Comparing morphological levelings constrained by different markers

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Abstract Morphological levelings are powerful operators and possess a number of desired properties for the construction of nonlinear scale space image representations. In this paper, a comparison between levelings constrained by different multiscale markers — namely, reconstruction openings, alternate sequential, isotropic and anisotropic diffusion filters — was performed. For such a comparison a relation between the scales of each marker was established. The evaluation of the simplified images was performed by both qualitative and quantitative measures. Results indicate the characteristics of each scale space representation.

Keywords: filtering, simplification, scale space representations, evaluation.

1. Introduction

Since objects in images belong, in the general case, not in a fixed but in many scales, the use of scale space image representations is of fundamental importance for a number of image analysis and computer vision tasks. The concept of Gaussian scale space goes back to the sixties and was first introduced by Iijima [18, 19]. In the western literature and following the ideas of Witkin [20], Koenderink [3] and Lindeberg [4] many possible ways to derive the Gaussian scale space were introduced and respectively many linear multi-scale operators were developed, all of which, though, present the same important drawback: image edges are blurred and new non-semantic objects may appear at coarse scales [11, 12, 20].

Nonlinear operators and nonlinear scale spaces have been studied and following the pioneering work of Perona and Malik [13] there has been a flurry of activity in partial differential equation and anisotropic diffusion filtering techniques [17]. Such approaches either based on diffusions during which the average luminance value is preserved, either based on geometry-driven diffusions, reduce the problems of isotropic filtering but do not eliminate them completely: spurious extrema may still appear [10].

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Advanced scale space morphological representations, the levelings, which have been introduced by Meyer [8] and further studied by Matheron [7] and Serra [14], overcome this drawback and possess a number of desired properties for the construction of such representations. Levelings, which are a general class of self-dual morphological filters, are powerful, do not displace contours through scales and are highly dominated by the structure of the markers used for their construction [5, 6, 8–10].

In this paper, a comparison of different operators — namely, the reconstruction openings, the alternate sequential, isotropic and anisotropic diffusion filters — was performed aiming to study the resulting simplified images and describe the characteristics of the each scale space representation. The comparison was based on both qualitative and quantitative evaluation. The later was focused on measurements about the extent of intensity variation and the structural similarity between the reference image and the leveling.

2. Morphological levelings

Following the definitions from [9], one can consider as f_x and f_y the values of a function f at the two pixels x and y and then define the relations: $f_y < f_x$ (f_y is lower than f_x), $f_y \geq f_x$ (f_y is greater or equal than f_x) and $f_y \equiv f_x$ (the similarity between f_x and f_y, which are at level). Based on these relations the zones in an image without inside contours (isophotes, i.e., contour lines with constant brightness values) will be called smooth flat zones.

Two pixels x, y are smoothly linked ($f_x \diamond f_y$) and may belong to the same R-flat zone of a function f if and only if there exists a series of pixels $\{x_0 = x, x_1, x_2, ..., x_n = y\}$ such that they satisfy a symmetrical relation $f_{x_i} \equiv f_{x_{i+1}}$. For equality $f_{x_i} = f_{x_{i+1}}$ the quasi-flat zones are flat. For the symmetrical relation between two neighboring pixels p and q, $f_p \approx f_q$ with $|f_p - f_q| \leq \lambda$, the quasi-flat zones are defined within a maximum difference (slope). Thus, a set X is a smooth zone of an image f if and only if the two values f_x and f_y are smoothly linked ($f_x \diamond f_y$) for any two pixels x and y in X .

Being able to compare the values of 'neighbouring pixels', the more general and powerful class of morphological filters the levelling can be defined. In general they are a particular class of images with fewer contours than a given image f. A function g is a leveling of a function f if and only if

$$f \wedge \delta g \leq g \leq f \vee \varepsilon g,$$

where δ is an extensive operator ($\delta g \geq g$) and ε an anti-extensive one ($\varepsilon g \leq g$).

