# Determining and Exploiting the Distribution Function of Wind Power Forecasting Error for the Economic Operation of Autonomous Power Systems

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***Abstract*--Many efforts have been presented in the bibliography for wind power forecasting in power systems and few of them have been used for autonomous power systems. The impact of knowing the distribution function of wind power forecasting error in the economic operation of a power system is studied in this paper. The papers proposes that the distribution of the wind power forecasting error of a specific tool can be easily derived if, for that model, an evaluation of its performance is made off-line comparing the forecasted values of the tool with the actual wind power values in the same horizon. The proposed methodology is applied to the autonomous power system of Crete. It is shown that the improvement of the performance of wind power forecasting tool has significant economic impact on the operation of autonomous power systems with increased wind power penetration. The obtained results for various levels of wind power production and load show that using only mean absolute percentage error (MAPE) leads to significant change in the estimation of the wind power to be shed to avoid technical limits violation, especially if the wind power forecasting tool presents underestimation of the actual production.**

***Index Terms*--Autonomous power systems, economic operation, normal distribution, spinning reserve, wind power forecasting.**

## I. Introduction

AUTONOMOUS power systems with increased wind power penetration have significant differences from interconnected power systems. First of all, due to no interconnection to other power system, the spinning reserve requirements are increased to account for the uncertainties in load and wind power estimations or equipment break down. Moreover, they present low minimum to maximum demand ratio and significantly larger frequency deviations with relatively small production or demand changes compared to interconnected power systems. Furthermore, quite often the installed thermal units have significant values of technical minimum that introduce problems in co-operation between thermal units and wind power making the operators disconnect some of the wind power production in order to avoid technical limits violations. The higher the uncertainty in estimation of load and wind, especially in islands with high wind power penetration, the more frequent this incident is expected to happen. Sometimes, in such power systems, operators may even consider totally unreliable the wind power production leading the system to operate with excessive spinning reserve increasing its operating cost.

Under such circumstances, efficient wind and load forecasting is rather important for the secure and economic operation of autonomous power systems. Advanced control software for such power systems, like MORE CARE [1], incorporate load and RES forecasting functions [2], unit commitment and economic dispatch functions as well as dynamic security assessment functions in a user-friendly human machine interface.

Some studies about the impact of improved wind power forecasting have been published recently [3]-[5], showing that improving wind power forecasting has significant economic savings for both the utilities and the owners of wind parks participating in the open market.

The developers of wind power forecasting models usually provide their end-users with the mean absolute percentage error (MAPE) index for their model expressed as a percentage of the installed wind power capacity. However, this index does not give very much information neither about the performance of the forecasting tools for different forecasting horizon nor about their performance for a variety of output values.

Some wind power forecasting tools also provide their users with a confidence interval of the forecast based on their own estimations [2], [6]. In such a case, the operators can estimate the range of the expected wind power production and thus the spinning reserve requirements to cope with the wind power production uncertainty.

In this paper, the impact of increased information on the evaluation of the wind power forecasting tool is examined as far as the economic operation of an autonomous power system is concerned. More specifically, the operation of an autonomous power system when only the MAPE information

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is available from the forecasting tool is compared with the operation when the operators are aware of the probability distribution function (pdf) of the forecasting errors for wind power production corresponding to the MAPE value.

The impact of improved wind power forecasting for an island power system is described in Section II. Actual data from the autonomous power system of Crete were used for the study as described in Section III. The proposed methodology is described in Section IV and results are presented in Section V. Conclusions are drawn in Section VI.

## II. Economic Impact of Wind Power Forecasting

The impact of improving wind power forecasting errors for the utility and the wind park owner has been studied in [3]-[5].

More specifically in [4], the impact of different forecasting errors for different utilities and wind power production levels is studied. The results show that accurate forecasts have the higher benefit for a utility that integrates wind power. The benefits for the utilities are reduced as the forecasting error accuracy is reduced. This reduction depends on the generation mix of the utility, the underestimation or overestimation of the wind power and the level of the wind power production. However, even for low wind power production, the economic benefits are significant.

In [5], a similar study for the importance of a short-term wind power prediction tool in the participation of wind power in electricity markets was conducted. In that study different levels of aggregations of wind parks and different accuracy levels of short term wind power forecasting have been used concluding that the income for wind park owners is increased when a short-term wind power forecasting is used even with average accuracy. This income increase depends on the accuracy of the prediction and the higher aggregation of wind parks.

Within the framework of ANEMOS project, the impact of wind power forecasting errors in the operation of a power system is under study [7].

