Check and calibration of a single GNSS receiver by using the VRS RTN positioning method

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ABSTRACT

The constant evolution of the demands of geodetic applications requires instruments, products and services with certified specifications and measurement accuracy. Particularly Global Navigation Satellite Systems (GNSS) are now closely approaching the same uncertainty as measurements provided by total stations, in current geodetic applications. In addition, the small occupation time required to measure points, leads to the increased use of them.

The technological advancement of GNSS receivers and the installation of many networks of permanent and continuously operated reference stations in the world have now resulted in the use of a single GNSS receiver to determine coordinates and geometric features worldwide.

This paper proposes a methodology for both, the check and calibration of a single GNSS receiver, using the VRS method, which is subset of the network RTK positioning methods (from now on mentioned as RTN methods).

In both procedures the appropriate statistical checks were carried out in order to conclude about the proper function of the receiver under check.

The methodology succeeds the results by using efficient number and type of observations and simple mathematical models. So it is convenient to be used by professionals in order to improve and ensure their products. Moreover it may consists as a new supplement of the ISO, as the existing ISO 17,123-8 (ISO 17123-8, 2007) requires two GNSS receivers in order to be implemented by using the real time kinematic method. It is worth mentioned that ISO 17,123-8 informs the user only for the internal accuracy, namely the precision of the base receiver which is under check.

Therefore the proposed methodology concludes not only about the precision of a single GNSS receiver being checked, but also about the provided accuracy due to the comparison with the true values.

1. Introduction

Nowadays the majority of GNSS measurement systems include a reference station, which is placed fixed at a particular and proper site, and a single rover GNSS receiver operated by the user. Continuously Operating Reference Stations (CORS) can take the place of a traditional base station used in differential GNSS positioning [9]. They can give an instant position to an accuracy of ± 20 mm [7].

The optimum maximum distance between a base GNSS station and a rover GNSS receiver should be about 10–15 km. This is due to the effect of the atmosphere on the GNSS signals as they travel from the satellite towards the earth. With the establishment of a network of reference stations, the distance between the base and the rover can be extended. The permanent stations can be spaced about 70 km apart, and by using at least 3 of them, the atmospheric effects can be modeled and corrected [20].

The advantages of using a permanent reference station network worldwide refer to the elimination of base station issues, the elimination of radio issues, the use of common coordinate system by all and the increment of the working range. So the distance dependent errors are greatly reduced and a larger area is covered with few reference stations. Also users need only one RTK enabled GNSS receiver in order to use the efficient, accurate, reliable and economical positioning methods, like VRS, MAC and Single base RTK [6,27,15,11].

Moreover the PPP-RTK methods enable single receiver users to compute their positions, with decimeter or centimeter accuracy, by using precise satellite orbits and clocks [23]. PPP-RTK methods require information about the satellite phase biases which is used in order to recover the integerness of the ambiguities, thus enabling single receiver ambiguity resolution, thereby reducing the convergence times as compared to that of the method PPP [28].
Only limitation of the use of continuously operating reference stations is the mobile coverage, in order to succeed the transmission of the corrections between the control center and the user [22].

Several countries have developed their own reference stations networks including Australia (CORSNet –NSW) [12], USA (NGS CORS) [29], Canada (CACS and Network RTK networks) [5], Dubai (DVRS) [1] and Sweden (SWEPOS) [13,17]. Each of these networks consists of an appropriate amount of stations. Many of these stations are participating in the International GNSS Service (IGS) network [2,4].

New state-of-the-art positioning techniques have been developed, using these permanent networks, such as the RTN method [6,24,25], which relates to real-time determination using a single receiver and one or more permanent reference stations. These stations may be real (Single Base) or virtual (VRS), where the station is automatically placed at the optimum point by the network processing center [11].