For the construction of levelings a class $Inter(g, f)$ of functions h is defined, which separates function g and the reference function f. For the function h we have that $h \in Inter(g, f)$ and so $g \wedge f \leq h \leq g \vee f$. Algorithmically and with the use of h, one can 'interpreter' above equation and construct levelings with the following pseudo-code: in cases where $\{h < f\}$, replace the values of h with $f \wedge \delta h$ and in cases where $\{h > f\}$, replace the values of h with $f \vee \varepsilon h$. Equally and in a unique parallel step we have that

$$h = (f \wedge \delta h) \vee \varepsilon h.$$

The algorithm is repeated until the above equation has been satisfied everywhere. This convergence is certain, since the replacements on the values of h are pointwise monotonic. Hence, levelings can be considered as transformations $\Lambda(f, h)$ where a marker h is transformed to a function g, which is a leveling of the reference signal f. Where $\{h < f\}$, h is increased as little as possible until a flat zone is created or function g reach the reference function f and where $\{h > f\}$, h is decreased as little as possible until a flat zone is created or function g reach the reference function f. This makes function g be flat on $\{g < f\}$ and $\{g > f\}$ and the procedure continues until convergence.

2.1 Scale space representations with morphological levelings

Levelings, which are a general class of morphological operators, are powerful and characterized by a number of desirable properties for the construction of nonlinear scale space representations. They satisfy the following properties [8–10]:

(i) the invariance by spatial translation,
(ii) the isotropy, invariance by rotation,
(iii) the invariance to a change of illumination,
(iv) the causality principle,
(v) the maximum principle, excluding the extreme case where g is completely flat.
In addition levelings:
(vi) do not produce new extrema at larger scales,
(vii) enlarge smooth zones,
(viii) they, also, create new smooth zones,
(ix) are particularly robust (strong morphological filters),
(x) do not displace edges.

Above properties make them a very useful simplification tool for a number of low level computer vision applications.

Different types of levelings can be constructed based on different types of extensive δ and anti-extensive ε operators. Based on a family of extensive dilations δ_i and the corresponding family of adjunct erosions ε_i, where $\delta_i < \delta_j$ and $\varepsilon_i > \varepsilon_j$ for $i > j$, multiscale levelings (a hierarchy of levelings) can be constructed [10]. Multiscale levelings can be, also, constructed when the reference function f is associated to a series of marker functions $\{h_1, h_2, ..., h_n\}$. The constructed levelings are respectively

$$g_1 = f, \; g_2 = \Lambda(f, h_1), \; g_3 = \Lambda(f, h_2), ..., \; g_{n+1} = \Lambda(f, h_n).$$

Thus, a series of simpler and simpler images, with fewer and fewer smooth zones are produced. Levelings can be associated to an arbitrary or an alternating family of marker functions. Examples with openings, closings, alternate sequential filters and isotropic markers can be found in [6, 9, 10] and two examples with anisotropic in [15] and [2]. Furthermore, for specific tasks one may take advantage of the knowledge of the scene and design accordingly the family of markers.

3. Comparing different markers for morphological levelings

In this paper, levelings constrained by four different families of markers are compared for the construction of nonlinear scale space representation. The marker functions are constructed by

1. a morphological *reconstruction opening* (RO) with a flat disk-shaped structuring element of radius r (scale parameter), which is the distance from the structuring element origin to the perimeter of the disk,

2. a morphological *alternate sequential filter* (ASF) with reconstruction openings and closings with the same structure element and scale parameter r, as above,

3. an *isotropic* Gaussian function (ISO) with scale parameter σ (standard deviation),

4. an *anisotropic* diffusion filtering (ADF) proposed by Alvarez et al. [1]:

$$\vartheta I(x, y)/\vartheta t = w(|G_\sigma * \nabla I|)|\nabla I| div(\nabla I/|\nabla I|),$$

   where $I(x, y)$ is the original image and t the scale parameter (iterations of the partial differential equation). The term $|\nabla I| div(\nabla I/|\nabla I|)$ diffuses the image $I(x, y)$ in the direction orthogonal to its gradient $|\nabla I|$ and does not diffuse it at all, in the direction of $|\nabla I|$. w is an 'edge-stopping' smooth and non-increasing function like: $w(k) = 1/(1 + k^2/K^2)$ with K a constant. In all cases in this paper, $K = 10$.

3.1 Relation between scales

All the above families of markers are controlled by a scale parameter. For the morphological operators, since scale refers to the same parameter, the comparison can be straight forward, but this is not the case for the levelings that are associated with the isotropic and the anisotropic families of markers.