The economic impact of wind power forecasting errors can be evaluated taking into account the wind power forecast uncertainty via the spinning reserve calculation as described in the following equation:

$$spining\_reserve = load\_reserve + wind\_reserves \tag{1}$$

where *load_reserve* stands for the uncertainty in the load estimation (a constant percentage of the actual load in this paper) and *wind_reserves* for the wind power production forecast errors.

For wind power forecast and *wind_reserves*, three cases are studied.

a. The system operator does not have any confidence for the wind forecast and maintains *wind_reserves* equal to the 100% of the expected wind power production. It should be noted that this practice is rather common in island power systems.
b. The system operator has at his disposal one satisfactory wind power forecast tool presenting 50% maximum forecasting error. In this case, the operator maintains *wind_reserves* at least equal with half the wind production.
c. The system operator has at his disposal a reliable wind forecast model and maintains *wind_reserves* equal to 20% of wind power production.

Results from the last 5 months of 2001 [3] and first 6 months of 2003 are summarized in Table I for the island system of Crete. More specifically, the cost reduction in actual prices (thousands of euros - k€) and in percentage (%) when comparing the cases a-c indicate that focusing on improving wind power forecasting may have significantly impact on the economic operation of the power system.

TABLE I
Impact of Improving Wind Power Forecasting Error

| Case | 2001 cost reduction | | 2003 cost reduction | |
|---|---|---|---|---|
| | **k€** | **%** | **k€** | **%** |
| a | 544.53 | 1.14 | 775.48 | 1.09 |
| b | 1362.34 | 3.00 | 3260.00 | 4.33 |
| c | 1906.00 | 4.00 | 4033.00 | 5.37 |

## III. Proposed Methodology

In island power systems in Greece, like Crete, the operator of the power system, Public Power Corporation (PPC), is obliged to buy the wind power produced at specific feed-in tariff price and thus the load to be dispatched to the thermal units is the demand minus the wind power production. The uncertainty in the load to be dispatched to the units comes from load and wind power forecasting error and the forced outage rate (FOR) of the installed units of the power system. In order to focus only on the impact that the increased information about forecasting errors may have, it is assumed that FOR is negligible.

It is assumed that for each time interval the load and wind power forecasting errors, *l_e(t)* and *w_e(t),* respectively, are both random variables with probability density functions (pdf) as examined in Sections III.B and III.C.

The total uncertainty for the load to be distributed to the thermal units for each interval, namely *l_u_e(t),* is due to the summation of load and wind power production errors and is a random variable given by equation (2):

$$l\_u\_e(t) = l\_e(t) + w\_e(t) \tag{2}$$

Both forecasting errors are statistically independent and thus the pdf of *l_e(t)* and *w_e(t)* can be easily convoluted as any pair of independent random variables to produce the *l_u_e(t)* pdf.

Having calculated this pdf, the operators can take confidence intervals for the expected uncertainty of the load to be distributed to the thermal units of the examined power system. These intervals can be used in the scheduling of the power system for the next few hours using the economic scheduling functions described in Section III.A.

In Section III.B, it is assumed that the only available information about load and wind power forecasting errors is their MAPE using convolution of normal-like pdf.

Similarly, if there is additional information about the pdf of the forecasting error, the methodology described in Section III.C provides the *l_u_e(t)* random variable pdf and its confidence interval.

*A. Economic Scheduling Functions*

Assuming that the actual load and the actual wind power time-series for the typical days, studied in Section IV are the outputs of the forecasting tools and using the information about the performance of the forecasting tools, an estimation of which units should be committed, unit commitment (UC), can be made.

The operators should meet the demand minus the wind power production taking into account the uncertainty of the forecasts. They should account for overestimation of wind power and underestimation of load to minimize the probability of insufficient capacity to meet the demand. Therefore, estimation of the possibility of such an event should be provided to them, to help them avoid such operating points.

In the following, we define as *perc(q,l_u_e(t))* the solution of the following equation :

$$F_{l\_u\_e(t)}(l\_u\_e(t))=q \tag{3}$$

meaning the $q$ percentile of the $F_{l\_u\_e(t)}(l\_u\_e(t))$ cumulative distribution function (cdf) of the *l_u_e(t)* pdf. To obtain the solution of (3), the methodology described in Sections III.B and III.C is used.

Having obtained the *l_u_e(t)* pdf, the equation (4) describes the minimum load that the units to be committed should be able to meet, *Load_to_units(t),* at each time-interval $t$:

$$Load\_to\_units(t)=perc(q,l\_u\_e(t))+load(t)-WP(t)) \tag{4}$$

where *load(t)* is the output of the load forecasting tool and *WP(t)* is the output of the wind power forecasting tool.

