The Influence of Deployment Parameters on 2D Wireless Sensor Network by Using a Centroidal Voronoi Tessellation Based Algorithm

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Abstract—The optimum spread of a Wireless Sensor Networks (WSNs) is the main requirement for all the modern applications that use WSNs for the monitoring and the observation of natural environment. The early warning systems e.g. for fire or deformation detection are the main fields that uses this technology as well as the monitoring of other environmental parameters as temperature, humidity, pollution and radiation. The spatial distribution of the sensors of a WSN must follow specific criteria. The equilateral triangle grid leads to the maximum coverage with the minimum number of sensors. Nevertheless, in most large-scale outdoor applications, achieving the ideal deployment geometry is hard or even impossible. The OptEval algorithm, using the Centroidal Voronoi Tessellation (CVT), achieves geometry as near as possible to the ideal one, minimizing the numbers of sensors needed, which subsequently means less cost for the entire network. A range - independent index that takes into account the geometry of the distributed sensors is used for the evaluation of the solution, comparing the random TIN, which the nodes of the network form, with the ideal geometry. This paper investigates the features of the above procedure in relation to the main parameters namely the density of points for supervision, the range of the sensor and the number of used sensors. This will provide an useful tool for the immediate assessment of the probable optimum solution, it can create directives for the operation and the efficiency of the deployment as well as for its cost. So, ninety different scenarios are formed for the same area with randomly distributed observation points, changing the parameters that affect the final result, i.e. the number of the points to be observed, the number of the available sensors and the radius of the sensors. The results follow a pattern which can be taken into consideration for future work. Also has emerged that the most crucial parameter is the range of the sensor which is used. Thus, the users of this method can assess the expected values of coverage percentage and the geometry of the final solution in order to deside for the implementation of the optimum deployment.

Index Terms—Wireless Sensor Network (WSN), Centroidal Voronoi Tessellation (CVT), Delaunay Triangulation, sensor range, sensor deployment, spatial coverage, assessment index.

1 INTRODUCTION

The optimum spread of a Wireless Sensor Networks (WSNs) is the main requirement for all the modern applications that use WSNs for the monitoring and the observation of natural environment. The early warning systems e.g. for fire or deformation detection are of the main fields that uses this technology as well as the monitoring of other environmental parameters as temperature, humidity, pollution and radiation. The optimization of such networks is of crucial significance in terms of both geographical and network coverage.

The simple geographical rule that must be followed for the planning of a WSN is: Maximum coverage with the minimum number of sensors. The ideal geometry attainment requires placing the sensors (nodes) in the equilateral grid positions. Usually, this gets impossible to be achieved. Either because the number of the sensors is extremely big and the deployment in such geometry would raise the network deployment cost or because the application itself determines that the nodes position can only be chosen among a set of predetermined positions.

In applications such as fire-detection projects, some thousands of sensors are needed. The placement of the sensors in equilateral grid would take too long in time. Moreover, the trees which are both the observation points and the possible deployment positions are distributed in random positions inside the area.

The optimum solution for the coverage problem of WSN nodes is the main requested by the scientific community. Computational geometry seems to be ideal for solving the multi-criteria problem of network coverage, as many solutions are based to its algorithms and applications.

A solution to the problem using Centroidal Voronoi Tessellation (CVT) was proposed in [1] and [2]. The OptEval algorithm [1], using the Centroidal Voronoi Tessellation (CVT), results in the nearest possible geometry to the ideal one, minimizing the numbers of sensors needed, which subsequently means lower cost for the entire network. Additionally an appropriate index, that takes into account the geometry of the distributed sensors, is used for the evaluation of the solution. The OptEval algorithm approximates the solution comparing the random TIN that the nodes of the network form, with the ideal geometry. There is the constraint that the nodes must be deployed in cer-
tain positions chosen from a list of possible ones. It was the first time that the coverage problem was approached using this part of computational geometry, although the same method was proposed by Zhou, Jin & Wu [3] for optimizing the network communication problem. An approach that uses Voronoi Diagram (VD) is proposed by Vieira et al [4]. According to this proposal, the potential deployment positions are a priori known. The sensors are considered to be placed at all points, and the corresponding VD is produced. Then, the point with the smallest polygon is removed, as the area is then supervised by the adjacent sensors. The process is repeated until all Voronoi Polygons reach a specified threshold. For a large number of sensors the algorithm becomes extremely time-consuming. Delaunay Triangulation (DT) is the basis for coverage algorithms. Wang & Medidi [5] propose a methodology to minimize energy consumption and achieve complete coverage of the area, but they study the ideal geometry scenario, as well. Vu & Li [6] improve the aforementioned algorithm studying the boundary effect, but they mainly focus on minimizing energy consumption. Another coverage proposal using DT is set by Wu, Lee, & Chung [7]. The idea of the gradual elimination of nodes through the Delaunay Triangulation with constraints (CDT) is proposed by Devaraj [8]. Another study has been conducted by Argany et al [9]. They gather and record different coverage algorithms for WSN. They focus on algorithms based on DT and VD, and propose a solution that uses Voronoi polygons based on spatial information (physical boundaries, DTM etc.).

