CALCULATION OF POWER FREQUENCY FIELDS FROM HIGH VOLTAGE OVERHEAD LINES IN RESIDENTIAL AREAS

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Abstract: Precise calculation of magnetic fields produced by overhead power lines is highly important in several research areas. Although there are many commercial software packages dedicated to estimate the power frequency fields under the overhead transmission lines, this paper deals with the analysis and application of two- and three-dimensional calculation methods. These methods based on the Biot-Savart law were employed for this purpose. Both methods were implemented in MATLAB program for the power frequency field calculation entering the necessary data of the transmission lines. The results obtained by the proposed computational methods were compared with on-site measurements performed in the vicinity of power transmission lines in residential areas. In the cases that the measurement results deviated from the computed results, a commercially available software package was also applied in field computations. The level of agreement between the two methods and the measurements observed in this study is discussed giving useful conclusions about the applicability of each method.

1 INTRODUCTION

Serious public concerns are expressed about biological interactions and the potential negative effects on human health as a consequence of exposure to extremely low frequency electromagnetic fields (ELF EMFs) [1]. In the case of ELF fields, attention is focused on the AC power transmission systems, especially on high voltage overhead lines in residential areas, because of their significant field level and the great number of people, who live in these areas.

Considering the importance of estimation of the levels of EMFs to which general public are exposed, this paper deals with the calculation of the power frequency magnetic fields from overhead power transmission lines in two and three dimensions. A method based on the Biot-Savart law for calculation of the magnetic field density is applied taking into account the power line specifications. This method was examined by measurements close to power lines, which were performed according to standard procedures. When an insufficient level of agreement between the calculation method and the measurements is observed a software package was used. Finally, significant outcomes for the proposed method were drawn from comparisons between measured and computed results.

2 CALCULATION METHOD

The rms value of resultant magnetic field generated by overhead power lines can be evaluated with a good approximation by applying two different approaches of the Biot-Savart law: i) the two dimensional technique assuming the power line's conductors as straight horizontal wires of infinite length and ii) the three dimensional technique considering the effect of power line's conductor sag [2-4]. The Biot-Savart law is formulated and applied for overhead power lines in two and three dimensions in below sections.

2.1 Biot–Savart law

The law of Biot and Savart is fundamental to electromagnetism and specifies the direction and strength of the magnetic field in the vicinity of a conductor carrying electric current. This method has an advantage because it is readily applicable to any complex structure and any current condition. Therefore, this method is relatively straightforward to use in three-phase systems where overhead power lines with various geometrical configurations and time-varying currents exist.

The magnetic field of a single current-carrying conductor at an arbitrary point P shown in Figure 1 can be calculated by the Biot-Savart's law:

$$\vec{B} = \mu_o \int_l I \frac{\vec{dl} \times \vec{r_o}}{4\pi r_o^3} \qquad (1)$$

where μ_o = 4πx10^{-7} Tm/A is the permeability of free space, I is the line current, $\vec{dl}$ is a differential element of the conductor in the direction of current and $\vec{r_o}$ is a distance vector from the source (x,y,z) to the field point P (x_o,y_o,z_o). In Cartesian coordinates, the vector quantities $\vec{r_o}$ and $\vec{dl}$ are:

$$\vec{r_o} = (x_o - x)\hat{\imath}_x + (y_o - y)\hat{\imath}_y + (z_o - z)\hat{\imath}_z \qquad (2)$$

$$\overrightarrow{dl} = \overrightarrow{dx}\hat{\imath}_x + \overrightarrow{dy}\hat{\imath}_y + \overrightarrow{dz}\hat{\imath}_z \qquad (3)$$

where $\hat{\imath}_x, \hat{\imath}_y, \hat{\imath}_z$ are the unit vectors along the direction x, y, z, respectively.

In this paper, the sinusoidal steady-state quantities at the fundamental power line frequency are represented here as phasors with the symbol " $\dot{}$ ", whereas " $\rightarrow$ " and " $\hat{}$ " indicates vector and unit vector, respectively.