The Virtual Reference Station concept is a method used to provide RTN correction data. It relies on the technique of creating GNSS reference station data for a virtual reference station. In this method, a network of reference stations is continually linked to a control center used to calculate regional area corrections across the network. A virtual reference station is then created from this information to be a few meters away from the initial position of the rover receiver. The rover interprets this data in the same manner as if it were from a single and real reference station [19].

The only limitation of the VRS method refer to the need of at least 3 reference stations at a close range around it (approximately 25 km), in order for the virtual station to be created. As the virtual station is created near the field of measurements, the issues of the atmospheric conditions which cause errors are eliminated.

In order to succeed sub-centimeter accuracy [26], it is necessary that the measurement system (including a single receiver) to be calibrated and functioning properly, as the manufacturer defines. Therefore a full procedure that ensures the proper function of a GNSS receiver is indispensable.

The proposed methodology consists of two procedures, the check and the calibration procedure. Each of them can be applied independently. The novelty of the method refers to the fact that it can conclude about the proper function of a single GNSS receiver, through the implementation of either the check or the calibration procedure. On the contrary as the proposed by the ISO methodology requires two GNSS receivers in order to obtain the measurements and inform the user about the precision but not about the accuracy of the provided coordinates.

2. Check procedure

The check procedure follows simple calculations and reveals the suitability of GNSS receivers for a geodetic application by determining the internal accuracy, namely the precision of the calculated coordinates via the VRS RTN positioning method. For the implementation of the check procedure the next steps should be followed.

Initially the connection to a permanent GNSS network and then the creation of a VRS station near the site of the receiver under check is necessary. The observations are developed in three series, each consisting of 5 sets. Each set of measurements includes the unique determination of the three-dimensional coordinates at an arbitrary point where the receiver is set. Moreover it should be taken into account the change in the geometry of the satellite formation in order to minimize the value of DOP [16]. Therefore, the observations may be carried out in all time periods of a day, where the different measurement series ought to be separated for at least 90 min.

So, a total of 15 measurements are implemented to complete the check procedure. The mathematical analysis is performed with 15 triplets of coordinate 15 x, 15 y and 15H.

The mean value of the 15 measurements for x, y and H is calculated as well as the standard deviation of the single measurement \( \hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_H \) according to the low of the propagation of errors (Eq. (1)).

The receiver passes the check procedure as long as the Eq. (2) are simultaneously valid.

\[
\hat{\sigma}_x = \pm \sqrt{\frac{\sigma_{\text{HORIZONTAL MAN}}^2 + \sigma_{\text{VERTICAL MAN}}^2}{n-1}} \\
\hat{\sigma}_y = \pm \sqrt{\frac{\sigma_{\text{HORIZONTAL MAN}}^2 + \sigma_{\text{VERTICAL MAN}}^2}{n-1}} \\
\hat{\sigma}_H = \pm \sqrt{\frac{\sigma_{\text{HORIZONTAL MAN}}^2 + \sigma_{\text{VERTICAL MAN}}^2}{n-1}}
\] (1)

\[
\hat{\sigma}_{x,v} \leq \sigma_{\text{HORIZONTAL MAN}} \text{HDOP,} \hat{\sigma}_{H} \leq \sigma_{\text{VERTICAL MAN}} \text{VDOP}
\] (2)

Also n equals to 15, HDOP, VDOP is the mean value of the corresponding indexes of the three measurements series, \( \sigma_{\text{HORIZONTAL MAN}} \) and \( \sigma_{\text{VERTICAL MAN}} \) are the given accuracies by the receiver’s specifications, as the manufacturer defines.

The simultaneous validation of the Eq. (2) ensures the precise measurements of the under check GNSS receiver in both the horizontal and the vertical direction.

3. Calibration procedure

The calibration of a GNSS receiver may be implemented by two different calculation methods, the simplified and the advanced. Each one can be applied independently and they lead to complementary information.

The simplified method needs simple calculations as the advanced requires a least square adjustment for a linear adaptation. Both procedures use the same initial observation data and they both conclude about the proper function of the GNSS receiver by the comparison to “true” values \( (x_R, y_R, H_R) \).