The highest values of $F_{l\_u\_e(t)}$ correspond to the case of overestimation of wind power and underestimation of load, the case that the operators would like to avoid. Therefore, $q$ in such a case is selected to be rather high, e.g. 97.5% of the forecasting error cases, meaning that the risk for the operators not to have sufficient committed capacity is less than 1-$q$.

Having calculated the load that the units should be able to meet, the UC problem is resolved using the following formulation for each time interval,

$$\begin{aligned} &Min(OC(t)) = \\ &Min\sum_{i=1}^{N}\left(u_{i,t}\cdot FC(Pg_i(t)) + u_{i,t}\cdot(u_{i,t-1})\cdot SUC_i(t)\right) \end{aligned} \tag{5}$$

where *OC(t)* is the operating cost, $FC(Pg_i(t))$ the fuel cost of the unit depending on its production $Pg_i(t)$, a quadratic cost function in our case, $u_{i,t}$ is the status (0 or 1) and $SUC_i(t)$ the start up cost of the $i$-th unit.

The minimization of (5) is subject to the following constraints:

$$\sum_{i=1}^{N} u_{i,t}\cdot P_i^{\max} \geq Load\_to\_units(t) \tag{6}$$

and

$$P_i^{\min} \leq Pg_i(t) \leq P_i^{\max} \tag{7}$$

where $P_i^{\min}$ and $P_i^{\max}$ are the technical minimum and maximum of the unit $i$, respectively.

For the solution of the UC problem, the priority list is used, which is a method commonly used for deriving solution of this problem for the utilities.

After the solution of the UC problem, the selection of the units to be committed might impose constraints in the operation of the wind parks. More specifically, if there is underestimation of the wind power production and overestimation of the load, then the load to be dispatched to the units will be significantly lower. There might be the case that the summation of the technical minima of the units selected to operate will be larger than the actual load to be served, as inequality (8) describes:

$$\sum_{j\in IN(t)} P_j^{\min} \geq load(t) - WP(t) + perc(p, l\_u\_e(t)) \tag{8}$$

where *IN(t)* is a set consisting of the committed units during the time interval $t$ and $p$ is the value for which the percentile is sought for and is usually a very small value e.g $p$=2.5%. Therefore, probability less than $p$% is the acceptable risk for violating the technical minima of the units. If inequality (8) is true, then some wind power should be shed in order to reduce the risk of violating technical minimum of the committed units. The new *WP(t)* in such a case is given by equation (9) and the rest is shed:

$$WP(t) = load(t) + perc(p, l\_u\_e(t)) - \sum_{j\in IN(t)} P_j^{\min} \tag{9}$$

Then the economic dispatch (ED) of the load to the committed units is performed. This problem is formulated as follows: minimize operating cost (10), meeting the load as described in (11), without violating the technical limits of the units described in (7):

$$\min \sum_{j\in IN(t)} FC(Pg_j(t)) \tag{10}$$

$$\sum_{j\in IN(t)} Pg_j(t) = load(t) - WP(t) \tag{11}$$

In order to solve the optimization problem formulated in (10) subject to constraints (7) and (11), the sequential quadratic programming (SQP) method has been utilized [8]-[9]. This method for the specific unit schedule guarantees the optimum dispatch of the load to the committed thermal units belonging to *IN(t),* if convex and concave cost functions are used, like the ones in our case study system.

### B. Uncertainty Management Knowing Only MAPE

The operators of a power system are usually aware of the MAPE of the forecasting errors for the load and wind power forecasting tools they use, especially after an evaluation of these tools has been made [10]. For load, MAPE is expressed as a percentage of the actual load and for wind power is expressed as a percentage of the installed wind power capacity for which the forecast is available, as described in (12) and (13).

$$e_r = \left(\frac{P_f - P_a}{P_i}\right) \tag{12}$$

$$MAPE = \frac{1}{N}\sum_{i=1}^{N}\left|e_r\right| \tag{13}$$

where $P_f$ stands for the forecasted value provided by the tool, $P_a$ is the actual value of load or wind, $P_i$ is either the actual load for load forecasting tool or the installed wind power capacity for the wind power forecasting tool and $N$ is the number of periods studied (here $N$=24). Negative values of $e_r$ mean underestimation of the real value, while positive values mean overestimation.

In many studies on error functions it is assumed that the error follows a Normal pdf around a mean value with certain standard deviation value. Knowing MAPE of forecasting errors without any further information and taking the above assumption into account, it can be assumed that absolute error follows a Normal pdf with mean value equal to MAPE and standard deviation equal to MAPE/3.09. The standard deviation value is based on the fact that the minimum expected value of each section of the pdf, is expected to be zero and that this value corresponds to the 99.9% of the cases, which according to the normal cumulative distribution function (cdf) tables is 3.09$\sigma$.

