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MULTI-OBJECTIVE OPTIMUM DESIGN OF 3D STRUCTURES UNDER STATIC AND SEISMIC LOADING CONDITIONS

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Abstract. *Almost every real world problem involves simultaneous optimization of several incommensurable and often competing objectives which constitutes a multi-objective optimization problem. In multi-objective optimization problems the optimal solution is not unique as in single-objective optimization problems. This paper is concerned with large-scale structural optimization of skeletal structures such, as space frames and trusses, under static and/or seismic loading conditions with multiple objectives. Combinatorial optimization methods and in particular algorithms based on evolution strategies are implemented for the solution of this type of problems. In treating seismic loading conditions a number of accelerograms are produced from the elastic design response spectrum of the region. These accelerograms constitute the multiple loading conditions under which the structures are optimally designed. This approach for treating seismic loading is compared with an approximate design approach, based on simplifications adopted by the seismic codes, in the framework of multi-objective optimization.*

1 Introduction

In single-objective optimization problems the optimal solution is usually clearly defined since it is the minimum one, this does not hold in real world problems having multiple and conflicting objectives. Instead of a single optimal solution, there is usually a set of alternative solutions, generally denoted as the set of Pareto optimal solutions. These solutions are optimal in the wider sense since no other solution in the search space is superior to them when all objectives are considered. In the absence of preference information, none of the corresponding trade-offs can be said to be better than the others. On the other hand, the search space can be too large and too complex, which is the usual case of real world problems, to be solved by the conventional deterministic optimizers. Thus, efficient optimization strategies are required able to deal with the multiple objectives and the complexity of the search space. Evolutionary Algorithms (EA) have several characteristics that are desirable for this kind of problems and most frequently outperform the deterministic optimizers such as gradient based optimization algorithms. There are some classical methods for dealing with the multi-objective optimization problems, such as the linear weighting method, the distance function method and the constraint method, that have to be combined with the optimization algorithm. The implementation of gradient based optimizers for this type of problems becomes even more cumbersome. The application of EA in multi-objective optimization problems has received considerable attention in the last five years due to this difficulty of conventional optimization techniques, to be extended to multi-objective optimization problems [1]. EA optimizers employ multiple individuals that can search simultaneously for multiple solutions. Using some modifications on the operators used by the EA optimizers the search process can be driven to a family of solutions representing the set of Pareto optimal solutions.

The performance of the proposed method for handling optimization problems with multiple objectives is examined in one space frame. For this test example both the rigorous approach and the simplified one with respect to the loading condition are implemented and their efficiency is compared in the framework of finding the optimum design of a structure under multiple objectives. In the context of the rigorous approach a number of artificial accelerograms are produced from the design response spectrum of the region for elastic structural response.

2 Single-objective Structural Optimization

In sizing optimization problems the aim is to minimize a single-objective function, usually the weight of the structure, under certain behavioral constraints on stress and displacements. The design variables are most frequently chosen to be dimensions of the cross-sectional areas of the members of the structure. Due to fabrication limitations the design variables are not continuous but discrete since cross-sections belong to a certain set. A discrete structural optimization problem can be formulated in the following form

$$\begin{aligned} &\min && f(s) \\ &\text{subject to} && g_j(s) \le 0 \quad j=1,...,k \\ & && s_i \in R^d, \quad i=1,...,n \end{aligned} \tag{1}$$

where R^d is a given set of discrete values and the design variables s_i (i=1,...,n) can take values only from this set.

In the optimal design of frames the constraints are the member stresses and nodal displacements or inter-storey drifts. For rigid frames with I-shapes, the stress constraints, under allowable stress design requirements specified by Eurocode 3 [2], are expressed by the following formula

$$\frac{N_{sd}}{Af_y/\gamma_{M1}} + \frac{M_{y,sd}}{W_{pl,y}f_y/\gamma_{M1}} + \frac{M_{z,sd}}{W_{pl,z}f_y/\gamma_{M1}} \le 1.0 \tag{2}$$

where N_{sd}, $M_{sd,y}$, $M_{sd,z}$ are the stress resultants, $W_{pl,y}$, $W_{pl,z}$ are the plastic first moment of inertia, and f_y is the yield stress. The safety factor γ_{M1} is a Eurocode 1 [3] box value usually taken as 1.10.