In most of the studies, only a couple of scenarios are described for each method, so it is difficult to understand and explain the reaction of each algorithm towards the change of the different parameters. Moreover apart from the lack of different scenarios, one can note the lack of an evaluation index to verify how good or how efficient a deployment is. Most of the indexes proposed compare the deployment method with other deployment methods, without providing a universal value for the spatial distribution of the sensors of a WSN [10].

A typical index is proposed by Chizari, Hosseini, & Poston, in their work [11]. They determine the percentage of the area covered by at least one sensor in relation to the whole area and the distances between the sensors. Furthermore, the sensors are separated in those that have large, adequate or small number of other sensors near them. The percentage of the supervised area is also used as an index by Vieira, Vieira, et al. [4]. But in both cases no information is given for the sub-areas that are not covered by any sensor or how well the deployment of the sensors approximates the ideal triangular grid, which results in the optimum area coverage.

Such an index is proposed in [1] and explained in [12]. The “g” index evaluates the geometry achieved in a WSN based on Delaunay Triangulation. The deployment positions are modeled as a Delaunay Triangulation and all scenarios are compared to the ideal equilateral triangle grid. The index “g” takes into account the geometry of all the triangles that are formed from the deployment positions. It is independent of the position or the orientation of each triangle. Furthermore, it allows the comparison of the triangles meshes created in scenarios with different sensing range. The metric is unique for each scenario, so the different scenarios are directly comparable. The proposed assessment methodology is easy to be programmed as it is based on tools and methods of computational geometry.

This paper examines 90 different scenarios, for the same area with randomly distributed observation points, changing the different parameters that affect the final result, such as the number of observation points, the number of the available sensors, the range of the sensors and the number of iterations of the Lloyd’s algorithm. Moreover, presents briefly the CDT theory, as well as the OptEval algorithm which is used for the optimization and the evaluation of the WSN. The results follow a pattern which can be taken into consideration for future work. Thus, the users can easily assess the values of coverage and the achieved geometry in order to decide for the implementation of the optimum deployment.

2 A BRIEF REVIEW OF THE DEPLOYMENT AND EVALUATION METHODOLOGIES

2.1 The deployment method

A CVT is a special Voronoi Diagram, where the generating point of each Voronoi cell is also its mean (i.e., center of mass) [3]. In other words, the generator of each CVT polygon must also be the centroid of each polygon. It approximates an ideal partition of the area, through the optimal allocation of the generators. According to Gersho’s conjecture, “as the number of generators increases, the optimum CVT will form a uniform partitioning of the space, with shapes that would result from the repetition of a single polytope. The shape of the polytope only depends on the spatial dimension”. In 2D the basic polygon is a regular hexagon [13]. Given a region $V \subset \mathbb{R}^N$, and a density function $\rho$, defined in $V$, the mass centroid $z^*$ of $V$ is defined by the equation 1.

$$z^* = \frac{\int_V y\rho(y)d(y)}{\int_V \rho(y)d(y)}$$

(1)

Given k points $z_i, i = 1, \ldots, k$, their associated Voronoi regions, $V_i; i = 1, \ldots, k$, can be defined. Moreover, given the Voronoi polygons, $V_i; i = 1, \ldots, k$ their mass centroids, $z_i^*, i = 1, \ldots, k$ can also be defined. What matters is that the generators of the polygons must be the mass centroids too: $z_i = z_i^*, i = 1, \ldots, k$.

