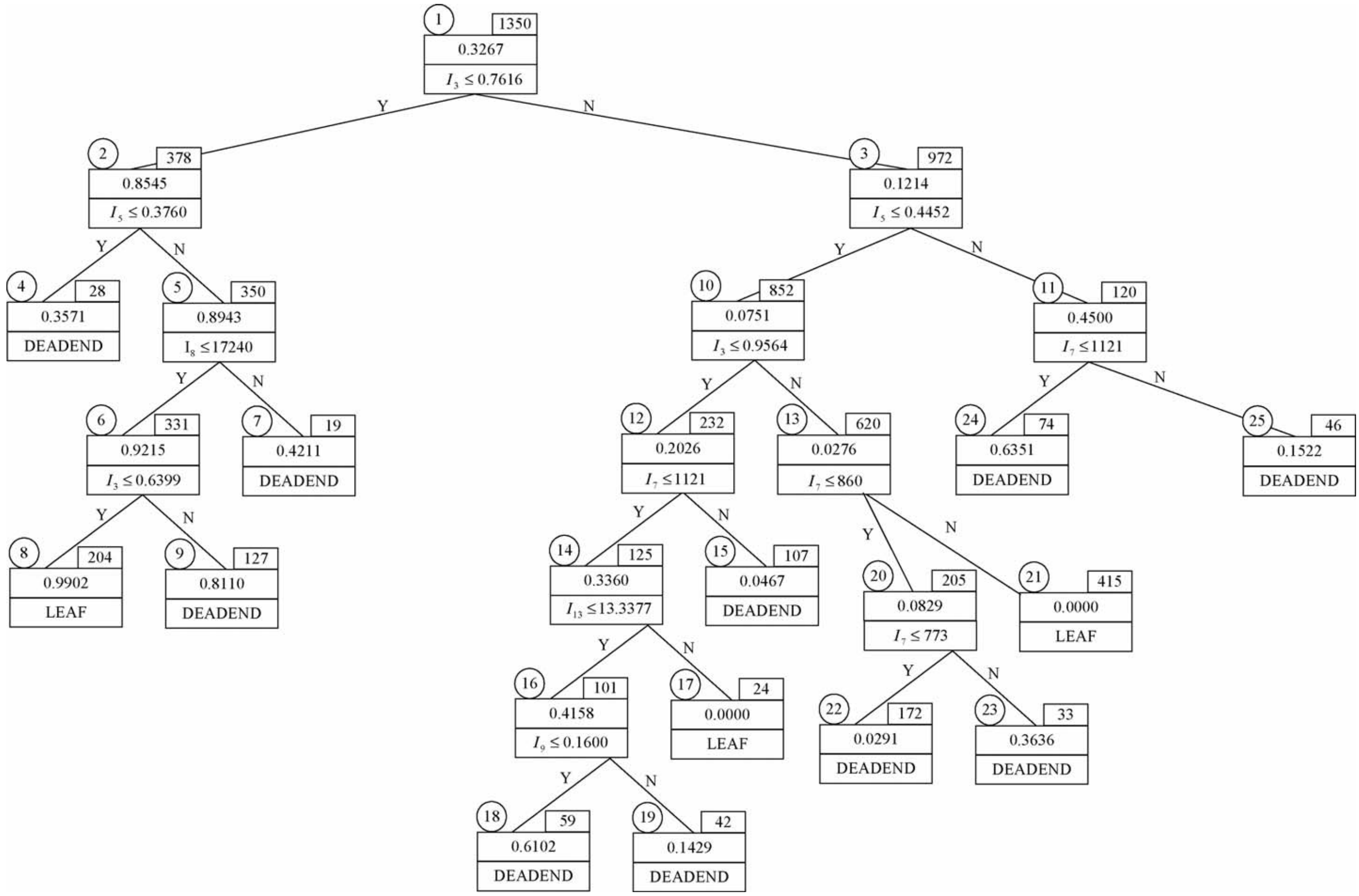


Fig. 1. Decision tree for the selection of the winding material.

are shown in Table 3. The winding material of the transformer is selected as follows:

- We start from node 1. The transformer has $I_3 = 0.94$, so the node 1 test $I_3 \leq 0.7616$ is not satisfied, so we lead to node 3.
- The transformer has $I_5 = 0.37$, so the node 3 test $I_5 \leq 0.4452$ is satisfied, so we lead to node 10.
- The transformer has $I_3 = 0.94$, so the node 10 test $I_3 \leq 0.9564$ is satisfied, so we lead to node 12.
- The transformer has $I_7 = 610$, so the node 12 test $I_7 \leq 1121$ is satisfied, so we lead to node 14.
- The transformer has $I_{13} = 15$, so the node 14 test $I_{13} \leq 13.3377$ is not satisfied, so we lead to node 17.
- Node 17 is a terminal node with CU index of 0.0, so 0% of transformers of this node are from CU winding. As a result,

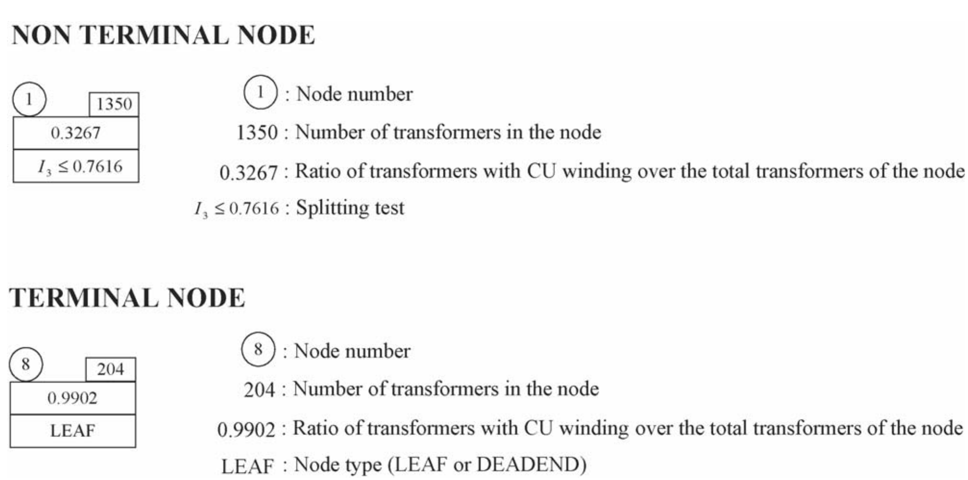


Fig. 2. Notation of the decision tree nodes.

Table 4
Calculation of the classification success rate of the decision tree

| Node number | Node type | CU index | Transformer designs | Correctly classified transformer designs | Classification success rate (%) |
|---|---|---|---|---|---|
| 4 | DEADEND | 0.3571 | 26 | 20 | 76.9 |
| 7 | DEADEND | 0.4211 | 18 | 13 | 72.2 |
| 8 | LEAF | 0.9902 | 196 | 195 | 99.5 |
| 9 | DEADEND | 0.8110 | 122 | 105 | 86.1 |
| 15 | DEADEND | 0.0467 | 103 | 101 | 98.1 |
| 17 | LEAF | 0.0000 | 23 | 23 | 100.0 |
| 18 | DEADEND | 0.6102 | 57 | 42 | 73.7 |
| 19 | DEADEND | 0.1429 | 41 | 38 | 92.7 |
| 21 | LEAF | 0.0000 | 398 | 398 | 100.0 |
| 22 | DEADEND | 0.0291 | 165 | 163 | 98.8 |
| 23 | DEADEND | 0.3636 | 32 | 26 | 81.3 |
| 24 | DEADEND | 0.6351 | 71 | 54 | 76.1 |
| 25 | DEADEND | 0.1522 | 44 | 41 | 93.2 |
| Total | | | 1296 | 1219 | 94.1% |

the decision tree of Fig. 1 estimates that the transformer of Table 3 has to be designed with AL winding.

### 5.3. Classification success rate

Following the methodology of Section 5.2, the winding material for all the 1296 FODs of the test set is estimated. The estimated winding material is compared to the actual winding material for all these 1296 FODs and this comparison provides the DT classification success rate. Table 4 shows that the success rate of the DT, tested with the independent test set, is 94%. The accuracy of the proposed method (94%) makes it very efficient for industrial use.

## 6. Conclusions

In this paper, a decision tree method is proposed for the selection of the winding material in power transformers. The basic steps in the application of the method to power transformers, like the generation of the learning set and test set, the selection of candidate attributes and the derivation of a characteristic decision tree are presented. The decision tree automatically selected the five most important attributes among the 13 candidate ones. With the learning set and test set used and for the selected candidate attributes, the decision tree method achieves a total classification success rate of 94%, which makes it very suitable for the selection of winding material in power transformers. The proposed method is very fast and effective, since it divides by two the effort of the transformer design engineer for the design of the optimum transformer.

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