Convex AC Optimal Power Flow Method for Definition of Size and Location of Battery Storage Systems in the Distribution Grid

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ABSTRACT

Majority of low carbon (LC) technologies such as photovoltaic (PV), electric vehicles (EV) and electric heat pumps (EHP) are and will be connected close to the final user at the low voltage (LV) level. These energy sources and consumers are assumed to have a low impact on the electricity grid, especially when their penetration level is low. Flexibility from these technologies is not stimulated as existing feed-in tariffs and static billing tariff systems do not stimulate intelligent price led production/consumption. The stochastic nature of LC technologies will, when reaching a significant share in the grid, result in undesirable events such as high spikes, overloading and under voltage sags. The basic assumption of the paper is that each consumer or producer is or will be led by market signals while the technical price signals for changing their production/consumption do not exist and are out of the scope of the paper. For each of the analyzed cases, the distribution system operator (DSO) is the one handling technical constraints and ensuring they are not violated.

The goal of the paper is to identify the role, size and placement (centralized, decentralized) of electricity storage owned by DSO. Consumption and production decisions are a result of Mixed Integer Nonlinear Programming (MINLP) economic dispatch and their impact is analyzed on typical Croatian distribution grids. As it will be shown, the battery storage reduce sudden demand or excess production spikes, voltage problems, equipment overloading and is, at the same time, flexible in terms of responding to market signals and different future scenarios. The analyses result in a techno-economic framework for short-term and long-term network planning of low carbon active distribution networks.

KEYWORDS

Battery Storage Systems, Distribution Network, Linearity, Renewable Energy Sources,

1. INTRODUCTION

Challenges brought by liberalization of electricity business environment, but also by the increasing integration of renewable energy sources (RES) driven by environmental goals, extend to all segments of future low carbon energy systems. In this new environment the single direction flow of electricity and information changes to interactive all system level energy interfaces. Putting aside potential positive impact of such approach, both economic and environmental, the variability and uncertainty of RES production also requires changes in basic concepts how equilibrium between generation and consumption is maintained. Energy storage systems (ESS) are often stipulated to be the crucial link in providing additional system security, reliability and flexibility to respond to changes that are still difficult to accurately forecast. And although larger storage units have been present in electric power systems for a long time, smaller distributed units are still characterized by relatively high initial investment cost and the economic viability by only energy arbitrage on a distribution level still does not exist. On the other hand, increasing integration of variable and uncertain generation on a domestic/district level creates new opportunities for ESS, not only for the final user but also for the distribution system operator (DSO). The role of ESS regardless of the ownership can, and probably will be, diverse and multiple in order to justify its investment. Despite this most papers focus on a single actor/stakeholder capability to exploit advantage of utilizing ESS, usually only through energy arbitrage; only a few recognize the advantage of multiple services ESS can offer to different system stakeholders. The challenges for future research lay in capturing the value of specific services and defining benefits of providing different services to multiple stakeholders. These service need to be compared with competing and/or complementing technologies such as demand response, electric vehicles, flexible generation and network enhancements resulting in a business case for investment into storage systems.

Size, type and location of battery storage systems in distribution network are discussed in a number of papers [1–10], however the existing research does not provide a unique conclusion or a framework for assessing benefits as the authors use different methodologies (ranging from heuristic methods due to complexity of the problem to mathematical programming techniques) and storage technologies (where some, such as CAES, do not seem as a likely solution at a distribution level). According to [11] ESS technologies are characterised by nine relevant feature out of which six are the physical ones. Storage systems are usually selected and optimized according to their power rating (MW), energy capacity (MWh) and location in the network. Power rating and energy capacity should be treated as separate technical characteristics of a specific ESS technology and both need to be defined and dimensioned separately taking into account their investment cost. The most common approach found in the literature is defining both discrete values and using exhaustive search methods to find the most favourable solution. The idea behind the work, presented in the following Sections, is to optimally decide on the size and position using direct method of mathematical programming. The concept, as elaborated in details in the following section, is based on the maximum ramp rate and energy required for provision of a specific service. By doing so the method is less restricted on the technology as the optimal result will be one suggesting which technology fits the desired characteristics the best. Siting and sizing of ESS cannot be defined separately from the operation of these units thus the proposed algorithm takes into account participation in the market as well as constraints and requirements defined by the distribution system operator.

It is difficult to argue there is a unified solution to the role, siting and sizing of the EES at the distribution level. However, the methodology presented in the following sections of this paper clearly defines benefits that ESS can bring to the future low carbon distribution system and defines a concept for determining the advantages of optimal dimension and placement of such units in the grid.

2. MODEL AND OPTIMIZATION PARAMETERS

Table I and Table II present the parameters and variables used in the Section 6 for modelling the distribution grid, distributed generation and storage units.

Table I Parameters of the optimization model

Parameter	Description
N	Total number of nodes
N_{line}	Total number of lines (branches)
N_{ESS}	Maximal number of ESS installed
Υ_{ESS}	Set of nodes where ESS can be installed
i	Counter referring to *i-th* node
t	Counter referring to *t-th* time step
$T_{\max}$	Time horizon of the simulation [hour]
τ	Simulation time step duration [hour]
$c_{el}(t)$	Electricity supply price [€/kWh]
$P_{load}(i,t)$	Active power load in node *i* in time step *t* [kW]
$Q_{load}(i,t)$	Reactive power load in node *i* in time step *t* [kvar]
R_{i-j}	Resistance of line between nodes *i* and *j* [Ω]
X_{i-j}	Reactance of line between nodes *i* and *j* [Ω]
Bsh_{i-j}	Susceptance of line between nodes *i* and *j* [S]
$\eta_{\text{charge}}(t,i)$	ESS charging efficiency
$\eta_{\text{discharge}}(t,i)$	ESS charging efficiency
$SOC(t,i)$	ESS State of Charge [kVAh]
$R_{ESS,up}(i)$	Ramp-up rate for ESS in node *i* [kW/h]
$R_{ESS,\text{down}}(i)$	Ramp-down rate for ESS in node *i* [kW/h]
$c_{ESS,P}$	ESS rated power installation cost [€/kW]
$c_{ESS,W}$	ESS rated capacity installation cost [€/kVAh]

Table II Decision variables of the optimization model

Parameter	Description
$P_{ESS}(i,t)$	ESS active power generation in node *i* in time step *t* [kW]
$P_{ESS,ch}(i,t)$	ESS charging active power in node *i* in time step *t* [kW]
$P_{ESS,dsch}(i,t)$	ESS discharging active power in node *i* in time step *t* [kW]
$P_{ESS\max}(i)$	Maximum charging or discharging power in node *i* [kW]
$Q_{ESS}(i,t)$	ESS reactive power generation in node *i* in time step *t* [kvar]
$W_{ESS}(i,t)$	Energy stored in ESS in node *i* in time step *t* [kVAh]
$\delta_{ESS}(i)$	Binary decision – ESS existence
$\Delta W_{ESS}(i)$	Difference between maximum and minimum ESS energy in node *i* [kVAh]
$P_{DG}(i,t)$	DG active power generation in node *i* in time step *t* [kW]
$Q_{DG}(i,t)$	DG reactive power generation in node *i* in time step *t* [kvar]
$P_{i-j}(t)$	Active power in line between nodes *i* and *j* in time step *t*[kW]
$Q_{i-j}(t)$	Reactive power in line between nodes *i* and *j* in time step *t*[kvar]
$U_i(t)$	Voltage in node *i* in time step *t*
$I_{i-j}(t)$	Current in line between nodes *i* and *j* in time step *t*
$v_i(t)$	Auxiliary substitute variable $v_i(t)=U_i(t)^2$
$i_{i-j}(t)$	Auxiliary substitute variable $i_{i-j}(t)=I_{i-j}(t)^2$

3. DISTRIBUTION GRID COMPONENTS AND MODELLING

Distribution network system is the largest and “final” part of the power system, delivering electricity to end users. A distribution system's network carries electricity from the transmission system and delivers it to consumers. Typically, the network would include medium-voltage (10 kV to 35 kV) power lines, transformer substations at several levels and low-voltage lines (less than 1 kV).

In general, there are two major types of distribution network layouts, radial or interconnected. A radial network is characterized by a single supply point and a central power line, a radial, branching out to supply the final consumer. This is typical for rural areas with isolated consumption points. Interconnected network layout is characteristic for urban areas, having multiple connections to other supply point. In normal operation these supply points are offline, or “open”; such configuration allows various configurations for the operating utility in cases of contingency. For this reason such layout is more reliable allowing continuous supply of consumers even in case of upstream network failures.

