

# USE OF A NEW MODEL TO REPRESENT HYDROCYCLONE CORRECTED-EFFICIENCY CURVES

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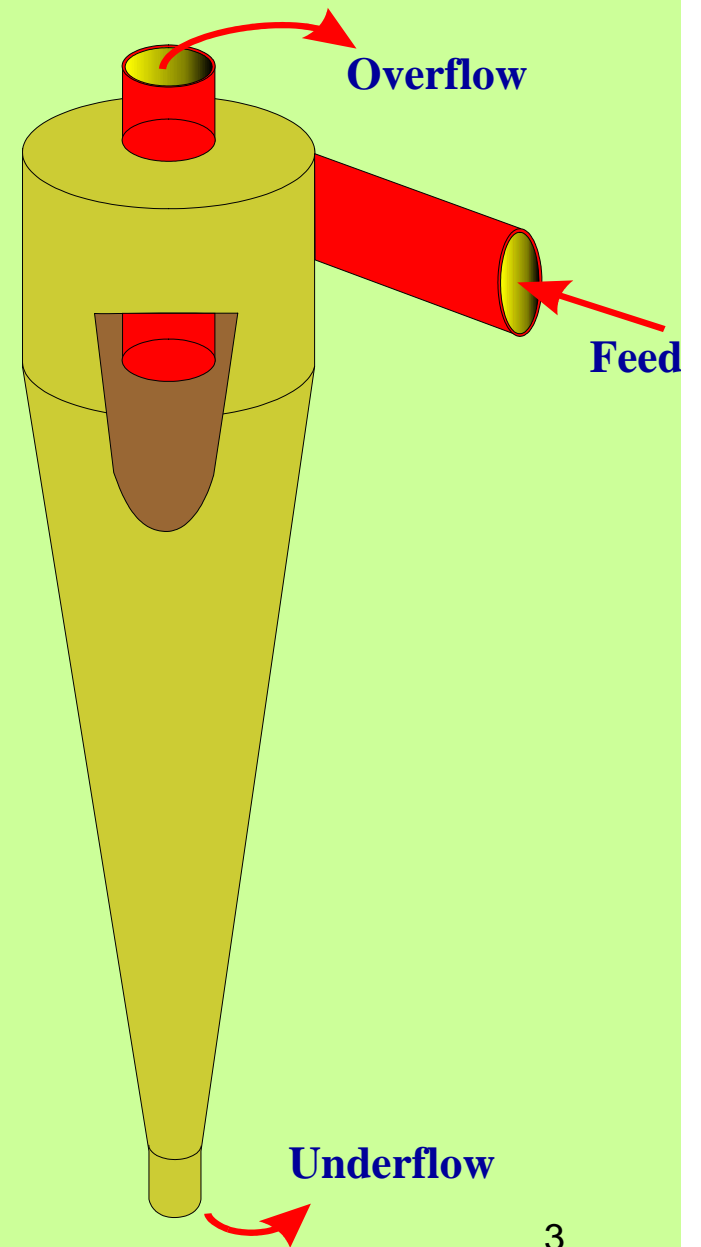
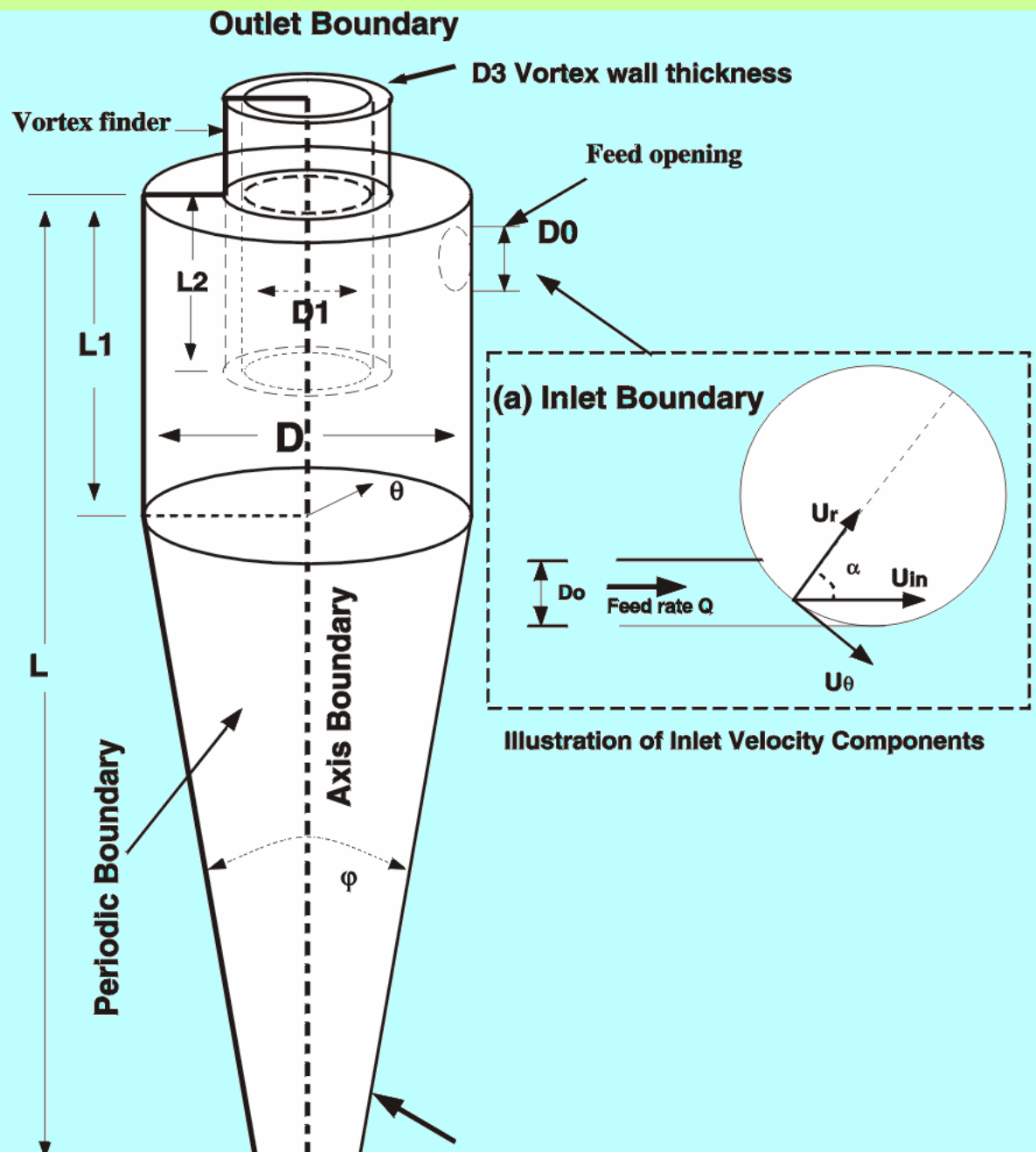
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# MAIN USES OF HYDROCYCLONES