3 Multi-objective Structural Optimization

In practical applications of structural optimization of 3D frames and trusses the material weight rarely gives a representative measure of the performance of the structure. In fact, several conflicting and incommensurable criteria usually exist in real-life design problems that have to be dealt with simultaneously. This situation forces the designer to look for a good compromise design between the conflicting requirements. These kinds of problems are called optimization problems with many objectives. The consideration of multi-objective optimization in its present sense originated towards the end of the 19^{th} century when Pareto presented the optimality concept in economic problems with several competing criteria. Since then, although many techniques have been developed in order to deal with multi-objective optimization problems the corresponding applications were confined to mathematical functions. The first applications in the field of structural optimization with multiple objectives appeared at the end of the seventies.

3.1 Criteria and conflict

The designer looking for the optimum design of a structure is faced with the question of selecting the most suitable criteria for measuring the economy, the strength, the serviceability or any other factor that affects the performance of a structure. Any quantity that has a direct influence on the performance of the structure can be considered as a criterion. On the other hand, those quantities that must satisfy only some imposed requirements are not criteria but they can be treated as constraints. Most of the structural optimization problems are treated with one single-objective usually the weight of the structure, subjected to some strength constraints. These constraints are set as equality or inequality constraints using some upper and lower limits. When there is a difficulty in selecting these limits, then these parameters are better treated as criteria.

One important basic property in the multicriterion formulation is the conflict that may or may not exist between the criteria. Only those quantities that are competing should be treated as independent criteria whereas the others can be combined into a single criterion to represent the whole group. The local conflict between two criteria can be defined as follows: The functions f_i and f_j are called locally collinear with no conflict at point s if there is c > 0 such that $\nabla f_i(s)=c\nabla f_j(s)$. Otherwise, the functions are called locally conflicting at s. According to this definition any two criteria are locally conflicting at a point of the design space if their maximum improvement is achieved in different directions. The global conflict between two criteria can be defined as follows: The functions f_i and f_j are called globally conflicting in the feasible region $\mathcal{F}$ of the design space when the two optimization problems $\min_{s\in\mathcal{F}} f_i(s)$ and $\min_{s\in\mathcal{F}} f_j(s)$ have different optimal solutions.

3.2 Formulation of a multiple objective optimization problem

In formulating an optimization problem the choice of the design variables, criteria and constraints represents undoubtedly the most important decision made by the engineer. In general the mathematical formulation of a multi-objective problem includes a set of n design variables, a set of m objective functions and a set of k constraint functions and can be defined as follows:

$$\begin{aligned} \min_{s\in\mathcal{F}} \quad & [f_1(s), f_2(s), \ldots, f_m(s)]^T \\ \text{subject to} \quad & g_j(s) \le 0 \quad j=1,...,k \\ & s_i \in R^d, \quad i=1,...,n \end{aligned} \tag{3}$$

where the vector s = $[s_1\ s_2\ ...\ s_n]^T$ represents a design variable vector and $\mathcal{F}$ is the feasible set in design space R^n. It is defined as the set of design variables that satisfy the constraint functions g(s) in the form:

$$\mathcal{F} = \{\ s \in R^n \mid g(s) \le 0\} \tag{4}$$

Usually there exists no unique point which would give an optimum for all m criteria simultaneously. Thus the common optimality condition used in single-objective optimization must be replaced by a new concept the so called Pareto optimum: A design vector $s^* \in \mathcal{F}$ is Pareto optimal for the problem of eq. (4) if and only if there exists no other design vector $s \in \mathcal{F}$ such that

$$\begin{gathered} f_i(s) \le f_i(s^*) \text{ for } i=1,2,...,m \\ \text{with} \\ f_j(s) < f_j(s^*) \end{gathered} \tag{5}$$

for at least one objective *j*.

3.3 Solving the multi-objective optimization problem

Classical methods for generating the Pareto optimal set combine the objectives into a single, parameterized objective function. Basically, this procedure is independent of the underlying optimization algorithm. Three previously used methods in the literature [4-6] are briefly discussed and are compared in this study with the proposed modified ES in terms of computational time and efficiency for treating multiobjective optimization problems.