Towards the establishment of such a scale relation, we have performed extensive experiments by applying these four operators to various scales, attempting first to understand the extent of their filtering effect and relate their result. Observing the extension of their smoothing result and looking forward, in general, to an approximate equivalence, the proposed relation values between the parameters of all four operators were chosen and are shown in Table 1. For example, when four different levelings of scale one are constructed, constrained by the four different families of markers, this means that a disk-shaped structure element with radius $r = 1$ will be used for the two morphological operators, a standard deviation of $\sigma = 1$ will be used for the Gaussian function (in a 5×5 kernel) and 100 iterations t will take place for the computation of the anisotropic marker.

Table 1. Establishing a relation between the scales of the different markers. Proposed values for scales: 1 to 7. The scale parameter for the morphological operators is the size (radius) of the disk-shaped structure element, for the isotropic marker the value of the standard deviation and for the anisotropic the number of the iterations of the partial differential equation.

Leveling's Scale	Values for the scale relation of the four different type of markers			
	Structure element's size r for RO and ASF	Isotropic diffusion		Anisotropic diffusion iterations t
		Standard deviation σ	Kernel size	
1	1	0.5	5×5	100
2	2	1	7×7	200
3	3	1.5	11×11	300
4	4	2	13×13	400
5	5	2.5	17×17	500
6	6	3	19×19	600
7	7	3.5	23×23	700

4. Results and discussion

Levelings (fixed levelings associated to an extensive dilation δ and the adjunct erosion ε) which were associated to RO, ASF, ISO and ADF families of markers, were compared. Two reference images were used for this comparison: an artificial test image and the cameraman test image. The artificial test image was a binary 'chessboard type' one, which was contaminated with both additive and salt and pepper noise. Half was black and half was white, forming a horizontal straight separation/ edge Figure 1.

In addition, for the comparison apart from the qualitative evaluation,

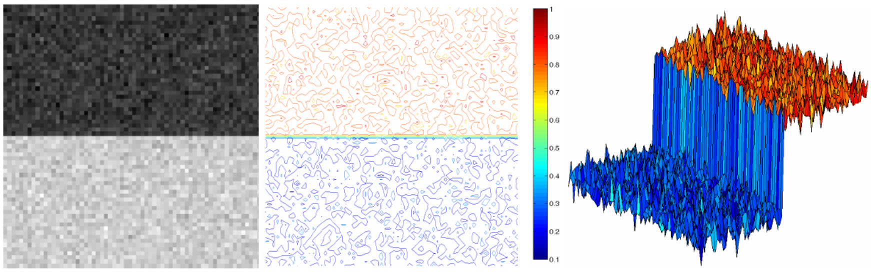

Figure 1. The artificial test image (left), its contours-isophotes, lines of constant brightness (middle) and its 3D representation, in which brightness values are proportional to surface's height.

a quantitative one also took place, based on three quantitative measures (RMS, NRMS, SSIM and NCD), which are described in Appendix.

4.1 Test image

In Figure 2, the resulting levelings (of scale 2) constrained by the four distinct families of markers (RO, ASF, ISO and ADF) are shown. The ASF lead to the most intensive filtering result producing large smooth zones. The RO simplified and at the same time enhanced abrupt brightness changes in a number of small regions. The ADF simplified the image and at the same time preserved regions with strong intensity changes, contrary to ISO. The above qualitative evaluation can be confirmed by the quantitative measures in Table 2. With the ASF, the resulting leveling yielded to the larger RMSE, NMSE values (its brightness values differ much from the original image) and to the smallest SSIM value, that confirms its lower structural similarity with the reference image. The leveling that was constrained by the ADF simplified the image but kept the closest relation with the reference image, regarding both i) the extent of variation in intensity values (RMSE and NMSE measures) and ii) their structural similarity (SSIM measure).

Furthermore, in order to compare, the resulting, from the different levelings simplification (scale 2), cross-sections along the y axis, are shown in Figure 2 (bottom right). First of all one can observe that all methods do not displace edges and in particular the 'black to white region' edge. The constrained by the ASF leveling differed most from the reference image intensities and the leveling constrained by the ISO simplified but at the same time smoothed the brightness values between the different image's zones. Moreover, the leveling constrained by the RO did enhanced the difference in intensities between image zones and those constrained by the ADF simplified and at the same time followed, more constantly, the changes in image intensity values.