- Classification (e.g. closed grinding circuits extremely efficient at fine separation sizes)
- De-sliming (clarification)
- De-gritting
- Thickening
- Sorting

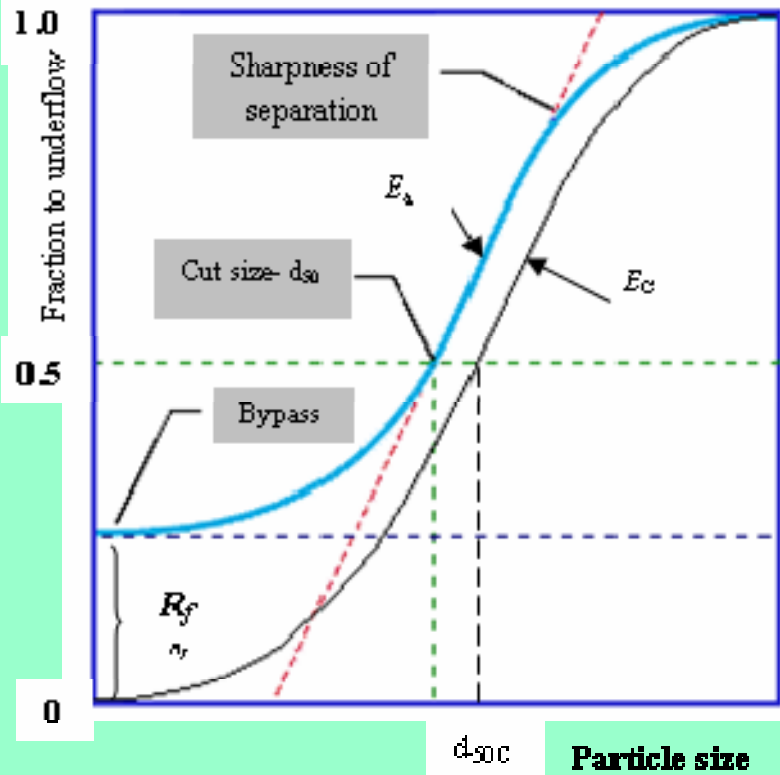
# Hydrocyclone geometrical characteristics



# CYCLONE EFFICIENCY

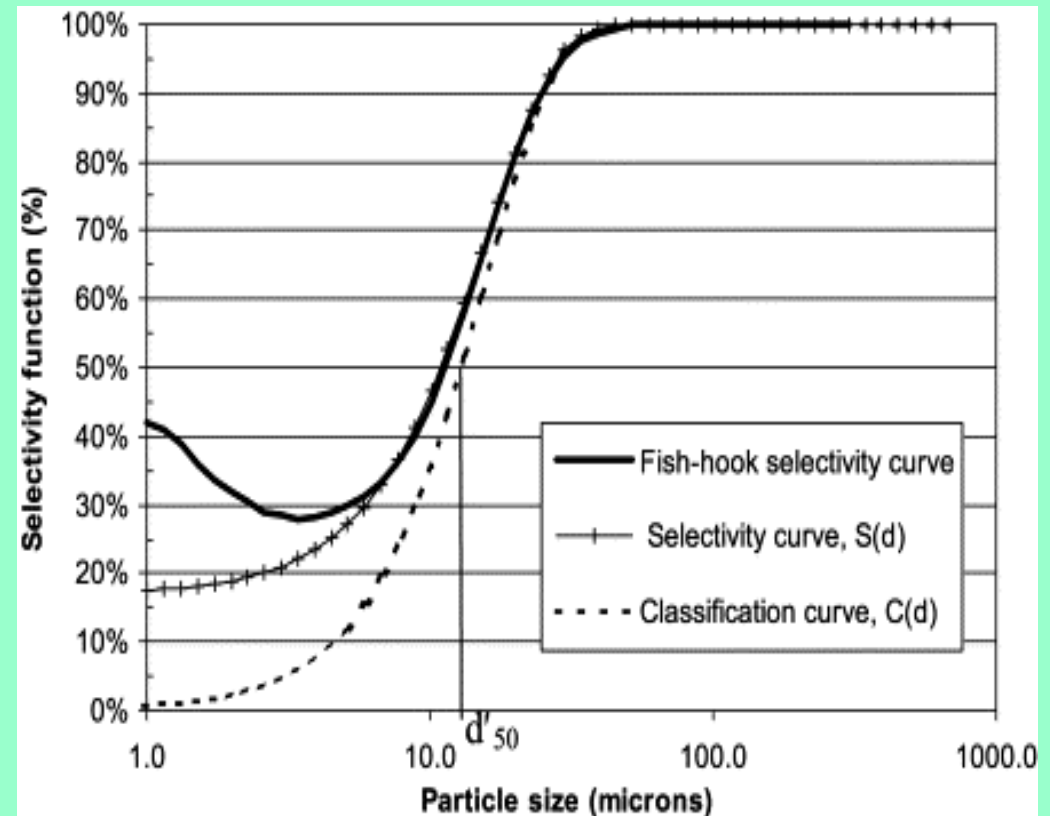
- The *partition* or *performance* curve is the method of representing the cyclone *efficiency*.
- The curve relates the *weight fraction* or *percentage* of **each size fraction** found in the feed, which reports to the **underflow** (coarse material).
- The **cut size** (separation size) or  $d_{50}$  is the mean size fraction for which, 50% of its particles in the feed reports to the underflow (equal chance of going either with the underflow or with the overflow).
- The *sharpness* of the separation depends on the slope of the *central section* of the partition curve.
- The closer to vertical is the slope, the higher is the efficiency.