3.3.1 Linear weighting method

The first method called the linear weighting method [6] combines all the objectives into a single scalar parameterized objective function by using weighting coefficients. If w_i, i=1,2,...,m are the weighting coefficients the problem of eq. (5) can be written as follows:

$$\min_{s\in\mathcal{F}} \sum_{i=1}^{m} w_i\, f_i(s) \tag{7}$$

with no loss of generality the following normalization of the weighting coefficients is employed

$$\sum_{i=1}^{m} w_i = 1 \tag{8}$$

By varying these weights it is now possible to generate the set of Pareto optima solutions for problem of eq. (5). The values of the weighting coefficients are adjusted according to the importance of each criterion. Every combination of those weighting coefficients correspond to a single Pareto optimal solution, thus, performing a set of optimization processes using different weighting coefficients it is possible to generate the full set of Pareto optimal solutions.

In real world problems different units correspond to different objectives leading to variations of some orders of magnitude between the values of the objectives. It is therefore advisable that the objectives should be normalized according the following expression:

$$\tilde{f}_i(s) = \frac{f_i(s) - f_{i\ min}}{f_{i\ max} - f_{i\ min}} \tag{9}$$

where the normalized objectives $\tilde{f}_i(s) \in [0,1]$, i = 1,2,...,m, use the same design space with the non normalized ones, while $f_{i\ min}$ and $f_{i\ max}$ are the minimum and maximum values of the objective function i.

3.3.2 Distance function method

The distance methods [4] are based on the minimization of the distance between the set of the objective function values and some chosen reference points belonging to the so called criterion space. Where as criterion space is defined the set of the objective function values that correspond to design vectors of the feasible domain. The resulting scalar problem is:

$$\min_{s \in \mathcal{F}} d_p(s) \tag{10}$$

where the distance function can be written as follows:

$$d_p(s) = \left\{ \sum_{i=1}^{m} w_i \left[f_i(s) - z_i \right]^p \right\}^{1/p} \tag{11}$$

and p is an integer number.

The reference point $z^{id} \in R^m$ that is selected by the designer is also called ideal or utopian point. A reference point that is frequently used is the following:

$$z^{id} = [\ f_{1\,min}\ \ f_{2\,min}\ \ ...\ \ f_{m\,min}\ ]^T \tag{12}$$

where $f_{i\ min}$ is the optimum solution of the single-objective optimization problem where the i-th objective function is treated as the unique objective. The normalization function eq. (8) for the weighting factors w_i is also used. In the case that p = ∞ eq. (10) is transformed to the minimax problem:

$$\min_{s \in \mathcal{F}} \max_{i} \left[w_i f_i(x) \right],\ i = 1, 2, ...,m \tag{13}$$

In the case of p=1 the formulation of the distance method is equivalent to the linear method when the reference point used is the zero $\hat{z} = 0$, while the case of p=2 the method is called weighted quadratic method.

3.3.3 Constraint method

According to this method the original multicriterion problem is replaced by a scalar problem where one criterion f_k is chosen as the objective function and all the other criteria are removed into the constraints [2]. By introducing parameters ε_i into these new constraints an additional feasible set is obtained:

$$\mathcal{F}_k\,(\,\varepsilon_i\,) = \{\ \mathbf{s} \in R^n\ |\ f_i(\mathbf{s}) \le \varepsilon_i\ ,\ i = 1, 2, ..., m\ \ \mu\varepsilon\ \ i \neq k\ \} \tag{14}$$

If the resulting feasible set is denoted by $\bar{\mathcal{F}}_k = \mathcal{F} \cap \mathcal{F}_k$ the parameterized scalar problem can be expressed as:

$$\min_{s \in \bar{\mathcal{F}}_k} f_k(s) \tag{15}$$

The constraint method gives the opportunity to obtain the full domain of optimum solutions, in the horizontal or vertical direction using one criterion as objective function and the other as constraint.

3.3.4 Modified Evolution strategies for multiobjective optimization

The three above mentioned methods for multi objective optimization have been used in the past with mathematical programming optimization algorithms where at each optimization step one design point was examined as an optimum design candidate. In order to locate the set of Pareto optimum solutions a family of optimization runs have to be executed. On the other hand, evolutionary algorithms work simultaneously with a population of design points, instead of a single design point, which constitute a population of optimum design candidates, in the space of design variables. Due to this characteristic, evolutionary algorithms have a great potential in finding multiple optima, in a single optimization run, which is very useful in Pareto optimization problems. Since the early nineties a number of researchers have suggested the use of evolutionary algorithms in multiobjective optimization problems [1].