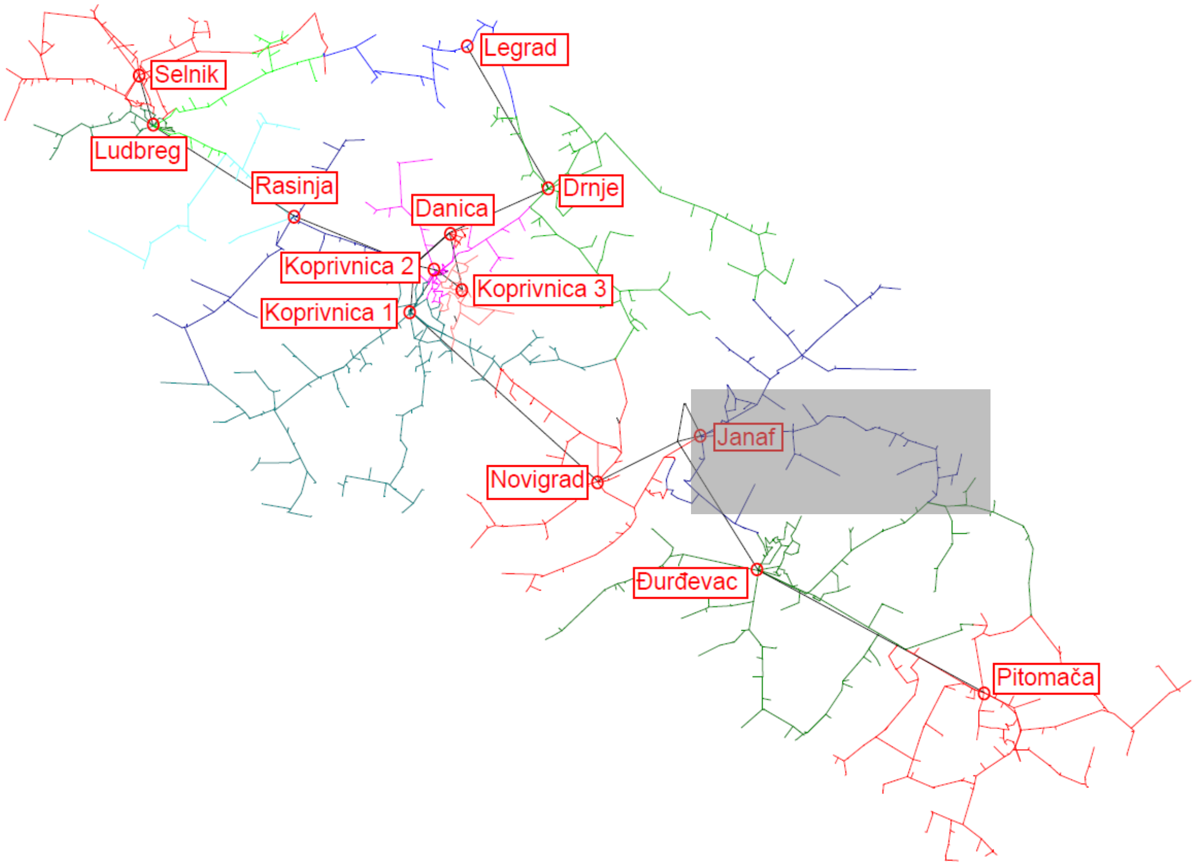

Figure 1 Neplan Model of Distribution Network DSO Koprivnica with gray rectangle marking analyzed Feeder Severovci

Network model, used in this paper and shown in Figure 1, is a part of distribution network in Croatia, more specifically in one of 21 Croatians distribution control areas (DCA) Koprivnica. The network is modelled in software Neplan for the purpose of planning the future 20 years of the Koprivnica DCA. The model is created from the existing geographic information system (GIS) and represents real, georeferenced distribution network with all its technical features. Part of this modelled network, feeder Severovci, is used for comparison with optimization algorithm and is shown with a grey rectangle in Figure 1. Details of the layout are shown in Figure 2.

Modelled feeder Severovci consists mainly of low cross section overhead lines as seen in Table III and Table IV.

Table III Optimization model data- Overhead lines summary

Type	mm^2	Length (km)
Al-Fe	25	28.85
Al-Fe	35	2.47

Table IV Optimization model data- Cable lines summary

Type	mm^2	Length (km)
XHE	70	1.70
XHP	95	0.08
XHE	150	0.47

Feeding node in optimization model network is 10 kV side of 35/10 kV power transformer in power station Janaf. In Neplan simulation and in optimization model, this node is set to 105% of its nominal voltage. Although power transformer in PS Janaf has on-load tap changer the voltage is set to fixed value to enable better comparison with the optimization model

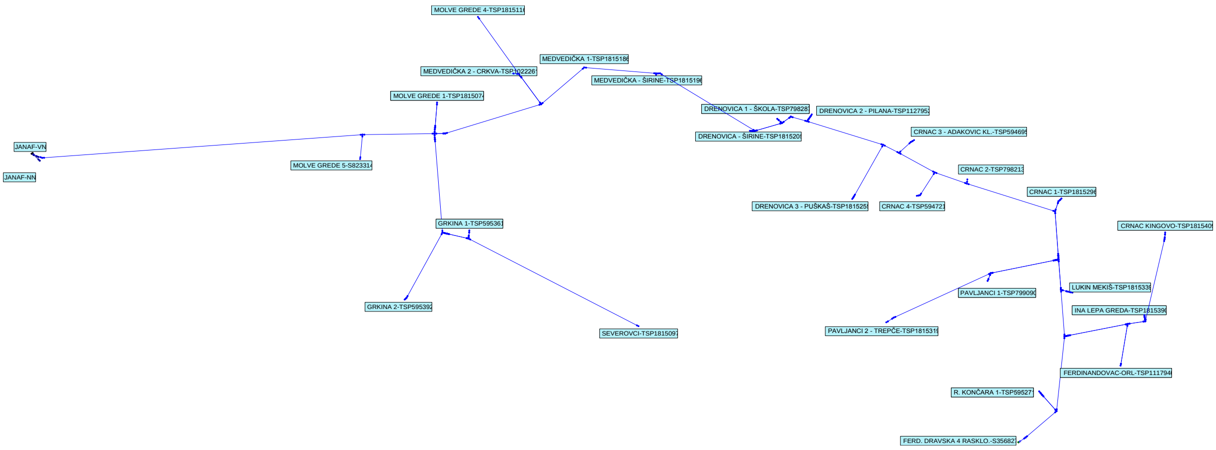

Figure 2 Neplan Model of Feeder Severovci

Model is taken from GIS and has many elements which would be omitted or approximated in usual modelling process. Examples are connection nodes between overhead line and cable or nodes which represent line switches. Resulting network has 77 nodes of which 26 represent power stations and are considered as possible location for ESS installation.

4. ELECTRICITY DEMAND

Electricity demand in Neplan is modelled with load profile curves, based on real measurements. In practice, except for measurement in feeding station, distribution network is often viewed as a "black box", with little known information of real time consumption in the downstream network. For this reason some approximations need to be made. Maximum measured power detected on feeder is divided on loads in its power station based on their nominal powers. Matching the scaling factors to incorporate network losses it is possible to approximate the load in each consumption node at a satisfactory level. Furthermore, DSO has information on the number and a type of consumers at each power station from their billing system. This data can be used for each consumption node and, using a profile based on measured curves for each consumer type (there are 9 typical curves defined by Croatian DSO) and their percentage in total load, reengineer a consumption profile for each consumer in the downstream distribution network. Resulting load for active and reactive power is shown in Figure 3. Most of the modelled loads have constant power factor 0.95.

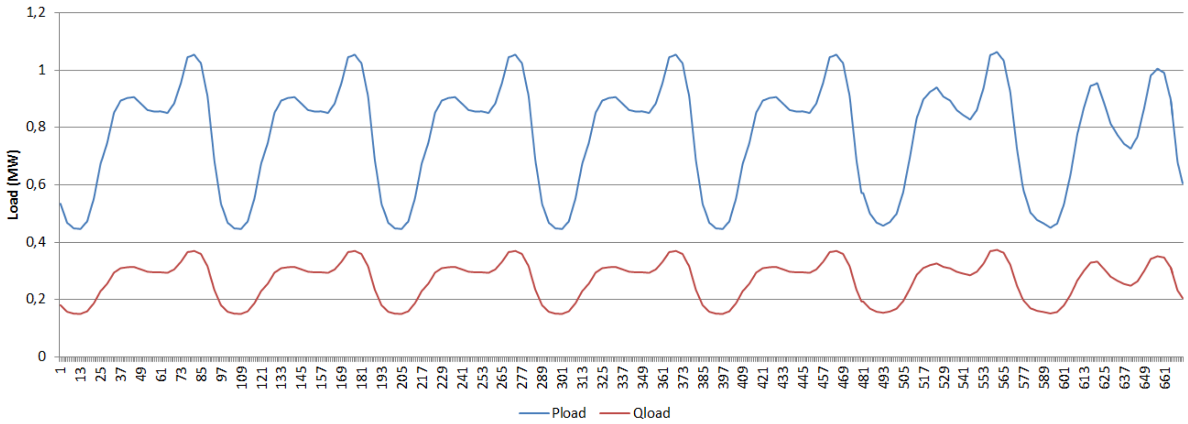

Figure 3 Cumulative active and reactive load at feeder Severovci

5. DISTRBUTED GENERATION

In this paper distributed generation (DG) is presented with photovoltaic (PV) generation. Normalized measurements presented in Figure 4 based on installed nominal power are used. Each power transformer station is assumed to have 30 kW installed PV generation. It results in 672.48 kW peak combined generation and total production of 36,171.72 kWh.