The elements involved in the calibration procedure are the reference and observed values \( (x_{\text{OBS}}, y_{\text{OBS}}, H_{\text{OBS}}) \) of the coordinates obtained for one pillar. The receiver under calibration is put on a pillar where the reference coordinates have been calculated. The connection to a permanent GNSS network and then the creation of a VRS station near the site is necessary.

The observations are implemented, as previously, in three series each consisting of 5 sets, where the different measurement series ought to be separated for at least 90 min. Each set of measurements includes the unique determination of the three-dimensional coordinates of the pillar. The mathematical analysis is performed with 15 triplets of coordinate values (15\( x_{\text{OBS}}, y_{\text{OBS}}, H_{\text{OBS}} \)), a total of 45 equations.

3.1. The simplified mathematical model

The simplified calibration procedure, calculates the difference between the reference \( (x_R, y_R, H_R) \) and the observed coordinates \( (x_{\text{OBS}}, y_{\text{OBS}}, H_{\text{OBS}}) \), (Eq. (3)).

\[
\Delta x_i = x_{\text{OBS}} - x_R \\
\Delta y_i = y_{\text{OBS}} - y_R \\
\Delta H_i = H_{\text{OBS}} - H_R
\]

\[i=1 \text{ to } 15\] (3)

The total expected error \( \sigma_{x,i}, \sigma_{y,i}, \sigma_{H,i} \) according to the low of the propagation error is:

\[
\sigma_{x,i} = \pm \sqrt{\sigma_{x_{\text{OBS}}}^2 + \sigma_{x_R}^2} \\
\sigma_{y,i} = \pm \sqrt{\sigma_{y_{\text{OBS}}}^2 + \sigma_{y_R}^2} \\
\sigma_{H,i} = \pm \sqrt{\sigma_{H_{\text{OBS}}}^2 + \sigma_{H_R}^2}
\] (4)

\[
\sigma_{x_{\text{OBS}}} = \sigma_{x_{\text{OBS}}} = \pm \sigma_{\text{HORIZONTAL MAN}} \text{HDOP}
\]

\[
\sigma_{y_{\text{OBS}}} = \sigma_{y_{\text{OBS}}} = \pm \sigma_{\text{VERTICAL MAN}} \text{VDOP}
\]

The observations’ expected error is the error according to the manufacturer charged with the corresponding mean DOP values during...
the measurements. The $\sigma_{xy}, \sigma_{yH}, \sigma_{h}$ is the achieved errors of the reference values. It is obvious that $\sigma_{xy}, \sigma_{yH}, \sigma_{h}$ as manufacturer provides the same accuracy for both the horizontal coordinates.

Thus the first check is that the max value of $\Delta x$, $\Delta y$, and $\Delta H$ should be simultaneously smaller than the expected errors $\sigma_{xy}, \sigma_{yH}, \sigma_{h}$.

$$\Delta x_{max} \leq \sigma_{xy}, \Delta y_{max} \leq \sigma_{yH}, \Delta H_{max} \leq \sigma_{h} \tag{5}$$

Also the conditions described in Eq. (6) should remain valid as regards the error of the single value of $\Delta x$, $\Delta y$, $\Delta H$, namely $\Delta x_{0}, \Delta y_{0}, \Delta H_{0}$.

The receiver passes the calibration procedure as long as the Eq. (6) remains also valid.

$$\Delta x = \pm \sqrt{\frac{\sum_{i=1}^{n} \Delta x_{i}^{2}}{n-1}} \leq \sigma_{xy}, \Delta y = \pm \sqrt{\frac{\sum_{i=1}^{n} \Delta y_{i}^{2}}{n-1}} \leq \sigma_{yH}, \Delta H = \pm \sqrt{\frac{\sum_{i=1}^{n} \Delta H_{i}^{2}}{n-1}} \leq \sigma_{h} \tag{6}$$

where $\Delta x_{0}, \Delta y_{0}, \Delta H_{0}$ are the achieved errors of the receiver under calibration, namely $\Delta x_{0}, \Delta y_{0}, \Delta H_{0}$.

