A Parallel Mixed Integer Programming-Finite Element Method Technique for Global Design Optimization of Power Transformers

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Abstract —Transformer design optimization is determined by minimizing the transformer cost taking into consideration constraints imposed both by international specifications and customer needs. The main purpose of this work is the development and validation of an optimization technique based on parallel mixed integer nonlinear programming methodology in conjunction with finite element method, in order to reach a global optimum design of wound core power transformers. This optimization method has been implemented into software able to provide a global feasible solution for every given set of initial values for the design variables, rendering it suitable for application in the industrial transformer design environment.

I. INTRODUCTION

Transformer design optimization seeks a constrained minimum cost solution by optimally setting the transformer geometry parameters and the relevant electrical and magnetic quantities considering that manufacturing materials are highly variable stock exchange commodities. In the literature, a number of different design methodologies have been proposed. A geometric programming technique is used in order to find an optimum transformer design, [1]. Furthermore, computer-aided design techniques are presented that include mathematical models employing analytical formulas, based on design constants and approximations for the calculation of the transformer parameters, [2]. Recent research considered the use of artificial intelligence in the optimum design of transformers, [3].

An important improvement in deterministic design optimization techniques can be implemented by the incorporation of numerical methods, [4], [5]. The novelty of this paper consists in the adoption of a 3-dimensional cost-effective transformer finite element model (FEM), which is directly linked to the proposed parallel mixed integer programming (MIP) technique, enhancing its ability to reach a global optimum while maintaining low execution times. The robustness of this method is verified through application to several transformer cases.

II. PROPOSED METHODOLOGY

A. Description of the Optimization Model

The goal of the proposed parallel mixed integer optimization process is to find a set of integer variables linked to a set of continuous variables that minimize the objective function (active part cost) and meet the restrictions imposed on the transformer design problem.

In order to deal with the aforementioned problem, a methodology is developed, in which all the decision variables can assume not only continuous values but also integer values. Several methods to solve mixed integer programs are examined and compared in terms of convergence rate and solution quality. One of these methods is the Branch and Bound algorithm, [6], which offers solution to MIP problems based on an implicit enumerative evaluation of all feasible solutions. The basic principle of the method is to relax the constraint that the variables must be integers, obtaining what is known as the linear relaxation of the original problem. The latter is solved with linear programming methods, and this solution is used to iteratively fix the values of the integer variables in a tree of sub-problems that terminates with the desired optimal integer solution.

The proposed parallel MIP operates as follows (Fig. 1): at the 1st stage, the upper and lower bounds of the design vector are selected in accordance with transformer rated power, defining the interval of the design variables. Afterwards, a set of subintervals is generated, randomly or manually by the user, and distributed into *n* parallel implementations of MIP. The initial values of each MIP derive from the mean value of the respective subinterval bounds. In doing so, local optima are avoided. The FEM-based cycle of iterations uses the parallel MIP solution as initial vector and eventually converges to global optimum.

B. Mathematical Formulation

A mixed integer nonlinear problem to optimize the transformer design is based on the minimization of the cost of the transformer active part:

$$Transformer\ Active\ Part\ Cost = \min_{x} \sum f(x) \tag{1}$$

subject to:

$$c(x) = \begin{bmatrix} g_1(load\ \ losses) \\ g_2(short\ \ circuit\ \ impedance) \\ g_3(no\ load\ \ losses) \\ g_4(thickness\ \ of\ \ the\ core\ \ leg) \\ g_5(G, D) \end{bmatrix} \le 0 \tag{2}$$

$$lb \le x \le ub \tag{3}$$

where:

x: the vector of the design variables, consisting of the number of low voltage turns, the type of the core magnetic steel (4 different types), the height of the core window *G*, the width of the core leg *D*, (Fig. 2) and the magnetic induction.

f(x): the sum of the cost of 1) the primary winding, 2) the secondary winding, 3) the magnetic circuit, and 4) the insulating papers.

c(x): the nonlinear inequalities of the transformer design problem (consisting of the constraint functions g_i involving the transformer performance parameters).

lb, ub: the lower and upper bounds on the design variables, *x*, so that a solution is found in the interval [*lb*, *ub*].