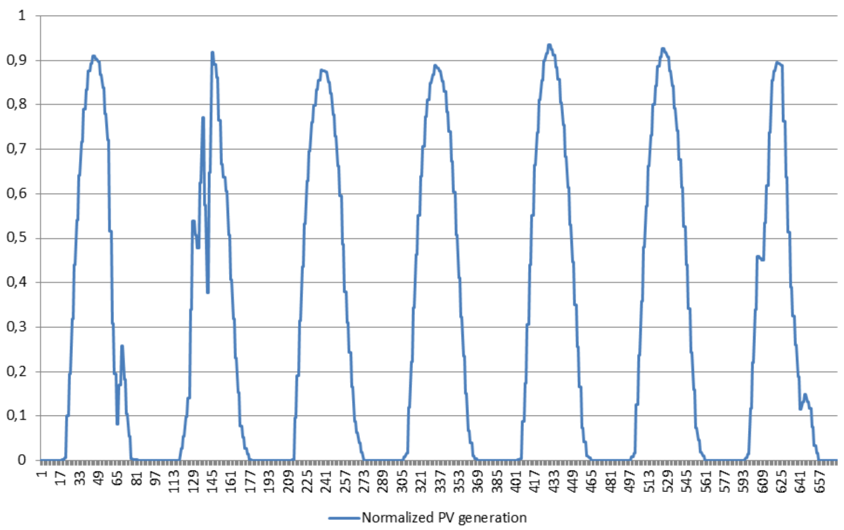

Figure 4 Normalized PV generation

6. OPTIMIZATION MODEL

Load flow with load profile calculation is solved for one week in 15 minutes resolution which results in 672 load flow calculations for each scenario. Resulting loadings are taken before transformer in power station and they include losses in them. In optimization model network, loads are including transformer and its load. Maximum loading in observed feeder is 1.063 MW and total active energy is 130,403 kWh

6.1 Second Order Conic Programming (SOCP) Optimal Power Flow

In this paper Mixed Integer Second Order Cone Programming (SOCP) is used to model and solve Optimal Power Flow (OPF) for distribution network. All variables are in per unit values. Same as in Neplan model, Load Flow calculations are solved for one week period in 15 minutes resolution, resulting in 672 load flow calculations for each scenario. Optimizations are made using FICO Xpress [12].

First Second Order Conic load flow formulation, known to authors, can be found in [13,14] and is later applied for OPF problem in [15]. In the mentioned papers, network analyses do not include distributed generation nor do they consider reverse power flows. The experience while modelling has shown that the model has problems with network loss calculation when applied on the network with reverse power flows. Losses can obtained negative values. The methodology in based on Cartesian complex number formulation and two substitutions; one which multiplies two voltages of connected nodes and sine of voltage angle difference, and the other with same voltages and cosine of angle difference. In this paper different formulation described in the following section is used.

6.2 Load flow equations

In this paper SOCP OPF model presented in [10] is adopted. Load Flow for two nodes *i* and *j* connected with impedance $Z_{i-j} = R_{i-j} + jX_{i-j}$ can be presented with equations (1) and (2). Impedance quadratic terms are neglected.

$$U_j^2(t) = U_i^2(t) - 2\left(P_{i-j}(t)R_{i-j} + Q_{i-j}(t)X_{i-j}\right) \tag{1}$$

$$I^2_{i-j}(t) \geq \frac{1}{U_i^2(t)}\left(P_{i-j}(t)^2 + Q_{i-j}(t)^2\right) \tag{2}$$

To linearize equation (1) and transform equation (2) to Second Order Cone constraint substitution (3) and (4) are introduced and voltage in equation (2) is approximated with 1 p.u..

$$i_{i-j}(t) = I_{i-j}^2(t) \tag{3}$$

$$v_i(t) = U_i^2(t) \tag{4}$$

This results in the following equations used in this paper:

$$v_j(t) = v_i(t) - 2\left(P_{i-j}(t)R_{i-j} + Q_{i-j}(t)X_{i-j}\right) \tag{5}$$

$$i_{i-j}(t) \geq P_{i-j}(t)^2 + Q_{i-j}(t)^2 \tag{6}$$

It is important to notice that using advanced interior point method solvers for (6) results in much better solutions than general nonlinear formulation in (2) avoiding the algorithm to results in local optimum points and give unexplainable/nonlogical results. However, an important relation between voltage and power losses is lost meaning that further research is required on how to improve this formulation without the loss of SOCP formulation.

6.3 *Distflow* equations

Simplified Distflow equations, introduced in [16,17] and further improved in [10,18], are used in this paper. Equations (7) and (8) are Kirchhoff current law for all lines in the network. Power going through the line is the sum of load, losses, shunts, distributed generation and ESS power at the end node and powers going from and to the end node. Equations (9) are important as they enable reverse power flows which can occur in networks with DG and ESS. Equation (10) describes lines' thermal limits.

$$P_{i-j}(t) = P_{load}(j) - P_{DG}(j,t) - P_{ESS}(j,t) + i_{i-j}(t) R_{i-j} + \sum_{j=m} P_{m-n} - \sum_{j=n} P_{m-n} \tag{7}$$

$$Q_{i-j}(t) = Q_{load}(j) - Q_{DG}(j,t) - Q_{ESS}(j,t) + i_{i-j}(t) X_{i-j} - v_i(t) Bsh_{i-j} + \sum_{j=m} Q_{m-n} - \sum_{j=n} Q_{m-n} \tag{8}$$

$$\begin{aligned} P_{i-j}(t) &\in (-\infty, +\infty) \\ Q_{i-j}(t) &\in (-\infty, +\infty) \end{aligned} \tag{9}$$

$$\begin{aligned} -P_{lineMAX} &\leq P_{i-j} \leq P_{lineMAX} \\ -Q_{lineMAX} &\leq Q_{i-j} \leq Q_{lineMAX} \end{aligned} \tag{10}$$

Load flow results obtained with this formulation are compared to Neplan results. Neplan uses extended Newton-Raphson numerical methods to solve load flow problem. Neplan results for node voltages for observed period are presented with Figure 5 and Xpress solutions are presented on Figure 6. Network nominal voltage is 10 kV.