Without having any additional information about the performance of these tools, it can be assumed that both wind power and load forecasting tools have 50% probability to overestimate the actual value and 50% to underestimate it. Thus, it can be considered that MAPE value is the absolute expected value for both the average overestimation and underestimation value. In the former case, the MAPE value is positive while in the latter is negative and the error distribution function follows a normal like pdf with two normal-like pdfs with half the magnitude of a Normal pdf.

A graphical representation of such a pdf is depicted in Fig. 1 and the pdf for $l\_e(t)$, $f_{l\_e(t)}$ is given by (14):

$$f_{l\_e(t)}(l\_e(t)) = 0.5\cdot N(l\_e(t), MAPE\cdot load(t), \frac{MAPE\cdot load(t)}{3.09}) + 0.5\cdot N(l\_e(t), -MAPE\cdot load(t), \frac{MAPE\cdot load(t)}{3.09}) \tag{14}$$

where $N(x,\mu,\sigma)$ is the normal pdf and MAPE refers to the MAPE of the forecasting error of load or wind, respectively using numerical representation of the percentage value. For wind power, only MAPE value changes and instead of load the wind power capacity can be used in equation (14).

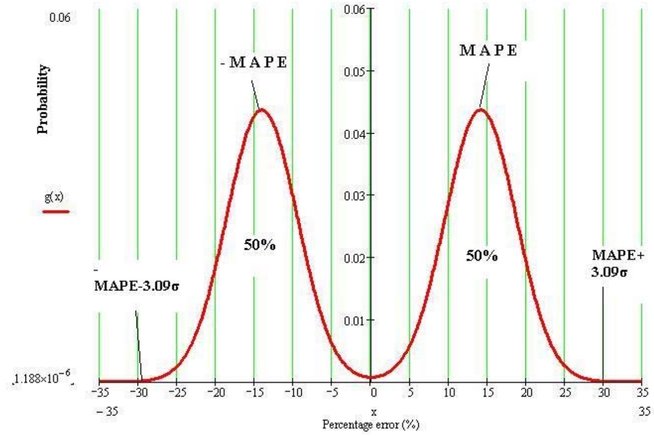


Fig. 1. Plot of the pdf considered for forecasting error when only MAPE is known.

The pdf for $l\_u\_e(t)$ comes from the convolution of the pdfs for $l\_e(t)$ and $w\_e(t)$ and for continuous variables is given by (15):

$$f_{l\_u\_e(t)}(l\_u\_e(t)) = \int_{-\infty}^{+\infty} f_{l\_e(t)}(l\_u\_e(t)-x)\cdot f_{w\_e(t)}(l\_u\_e(t))dx \tag{15}$$

Replacing (14) in (15), the pdf for $l\_u\_e(t)$ is calculated which consists of 4 adders each derived by the convolution of 4 normal pdfs with 25% of the magnitude of a normal pdf .The convolution of two random variables following normal pdf is, according to the probability theory [11], a Normal pdf with the following parameters:

1. As *mean value*, the summation of the mean values of the random variables, $\mu = \mu_1 + \mu_2$, and
2. *Standard deviation* given by the formula: $\sigma^2 = \sigma_1^2 + \sigma_2^2$

where $\sigma_1$ and $\sigma_2$ are the standard deviation of each random variable.

Thus, the $l\_u\_e(t)$ pdf consists of four adders of Normal-like pdf with 25% of the nominal magnitude as described in (16):

$$f_{l\_u\_e(t)}(l\_u\_e(t)) = 0.25\cdot N(l\_u\_e(t),\mu_1(t),\sigma(t)) + 0.25\cdot N(l\_u\_e(t),\mu_2(t),\sigma(t)) + 0.25\cdot N(l\_u\_e(t)\mu_3(t),\sigma(t)) + 0.25\cdot N(l\_u\_e(t),\mu_4(t),\sigma(t)) \tag{16}$$

where $\sigma^2(t) = \left(\frac{MAPE\cdot load(t)}{3.09}\right)^2 + \left(\frac{MAPE\cdot wind\_cap}{3.09}\right)^2$

and

$$\mu_1(t) = MAPE\_l\cdot load(t) + MAPE\_w\cdot ins\_cap$$
$$\mu_2(t) = MAPE\_l\cdot load(t) - MAPE\_w\cdot ins\_cap$$
$$\mu_3(t) = -MAPE\_l\cdot load(t) + MAPE\_w\cdot ins\_cap$$
$$\mu_4(t) = -MAPE\_l\cdot load(t) - MAPE\_w\cdot ins\_cap$$

where *MAPE_l* and *MAPE_w* stand for the MAPE of the load and wind forecasting respectively and *ins_cap* is the installed wind power capacity.