This partition is called Centroidal Voronoi Tessellation (CVT). In the special situation that the density function is constant and uniform, the CVT tends to consists from regular hexagons [2]. In literature several algorithms for CVT construction are recorded, with most common the Lloyd’s algorithm [14]. Other algorithms are the algorithm of MacQueen, the iterative method of Newton, and hybrid approaches of them [14], [15].
In limited field applications where the sensors to be deployed are few, the deployment can be done to the positions arising after the CVT construction. These positions are the center of mass of the polygons and they do not refer to specific points of the original dataset. There are cases that the deployment positions must belong to the original set. Therefore result additional constraints [1], [2].

The solution is approached in two phases:

Given the coordinates of the points to be observed (which are the candidate deployment positions, simultaneously), the convex hull is determined [16], [17]. Then according to the number of sensors, the sensing range and the termination condition for the Lloyd’s algorithm the theoretical positions of the sensors are determined, i.e. the CVT generators.

Finally the actual deployment positions are determined, by finding the nearest neighbors that belong to the original dataset and moving the theoretical points to the closest real position.

2.2 Evaluation method

The fact that the equilateral triangle consists of three edges equal, leads to a specific property, using measures of dispersion from the science of statistics [18]. The standard deviation of the mean of its edges, is equal to zero. The smaller the standard deviation is, the closer the random triangle is to the equilateral one [12].

The use of the mean value $\sigma_0$ of the standard deviations of the edges of each triangle of the TIN, shows how well the triangles that are formed adapt to the regular triangular grid. When the number of the triangles is increased or decreased (e.g. when the number of sensors changes), the index changes and the results are directly comparable. The problem arises when comparing scenarios involving sensors with a different sensing range $R$. Then, in these cases the index to be used will be:

$$g = \frac{\sigma_0}{R}$$  \hspace{1cm} (2)

Thus the estimation is independent of the sensor’s range. The smaller values the index “$g$” gets, the better solution is achieved.

For the final positions of the sensors, the corresponding Delaunay Triangulation is constructed [19], choosing the appropriate algorithm [20], [21] and the produced TIN is compared with the equilateral triangle grid (Figure 1).

3 INVESTIGATION OF THE DEPLOYMENT PARAMETERS

The main goal is to investigate the features of the above procedure in relation to the main parameters namely the density of points for supervision, the range of the sensor and the number of used sensors. This provide an useful tool for the immediate assessment of the probable optimum solution and to create directives for the operation and efficiency of the deployment and the evaluation algorithms.

In order to detect the influence of the above mentioned parameters different scenarios were created and tested. These scenarios include different number of available points for supervision, different number of sensors and various sensing ranges $R$, in order to find if the solutions follow specific patterns and to compare them. The dataset that is used consists of randomly generated points. In a real-life scenario, the coordinates or the positions of the points would have been obtained from measurements.

An area of 500m x 500m was chosen and three basic scenarios were created concerning three different types of observation point densities: low, medium and high. For each case 3 different range sensors (20m, 30m and 40m) were used. Taking into consideration the sensor range, the minimum number of required sensor comes out as the ratio of the examined area (for this case 250,000m$^2$) to the area that every sensor covers. For the range of 20m the denominator is equal to 1256m$^2$ namely min 200 sensors, for the range of 30m is equal to 2827m$^2$ namely min 90 sensors and for the range of 40m is equal to 5026m$^2$ namely min 50 sensors. Finally, for each one of the different cases, the coverage of the area was examined for a different number of sensors (10 different cases). Starting from the minimum number of required sensors and gradually increasing their number, up to about 50% more. The 90 different scenarios are presented in Figure 2.

![Fig. 1. Comparison of the vertexes of a resulted TIN vs equilateral triangle grid [12]](image1)

![Fig. 2. The different examined scenarios](image2)
Fig. 3. Deployment for 2000 points (a, c) & 5000 points (b, d), for R=20m and N=270

Fig. 4. Coverage percentage achieved in relation to different density areas the number of sensors used for different R

For each case, the number of points in the area which are not observed by any sensor namely the coverage percentage of the
In all cases examined, even if the minimum number of sensors is used, coverage percentage of 86% - 90% is achieved. On the other hand, no full coverage is achieved even if 50% more sensors than the minimum are used. The maximum coverage achieved is about 99% which is assessed as satisfying. Most unsupervised points are concentrated to the outer boundary of the area. In any case, if a small subarea remains unsupervised due to its geometry can be treated autonomously and the sensors can be deployed manually. This depends on the significance of the application and the cost of its failure as well as the cost of the additional number of sensors in the total cost of the network.