In our study the method of Evolution Strategies (ES) is applied for the first time for structural multi-objective optimization problems. To this purpose some changes have to be made in the random operators in order to guide the convergence to a population that represent the set of Pareto optimal solutions. These changes refer to (i) the selection of the parent population at each generation that has to be modified in order to guide the search procedure towards the set of Pareto optimum solutions, and (ii) the presentation from convergence to a single design point, and preserve diversity in the population in every generation step. The first demand is possible to be fulfilled using random selection of the objective according to which the individual will be chosen for reproduction. While in order to preserve diversity in the population and fulfil the second requirement, the fitness sharing is implemented. The idea behind sharing is to degrade those individuals that are represented in higher percentage in the population. The modified objectives after sharing are the following:

$$f'_i(s) = \frac{f_i(s)}{\sum_h sh(d(s,h))} \tag{16}$$

where the sharing function used in the current study is the following:

$$sh(d(s,h)) = \begin{cases} 1 - \left(\dfrac{d(s,h)}{\sigma_{share}} \right)^{\alpha} & \text{if } d(s,h) < \sigma_{share} \\ 0 & \text{otherwise} \end{cases} \tag{17}$$

The distance function used is in the objective space:

$$d(s,h) = \left\| f(s) - f(h) \right\| \tag{18}$$

4 Structural design under seismic loading

The equations of equilibrium for a finite element system in motion can be written in the usual form

$$M(s_i)\ddot{u}_t + C(s_i)\dot{u}_t + K(s_i)u_t = R_t \tag{19}$$

where $M(s_i)$, $C(s_i)$, and $K(s_i)$ are the mass, damping and stiffness matrices for the i-th design vector s_i; R_t is the external load vector, while $u, \dot{u}$ and $\ddot{u}$ are the displacement, velocity, and acceleration vectors, respectively of the finite element assemblage. The solution methods of response spectrum modal analysis, which is based on the mode superposition approach and direct integration of the equations of motion will be considered in this work.

5 Solution of the optimization problem

There are three types of algorithms belonging to the class of evolutionary computation that imitate nature by using biological methodologies in order to find the optimum solution of a problem: (i) evolutionary programming (EP), (ii) genetic algorithms (GAs) and (iii) evolution strategies (ESs). Their main difference is that GAs deal with bit-strings of fixed sizes, ES with real vectors and EP with finite state automata. GAs basic assumption is that the optimal solution can be found by assembling building blocks, i.e. partial pieces of solutions, while ESs and EP simply ensure the emergence of the best solutions. The most important consequence of this different approach is related to the recombination operator, viewed as essential for GA, as potentially useful for ES and as possibly harmful for EP. The modern tendencies seem to follow combinations of the two approaches, since GA users have turned to real number representations when dealing with real numbers following experimental results or heuristic demonstrations, whereas ES users have included recombination as a standard operator, and have designed special operators for non real-valued problems.

5.1 ES in multi-objective structural optimization problems

The application of evolutionary algorithms in multi-objective optimization problems has attracted the interest of a number of researchers in the last five years due to the difficulty of conventional optimization techniques, such as gradient based methods, to be extended to multi-objective optimization problems. EA, however, have been recognized to be more appropriate to multi-objective optimization problems since early in their development. Multiple individuals can search for multiple solutions simultaneously, taking advantage of any similarities available in the family of possible solutions to the problem.

In the first implementation where the classical methods are used, the optimization procedure, in order to generate a set of Pareto optimal solutions, initiates with a set of parent design vectors needed by the ES optimizer and a set of weighting coefficients for the combination of all objectives into a single scalar parameterized objective function. These weighting coefficients are not set by the designer but are being systematically varied by the optimizer after a Pareto optimal solution has been achieved. There is an outer loop which systematically varies the parameters of the parameterized objective function, and is called decision making loop. The inner loop is the classical ES process, starting with a set of parent vectors. If any of these parent vectors gives an infeasible design then this parent vector is modified until it becomes feasible. Subsequently, the offsprings are generated and checked if they are in the feasible region. According to $(\mu+\lambda)$ selection scheme in every generation the values of the objective function of the parent and the offspring vectors are compared and the worst vectors are rejected, while the remaining ones are considered to be the parent vectors of the new generation. On the other hand, according to (μ,λ) selection scheme only the offspring vectors of each generation are used to produce the new generation. This procedure is repeated until the chosen termination criterion is satisfied. The number of parents and offsprings involved affects the computational efficiency of the multi-membered ES discussed in this work. It has been observed that when the values of μ and λ are equal to the number of the design variables produce better results.