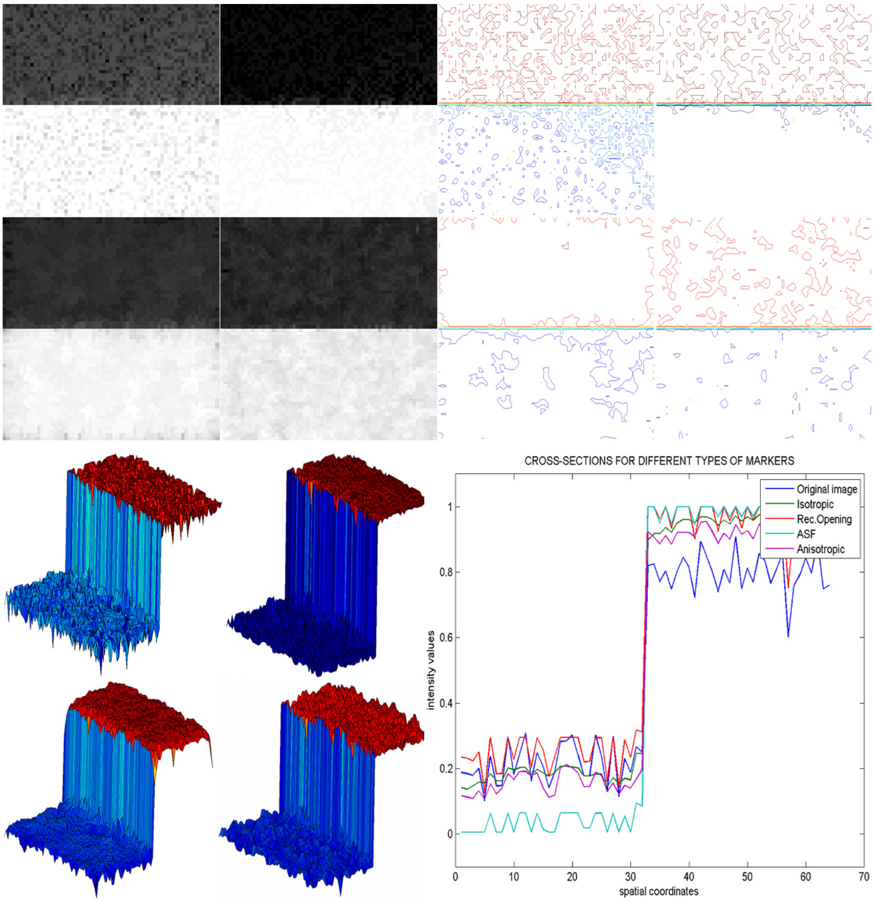

Figure 2. Top Left : Resulting levelings of scale 2 constrained by the four families of markers (RO: top left, ASF: top right, ISO: bottom left and ADF: bottom right). Top Right: The contours of the resulting levelings. Bottom Left: 3D representations, in which brightness values are proportional to surface height. Bottom Right: Line plot with the cross-sections along the y image axis of the compared levelings.

4.2 Cameraman image

In Figure 3 (left five images), the resulting levelings (scale 4) constrained by the four (RO, ASF, ISO and ADF) families of markers, are shown which were applied to the cameraman test image. ASF lead to the most intensive filtering result producing large smooth zones. In particular, it suppressed regional extrema in regions with proportional size to the structure element (like the top of the two buildings, in the right center of the image). Similarly, the RO marker was robust in flattening bright regions with proportional size with the structure element (like the bright values in the top of the two build-

Table 2. Quantitative evaluation of the resulting levelings by RO, ADF, ISO and ADF markers. Results for the artificial test image (scale 2) and the cameraman test image (scale 4 and scale 7) are presented. In general, the leveling that was constrained by the ADF, did simplified the images and at the same time scored better in all quantitative measures, indicating that it preserves effectively changes in intensities and respects more efficiently the structural similarity with the reference image.