An important outcome is about the influence of the increment of the number of sensors to the coverage percentage. The improvement of the coverage is slightly in relation to the number of sensors for the low range sensors. There is a “crucial” number of sensors about 30% up of the minimum number where above this no significant increment occurs in the area coverage percentage. Thus, the cost of the network deployment increases without improving the desired solution significantly. On the contrary as longer is the range of the sensor every increment in their number leads to significant improvement in the area coverage.

Moreover, the most effective sensor in terms of the coverage percentage is the one with the larger radius R. For the same number of sensors, the coverage percentage in the different densities (low, medium and high) is less varied than the other two types of sensors. Therefore, it could be used in areas with unequal point density, without having to separate it into individual sections, for which a separate case study should be done.

In order to achieve the same coverage percentage for any density area, the number of 30m range sensors which are needed is double than the 40m range sensors as the necessary number of 20m range sensors is five-fold. This is a valuable conclusion related to the cost of the different range sensors and for the total cost of the network implementation.

Figure 5 shows the change of the g index for each scenario. Taking a closer look to the figure 5, some ascertainments are derived.

The g index values fluctuate from 0.22 to 0.47 for all the scenarios. The higher range sensor achieves the best g indexes in all cases. Also the fluctuation of g for the 40m range sensor is about 0.1 as the fluctuation is 0.2 for the 20m and 30m sensors. That means that the longer the sensor range the more reliable
and stable results are achieved. Also the number of 30m range sensors which is needed is double than the 40m range sensors and the number of 20m range sensors is six-fold than the 40m range sensors in order to succeed the same quality deployment.

Moreover when the number of sensors increase, the g index decreases. Therefore the better adjustment is achieved. Also as the density of points in the area increases, the g index decreases.

Both conclusions are expected as a big number of sensors implies a more even distribution of these in the space and a higher density of supervised points, implies by definition that during the construction of the CVT, polygons emerging to regular hexagons. In addition, when moving the theoretical position to the nearest real one, the displacement will be smaller in areas with higher point density. Thus, the final triangle TIN in the actual positions will differ slightly from the ideal grid that would create CVT centroids.

4 Conclusions

Wireless Sensor Networks are increasingly used to support a wide variety of applications, such as environmental or structural monitoring. In all cases, maximum geometrical coverage with the minimum number of sensors is required. OptEval algorithm offers a geometrical solution to the problem, based on the properties of Centroidal Voronoi Tessellation. This ensures that the solution given is the best for the given geometry (points’ distribution, number of sensors, sensing range). Each sensor is placed as far away as possible from its neighbors.

The g index is a range-independent index which takes into account the geometry of triangles that are formed from the deployment positions. The metric is unique for each scenario, so different scenarios are directly comparable.

In order to propose some standard directives for such kind of sensors’ deployment, a total of 90 different scenarios have been carried out for an area with specific dimensions but different points’ density, number of sensors and sensor radii.

The analysis of the results shows that the increment of the number of the sensors results in a reduction of the g index. This implies that the random triangle mesh is adjusted better to the equatorial triangle grid. Also the higher density of the points in the area, results in the declination of the index g. On the other hand, the deployment of the sensors in areas with low spatial density, achieves high coverage percentage, but not necessarily an ideal geometry. The high density area is more difficult to be totally covered.

The investigation also concludes that the main parameter is the range of the sensor. The longer the range is, the best results can be achieved. For double sensor range about the one fifth of sensors are needed in order to achieve the same results both in the area coverage and the g index. This helps a lot in the a-priori cost determination of the network implementation, as it is the most crucial parameter.

Also for long range sensor, the coverage percentage in the different densities (low, medium and high) is less varied for the same number of sensors. Therefore; it could be used in areas with unequal point density, without having to separate it into individual sections, for which a separate case study should be done.

Consequently, according to the results the users of this method can easily assess the expected values of coverage percentage and the geometry of the final solution and to deside about the final network implementation taking in to consideration the total cost in relation mainly to the range of the sensors that will be used. For future work it will be very useful and interesting concave polygons and polygons with holes or buffer zones to be examined as these area cases are often appeared in a real-life scenario.

References