MODELING THE *SPECIFIC GRINDING ENERGY* AND BALL-MILL SCALEUP

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MODELLING THE SPECIFIC GRINDING ENERGY AND BALL-MILL SCALEUP

Ball-mill scale up (**Bond's Law**)

Data:

- Bond work index $\boldsymbol{w_i}$
- Feed $\boldsymbol{D_f}$ and product $\boldsymbol{d}$ size (both 80% cumulative passing)

Result: → The specific grinding energy $\boldsymbol{w}$

Mill power draw P = wT*, where* ***T*** *the mill capacity* → Mill dimensions (from Tables or charts)

Ball-mill scale up *Continued*

Denver method

Denver slide rule (*circular nomograph*)

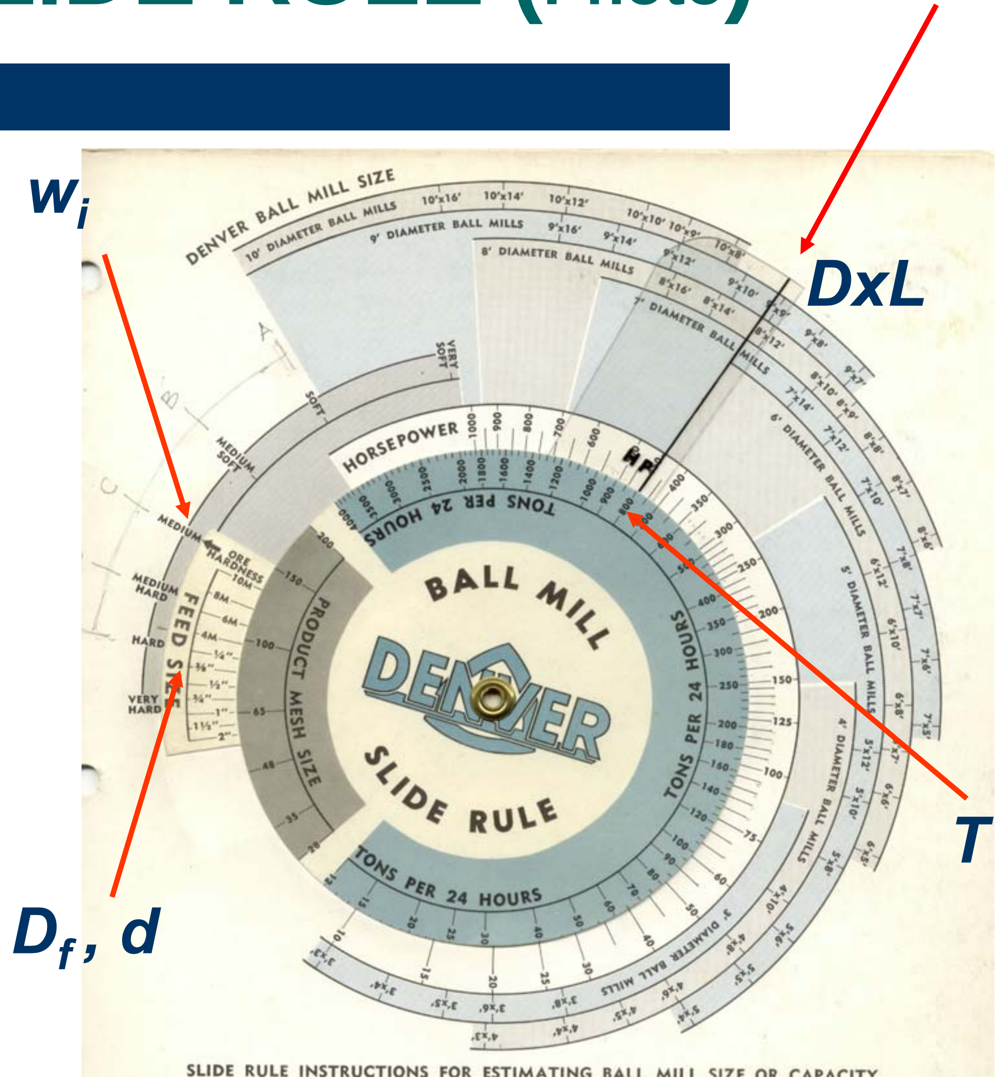
Data (necessary):