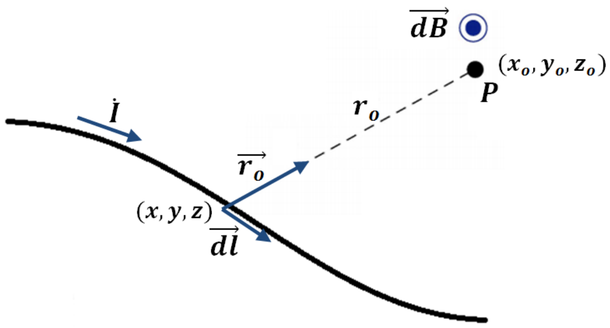


Figure 1: Application of the Biot-Savart law.

2.2 Two-dimensional analysis

In 2-D analysis, the magnetic field computation technique assumes that power transmission lines are straight horizontal conductors of infinite length, parallel to a flat ground, and parallel to each other. The influence of the line's sag is neglected or modelled by introducing an effective height for the horizontal line. The proposed 2-D calculation method is described below.

The magnetic field at point P generated by an infinite straight conductor L carrying a current $\dot{I}$ placed along the x-axis, as shown in Figure 2, can be evaluated by (1) with infinite intervals of integration (-∞,∞). The cross product is given by:

$$\overrightarrow{dl} \times \vec{r_o} = \left[(y_o - y)\hat{\imath}_z - (z_o - z)\hat{\imath}_y\right] dx \qquad (4)$$

Replacing (4) in the expression of the Biot-Savart law, the integration yields:

$$\vec{B} = \overrightarrow{B_y} + \overrightarrow{B_z} = -\mu_o \dot{I}\frac{(z_o - z)}{2\pi r_o^2}\hat{\imath}_y + \mu_o \dot{I}\frac{(y_o - y)}{2\pi r_o^2}\hat{\imath}_z \qquad (5)$$

where the current $\dot{I}$ oriented as the x-axis, whereas the magnetic field B in the field point P lies in the y-z plane.

The (5) is the well known expression for the magnetic flux density of an infinitely long straight conductor, called "Ampere's circuital law" and can be written in the form:

$$\vec{B} = \mu_o \dot{I}\frac{\hat{\Phi}}{2\pi r_o} \qquad (6)$$

where $\hat{\varphi}$ is a unit vector perpendicular to the plane determined by the observation point and the line of the conductor (Figure 2).

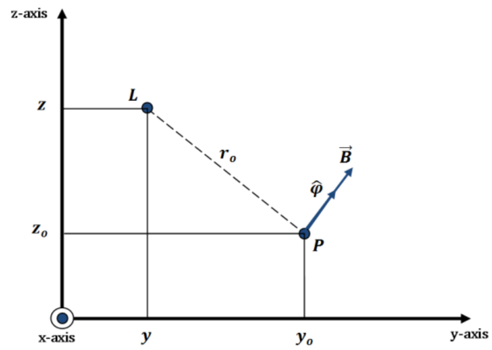


Figure 2: Magnetic field of an infinite straight conductor

The rms field value at the point P can be calculated with:

$$B = \sqrt{{B_y}^2 + {B_z}^2} \qquad (7)$$

where B_x and B_y the rms values of spatial components of magnetic field.

The influence of multiple conductors will be taken into account using superposition principle. Considering a three-phase system with straight parallel conductors of infinite length (see Figure 3), the resulting magnetic field can be expressed as:

$$\vec{B} = \sum_{k=1}^{3} \mu_o \dot{I}_k \frac{\hat{\Phi}_k}{2\pi r_{ok}} \qquad (8)$$

where the line currents $\dot{I}_\kappa$ of the symmetrically loaded overhead power line are described by:

$$\dot{I}_\kappa = \dot{I}_o[\cos(\omega t - \theta_k) + \mathrm{j}\sin(\omega t - \theta_k)] \qquad (9)$$

where $\dot{I}_o$ is the rms value of the line current, ω is angular frequency and θ_k is the phase angles. Since a three-phase system is examined, the phase angles of the load current in each phase differ 120 degrees.