The advanced mathematical model

The advanced calibration model is based on the Eq. (7). It is implemented separately for each coordinate, by a least squares adjustment in order to compare the measured values $[\text{meas}]$ to the reference values $[\text{ref}]$ for each coordinate.

$$\text{ref} = a \cdot \text{meas} + b \tag{7}$$

Thus 15 equations like the Eq. (7) are formed for each coordinate. A least squares adjustment is carried out considering all the observations equally weighted [8]. The goal is to calculate the values of the unknown systematic $a$ and random error $b$.

Since the calculated parameters are statistically significant, the accuracy of the receiver is concluded by the systematic $\sigma_{a}$ and random error $\sigma_{b}$ of the adjustment refers to the random error [18].

So, $\sigma_{\text{calibration}}$ is calculated as the total error which is given in Eq. (8).

$$\sigma_{\text{calibration}(X)} = \pm \sqrt{\sigma_{\text{calibration}(xy)}^{2} + b_{x}^{2}}$$

$$\sigma_{\text{calibration}(Y)} = \pm \sqrt{\sigma_{\text{calibration}(y)}^{2} + b_{y}^{2}}$$

$$\sigma_{\text{calibration}(H)} = \pm \sqrt{\sigma_{\text{calibration}(yH)}^{2} + b_{h}^{2}} \tag{8}$$

Therefore $\sigma_{\text{calibration}}$ for each one of the coordinates should be less than the required uncertainty of the geodetic applications $\sigma_{\text{required}}$ where the receiver under calibration will be used, as described in Eq. (9).

$$\sigma_{\text{calibration}(X)} \leq \sigma_{\text{calibration}(Y)} \leq \sigma_{\text{calibration}(H)} \leq \sigma_{\text{required}} \tag{9}$$

Also the conditions described in Eq. (10) should remain valid as regards the random errors of each one of the coordinates.

$$\sigma_{a} \leq \sigma_{\text{calibration}(X)}, \sigma_{a} \leq \sigma_{\text{calibration}(Y)}, \sigma_{a} \leq \sigma_{\text{calibration}(H)} \tag{10}$$

It is obvious that the advanced calibration model requires a little more complicated calculations. However it is advised to be used mainly when the simplified model concludes into not successful results. Therefore the advanced model can detect the exact problem of the under calibration receiver, namely the values of the systematic or random error.

3.3. The reference values acquisition

The reference “true” values of the coordinates $x$, $y$ and $H$ of a pillar are indispensable for the implementation of the calibration procedure. These true values should be calculated by a separate procedure by using a certified GNSS receiver. The reference values could be determined by using the same positioning method, (i.e. VRS RTK).

Throughout the process of determining the reference values, the rules of the repeatability and proper conduct of measurements, should be followed at regular intervals. As it is stated above, in order to take into account the change in the geometry of the satellite formation and to minimize the value of DOP the observations must be carried out in different measurement series, which ought to be separated for at least 90 min [16, 21].

In order to approximate the optimal number of observations is decided to use the precision for coordinates’ determination according to the receivers’ manufacturer.

The determination of the reference values should be at least one order more precise [3] than the observations. So the number of observations (c) is chosen, in order to fulfill the requirements according to the nominal accuracy of the receiver under calibration.

Considering that the measurements are equally weighted, the number of observations (c) will improve the uncertainty as an optimization $\sigma_{\text{a-priori}}$ can be calculated as described in Eq. (11).

$$\sigma_{\text{a-priori}} = \frac{\sigma_{\text{MANUFACTURER}}}{\sqrt{c}} \tag{11}$$

Fig. 1 presents the a priori errors of the reference values determination in relation to the number (c) of independent observations. In this example the uncertainty $\sigma_{\text{MANUFACTURER}}$ is assumed to be $\pm 5$ mm, $\pm 10$ mm, $\pm 15$ mm and $\pm 20$ mm. Fig. 1 shows that at least 60 observations are required to determine reliable reference values for all the cases. Further increase of the number of observations does not result in considerable improvement of the optimization $\sigma_{\text{a-priori}}$.