	Quantitative measures	Markers			
		RO	ASF	ISO	ADF
Test image scale 2	RMSE	0.126	0.187	0.118	0.103
	NMSE	0.046	0.101	0.040	0.031
	SSIM	0.9981	0.9956	0.9984	0.9989
Cameraman scale 4	RMSE	13.748	15.649	10.132	4.325
	NMSE	0.011	0.014	0.006	0.001
	SSIM	0.923	0.847	0.904	0.933
Cameraman scale 7	RMSE	20.963	22.652	13.108	4.650
	NMSE	0.024	0.029	0.010	0.001
	SSIM	0.851	0.757	0.866	0.925
Crop of cameraman scale 7	RMSE	30.914	31.943	17.831	2.870
	NMSE	0.053	0.057	0.018	0.001
	SSIM	0.831	0.772	0.891	0.983

ings in the right center of the image). The ADF marker lead to a simplified image, that is characterized by the preserved level of contrast between the different image flattened zones, contrary to ISO and the other morphological markers. The above qualitative evaluation can be confirmed by the quantitative measures in Table 2 (cameraman image, scale 4). Constrained by the RO and ASF, the resulting levelings yielded to the broadest intensity variations, since their brightness values differ much from the original image (RMSE and NMSE measures). In addition, the ASF and the ISO markers produced levelings with the smallest structural similarity with the original image (SSIM). The leveling that was constrained by the ADF, simplified the cameraman image and scored better in all quantitative measures, indicating that it preserves effectively changes in intensities and at the same time in the structural similarity with the reference image.

Furthermore, in Figure 4 (left plot), the cross-sections along a part of the y axis, are shown, from the levelings (scale 4), which were constrained by the RO (red line), the ASF (turquoise line), the ISO (green line) and the ADF (purple line) families of markers. All methods did not displace image edges. The ASF and RO markers resulted into the most extended simplification and drew most away from the reference image intensities. The ADF markers simplified the reference image and at the same time followed more constantly, than all the other markers, intensity changes between the

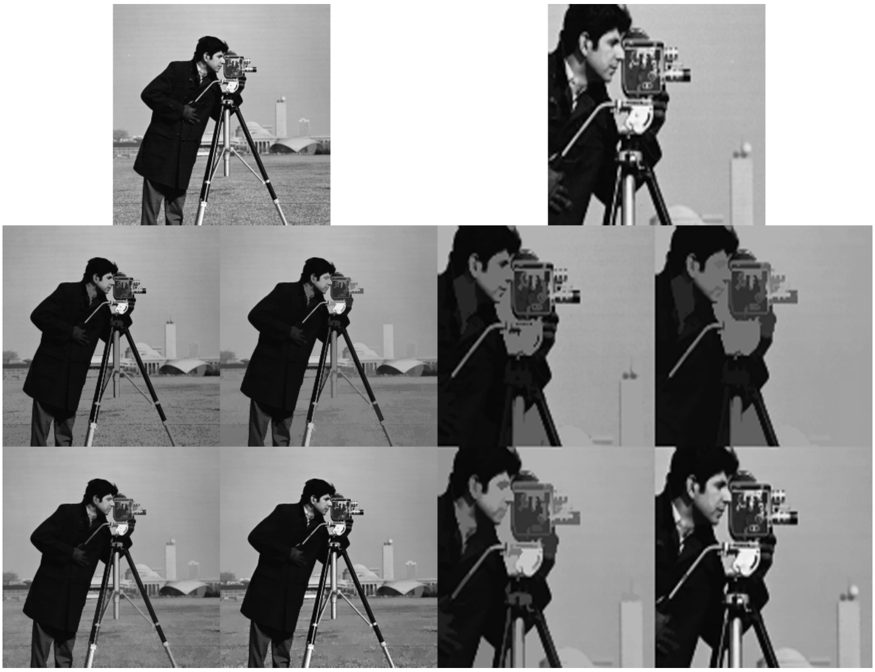

Figure 3. Left five images: The reference cameraman test image (top) and the levelings (scale 4) constrained by the four families of markers (RO: top left, ASF: top right), ISO: bottom left and ADF: bottom right). Right five images: A crop of the cameraman test image and the resulting levelings of scale 7.

different image zones, due to its edge preserving nature.

In addition, in Figure 3 (five images on the right), the resulting levelings of scale 7, are shown for a crop of the cameraman image. ASF lead to the most intensive filtering result producing large smooth zones and in particular, eliminated objects with a size proportional to the structure element. Similarly, the RO marker was robust in flattening and in eliminating bright objects with a size proportional to the structure element. The ADF marker lead to a simplified image, that is characterized by the preserved level of contrast between the different image flattened zones, opposite to ISO and the other morphological markers. The resulting leveling from the ADF marker is a simplified version of the reference image on which the edges and the contrast have been preserved.