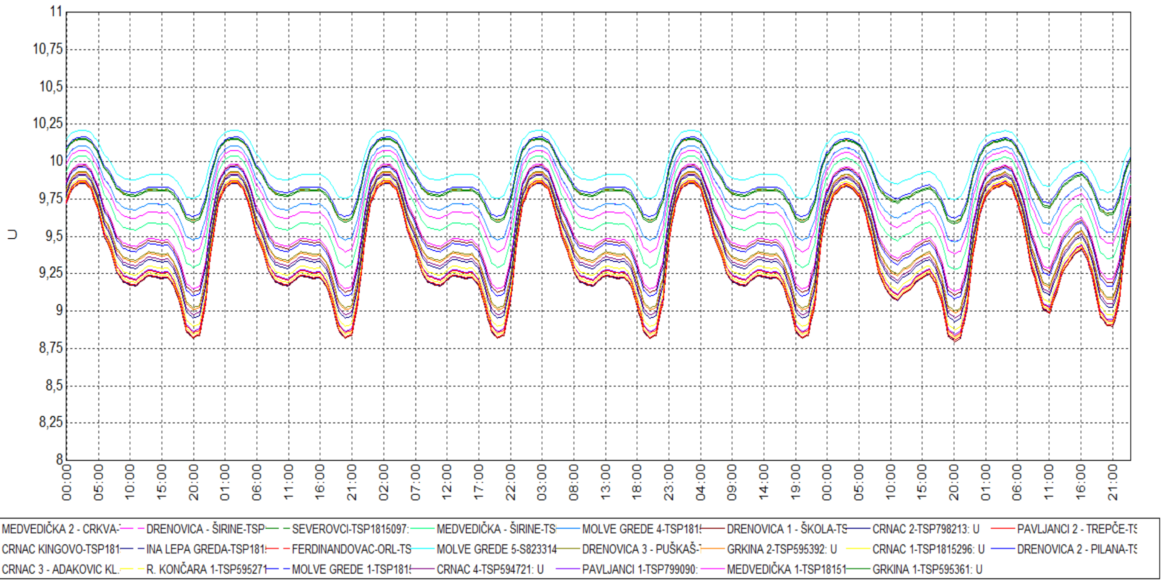

Figure 5 Neplan load flow results - node voltages - initial network

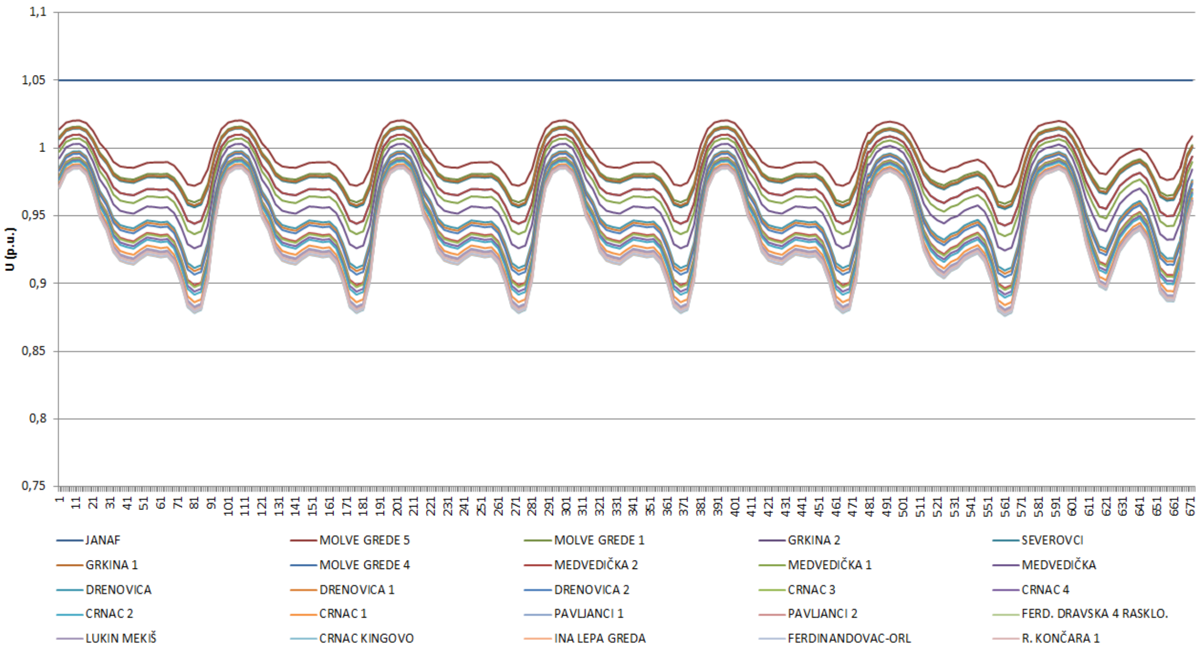

Figure 6 Xpress load flow results - node voltages - initial network

To further present obtained results, for the node with lowest voltage (Power Station Crnac Kingovo), Neplan and Xpress results and their difference are presented in Figure 7. Maximum difference in observed period was 0.36%. Voltage results obrained by Xpress are for all nodes a bit lower then Neplan results.

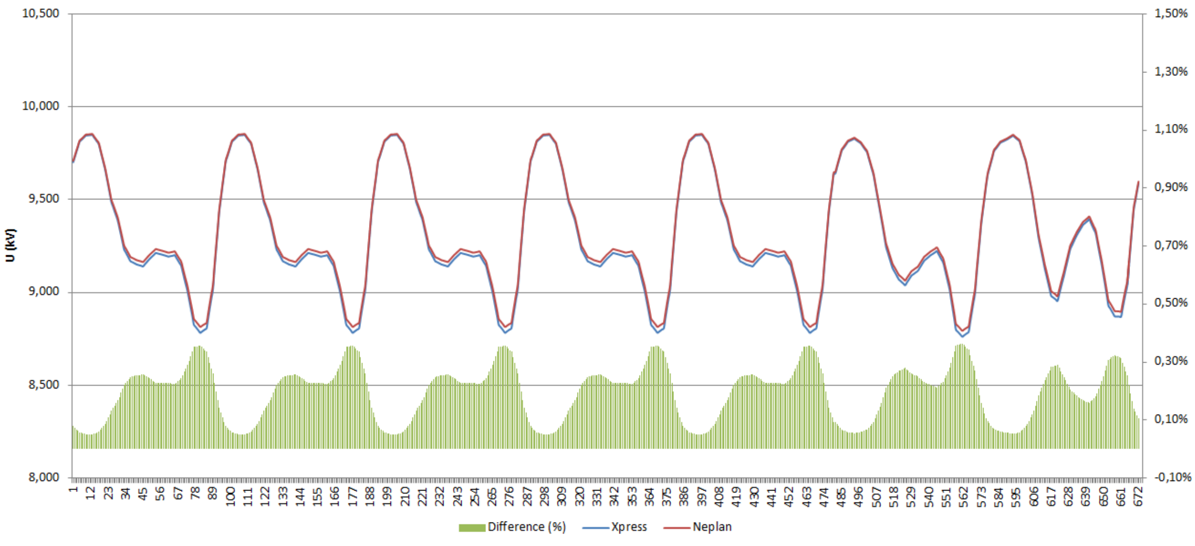

Figure 7 Results comparison for node with lowest voltage (Crnac Kingovo)

Power quality requirements, defined by Distribution System Operator, demand voltages in the entire network to be within ±10% range. This, as seen in Figure 6, is not true for some nodes which are far from the feeding power station in high load periods. To solve this technical issue, DSO would invest in network reinforcement, building an additional line or reinforcing the existing ones. An alternative to this is to try to control the load in the selected area, using a form of flexible demand response. Another solution, easy to implement, is the installation of ESS, presented in this paper.

6.4 ESS equations

In this paper ESS power and energy rating can be determined simultaneously with ESS location using Mixed Integer formulation. ESS constraints are presented with equations (11)-(21).

$$\begin{aligned} &P_{ESS}\left(i,t\right) \in \left(-\infty,+\infty\right) \\ &Q_{ESS}\left(i,t\right) \in \left(-\infty,+\infty\right) \end{aligned} \tag{11}$$

$$Q_{ESS}\left(i,t\right) = 0 \tag{12}$$

Equations (11) enable ESS charging and discharging. This is the main difference in modelling between ESS and DG. DG can only inject power in the network. In this paper ESS does not produce reactive power which is presented in Eq (12). ESS reactive power modelling will be included in future work.

$$P_{ESS}\left(i,t\right) = 0 \mid i \in N \setminus \Upsilon \tag{13}$$

$$\sum_{i\in\Upsilon} \delta_i \le N_{ESS} \tag{14}$$

Equation (13) prevents ESS installation in nodes which are not power stations. Because network is transferred directly from GIS many structural nodes are part of the network. Also, branching nodes in radial networks, which are not power stations, are excluded as possible ESS installation sites. With (14) it is possible to limit upper number of ESS sites in the solution. This is important from the perspective of algorithm complexity, reducing the feasible area of the algorithm to only acceptable and logical solutions.

$$P_{ESS\,\min}\delta\left(i\right) \le P_{ESS}\left(i,t\right) \le P_{ESS\,\max}\delta\left(i\right) \tag{15}$$

$$W_{\min}\left(i\right)\delta\left(i\right) \le W\left(i,t\right) \le W_{\max}\left(i\right)\delta\left(i\right) \tag{16}$$

$$P_{ESS}\left(i,t\right) = P_{ESS,ch}\left(i,t\right) - P_{ESS,dsch}\left(i,t\right) \mid \; P_{ESS,ch}\left(i,t\right) \ge 0, P_{ESS,\mathrm{ds}ch}\left(i,t\right) \ge 0 \tag{17}$$

With (15) and (16) ESS power and energy are set to zero if node *i* is not selected as ESS installation node. Equation (17) is used to make a distinction between charging and discharging power. This method is often used in linear programming where solution is found on the state space vertices but it has shown acceptable behaviour in SOCP formulation.

$$W_{ESS}\left(i,t+1\right) = W_{ESS}\left(i,t\right) + \left(\eta_{\mathrm{charge}} P_{ESS,ch} - \eta_{\mathrm{discharge}} P_{ESS,\mathrm{ds}ch}\right)\tau \tag{18}$$

Equation (18) defines ESS State of Charge. Presented model uses 15 minutes time step; for this reason $\tau = 0.25h$. It should be noticed that any time step can be used, depending on the environment modelled.

In modelling, power and energy capacities are a result of optimization, avoiding firm predefinition limitation of upper bounds. This is done by using the following two equations:

$$P_{ESS\,\max}\left(i\right) = \max\left\{\mathrm{P_{ESS,ch}}\left(i,\mathrm{t}\right), \mathrm{P_{ESS,dsch}}\left(i,\mathrm{t}\right)\right\}, \forall i \in N, \forall t \in T \tag{19}$$

$$W_{ESS}\left(i\right) = 1.25\left(\max\left\{WESS(i,t)\right\} - \min\left\{WESS(i,t)\right\}\right), \forall i \in N, \forall t \in T \tag{20}$$

Equation (19) defines rated power at node *i* as maximum between charging and discharging power in all observed times. In a similar way equation (20) determines upper and lower battery energy levels and determines BESS capacity rating from these values. It is multiplied by 1.25 because due to recommendations to use only 80% of nominal BESS rating.

$$\begin{aligned} P_{ESS}\left(i,t+1\right)-P_{ESS}\left(i,t\right) &\leq R_{ESS,up} \\ P_{ESS}\left(i,t+1\right)-P_{ESS}\left(i,t\right) &\geq -R_{ESS,down} \end{aligned} \tag{21}$$

Equation (21) enables setting of up and down ramp rate which is required by some ESS technologies. However, more of the battery technologies are characterized by fast power response capabilities and these constraints can often be neglected, even for very short simulation periods.