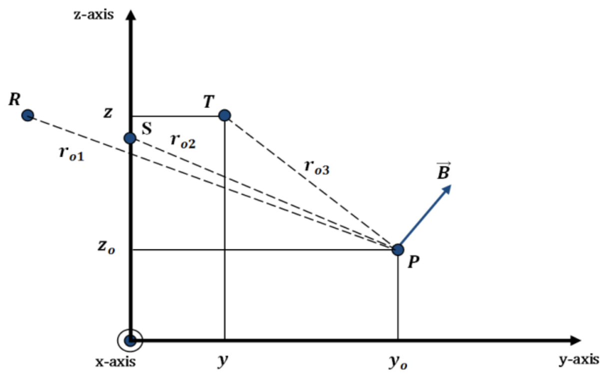


Figure 3: Magnetic field of three straight, parallel infinitely long conductors

As it is mentioned, it is assumed in the calculations of magnetic fields generated by transmission line that the transmission line conductors run in parallel with flat ground, and that the line conductors carry a three-phase balanced current. Therefore, the resultant magnetic field at ground level is:

$$\vec{B} = \overrightarrow{B_y} + \overrightarrow{B_z} = B_y\hat{\iota}_y + B_z\hat{\iota}_z \quad (10)$$

with vertical and horizontal components:

$$\overrightarrow{B_y} = (B_{ry} + B_{sy} + B_{ty})\hat{\iota}_y \quad (11)$$

$$\overrightarrow{B_z} = (B_{rz} + B_{sz} + B_{tz})\hat{\iota}_z \quad (12)$$

where B_{ry}, B_{rz}, B_{sy}, B_{sz}, B_{ty} and B_{tz} are the horizontal and vertical components of phase R, S and T, respectively. These components can be evaluated by (5) replacing the corresponding geometrical coordinates for each phase.

2.3 Three-dimensional analysis

In order to calculate accurately the magnetic field of transmission lines at any point above the ground, the influence of the conductors' sag and the different heights of the towers must be taken into account. The proposed 3-D estimation technique of the magnetic field from transmission lines of various configurations is presented.

Although the conductor sag can be exactly characterized by the equation of a catenary, it also can be very well approximated with the equation of a parabola. Consequently, an overhead power line, as shown in Figure 4, can be analyzed in two planes and described by the following equations:

$$z = h_{min} + \frac{x^2}{2C} \quad (13)$$

$$y = \frac{y_{t_2} - y_{t_1}}{x_{t_2} - x_{t_1}}x + \frac{x_{t_2}y_{t_1} - x_{t_1}y_{t_2}}{x_{t_2} - x_{t_1}} \quad (14)$$

where $(x_{t_1}, y_{t_1}, z_{t_1})$ and $(x_{t_2}, y_{t_2}, z_{t_2})$ are the points of suspension, h_{min} is the height of the lowest point above the ground and C is the solution of the system of the (13) and (14). The parameter C defines the shape of the parabola and also can be written as:

$$C = \frac{l}{4S} = \frac{\sqrt{L^2 - (y_{t_1} - y_{t_2})^2}}{4S} \quad (15)$$

where $S = z_{t_1} - h_{min}$ is the sag of the conductor and L is the distance between the points of suspension (span).

Considering the differential element (3), it can be can be determined as:

$$\overrightarrow{dl} = \overrightarrow{dx}\left(\hat{\iota}_x + \frac{dy}{dx}\hat{\iota}_y + \frac{dz}{dx}\hat{\iota}_z\right) \quad (16)$$