The values of $\mu_1(t)$-$\mu_4(t)$ are given in descending order, meaning that the higher underestimation of the load to be distributed to the units is in the first adder of (16) and the highest overestimation of the load to be dispatched to the units is expected in the fourth adder of (16). The former corresponds to the necessary spinning reserve to account for load and wind power forecasting errors and the latter for the case of possible disconnection of wind parks.

Thus, the *p* value percentile of the cdf of *l_u_e(t)* is expected to be in the fourth adder of (16) and the *q* percentile in the first section of (16). For the cdf of a normal pdf, the *p* and *q* percentiles can be given using the corresponding cdf tables for values *p*/0.25 and (1-*q*)/0.25 since each of the first and the fourth section of (16) corresponds only to 25% of the normal pdf.

Thus *p* and *q* percentiles in the former case are given by the following formulas:

$$perc(q, l\_u\_e(t)) = \mu_1(t) + k\_q \cdot \sigma(t) \tag{17}$$

$$perc(p, l\_u\_e(t)) = \mu_4(t) - k\_p \cdot \sigma(t) \tag{18}$$

where *k_q* and *k_p* are the percentile points coming from the standard normal pdf tables.

Such estimation can be valid only if the third section does not contribute to the first *p%* of the cdf *l_u_e(t)* more than a small value e.g 0.1%, so that such contribution can be regarded negligible. Similarly, for the second section of (16) and for the *q%* of the cdf of *l_u_e(t).* Otherwise, the pdf described by (16) is easier to be discrete and then consider the confidence intervals on it.

*C. Having studied absolute values of errors*

An evaluation of the a forecasting tool can be performed off-line using past actual time-series and comparing them with the forecasts provided by the forecasting tool using equation (12).

Using this methodology a distribution of wind power forecasting errors was derived for the wind power forecasting tool of the MORE CARE project [2]. It was observed that forecasting errors were not symmetrical. This means that the forecasting model may present larger errors either in the overestimation or the underestimation of the load or wind production, in our case underestimation of the wind power production.

If MAPE is only provided to the operators of the power system, an easy solution is to assume a semi-normal pdf, but as shown in Fig. 2, for the same MAPE, there is significant difference in the values of forecasting errors and their probability to occur. For instance zero forecasting error is expect to be more frequent than in normal-like pdf of Section III.B whille some extreme forecasting error values are expected as well with higher probability compared to Section III.B.

For such a case, the methodology of Section III.B for wind power forecasting error cannot be used. Instead, a discrete pdf for *w_e(t)* random variable is used given by (19):

$$f_{w\_e}(w\_e(t)) = \sum_{k=1}^{m} G_k \cdot \delta(w\_e(t) - H_k) \tag{19}$$

where *m* is the number of impulses for the discrete pdf, $G_k$ is the value of each bin, and $H_k$ the bin of wind power forecast error. For *l_e(t)*, the same pdf as described by (14) in Section III.B is used.

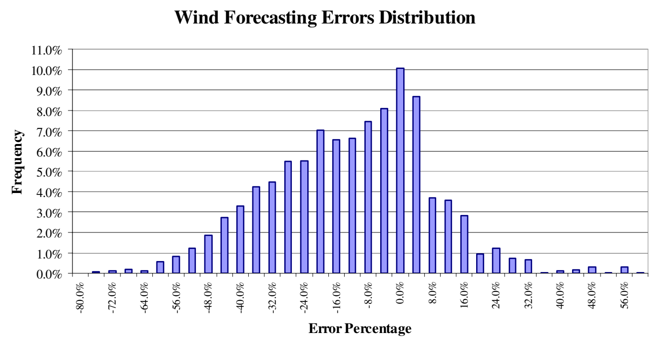


Fig. 2. Distribution of wind power forecasting errors.

The convolution of pdf of two independent random variables, one with discrete pdf and one with normal pdf, is generally given by (20):

$$f_w(w) = f_N(x_N) f_D(x_D) = \sum_{k=1}^{t} \frac{G_k}{\sqrt{(2\pi)}\,\sigma_N} \cdot exp\left(-\frac{(w-\mu_{wk})^2}{2\cdot\sigma_N^2}\right) \tag{20}$$

where  is the standard deviation of the normal pdf, and $_{wk}$= $_N+H_k$ [11].