Quantitative measures in Table 2 (crop of cameraman image, scale 7), indicate that the RO and the ASF levelings resulted into the broadest intensity variations (RMSE and NMSE measures). The ASF, the RO and the ISO markers produced levelings with the smallest structural similarity with the original image (SSIM). The leveling that was constrained by the ADF, scored by far better in all the quantitative measures, indicating that it preserves effectively changes in intensities and at the same time in the

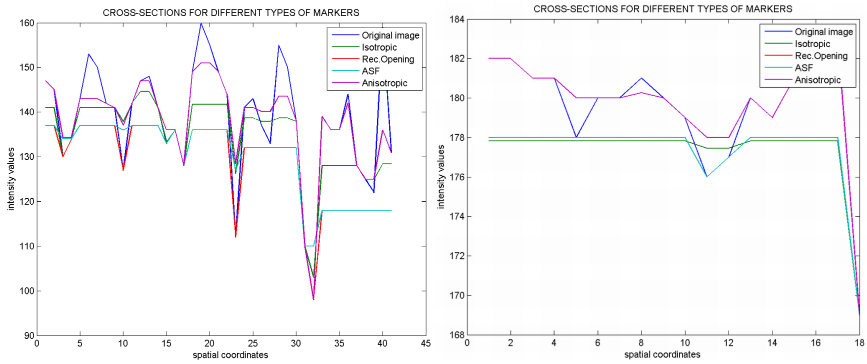

Figure 4. Line plot with the cross-sections of the different levelings for the cameraman image (scale 4, left) and for a crop of the same image (scale 7, right). The simplified brightness values of the different levelings-along a part of the y axis- are shown, which were constrained by RO (red line), ASF (turquoise line), ISO (green line) and ADF (purple line). All methods did not displace image edges. The ASF and RO markers resulted into the most extended simplification and drew most away from the reference image intensities. The ADF markers simplified less and at the same time followed more constantly, than all the other markers, the image intensity changes between the different zones, due to its edge preserving nature.

structural similarity with the reference image. Finally, in Figure 4 (right), cross-sections along the y axis of the crop of the cameraman image, are shown, from the levelings (scale 7). The ADF marker was the only one which did preserve small changes between the different image zones, due to its edge preserving nature.

5. Conclusions and future perspectives

In this paper, a framework for the comparison of levelings that were constrained by different markers was developed, through the introduction of a relation between the scale parameters of all markers. Four different families of markers were evaluated both by a qualitative and a quantitative comparison of their resulting simplification. The evaluation of the different families of markers concluded to the following points:

- The ASF and RO markers resulted into the most extended simplification and differed most from the reference image intensities.

- The ADF markers yielded to a simplified version of the reference image which followed more constantly, than all the other cases, the intensity changes between the different image zones, due to its edge preserving nature.

- The leveling that was constrained by the ADF, scored by far better (in terms of keeping small the extent of intensity variations and high the structural similarity with the reference image) in all the quantitative measures.

- ASF (and resp. RO) levelings eliminated objects (resp. bright objects) with a size proportional to their structure element. Similarly, the ISO leveling eliminated — with an isotropic diffusion procedure which, contrary to ADF, blurs image edges — objects according to the standard deviation value.
- The ADF marker lead to a simplified image, which is characterized by the preserved level of contrast between the different image flattened zones, contrary to ISO and the other morphological markers.

Subjects for further research are the establishment of an axiomatic relation between the scales of different markers and their evaluation for specific computer vision tasks like the segmentation and the extraction of specific objects.

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Appendix

Objective methods for assessing perceptual image quality traditionally attempted to quantify the visibility of errors (differences) between a processed image and a reference image using a variety of known properties of the human visual system. In this regard the simplest and most widely used quality metrics are the root mean squared error (RMSE) and the normalized mean square error (NMSE). RMSE is computed by averaging the squared intensity differences of the processed and reference image pixels and NMSE normalized to a range between 0 and 1. Both measures give a quantitative sense for the extent of variation between the intensity values of the two compared images forming a kind of a generalized standard deviation measure. RMSE and NMSE are appealing because they are simple to calculate, have clear physical meanings, and are mathematically convenient in the context of optimization. But they are not very well matched to perceived visual quality [16]. Hence, it has been also adopted a recently proposed alternative complementary quality measure of the structural similarity (SSIM) between two images, which compares local patterns of pixel intensities that have been normalized for luminance and contrast [16]. The above three quality measures (RMSE, NMSE and SSIM) are aiming to an objective image quality assessment of the achieved results.