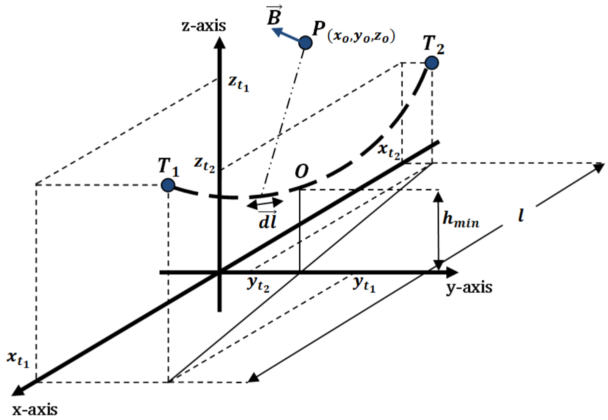


Figure 4: Magnetic field of a single-conductor line with sag in three dimensions

Also, the first-order differential equations can be written as:

$$\frac{dz}{dx} = \frac{x}{C} \quad (17)$$

$$\frac{dy}{dx} = \frac{y_{t_2} - y_{t_1}}{x_{t_2} - x_{t_1}} \quad (18)$$

If these differential equations are replaced into expression (16), it is obtained:

$$\overrightarrow{dl} = \overrightarrow{dx}\left(\hat{\iota}_x + \frac{y_{t_2} - y_{t_1}}{x_{t_2} - x_{t_1}}\hat{\iota}_y + \frac{x}{C}\hat{\iota}_z\right) \quad (19)$$

Substituting (2) and (19) into (1), and carrying out the cross product, the magnetic flux density vector becomes:

$$\vec{B} = \overrightarrow{B_x} + \overrightarrow{B_y} + \overrightarrow{B_z} = \int_{x_2}^{x_1} (b_x\hat{\iota}_x + b_y\hat{\iota}_y + b_z\hat{\iota}_z)dx \quad (20)$$

where:

$$b_x = \frac{\mu_o \dot{I}\left[\frac{y_{t_2} - y_{t_1}}{x_{t_2} - x_{t_1}}(z_o - z) - \frac{x}{C}(y_o - y)\right]}{4\pi r_o^3} \quad (21)$$

$$b_y = \frac{\mu_o \dot{I}\left[\frac{x}{C}(x_o - x) - (z_o - z)\right]}{4\pi r_o^3} \quad (22)$$

$$b_z = \frac{\mu_o \dot{I}\left[(y_o - y) - \frac{y_{t_2} - y_{t_1}}{x_{t_2} - x_{t_1}}(x_o - x)\right]}{4\pi r_o^3} \quad (23)$$

The total rms value B at the same point can be also defined by compositing the components:

$$B = \sqrt{{B_x}^2 + {B_y}^2 + {B_z}^2} \quad (24)$$

Considering the (20), the magnetic flux density vector can be presented for three phase systems in the form:

$$\vec{B} = \sum_{k=1}^{3} \int_{x_2}^{x_1} (b_x \hat{\iota}_x + b_y \hat{\iota}_y + b_z \hat{\iota}_z) dx \qquad (25)$$

Similarly, the total rms value B can be calculated for three phase systems with (24).

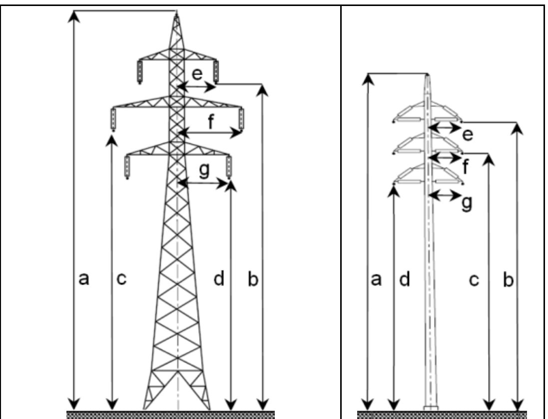
Finally, it is noted that the following assumptions have been made for both 2-D and 3-D analysis: (a) There are no harmonics on the load currents of line, (b) the influence of the image currents has been neglected, (c) the effect of neighboring spans has been ignored and (d) the magnetic field has been evaluated in the absence of ferromagnetic materials.