Two ES algorithms for multi-objective structural optimization applications under seismic loading are compared and tested in the subsequent section:

(i) The ES algorithm combined with the classical methods which can be stated as follows:

Outer loop - Decision making loop
Set the parameters w_i of the parameterized objective function
Inner loop - ES loop
1. *Selection step :* selection of s_i (i = 1,2,...,μ) parent vectors of the design variables
2. *Analysis step :* solve $M(s_i)\ddot{u} + C(s_i)\dot{u} + K(s_i)u = R(t)$ (i=1,2,...,μ)
3. *Evaluation of parameterized objective function*
4. *Constraints check :* all parent vectors become feasible
5. *Offspring generation :* generate s_j, (j=1,2,...,λ) offspring vectors of the design variables
6. *Analysis step :* solve $M(s_j)\ddot{u} + C(s_j)\dot{u} + K(s_j)u = R(t)$ (j=1,2,...,λ)
7. *Evaluation of the parameterized objective function*
8. *Constraints check :* if satisfied continue, else change s_j and go to *step 5*

9. *Selection step :* selection of the next generation parents according to (μ+λ) or (μ,λ) selection schemes
10. *Convergence check :* If satisfied stop, else go to *step 5*

End of Inner loop
End of Outer loop

(ii) The modified ES algorithm (ESMO) as described in section 3.3.4 which can be stated as follows:

1. *Selection step :* selection of s_i (i = 1,2,...,μ) parent vectors of the design variables
2. *Analysis step :* solve $M(s_i)\ddot{u} + C(s_i)\dot{u} + K(s_i)u = R(t)$ (i=1,2,...,μ)
3. *Evaluation of the objective functions*
4. *Constraints check :* all parent vectors become feasible
5. *Offspring generation :* generate s_j, (j=1,2,...,λ) offspring vectors of the design variables
6. *Analysis step :* solve $M(s_j)\ddot{u} + C(s_j)\dot{u} + K(s_j)u = R(t)$ (j=1,2,...,λ)
7. *Evaluation of the objective functions*
8. *Constraints check :* if satisfied continue, else change s_j and go to *step 5*
9. *Selection step :* random selection of the potential objective for the each individual and selection of the next generation parents according to (μ+λ) or (μ,λ) selection schemes
10. *Fitness sharing*
11. *Convergence check :* If satisfied stop, else go to *step 5*

6 Numerical results

The performance of the multi-objective optimization methods discussed in this paper is investigated in two benchmark test examples: A six storey space frame and a multi-layered space truss. The following abbreviations are used in this section: *DTI* refers to the Newmark Direct time Integration method. *RSMA* refers to the Response Spectrum Modal Analysis. LWM refers to the Linear Weighting method for treating multi-objective optimization problems. DFM refers to the Distance Function method for treating multi-objective optimization problems. CM refers to the Constraint method for treating multi-objective optimization problems. ESMO refers to the proposed Evolution Strategies for treating Multi-objective Optimization problems.

The objective functions considered for this problem are the weight of the structure and the maximum displacement. The constraints are imposed on the inter-storey drifts and for each element group on the maximum stress constraint of Eqs. (2) under a combination of axial force and bending moments. The space frame consists of 63 elements with 180 degrees of freedom as shown in Figure 2. The length of the beams and the columns are L_1=7.32 m and the columns L_2=3.66 m, respectively. The structure is loaded with a 19.16 kPa gravity load on all floor levels and a static lateral load of 109 kN applied at each node in the front elevation along the z direction. The element members are divided into 5 groups, each one having two design variables resulting in ten total design variables. The cross section of each member is assumed to be a I-shape and for each member two design variables are considered as shown in Figure 1. The modulus of elasticity is 200 GPa and the yield stress is σ_y=250 MPa.