7. RESULTS

This section presents the results of the optimal sitting and sizing problem in the observed feeder in the distribution network. The impact of added storage is quantified and assessed.

7.1 ESS impact

Battery ESS (BESS) in this paper is modelled over a set of scenarios with the goal of creating benefits at two levels: maintaining distribution network parameters within the set constraints helping to minimize network losses and maximizing profit by participating in a day ahead market. Table V presents scenarios analyzed in this paper. In all scenarios considering DG, multiple ESS locations were enabled in order not to favour a specific location and to enable a better solution. The solution in which a number of ESS nodes is limited can be considered a subset of the solution where all nodes can be selected as ESS installation nodes.

Table V Scenario overview

Scenario name	DG	Market	No. of ESS	Objective function	Limits
A	No	No	0	Load flow	-
B	No	No	1	min (investment + losses)	Voltage
C	No	No	N	min (investment + losses)	Voltage
D	Yes	No	N	min (investment + losses)	Voltage
E	Yes	Yes	N	max (profit(ESS) – losses)	Voltage
F	Yes	Yes	N	max (profit(ESS) – losses)	Voltage, ESS power and capacity less or equal to scenario C solution
G	Yes	Yes	N	max (profit(ESS+DG))	Voltage; ESS charged from DG, daily SOC
H	Yes	Yes	N	max (profit(ESS+DG) – losses)	Voltage; ESS charged from DG, daily SOC
I	Yes	Yes	N	max (profit(DG+ESS) - losses – investment)	Voltage; ESS charged from DG, daily SOC

Network voltage constraints are presented with the equation (22). Capacity and power constraints in scenario F are using the following logic; to satisfy network constraints some capacity should be installed. Is it possible to also maximize profit by using this capacity. In last three scenarios ESS can be charged only from DG and discharge power is unconstraint. BESS SOC in those scenarios should be equal the initial one at the beginning of every day.

$$0.9^2 \le v_i(t) \le 1.1^2 \tag{22}$$

First scenario is used to verify load flow calculation to those obtained by commercial product Neplan and although loss minimization is used to run the solver and find solution, the model does not have freedom to find any other solution. It must be stressed that voltage limits were relaxed in initial load flow results because there is no feasible solution within allowed range. Scenarios B, C and D do not include day ahead market and in them network losses and investment costs are minimized according to equation (23). Energy range for every BESS is multiplied with 1.25 to determine optimal BESS energy capacity in which SOC is between 20% and 100%.

$$\min \sum_{t\in T} P_{loss} c_{LOSS} + \sum_{i\in N} P_{ESS\max}(i) c_{ESS,\mathrm{P}} + \sum_{i\in N} 1.25\Delta W_{ESS}(i) c_{ESS,\mathrm{W}} \tag{23}$$

BESS power and capacity prices are given in Table VI. Prices are for lead – acid battery from [19]. Those prices are also used in [10]. It should be noted these values are pessimistic as many reports suggest in the upcoming years, even by the end of 2015, the prices might be even 10 times lower [20].

Table VI ESS installation cost

Technology	$C_{ESS,P}$ (€/kW)	$C_{ESS,W}$ (€/kWh)
BESS	1,000	4,000

In scenarios E to I day ahead prices [21], shown on Figure 8, are used. Fixed cost for losses of 62.9 EUR/MWh are used [22].

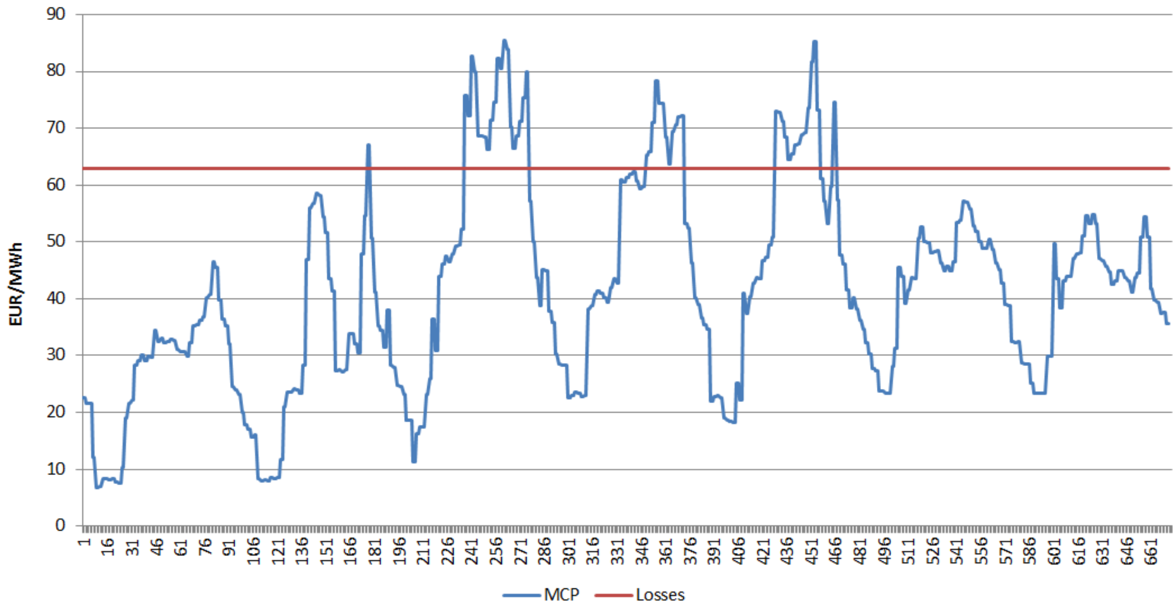

Figure 8 Day ahead market clearing price and losses cost

In scenarios E and F objective function (24) is used. The goal is to make profit from ESS participation in day-ahead market and minimize losses in the same time.

$$\max \sum_{i\in N}\sum_{t\in T} P_{ESS}(i,t)\,c_{DA}(t) - \sum_{t\in T} P_{loss}(t)\,c_{loss}(t) \tag{24}$$

In scenarios G to I distributed generation can store energy in ESS and sell it in more profitable moment. In those scenarios ESS cannot store energy taken from the network but its discharge power is not limited.

Scenario G, as presented in objective function (25) maximizes profit of such ESS-DG system without losses consideration.

$$\max \sum_{i\in N}\sum_{t\in T} P_{ESS-DG}(i,t)\,c_{DA}(t) \tag{25}$$

Additionally on previous one, scenario H also minimizes network losses as presented with (26).

$$\max \sum_{i\in N}\sum_{t\in T} P_{ESS-DG}(i,t)\,c_{DA}(t) - \sum_{t\in T} P_{loss}(t)\,c_{loss}(t) \tag{26}$$

Scenario H maximizes profit but minimizes losses and investment cost as seen in (27). Such formulation is possible only because of (19) and (20). Usually exhaustive search with predetermined ESS power and capacity ratings is conducted.

$$\begin{aligned}\max & \sum_{i\in N}\sum_{t\in T} P_{ESS-DG}(i,t)\,c_{DA}(t) - \sum_{t\in T} P_{loss}(t)\,c_{loss}(t) \\ & -\sum_{i\in N} P_{ESS\max}(i)\,c_{ESS,P}(t) - \sum_{i\in N} 1.25 W_{ESS\max}(i)\,c_{ESS,\mathrm{W}}(t)\end{aligned} \tag{27}$$

Table VII Results summary

Scenario name	**P_{ESS} (kW)**	**W_{ESS} (kWh)**	**Investment ESS (EUR)**	**Losses (EUR/y)**	**Profit ESS (EUR/y)**	**Profit DG/DG-ESS (EUR/y)**
A	0.00	0.00	0.00	44,166.66	-	-
B	84.90	318.51	1,358,932.00	43,724.62	2,315.86	-
C	84.70	318.00	1,356,700.00	43,407.47	2,350.81	-
D	84.75	289.24	1,241,686.02	26,049.51	-781.42	92,953.17
E	3,612.20	17,254.26	72,629,261.00	45,312.77	587,703.27	92,953.17
F	84.80	289.23	1,241,738.80	26,648.78	2,724.81	92,953.17
G	7,018.43	6,029.85	31,137,833.33	213,039.37	-	106,746.62
H	2,127.23	3,046.91	14,314,870.65	27,370.81	-	102,699.45
I	84.75	289.24	1,241,687.52	26,255.34	-	93,252.54

Based on the data above, annual benefit and Net Present Value (NPV) for 15 years and 5% discount rate is calculated. Results are shown in Table VIII. Initial state (scenario A) is used as reference point. Benefits for network losses are difference between scenario losses and losses in scenario A. Benefits from ESS are profit obtained in day ahead market. For scenarios B-D day ahead prices are used. In scenarios G-I ESS generation is combined with

DG and benefits are calculated as a difference between non optimized DG generation (as in scenarios D-F) and optimized DG-ESS generation.