- Feed size D_f and product size d
- Given ore hardness (soft, medium or hard)
- Given capacity T (short ton/h)

Result: → The mill power draw ***P***, *which corresponds to a particular ball-mill size*

DENVER SLIDE RULE (Photo)

Determination of the ball-mill dimensions (***D* x *L***) with the help of the Denver slide rule (e.g. for this particular case 9'x10'≈2.74 m x 3.05 m)



Mill *Power Draw* Calculation (*Equations*)

- Arbiter and Harris (1980)
- Rowland and Kjos (1980)
- Harris and Arbiter (1982)
- Dor and Bassarear (1982)
- Rowland (1982)
- Turner (1982)
- Austin et al. (1992)

Researchers

- Nordberg, Morgardshammar (***manufacturers***)

EQUATIONS GIVING THE MILL POWER DRAW, *P*

The mill power draw ***P*** is a function of:

1. The fraction of mill filling f_L
2. The fraction of the mill critical speed f_c
3. The apparent specific gravity of the charge ρ and
4. The mill dimensions (diameter ***D***, length ***L***)

$$P = f(f_L, f_C, \rho, D, L)$$

Prediction of the Mill Power Draw (recent contribution; *Morell ,1996*)

- The **C-Model** (theoretical)

  Based on the way the mill charge moves inside the mill

- The simplified **E-Model** (empirical)

  Based on the former but it contains fewer and simpler equations

  Result: → Extensive database of power drawn from ***Ball-, autogenous*** *and* ***semi-autogenous*** **mills**

Present paper

An effort to develop simple and efficient models for:

- the calculation of the ***specific grinding energy*** ***w*** and
- the determination of the ***mill power draw*** ***P*** (as a function of the Bond work index w_i)

Use of the above models:

For ***ball-mill scale-up*** purposes.

Model giving the *specific grinding energy* w

- Applying multiple linear regression analysis to sets of data (***w***, ***D$_f$*** and ***d***) taken from the ***Denver slide rule***, the equation derived is of the general form:

$$w = k \cdot f(D_f, d)$$

Models giving the *specific grinding energy w*

The equations derived for the various types of ore referring to its hardness are:

$$w_s = 0.671 D_f^{\,0.193} d^{-0.962}$$ **(Soft ore)**

$$w_m = 1.290 D_f^{\,0.193} d^{-0.962}$$ **(Medium ore)**

$$w_h = 1.961 D_f^{\,0.193} d^{-0.962}$$ **(Hard ore)**

Introduction of the Bond work index w_i to the above equations

Table. Relationship between the Bond work index (w_i) and the ore hardness designated by Denver

Denver ore-hardness	*Bond work index (w_i),* kWh/short ton	*Coefficients k*
Soft ore	*6.5*	***0.671***
Medium ore	*12.0*	***1.290***
Hard ore	*18.0*	***1.961***