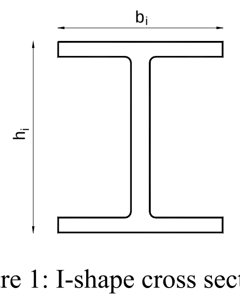

Figure 1: I-shape cross section

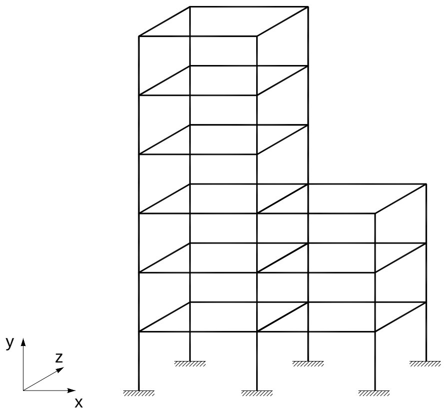

Figure 2: Six storey space frame

The Pareto optimal set of solutions was first computed with the LWM. The performance of this method for the case of seeking the simultaneous minimization of weight and maximum displacement is depicted in Figures 3 and 4 for both static and seismic loading conditions. In Figures 3 and 4 the performance of the DFM and ESMO methods are also presented. For the case of the DFM the zero (0) point was considered as the utopian point, while four different schemes of the DFM were examined. p=1: equivalent to the LWM, p=2: called quadratic LWM and p=8: equivalent to the p=∞.

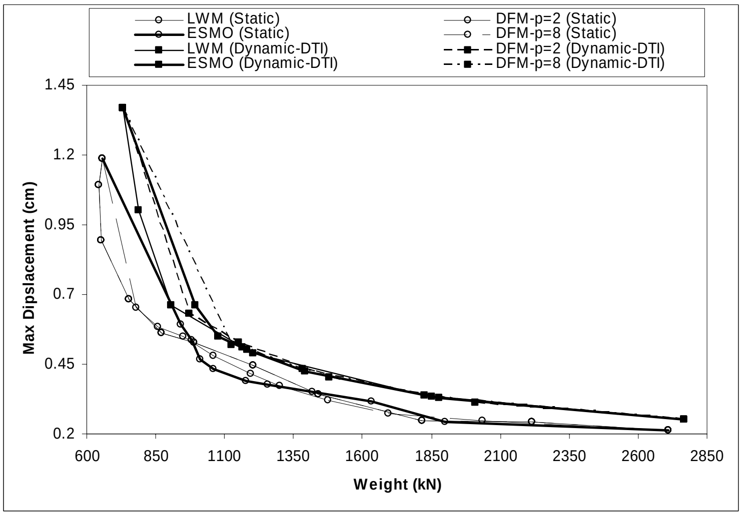

Figure 3: Six Storey frame: Performance of the methods for static and combined static and seismic loading conditions

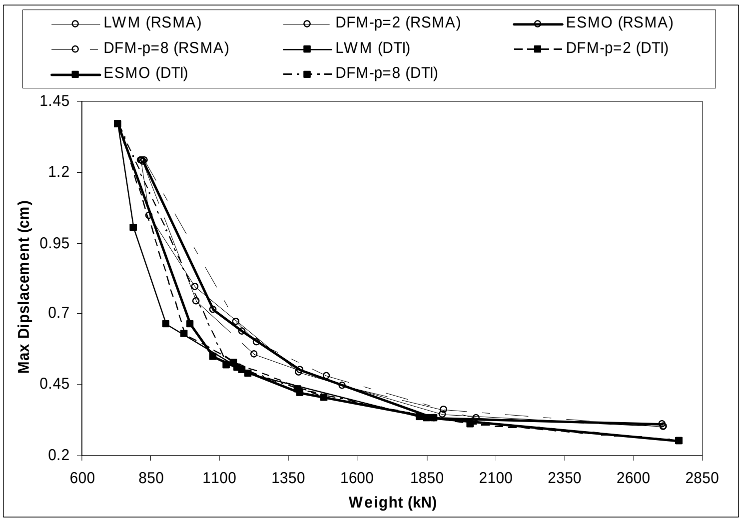

Figure 4: Six Storey frame: Performance of the methods for combined static and seismic loading conditions

6 Conclusions

Evolution Strategies can be considered as an efficient tool for multi-objective design optimization of structural problems such as space frames under static and seismic loading conditions. The ESMO method compared to the linear weighing method and the constraint method appears to be robust and reliable for treating multi-objective structural optimization problems giving almost identical results. The generalization of the linear weighing method for p>1 called the distance function method is also examined in this study. The results obtained by the distance function method were somewhat different than those taken by the other two methods, while for large values of p it produces either too close or disperse points in the Pareto sets.

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