Table VIII Benefit summary and NPV

Scenario name	Investment ESS (EUR)	Benefit (EUR/y)	NPV (EUR)	NPV recent prices (EUR)
A	0.00	0,00	0.00	0.00
B	1,358,932.00	2,757.90	-1,330,305.97	-194,279.22
C	1,356,700.00	3,110.01	-1,324,419.19	-190,169.19
D	1,241,686.02	17,335.73	-1,061,747.12	-33,950.63
E	72,629,261.00	586,557.17	-66,540,998.16	-4,506,321.73
F	1,241,738.80	20,242.69	-1,031,626.63	-3,858.18
G	31,137,833.33	-155,079.26	-32,747,503.05	-13,946,268.54
H	14,314,870.65	26,542.13	-14,039,372.44	-3,829,421.41
I	1,241,687.52	18,210.68	-1,052,666.84	-1,052,666.84

In all scenarios NPV values are negative which means that investment in ESS with investment cost as in Table VI is not profitable by only using energy arbitrage. Although NPV shows negative results in all scenarios network parameters are kept within a limit which is not the case in the initial solution indicating the need to investment into the grid to comply with the Grid Code requirements. However, the analysis does not include investments into new cables, overhead lines or other equipment as such investments are location, type and corridor specific. ESS can additionally participate in different ancillary markets and have multiple simultaneous roles [19,23,24]. Further research on this topic is required.

NPV with more recent ESS investment prices, taken from [20], are calculated in Table VIII. Prices taken are for Li-ion BESS technology and they are 1500 EUR/kW and 300 EUR/kWh. In scenarios B, C, D and I, which have investment cost minimization in objective function, it is not completely correct to apply new prices. However, the new prices are applied on previously obtained solutions providing only a general idea on potential benefits gained by price reduction.

7.2 Network voltages

Network voltages for all scenarios, except for initial state, are within allowed range. Voltage profiles for scenarios B to D are similar and are presented in Figure 9. Voltage profile pattern changes for scenario F, as seen in Figure 10., characterized by faster changes during specific time periods when DG and ESS generation covering local load and resulting in minimal voltage drop between power stations. For scenario G, with maximum ESS power installed, voltages are shown in Figure 11. It can be seen that the upper voltage limit is the critical constraint for ESS power determination. With favourable ratio between lowest and highest price on a day ahead market and without technical constraint the optimal size of the BESS would be much higher.

If requested and properly incentivized, ESS can be used to eliminate sudden voltage deviations and sags. The model presented in Section 6 can be easily expanded to include additional constraints.

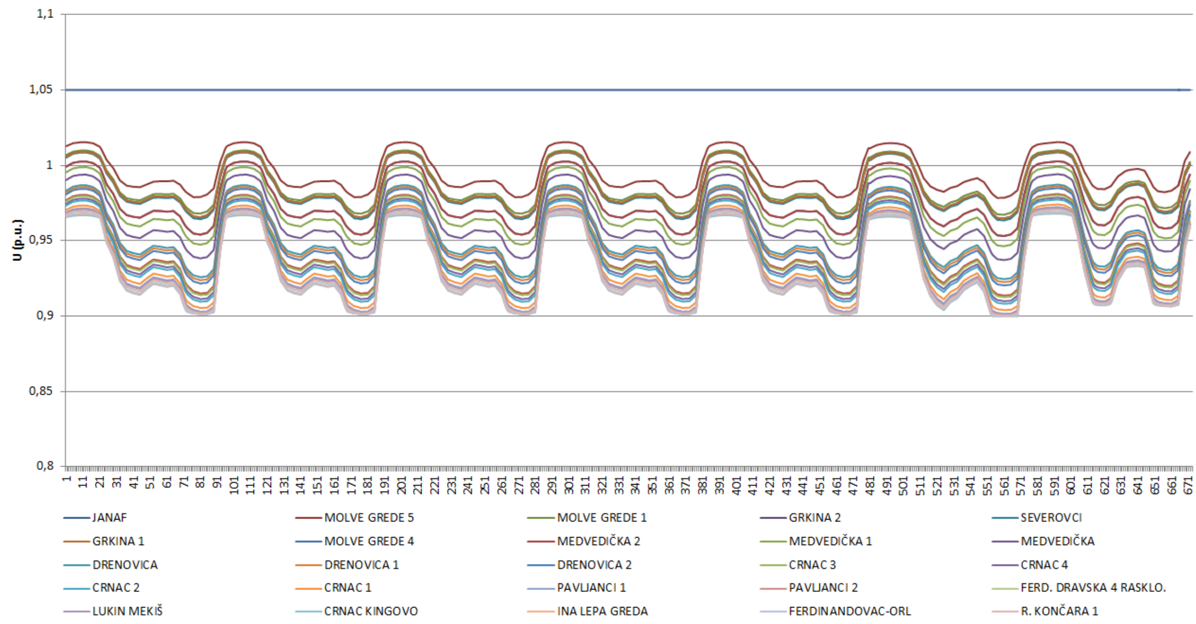

Figure 9 Voltages – Scenario D

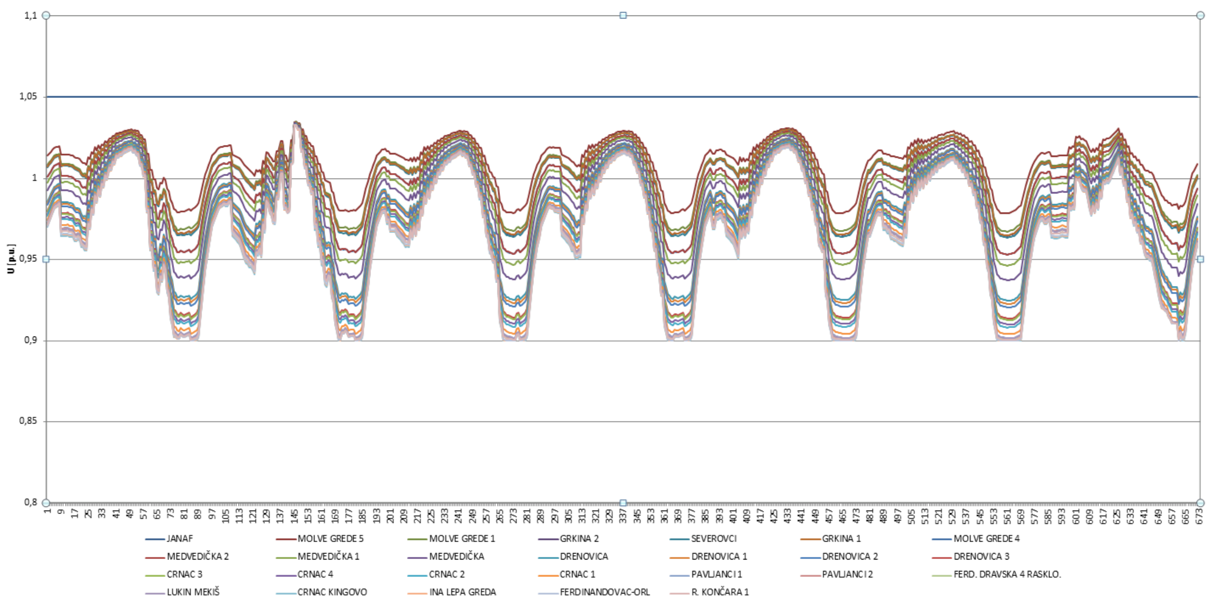

Figure 10 Voltages - Scenario F

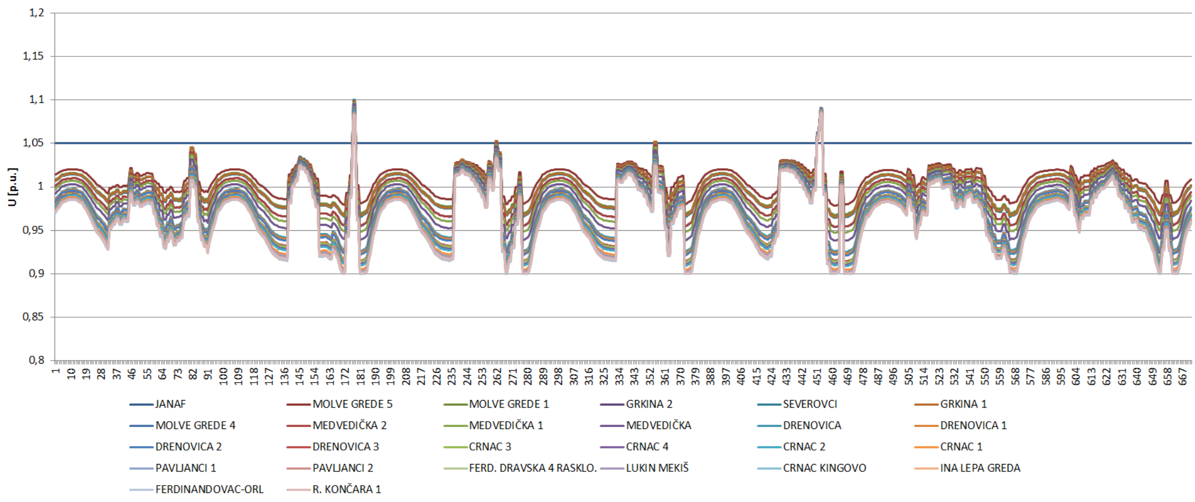

Figure 11 Voltages - Scenario G

7.3 Network losses

Table IX Network losses

Scenario name	Losses (kwh/w)	Difference %
A	13,503.32	0.00
B	13,368.17	1.00
C	13,271.21	1.72
D	7,964.26	41.02
E	13,853.73	-2.59
F	8,147.48	39.66
G	65,133.72	-382.35
H	8,368.23	38.03
I	8,027.19	40.55

Figure 12 shows dynamic network losses for scenarios A to D. Scenarios B and C have similar pattern as initial scenario A. They charge ESS in periods of low demand and discharge it in high loading periods to exploit square dependency between connection loading and losses. In scenario D the influence of DG can be noticed during daylight time. Energy stored in the ESS is discharged during high loading hours and during those hours power loss profile is similar to those in previous scenarios. Total losses are on the other hand around 40% lower.