Taking into account the above characteristic of the normal pdf in (20) and equation (14), the pdf for *l_u_e(t)* is given by (21):

$$f_{l\_u\_e(t)}(l\_u\_e(t)) = \sum_{k=1}^{m} \frac{0.5\cdot G_k}{\sqrt{(2\pi)}\,\sigma_N} \cdot \exp\left(-\frac{(l\_u\_e(t) - \mu_{w1k})^2}{2\cdot\sigma_N^2}\right) + \sum_{k=1}^{m} \frac{0.5\cdot G_k}{\sqrt{(2\pi)}\,\sigma_N} \cdot \exp\left(-\frac{(l\_u\_e(t) - \mu_{w2k})^2}{2\cdot\sigma_N^2}\right) \tag{21}$$

where $\sigma_N(t) = \frac{MAPE\_l \cdot load(t)}{3.09}$ and

$$\mu_{w1k}(t) = H_k + MAPE\_l \cdot load(t)$$

$$\mu_{w2k}(t) = H_k - MAPE\_l \cdot load(t)$$

Similarly, as in Section III.B, the user-defined confidence interval should be calculated for the *l_u_e(t)* random variable. For this calculation, discrete values for the pdf described in (21) for each time interval are considered and the necessary percentile points are then calculated.

## IV. Case Study Network

*A. General Description of the Power System*

The Power System of Crete is the largest isolated system in Greece, with the highest increase in energy demand (8%). The peak demand during 2004 was 540 MW and since 2000 the annual wind power penetration is around 10%. The instantaneous wind power penetration has reached 39% during some valley hours in winter and early spring [12]. Public Power Corporation is the operator of this power system and has the obligation to buy at specific price (90% of the retail low voltage price) the energy produced by the wind park installations. This legal framework is beneficial for both the operator of the power system [13] as well as the private sector investors that can timely payback their investments [14].

The installed wind power capacity currently (December 2005) on the island is 105.15 MW in 12 wind parks, mainly at the Sitia region, on the eastern part of the island. Of these units, 37.8 MW have been installed since 2001.

Moreover, on the island, 25 units with different technology and response characteristics (diesel, gas turbines, steam turbines and one combined cycle unit) in three power plants with total installed capacity 690 MW have been installed.

For the year of study, 2000, the peak demand was 435 MW and the consumed energy was 2078 GWh. 20 thermal units were installed with total capacity 490.3 MW and the installed wind power capacity was 67.35 MW.

The importance of wind power forecasting and estimation of its confidence interval is significant either during low load periods or during high load periods. In the former case, when slow response units, steam turbines and combined cycle units operate, there is the danger of low loading operation and perhaps disconnection of wind power to avoid violation of the technical minima of the units. In the latter case, overestimation of wind power during such periods may lead to temporary lack of energy if the necessary units are not timely committed to meet the increased demand.

### *B. Selected days for the study*

For our study in Section V, combinations of low, medium and high demand and wind power production have been considered, totally 9 selected days for the year 2000 as Table II describes. In such a way, the impact of various levels of wind power production and demand can be studied. The load time-series are classified according to the daily demand as follows:

1. *High load (HL)*: above 6100 MWh daily. Summer days have typical consumption of this level.
2. *Medium load (ML)*: between 5250-6100 MWh daily. Many days per year have similar consumption.
3. *Low load (LL)*: below 5250 MWh daily. Typical days of that kind are during March, April, and November and during winter Sundays.

The wind power production time-series can be classified according to the following criteria:

1. *High wind power production (HWP)*: above 40 MW for 75-80% of the day.
2. *Medium wind power production (MWP)*: production between 17-39 MW for 75-80 % of the day.
3. *Low wind power production (LWP)*: Production below 17 MW and most of the hours, about 75-80% of the day, below 10 MW.

For the days of Table II, actual hourly time-series for demand and wind power production have been considered for the analysis in Section V.

TABLE II
TYPICAL DAYS USED

| Date | Total daily demand (MWh) | Average Wind Power Production (MW) | Characterization |
|---|---|---|---|
| Day 1 | 7269.8 | 55.2 | HL, HWP |
| Day 2 | 7396.5 | 26.6 | HL, MWP |
| Day 3 | 8006.1 | 7.5 | HL, LWP |
| Day 4 | 5589.1 | 37.3 | ML, HWP |
| Day 5 | 5421.1 | 24.1 | ML, MWP |
| Day 6 | 5847.4 | 5.0 | ML, LWP |
| Day 7 | 5252.7 | 40.7 | LL, HWP |
| Day 8 | 4997.3 | 23.7 | LL, MWP |
| Day 9 | 5197.4 | 1.3 | LL, LWP |

### *C. Forecasting tools characteristics*

In our study, the MAPE error of the MORE CARE load and wind power forecasting functions has been used. More specifically, the fuzzy-neural load forecasting tool and the fuzzy-neural wind power forecasting tool with meteorogical information have been used. The values of MAPE for both forecasting tools are given in Table III.