So for the acquisition of reference values it is recommended to measure with a certified GNSS receiver, at least 3 phases of measurements where every one of them consists of (c = 20) determinations of the pillar’s coordinates. Each determination should last from 30 s to 1 min (occupation time). It is proved that when time exceeds the one minute, does not contribute more to reducing the uncertainty of the coordinates determination [14].

The mean values $\bar{x}, \bar{y}, \bar{z}$ of the reference coordinates as well as their a posteriori error $\sigma_{x}, \sigma_{y}, \sigma_{z}$ is given by Eqs. (12) and (13) correspondingly.

$$\bar{x} = \frac{\sum_{i=1}^{c} x_{i}}{3c}, \bar{y} = \frac{\sum_{i=1}^{c} y_{i}}{3c}, \bar{z} = \frac{\sum_{i=1}^{c} z_{i}}{3c} \tag{12}$$

$$\sigma_{x} = \sigma = \pm \sqrt{\frac{\sum_{i=1}^{c} (x_{i}-\bar{x})^{2}}{3c(3c-1)}}, \sigma_{y} = \pm \sqrt{\frac{\sum_{i=1}^{c} (y_{i}-\bar{y})^{2}}{3c(3c-1)}}, \sigma_{z} = \pm \sqrt{\frac{\sum_{i=1}^{c} (z_{i}-\bar{z})^{2}}{3c(3c-1)}} \tag{13}$$

Fig. 1. Determination of the required measurements for the reference values.
If the pillar and the continuously operated permanent station’s network which is used for the calibration procedure remain the same for several implementations, it is recommended the determination of the reference values at regular time intervals. The reputation of the above procedure in concrete time intervals will lead to further elimination of the standard deviation of the reference values so as to be considered as “true” coordinates.

4. Experimental application

An application was carried out for both the check and calibration procedures at a pillar where the under check or calibration receiver is set by using forced centering facilities. For both procedures the measurement system consists of the single GNSS receiver and the virtual reference station which is created at a nearby location. The virtual station is created by utilising the Hellenic Positioning System Permanent Reference Stations network (HEPOS). It is mentioned that the nearest physical reference station (098A) of this network to the control base, is positioned at approximately 4.5 km.

As regards the check and calibration procedures, one GNSS receiver, Trimble R8s is tested. The nominal horizontal ($\sigma_x = \sigma_y = \pm 6$ mm) and vertical accuracy ($\sigma_z = \pm 15$ mm $\pm 0.5$ ppm) for VRS RTN positioning method are defined by the manufacturer.

The coordinates and their determination errors are referred to the Hellenic projection system (Greek Grid - EGS87).

The resulted data used for both procedures are shown in Table 1. The measurements consist of the determination of the 15 triplets of coordinates for the pillar.

Also the mean values of the DOP indexes, which occurred at each one of the implemented series, during of measurements are included in Table 1.

The overall time for the measurements was approximately 4 h. Each of the three different measurement series last about 15 min and 90 min is the time internal between them. The DOP indexes fluctuate from 1.2 to 2.4, so they are accessed satisfying.