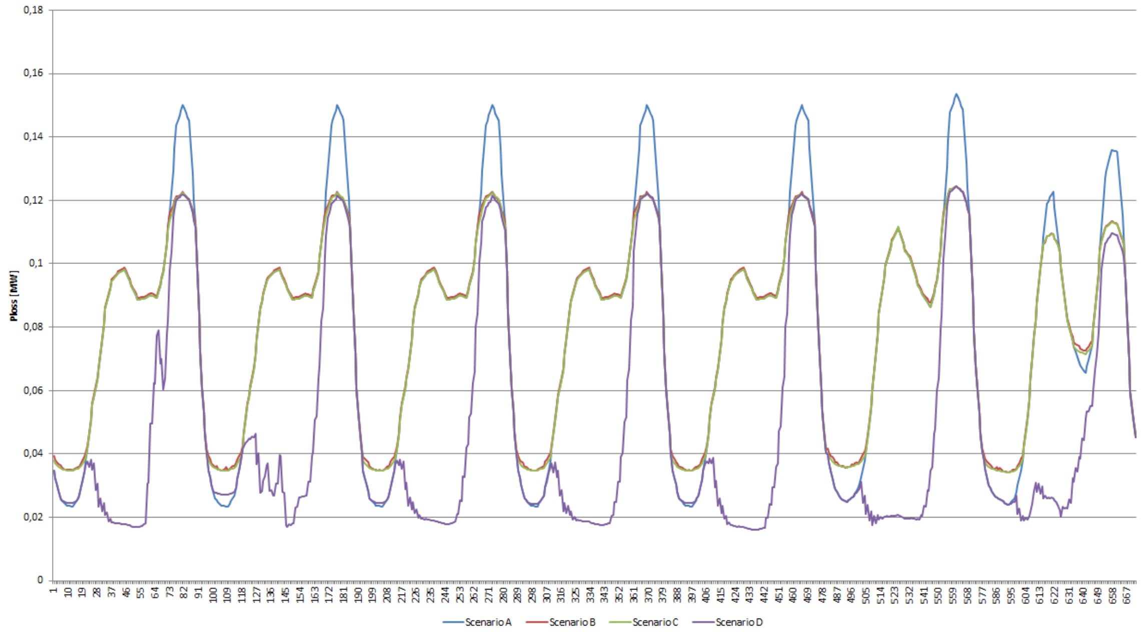

Figure 12 Network losses - scenarios A, B, C and D

In Figure 13 losses for scenarios which maximize profit and minimize losses are presented. Losses from initial scenario A are used as a reference for comparison. While scenario E losses are in the same range as the ones without installed DG, in scenario F it can be notice a 40% reduction compared to scenario C. The reason for comparing those particular scenarios can be explained with similar EES size and power, however adding distributed units in scenario F implies a possible increase in power losses. Optimal management of DG and EES actually negates this and helps the DSO to reduce them even further.

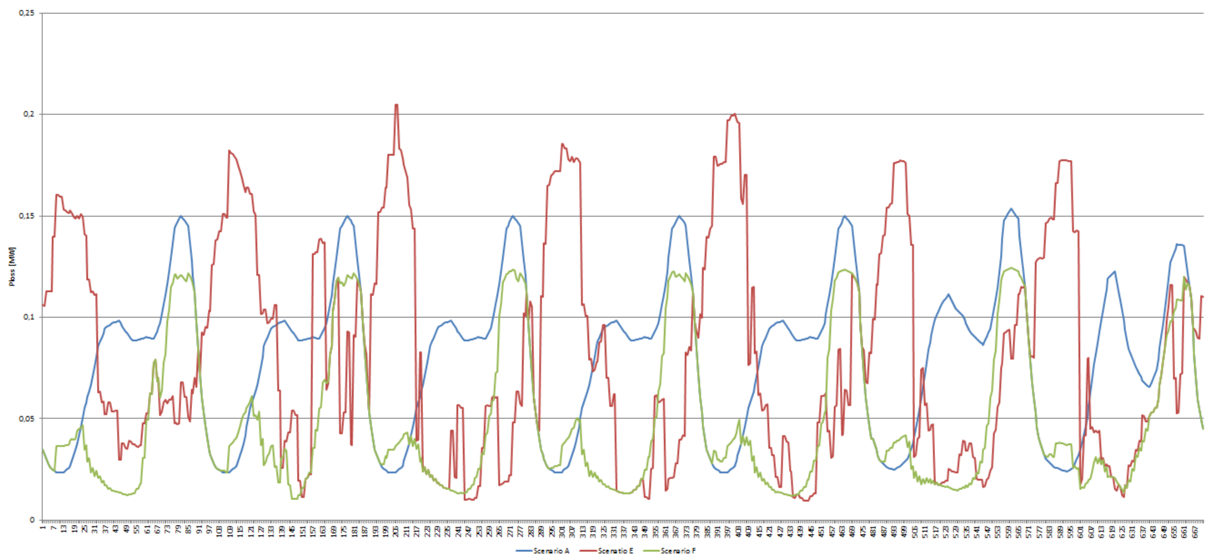

Figure 13 Network losses - scenarios E and F

In Figure 14 distribution network losses for scenarios G to I are presented. As the objective function of scenario G does not have minimization of losses included they reach high values, up to 5 MW. This is the result of injecting most of the produced power during the daily peaking prices. Although scenarios H and I have similar network losses (around 40% less then in the initial solution) scenario H has much larger ESS-DG installations. Similar to scenario D they exploit DG generation during daylight hours and lower total losses by discharging accumulated energy during high loading hours.

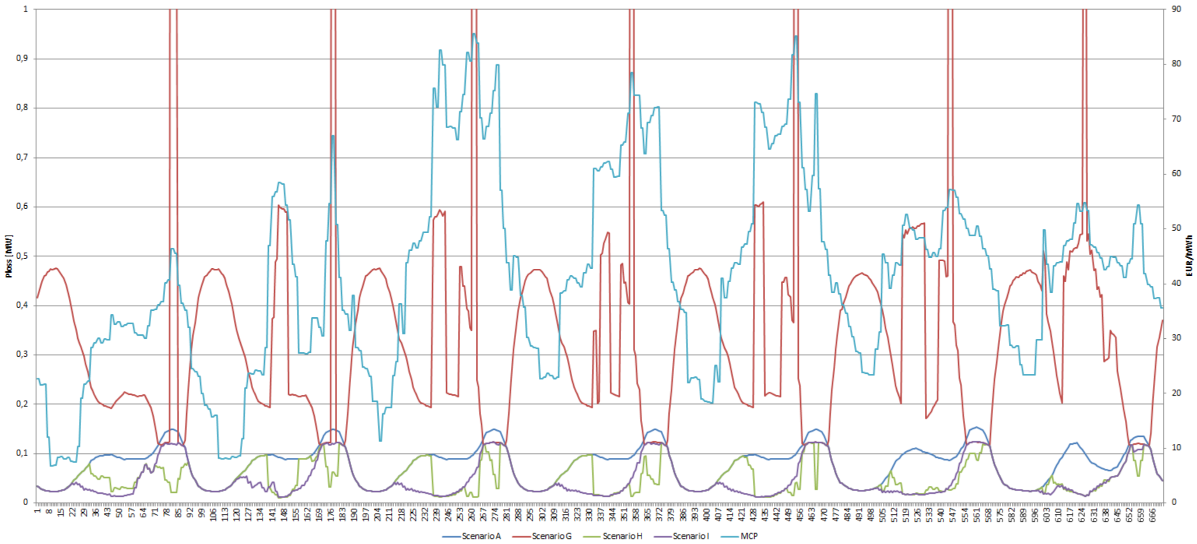

Figure 14 Network losses - scenarios G, H and I

8. POWER GENERATION

Figure 15 shows charging/discharging behaviour of EES for scenarios minimizing power losses and investments; scenarios B, C and D (discharging values are positive, and charging values are negative). In cases without DG, scenarios B and D, ESS is charged during the night and discharged during high loading periods. In scenario D, when DG is introduced, the algorithm minimizes losses by charging EES during high PV generation periods resulting in time shift for charge/discharge patterns (presented as green line).