TABLE III
MAPE FORECASTING ERRORS FOR LOAD AND WIND

| | |
|---|---|
| Load forecasting (MAPE_l) | 7.07% |
| Wind power forecasting (MAPE_w) | 14.06% |

The installed wind power capacity, *ins_cap* is 67.35 MW. Two confidence intervals of 95% (with $p$=2.5% and $q$=97.5%) and 99% (with $p$=0.5% and $q$=99.5%) have been considered. In the former case $k\_p$=$k\_q$=1.28 and in the latter case $k\_p$=$k\_q$=2.06. Results from the analysis of the typical days are provided in Section V.

## V. RESULTS

Using actual wind power and load time-series for the selected days of Table II, the operating cost and the expected wind power shedding per day was calculated for both 95% and 99% confidence interval. For both cases the load forecasting error, $l\_e(t)$, pdf is given by (14).

In Section V.A the results from considering MAPE for wind power forecasting error are provided while in Section V.B the results from considering discrete pdf for wind power forecasting with 36 bins are derived.

### *A. Semi-normal Distribution for Load and Wind Forecasting Error*

The results for 95% and 99% confidence interval are shown in Tables IV and V, respectively.

The operating cost is increased when $q$=99.5% compared to $q$=97.5% because more spinning reserve is required to be maintained by the operators and thus more units should operate to meet the same load resulting in lower efficiency operating points and increased no-load fuel cost. The cost increase is 0.98% and is higher, more than 1.2%, during high wind power periods.

The wind power shed is increased by 48 % of the total wind power to be shed when $p$=0.5 % compared to $p$=2.5 %, especially during low or medium load and high wind power production. Such a case is then more probable since more units are committed to meet the 99% confidence interval constraint for the same actual load as the comparison of the column units committed in Tables IV and V indicate.

This wind power shedding has also impact in further increasing the operating cost during periods of high wind power production and low to medium load due to the decrease in the wind power production.

TABLE IV
RESULTS WHEN ONLY MAPE INFORMATION IS AVAILABLE AND 95% CONFIDENCE INTERVAL IS CONSIDERED

| Date | Operating cost (€) | Wind power Shed (MWh) | Units Committed |
|---|---|---|---|
| Day 1 | 470823.5 | 0 | 284 |
| Day 2 | 552821.0 | 0 | 299 |
| Day 3 | 672429.5 | 0 | 341 |
| Day 4 | 359808.2 | 11.19 | 247 |
| Day 5 | 375006.4 | 0 | 225 |
| Day 6 | 441320.8 | 0 | 268 |
| Day 7 | 333776.3 | 50.39 | 228 |
| Day 8 | 345785.6 | 0 | 219 |
| Day 9 | 389430.3 | 0 | 251 |

TABLE V
RESULTS WHEN ONLY MAPE INFORMATION IS AVAILABLE AND 99% CONFIDENCE INTERVAL IS CONSIDERED

| Date | Operating cost (€) | Wind power Shed (MWh) | Units Committed |
|---|---|---|---|
| Day 1 | 477508.5 | 0 | 290 |
| Day 2 | 558085.9 | 0 | 304 |
| Day 3 | 677803.2 | 0 | 348 |
| Day 4 | 364273.9 | 21.87 | 256 |
| Day 5 | 375693.0 | 0 | 230 |
| Day 6 | 446851.9 | 0 | 272 |
| Day 7 | 338664.7 | 69.51 | 228 |
| Day 8 | 347114.4 | 0 | 224 |
| Day 9 | 394007.7 | 0 | 253 |

### *B. Discrete Pdf For Wind Power Forecasting*

If the wind power forecasting pdf is available, the results for 95% and 99% confidence interval are shown in Tables VI and VII, respectively.

Similar conclusions as in Section V.A can be drawn for the impact of confidence interval in the operation of the power system. The cost increase is about 0.98% ranging daily between 0.3%-1.56% and significant increase in wind power shedding, 52%, is noticed.