## Partition Curve



**Comparison  
between typical  
partition curves  
and the fish-  
hook effect**

**Typical hydrocyclone  
partition curves (actual  $E_a$   
and corrected  $E_c$  efficiency  
curves)**



## CYCLONE EFFICIENCY (continued)

- The slope of the partition curve can be approximated from the below given equation ( $d_{75}$  and  $d_{50}$  are the particle sizes on the curve with 75% and 25% of the feed in the underflow).
- The efficiency of the separation is called also **imperfection**  $I$  and is given from the same equation.

$$I = \frac{d_{75} - d_{25}}{2d_{50}}$$

## RELATIONSHIP BETWEEN actual efficiency $E_a$ and corrected efficiency $E_c$

- In many mathematical models of hydrocyclones the term (mean size fraction)  $d_{50c}$  is used, since it is assumed that solids from all size fractions are entrained in the coarse product due to **short-circuiting**, in direct proportion to the fraction of feed water reporting to the underflow.
- The relationship between  $E_a$  (separation size  $d_{50}$ ) and  $E_c(d_{50c})$  is given from

$$E_c = \frac{E_a - R_f}{1 - R_f}$$

# Models used for the corrected efficiency $E_c$ curves

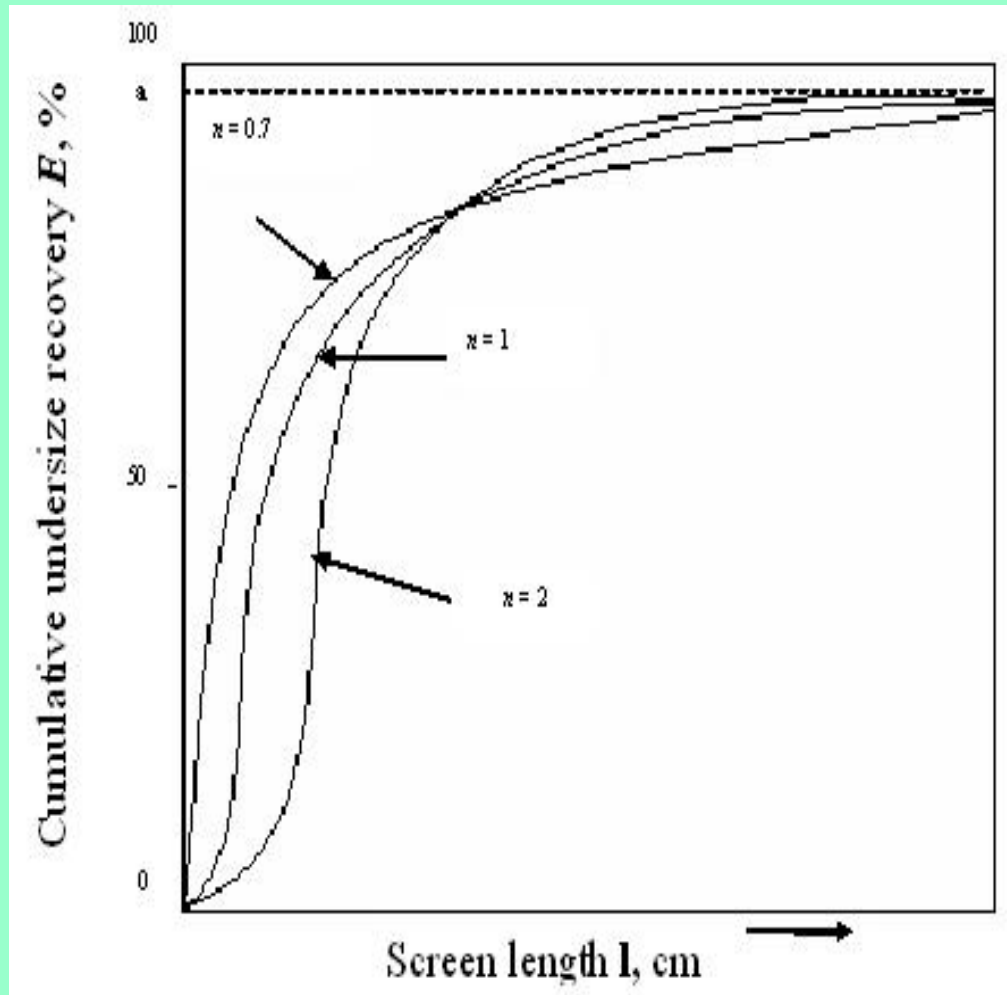
$$E_c = \frac{e^{[a(d/d_{50c})]} - 1}{e^{[a(d/d_{50c})]} + e^a - 2} \quad (\text{Lynch model, 1965})$$