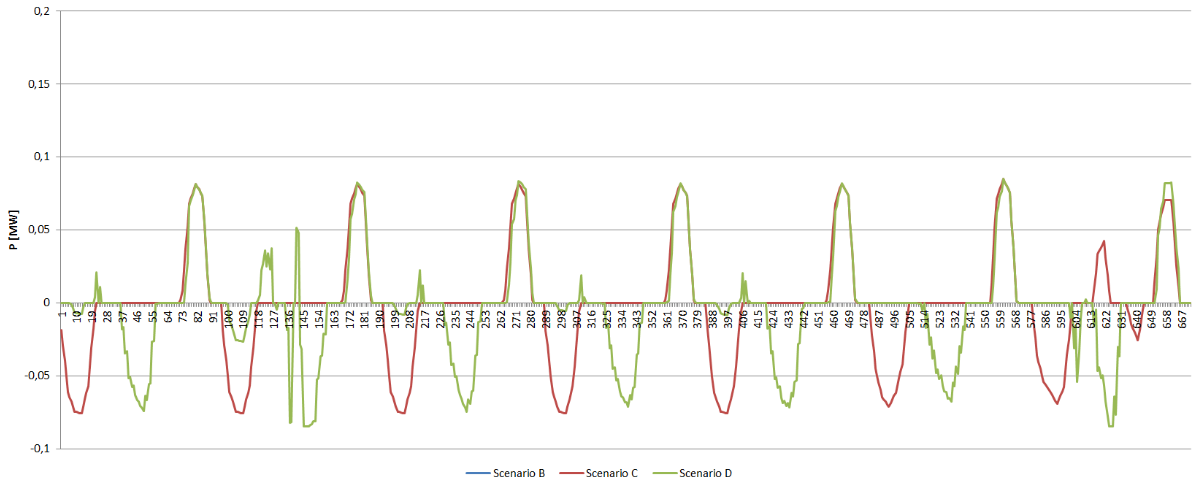

Figure 15 Total ESS generation - scenarios B, C and D

For cases E and F driven by profit maximization and minimization of network losses, ESS stores energy during low prices and returns it into the grid during high prices. Optimal solution obtained for in scenario E has much larger storage capacity as power and capacity of EES in scenario F are limited by constraints inherited from scenario C.

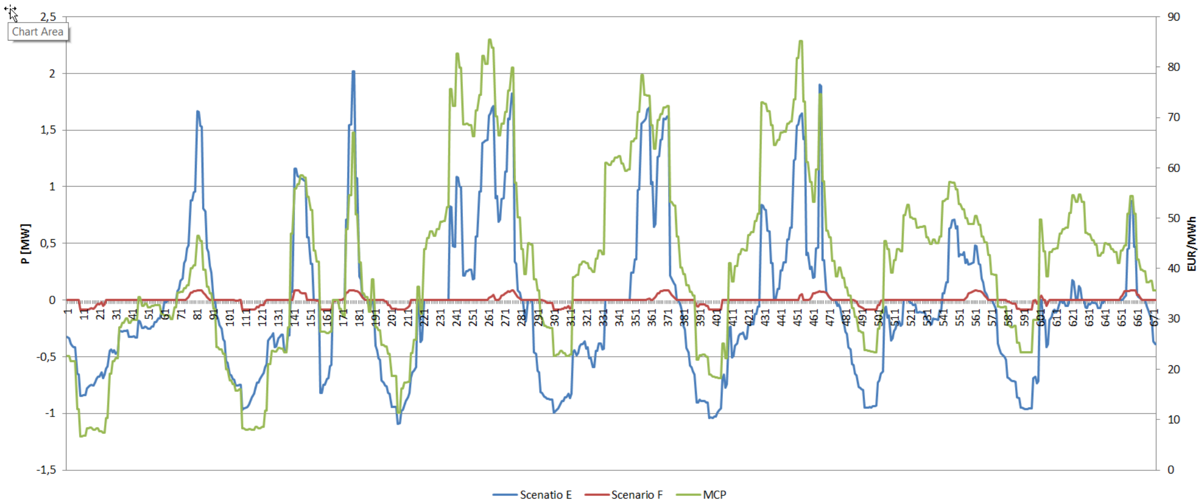

Figure 16 Total ESS generation - scenarios E and F

In scenarios G, H and I where ESS-DG combination can only discharge power into the network (EES can only be charged from DG, not from the grid) all values in Figure 17 are positive. For scenario G where only maximum profit is optimized, producer stores energy the entire day only to discharge maximum power in the periods with maximum prices; this can be seen as distinct daily discharge spikes. As seen in previous section with network losses, this behaviour has very negative impact on them and generally on network loading. Discharge patterns for scenario H are similar to the previous case during maximum prices but network losses minimization distributes this over multiple time periods avoiding spikes and by that not increasing losses. From Figure 17 it seems like there is a MCP threshold after which ESS-DG starts discharging power in the network, example period 77, period 175 etc. Scenario I has a goal to minimize investment; meaning it operates the EES-DG not to create sudden spikes which would consequently result in higher investment cost. It basically does not store too much PV generation to minimize investment in storage capacity and it generates energy during high PV production hours and, in addition, during high evening demand hours.

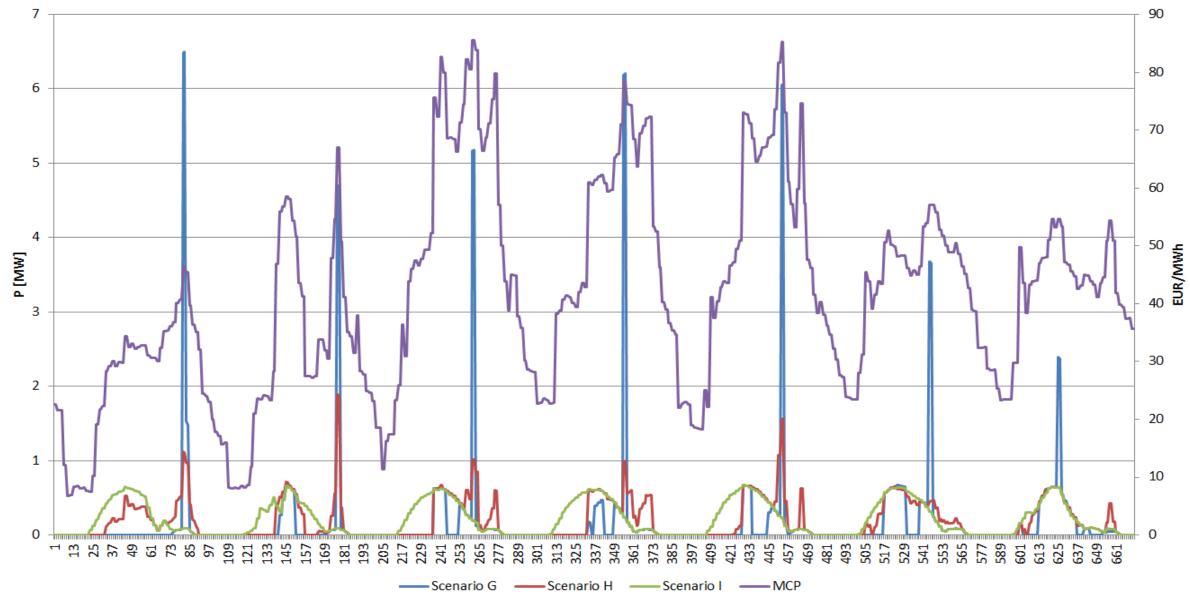

Figure 17 Total ESS-DG generation - scenarios G, H and I

9. CONCLUSION AND FUTURE WORK

The paper attempts to provide a comprehensive modelling framework for definition of role, size and location of electricity storages in the distribution grid. It does so by taking into account technical constraints of the technologies, characteristics and constraints of the distribution grid, investment costs and possibilities for providing different services to multiple stakeholders, in case for BESS and/or DG owner and to DSO. The contribution of the proposed concept is in creating a non bias model using direct modelling method, not predefining optimization values or suggesting a specific storage technology. To achieve this the paper presents an adjusted SOCP network model and MINLP based UC algorithm.

Presented results confirm that current BESS technologies do not justify their investment by utilization of energy arbitrage only. On the other hand, by analyzing optimal size, location and operation over a set of scenarios, they demonstrate what additional benefits BESS brings into the future low carbon distribution grids under various control concepts. As the goal of the paper is not to create an investment framework, the results do not capture financial benefits of avoiding investments into the distribution grid by BESS installation.

Future work will be focused on improving the model presented in this paper, particularly on the capability of BESS to contribute to reactive power stability. In addition, different distribution grid types have different requirements and further research is needed to asses these requirements.

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