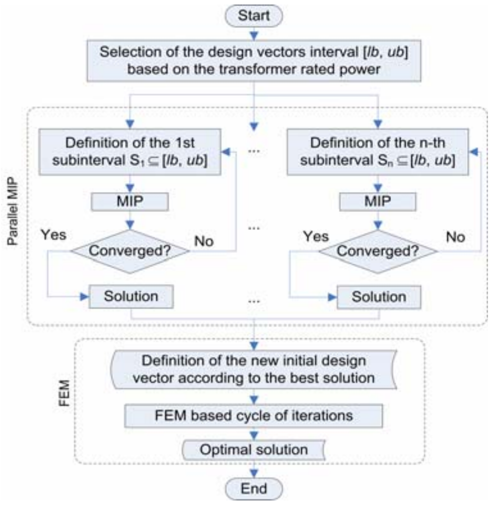

Fig. 1. Flowchart of the proposed technique.

C. Finite Element Model (FEM)

Fig. 2 illustrates the FEM model adopted in the development of the optimization technique, [7]. It comprises the low and high voltage windings of one phase, as well as the iron cores that surround them. Due to the symmetries of the problem, the solution domain is reduced to one fourth of the device. The integration of the FEM model into the optimization algorithm is realized as follows (Fig. 1): the optimal solution provided by the parallel MIP method is used as the initial vector for the design variables and a new cycle of iterations is performed, reducing the design variables only to the continuous ones (the remaining integer variables are considered equal to the optimal value provided by the previous iterations). During this cycle, FEM is used for the calculation of the transformer characteristics (unlike the analytical formulas used in the parallel MIP cycles depicted in Fig. 1) in order to provide more accurate results and better convergence to the optimal solution.

III. RESULTS AND DISCUSSION

In this Section, the results of the proposed optimization method on an 800 kVA transformer are presented. The method seeks the optimum core constructional parameters (continuous variables) (Fig. 2), the optimum magnetic induction (continuous variable), the type of the core magnetic steel, namely M4 0.27, MOH 0.27, MOH 0.23, ZDMH90 0.23 (each of the 4 types are represented by an integer identification number), and the optimum number of turns (integer variable). Fig. 3 shows the convergence history of the optimization methodology concerning the transformer active part cost, resulting in an optimum (minimum) cost of 4117 euros (last cycle of iterations, after *n*=7 parallel MIP cycles). The respective minimum cost provided by the manufacturer heuristic methodology, [2], for this test case is equal to 4216 euros. In other words, the use of the proposed methodology converges to an optimum solution that is 2.35% better than the one provided by the manufacturer. Apart from the better convergence of the proposed algorithm, the difference between the optimum values of the two methods is due to the difference in the permissible range of the design variables used in the heuristic algorithm, which is confined to discrete steps of the variables instead of the complete interval [*lb*, *ub*].

IV. CONCLUSION

The development of the proposed parallel mixed integer-programming technique using the Branch and Bound algorithm and FEM method is very effective because of its robustness, its high execution speed and its ability to search the large solution space. It may be noted that the global optimum obtained is not approached significantly by continuous variable consideration techniques. Additional results together with more implementation details that will be included in the full paper will further validate the robustness of the proposed method and the generality of its applicability to the power transformer design optimization problem.

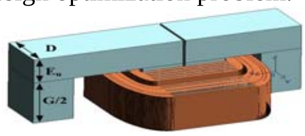

Fig. 2. FEM model and core constructional parameters (G: height of the core window, D: width of the core leg, Eu: thickness of the core leg).

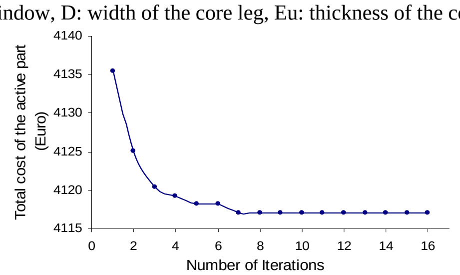

Fig. 3. Convergence characteristics of the proposed technique.

V. ACKNOWLEDGEMENTS

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