Relationship between the Bond work index (w_i) and the *Coefficients k*

$$k = 0.106\, w_i$$

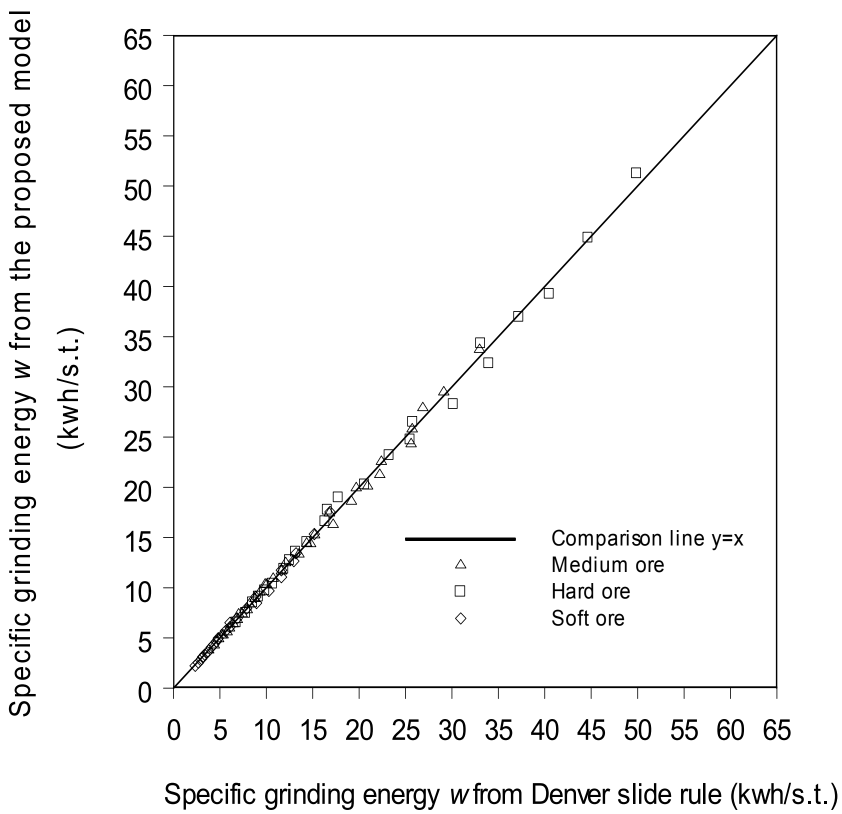
Thus, the *specific grinding energy* *w* is given now from the general expression:

$$w = 0.106 w_i D_f^{\,0.193} d^{\,-0.962} \quad \text{(kWh/short ton)}$$

$$w = 0.1169 w_i D_f^{\,0.193} d^{\,-0.962} \quad \text{(kWh/t)}$$

Comparison between the *specific grinding energy* (w) from the various methods

- *In the next Figure a comparison is made between the specific grinding energy (w) values, which are determined from Denver slide rule, and those calculated from the proposed model.*
- *The distribution of the points around the line proves the good agreement of the results obtained from the models and those obtained from the Denver slide rule .*



POWER DRAW P OF DENVER BALL-MILLS

- With application of linear least squares regression to 45 pairs (*P*, $D^{2.5} \times L$) obtained from Denver slide rule, the linear equation, without the constant term (*y*=*bx*), is:

$$P = 12.767 D^{2.5} L$$ (hp) or

$$P = 9.524 D^{2.5} L$$ (kw)

But since $P = k\rho f_L (1 - f_L) f_c D^{2.5} L$, and

for ρ= 4.9455 short ton/m^3 f_L = 0.45 and f_c = 0.75

POWER DRAW P OF DENVER BALL-MILLS

Then, $\rho f_L(1-f_L)f_c = 0.918$

Therefore, k = 12.767 / 0.9180 = 13.91

$$P = 13.91\rho f_L(1-f_L)f_c D^{2.5} L \qquad \text{(hp)}$$

$$P = 10.38\rho f_L(1-f_L)f_c D^{2.5} L \qquad \text{(kw)}$$

Prediction of the mill dimensions

- *for a given Bond work index* $\boldsymbol{w_i}$,
- *feed size* $\boldsymbol{D_f}$ *product size* ***d*** *and*
- *for capacity* ***T***

It is known that, ***P = w T***, (kW) , where ***w*** is the *specific grinding energy* (kWh/short ton) and *T* is the required mill capacity (short ton/h)

Thus,
$$P = 0.106 w_i D_f^{\ 0.193} d^{-0.962} T$$

Prediction of the mill dimensions

- The *ball-mill dimensions* (internal mill diameter ***D*** and length ***L***), for a given (***L/D***) ratio, for feed size D_f and product size ***d*** (mm), for a known Bond work index w_i (kWh/short ton) and for desirable capacity ***T*** (short ton/h), can be calculated from:

$$D^{3.5}\left(L/D\right) = \left(1/10.38\times 0.9180\right)0.106 D_f^{\;0.193} d^{-0.962} T$$

Comparison between the ball-mill *power draw* from the various methods

- In the next Figure a comparison is made between the ball-mill *power draw* values determined from the Denver slide rule, and those calculated from the proposed model.
- From the distribution of the points around the *line of comparison* the good agreement of the results received is obvious.