$$E_c = 1 - e^{\left[-0.6931(d/d_{50c})^m\right]} \quad (\text{Plitt-Reid model, 1971})$$

$$E_c = 1 - \left[1 - (d/d_{\max})^m\right]^r \quad (\text{Harris model, 1972})$$



# DERIVATION OF THE NEW MODEL



**Model used**

$$E = ae^{-\left(\frac{b}{l^n}\right)}$$

# NEW MODEL

- Equation can be suitably modified to give:

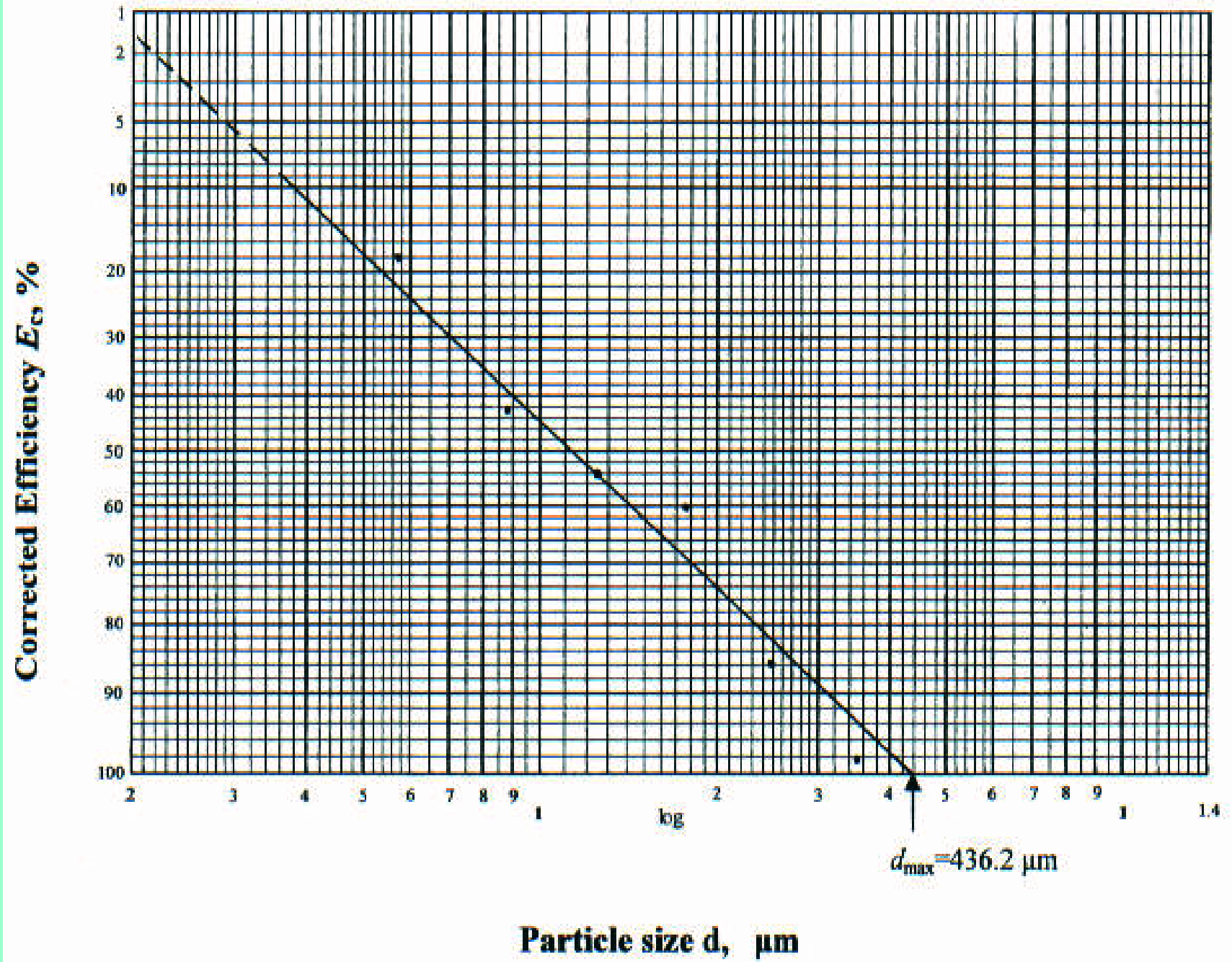
$$E_c / 1.359 = e^{-(d_{50c}/d)^n}$$

$$A = e^{-(d_{50c}/d)^n}$$

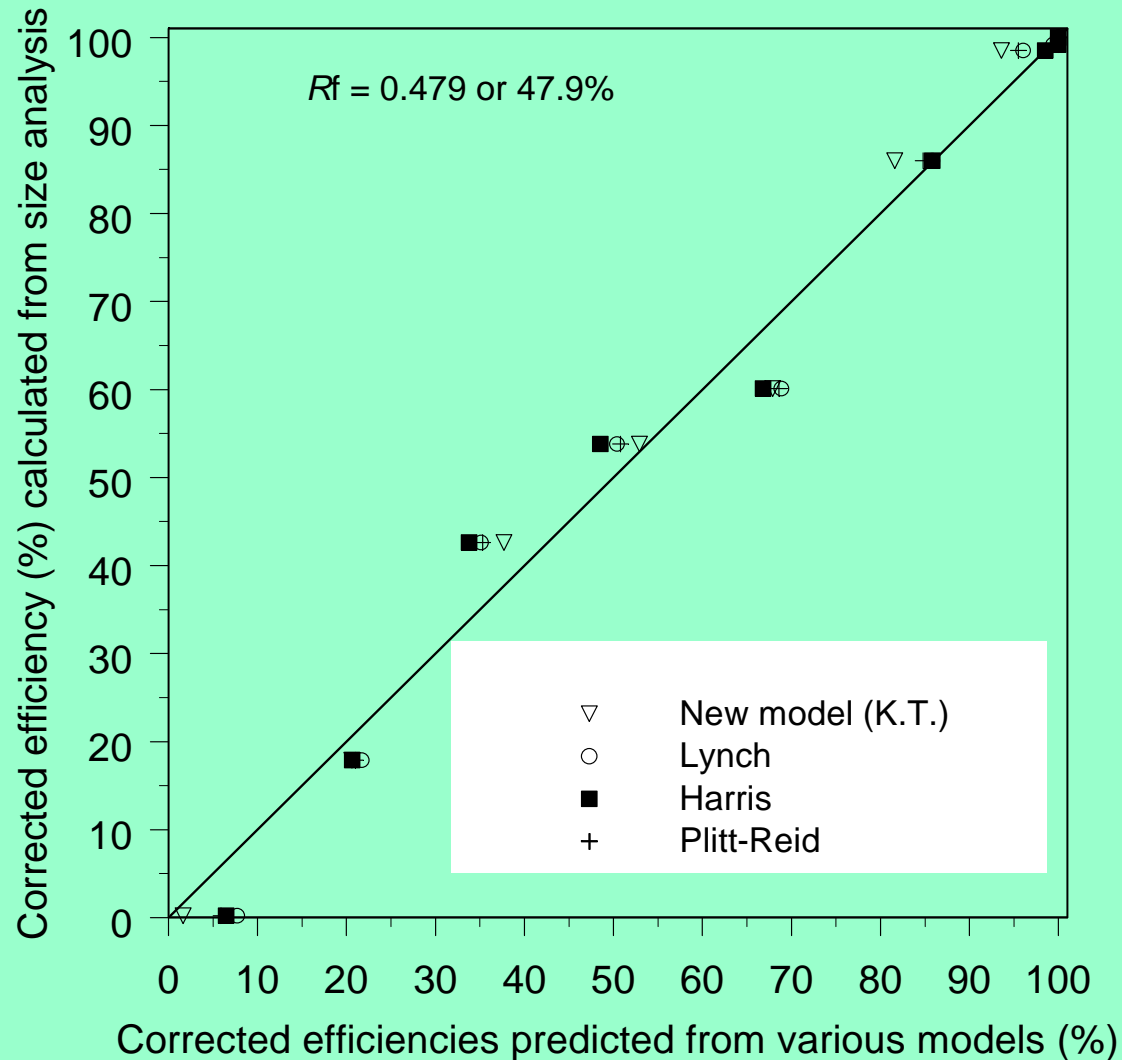
# OBSERVATIONS ON THE MODEL

- The model is a *modified* Rosin-Rammler equation. It is clear that, when  $d=d_{50c}$  then  $A=0.3679$  or 36.79%, which corresponds to  $E_c=0.5$  or 50%. Similarly, when  $E_c=1$  or 100%, then  $A=0.7358$  or 73.58 %. Taking into account the above observations, the ordinate (y-axis) of a Rosin-Rammler graph was modified, putting in the points of 36.79% and 73.58% retained, the values 50% and 100% for  $E_c$ , respectively.