For the check procedure the standard errors of coordinates are calculated, therefore the Eq. (2) can be applied, where:

$$\hat{\Delta}x = \pm \sqrt{\hat{\sigma}_{x}^2 + \hat{\sigma}_{y}^2} = \pm 6.1 \text{mm}$$
$$\hat{\Delta}y \leq \sigma_{\text{Horizontal, MAX}} \cdot \text{HDOP} \Rightarrow \hat{\sigma}_{x} \leq 6 \text{mm} \leq \pm 8.1 \text{mm} = \pm 13 \text{mm}$$
$$\hat{\Delta}z \leq \sigma_{\text{Vertical, MAX}} \cdot \text{VDOP} \Rightarrow \hat{\sigma}_{y} \leq 8 \text{mm} \leq \pm 15.8 \text{mm} = \pm 27 \text{mm}$$

All individual uncertainties satisfy the check conditions as described in Eq. (2). So the receiver under check can perform observations according to the uncertainties specified by the above Eqs. (14) and (15).

Table 1
The 15 triplets of x, y and H used for the check and calibration procedures.

<table>
<thead>
<tr>
<th>Number</th>
<th>Series</th>
<th>Set</th>
<th>x (m)</th>
<th>y (m)</th>
<th>H (m)</th>
<th>Mean DOP</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1st</td>
<td>1</td>
<td>480357.526</td>
<td>4202800.597</td>
<td>207.999</td>
<td>PDOP = 2.1</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>2</td>
<td>480357.526</td>
<td>4202800.597</td>
<td>207.997</td>
<td></td>
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<td>4202800.597</td>
<td>207.997</td>
<td></td>
</tr>
<tr>
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<td></td>
<td>4</td>
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<td>4202800.597</td>
<td>207.987</td>
<td></td>
</tr>
<tr>
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<td></td>
<td>5</td>
<td>480357.526</td>
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<td>VDOP = 1.9</td>
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<td>6</td>
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<td>PDOP = 2.4</td>
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<td>208.007</td>
<td></td>
</tr>
<tr>
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<td></td>
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<td>4202800.593</td>
<td>208.006</td>
<td></td>
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<td>4202800.592</td>
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<td>VDOP = 1.9</td>
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<tr>
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<td></td>
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<td>4202800.591</td>
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<td>VDOP = 1.6</td>
</tr>
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</table>

Table 2
Reference values and their determination errors.

<table>
<thead>
<tr>
<th>x (m)</th>
<th>$\sigma_x$ (mm)</th>
<th>y (m)</th>
<th>$\sigma_y$ (mm)</th>
<th>H (m)</th>
<th>$\sigma_H$ (mm)</th>
<th>Mean DOP</th>
</tr>
</thead>
<tbody>
<tr>
<td>480537.253</td>
<td>± 0.5</td>
<td>4202800.594</td>
<td>± 0.5</td>
<td>207.998</td>
<td>± 1.4</td>
<td>2.4</td>
</tr>
</tbody>
</table>

For the calibration procedure the reference values ought to be provided. Therefore the reference values of the pillar were calculated from the measurements provided by a certified GNSS receiver, by using the VRS RTN method. These measurements were formed by three different phases where a time delay of 90 min is mandatory, in order to include the change of the geometry of the satellites into the research. So a total of 60 individual measurements were implemented for the acquisition of the reference values for the used pillar. In Table 2 the reference coordinates are shown, according to Eqs. (12) and (13). Also the mean value of the PDOP indexes is calculated from the three phases of measurements.

As regards the simplified mathematical model the differences between the reference and the observed coordinates are calculated by using the Eq. (3). The differences fluctuate horizontally from ±1 mm to ±7 mm as vertically from ±2 mm to ±15 mm. As the total expected errors are calculated the Eq. (5) can be applied.

$$\Delta x_{\text{max}} \leq \sigma_x \Rightarrow 7 \text{mm} \leq \pm 13 \text{mm}$$
$$\Delta y_{\text{max}} \leq \sigma_y \Rightarrow 7 \text{mm} \leq \pm 13 \text{mm}$$
$$\Delta z_{\text{max}} \leq \sigma_z \Rightarrow 15 \text{mm} \leq \pm 27 \text{mm}$$