TABLE VI
RESULTS WHEN WIND POWER FORECASTING PDF IS AVAILABLE AND 95% CONFIDENCE INTERVAL IS CONSIDERED

| Date | Operating Cost (€) | Wind power Shed (MWh) | Units Committed |
|---|---|---|---|
| Day 1 | 469113.9 | 0 | 282 |
| Day 2 | 552821.0 | 0 | 299 |
| Day 3 | 674078.6 | 0 | 341 |
| Day 4 | 364087.4 | 56.79 | 242 |
| Day 5 | 377503.9 | 28.16 | 229 |
| Day 6 | 441320.8 | 0 | 268 |
| Day 7 | 340896.5 | 133.26 | 228 |
| Day 8 | 346236.8 | 2.74 | 219 |
| Day 9 | 389194.3 | 0 | 251 |

TABLE VII
RESULTS WHEN WIND POWER FORECASTING PDF IS AVAILABLE AND 99% CONFIDENCE INTERVAL IS CONSIDERED

| Date | Cost (€) | Wind power Shed (MWh) | Units Committed |
|---|---|---|---|
| Day 1 | 475656.03 | 0 | 289 |
| Day 2 | 561095.53 | 0 | 308 |
| Day 3 | 676111.10 | 0 | 342 |
| Day 4 | 366750.60 | 73.45 | 253 |
| Day 5 | 380329.80 | 68.06 | 230 |
| Day 6 | 444918.60 | 0 | 271 |
| Day 7 | 346217.92 | 157.08 | 240 |
| Day 8 | 348812.47 | 31.89 | 225 |
| Day 9 | 394252.35 | 5.62 | 255 |

### *C. Comparison of Cases Studied in Sections V.A and V.B*

This Section compares the results between the cases A and B that have been presented in Sections V.A and V.B, respectively.

It should be noticed that although the same MAPE for the wind power forecasting error for the cases A and B is calculated, in the pdf depicted in Fig. 2 more frequent underestimation is noted in comparison to the normal pdf, creating increased uncertainty in the wind power production during low demand periods. Due to this fact, there is significant increase in the wind power shed in case B compared to the case A, even during medium load periods. As expected, this wind power shed is even higher, if 99% confidence interval is considered.

Comparing operation with 95% confidence interval in cases A and B, it is concluded that the operating cost is higher in the latter case by 0.35% on average. The highest percentage cost increase is during the days with wind power shedding with the highest increase about 2% on the LL, HWP day.

However, during high demand and high wind power period, this cost is reduced by 0.4%, since the performance of the wind power tool mitigates the impact of underestimation of the load forecasting error, which is much higher during the high demand periods. This can be also noted, if the 99% interval is used even during medium wind power at high load period. During low wind power periods, such an impact is minimized. This cost change by 0.35% in both 95% and 99% is mainly due to the wind power shedding that is significantly increased for these 9 days, 159 MWh for 95% confidence interval and 227 MWh for 99% confidence interval.

## VI. CONCLUSIONS

The impact of knowing the distribution function of wind power forecasting error in the economic operation of a power system is studied in this paper. It was shown that the improvement of the performance of the wind power forecasting tools has significant economic impact on the operation of autonomous power systems with increased wind power penetration. Thus, further research on the area can help in the further decrease of the operating cost and the increased confidence of the operators on the wind power.

From the obtained results, it can be concluded that if the values for spinning reserve for 99% confidence interval are reduced to the ones of 95% confidence interval by improving the forecasting modules performance, significant cost reduction is noted ranging between 0.2%-1.6% with an average value of 0.98%. Furthermore, the reduction of the value corresponding to the 0.5% percentile, to the value corresponding to the 2.5% percentile of the demand to be distributed to the units, significantly helps in the reduction of the wind power shed to avoid technical minimum violations. This can be even more decreased not only because of the less frequent overestimation of the load to be dispatched to the units but also due to the reduced number of necessary units to be committed for meeting the demand uncertainty. Such a reduction in wind power shedding reduces the operating cost during high wind power penetration periods and low or medium load.

This paper has also shown that for the same MAPE forecasting errors, different probability density functions can correspond to it. MAPE can only provide a rough estimation of the extreme forecasting errors of the forecasting tool and their probability to occur and not the exact value for estimating the confidence intervals of the forecasting errors. Thus, additional information on the forecasting model should be provided either in confidence interval format, or as a distribution of the forecasting errors, after a detailed off-line evaluation of the model to reduce the operating risk of the system.

Using the proposed methodology in this paper, a confidence interval for the load to be dispatched to the units can be easily obtained based on past evaluation of the forecasting models. The above analysis proves that there may exist significant change in the estimation of the wind power to be shed or even in the operating cost, if a different probability density function with the same MAPE with a normal pdf is used for the wind power forecasting tool.

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## VIII. Biographies

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