$$A = e^{-\left(d_{50c}/d\right)^n}$$

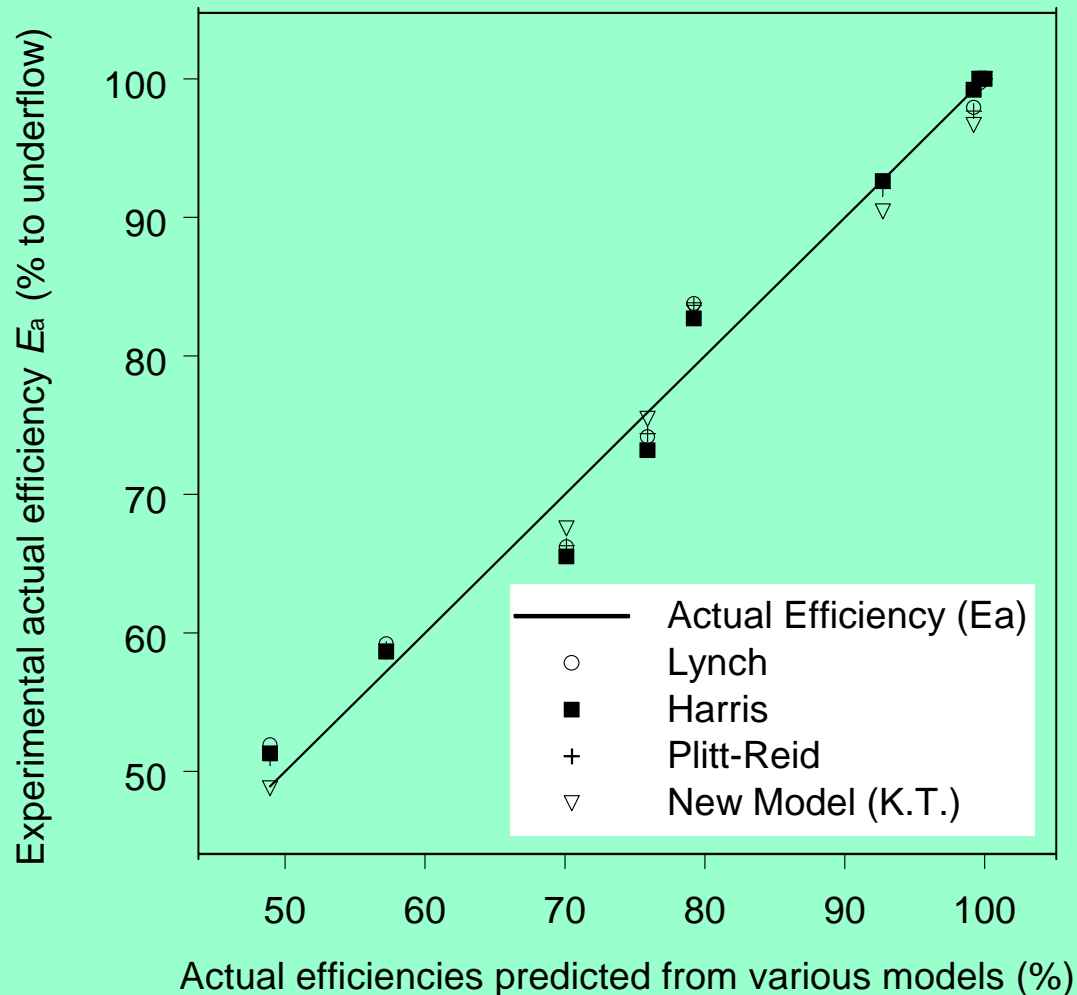


# RESULTS



Comparison of the corrected efficiency  $E_c$ , calculated from the experimental actual efficiency  $E_a$ , with the corrected efficiencies predicted from the various models. Solid line corresponds to  $y=x$ .

# RESULTS (continued)



Comparison of the actual efficiencies predicted from various models to the actual efficiency ( $E_a$ ) from size analysis. Solid line corresponds to  $y=x$ .

# COMPARISON between the various models

<i>Model</i>	<b>Lynch</b>	<b>Plitt-Reid</b>	<b>Harris</b>	<b>New model</b>
Values of the parameters	$d_{50c} = 123 \mu\text{m}$	$d_{50c} = 122 \mu\text{m}$	$d_{\text{max}} = 433.1 \mu\text{m}$	$d_{50c} = 116 \mu\text{m}$
	<b><math>a = 1.602</math></b>	<b><math>m = 1.42</math></b>	<b><math>m = 1.263</math></b>	<b><math>n = 0.892</math></b>
			<b><math>r = 2.878</math></b>	
Method of prediction	Non-linear regression	Simple linear regression & graphically	Non-linear regression & graphically (very complicated)	<b>Simple linear regression &amp; graphically</b>

# CONCLUSIONS

- The model is a powerful *two-parameter* model.
- Its parameters describing the performance of a classifier can be **mathematically and graphically** obtained with **accuracy comparable** to that presented by the already known models.
- It can be used as an alternative tool or in parallel with the already applied models for the prediction of  $d_{50c}$ ,  $d_{max}$  and  $d_{50}$  (actual separation size).
- $d_{50}$  (28.3  $\mu\text{m}$ ) predicted (from the new model) is **closer** to the experimental one (**>22  $\mu\text{m}$** ), than those predicted from the other models (from 11.83 to 16.7  $\mu\text{m}$ ). Probably this is due to the **superior fitting capability** of the proposed model for the fine size fractions.



## CONCLUSIONS (continued)

- It can also be thought as an advantage of the proposed model that  $E_c$  is predicted to be **1.359** or **135.9%** at infinite particle size, whereas  $E_c = 1$  or **100%** at a finite particle size  $d_{\max}$ , as it actually happens in wet classification.
- The proposed model is in most cases **reliable** and **adequate** for the representation of the classifier efficiency (corrected and afterwards actual).
- It needs further testing for its applicability to other classification tests.
- It proved to be valid for the cases examined here.

*THANK YOU*