Also the receiver passes the calibration procedure if the Eq. (6) are valid, namely:

$$\hat{\sigma}_x \leq \sigma_x \Rightarrow \pm 8 \text{mm} \leq \pm 13 \text{mm}$$
$$\hat{\sigma}_y \leq \sigma_y \Rightarrow \pm 7 \text{mm} \leq \pm 13 \text{mm}$$
$$\hat{\sigma}_z \leq \sigma_z \Rightarrow \pm 12 \text{mm} \leq \pm 27 \text{mm}$$

As regards the advanced mathematical model, a total of 15 observation equations were formed according to Eq. (7), and the matrices referring to the least-square adjustment were calculated. The freedom degree of the adjustment was 13. The results are checked for their statistical significance. Then the systematic (b) and the random ($\sigma_0$) errors come out as well as the adaptation equation for each one of the coordinates:

$$\text{ref}(x) = 1.0001 \text{-} \text{meas} \cdot 2.2 \text{(mm)}$$
$$\text{ref}(y) = 0.9997 \text{-} \text{meas} \cdot 3.1 \text{(mm)}$$
$$\text{ref}(z) = 1.0004 \text{-} \text{meas} \cdot 3.9 \text{(mm)}$$

Also the relations formed by the Eq. (10) are valid.

All individual uncertainties satisfy the test conditions as described in Eqs (8) and (10). So the receiver under calibration can perform observations without inserting statistical significant errors.

5. Discussions

Both the check and calibration procedures can be used in order to conclude about the proper operation or suitability for geodetic applications, of a single GNSS receiver, using the VRS positioning method. Today the VRS method is the well-spread method for any application.
Particular features of the methodology are:

- The utilization of a continuously operating permanent reference stations network.
- The small number of measurements needed to deliver reliable results.
- The involvement of the DOP precision indicators in the statistical analysis.
- The consideration of the role of repeatability of measurements at specific time intervals.
- Depending on the user’s needs, the check or calibration process can be selected.
- The ability of the independent implementation of either procedure, since none of them is a subset of the other.
- The comprehensive conclusion on the operation of a single GNSS receiver.

The time series of the true values can also be used for initial tests of the quality of the measurements made by the GNSS receiver which is under calibration. Their preliminary comparison with the coordinates obtained by each measurement series individually can provide a first check of possible gross errors. It is estimated that the total time for the measurements acquisition is 4 h, where the two intervals of 90 min between the 3 different measurement series, are included.

The above proposed procedures can also be implemented by using the Single Base RTK method, if there is any actual reference station at a close distance (1–5 km) from the site. Moreover, it is recommended that the distance between the permanent or VRS station and the check point (pillar), to be similar as the baselines that the receiver will measure at the geodetic applications. It is also suggested to apply the proposed methodology exclusively with the VRS technique, in cases where the nearest permanent station is located at more than 25 km [24,25].

The only limitation for the implementation of the methodology with the VRS method refers to the need of a permanent network, with at least 3 stations around the site at a range of 35 km.

6. Conclusions

The proposed methodology meets the requirements of any user and concludes for the proper operation of a single GNSS receiver, by using efficient number and type of observations and simple mathematical models. So it is convenient to be used by professionals in order to improve and ensure their products. The VRS RTK positioning method is widely used in current geodetic applications due to the ease of provided measurements, as long as at least three permanent stations are available at a close range from the measurement’s site.

The proposed methodology concludes not only about the precision of GNSS receiver being checked but also about the accuracy that it provides by the comparison to the true values. The result of the simplified calibration process of a receiver is to draw a conclusion on the measurement uncertainty that can be achieved with this measurement system (a single GNSS receiver and virtual reference station VRS). Also, through the advanced process, the results are concerned the system scale and the systematic and random errors that may be provided by the receiver.

By combining both procedures included in the overall methodology, it is possible to determine the internal accuracy (check procedure) and the external accuracy (calibration procedure) of the receiver.

The proposed check and calibration could constitute a supplement to the ISO 17,123-8 [10] regarding the proper function of a single GNSS receiver.

References