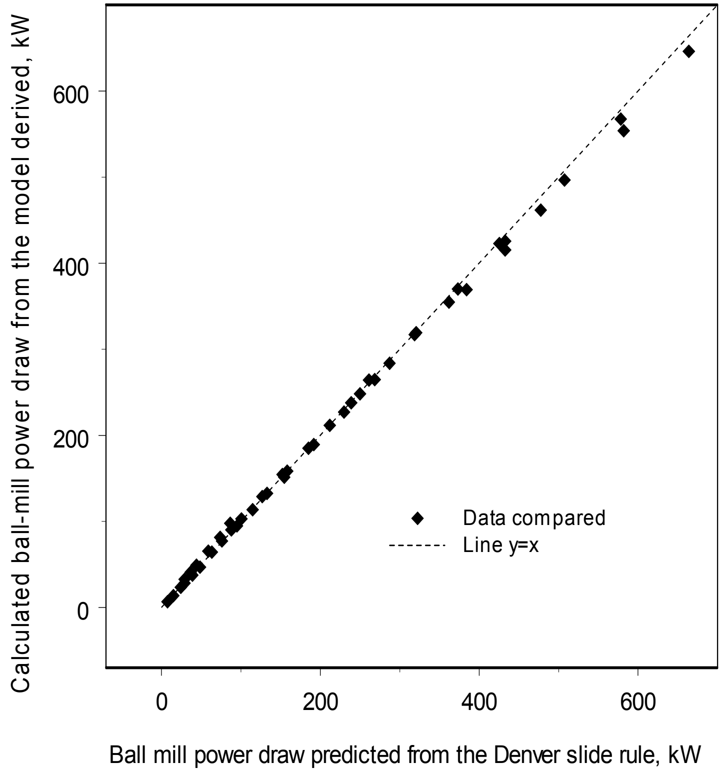


Fig. 2. Comparison of the ball mill power draw from the Denver slide rule and the proposed model. Dashed line corresponds to y=x.

COMPARISON BETWEEN THE *BOND* (CLASSIC) AND THE *PROPOSED METHODS*

Example	MILL DIMENSIONS (*D* x *L*)	
	BOND METHOD	***PROPOSED*** METHOD (DENVER)
1	**D = 3.93 m, L = 5.79 m**	**D = 3.97 m, L = 5.96 m**
2	**D = 4.85 m, L = 6.10 m**	**D = 4.75 m, L = 5.94 m**
3	**D = 3.05 m, L = 5.83 m**	**D = 3.10 m, L = 5.95 m**
4	**D = 5.49 m, L = 8.45 m**	**D = 5.33 m, L = 8.21 m**

CONCLUSIONS

In the present work, equations were derived, giving the *specific grinding energy* w as a function of:

- the feed $\boldsymbol{D_f}$ (mm) and product size $\boldsymbol{d}$ (mm) (both 80% cumulative passing) and the Bond work index $\boldsymbol{w_i}$ (kWh/short ton) or,
- as a function of the size reduction ratio

$\boldsymbol{R = D_f/d}$, of the Bond work index $\boldsymbol{w_i}$ and the product size $\boldsymbol{d}$,

CONCLUSIONS *Continued*

In the present work, equations were also derived, giving:

- the ball-mill *power draw* ***P*** as a function of its dimensions: internal mill diameter ***D*** and length ***L***,
- the ball-mill power draw ***P*** as a function of the feed D_f (mm) and the product size ***d*** (mm), the Bond work index w_i (kWh/short ton) and the mill throughput ***T*** (short ton/h),
- the ball-mill dimensions (*D* and *L*), when not only D_f, ***d***, w_i *and* ***T***, but also the mill operating conditions (ρ, f_L, and f_c) are known and assuming the value of the (L/D) ratio.

CONCLUSIONS *Continued*

- From this work it was shown that the proposed equations approach very well the values calculated with the help of the Denver slide rule. They represent the *mathematical expression* of the Denver slide rule, which is not always available.
- It was additionally shown that the ball-mill dimensions predicted from the above methodology are *almost equal* to those of the Bond method. This fact is very important, because the various corrections, associated with the Bond methodology, are not necessary and the model developed can be used as an alternative method for ball-mill scale-up purposes.

THANK YOU VERY MUCH

For your attention!!!