3 MEASUREMENTS AND CALCULATIONS

3.1 Examples of measurements

Power frequency magnetic fields from overhead transmission lines have been assessed with calculations or measurement procedures in many papers [2-4]. In order to compare the measurements results and the corresponding calculation results obtained from the proposed methods, measurements of power frequency magnetic fields in the vicinity of 150 kV double circuit power lines were performed in living environments. Table 1 details the configuration of the 150 kV double circuit power lines used in Greece and the coordinates of the conductors.

Table 1: Conductor configuration of 150kV double circuit transmission lines



	a) conventional				b) compact
Types	S4	R4	T4	Z4	-
a (m)	33,01	34,27	32,35	33,90	30.05
b (m)	27,75	28,45	26,80	26,80	25,55
c (m)	23,85	24,20	22,90	22,90	22,75
d (m)	19,95	19,95	19,00	19,00	19,95
e (m)	3,15	3,65	3,50	4,00	2,30
f (m)	5,15	5,65	5,50	6,35	2,30
g (m)	4,05	4,55	4,40	5,10	2,30

Power frequency field measurements were carried out according to the IEC 61786:1998, IEC 62110:2009 and IEEE 644-1994 Standards [5-7]. Specifically, the magnetic flux density were measured at a height of 1,0 m above ground level at points of interest at selected intervals within the area enclosed by two towers and their respective right-of-way (ROW). At least one lateral profile of the rms magnetic flux density in a direction normal to the power line was measured at each measurement area.

The measurements of the rms values of the resultant magnetic field were made by selecting band-pass mode at the fundamental frequency of 50 Hz, since the harmonic content is sufficiently small near power lines. The three-axis instrument used in measurements met the requirements of standards mentioned before and indicated the rms value of the power frequency magnetic field with uncertainty of ±3%. Nevertheless, the total uncertainty of magnetic field measurements near power lines in residential areas was assessed by considering not only all available equipment data, but also contributions that depend on the measurement procedures, environmental conditions and characteristics of the field source, which are considered to increase the error of measurement. A description of the measurement uncertainty evaluation for power frequency field measurements are presented in [8].

Magnetic field measurements were made at three measurement areas and grouped in case studies as follows:

1) Case A: Measurements were performed in urban area, where the power transmission line of compact configuration is specified by equal heights of the towers and parallel conductors to each other. The measurements were made close to the tower.

2) Case B: Measurements were performed in a park, where the power transmission line of conventional configuration is specified by equal heights of the towers and parallel conductors to each other. The lateral profile was measured at the maximal sag position.

3) Case C: Measurements were performed in suburbs, where the power transmission line of conventional configuration is specified by unequal heights of the towers and the transmission line conductors are not parallel to each other. The magnetic field values were measured at several distances of the span.

3.2 Computational results

The authors developed a software on the basis of calculation methods described above. The methods are numerically solved by MATLAB program using the necessary parameters such as the configuration of the transmission lines, the span length, the sag of the conductors and the operating current. The program allows calculating rms value of magnetic field of power frequency for any point of space. Therefore, the 2-D and 3-D calculation techniques were validated comparing the computation results with the experimental ones provided in above section. These three case studies are examined in the below sub-sections:

3.2.1 Case A In case A the magnetic field was calculated with the 2-D method because the lateral profile of the magnetic field was measured close to tower. The transmission line is substituted for three straight conductors at the maximum height since the minimal sag position occurred there. The current condition used for the calculation is balanced current of 124 A and 78 A in each circuit. The transmission line parameters are also given in Table 2.

Table 2: Transmission line parameters (Case A)

Case A		
Span length (m)	135	
Ground clearance (m)	19	
Sag (m)	1	
Load current (A)	1st circuit	2st circuit
	120,2-125,6	74,5-80,6
Line configuration	Compact	
Type of pylons	1st tower	2st tower
	-	-

Figure 5 shows both the measured and computed results. As it can be seen, a satisfactory agreement between the computational results and measurements is obtained. The difference with the measurements was less than 5% except from three points where the difference exceeds 10%.

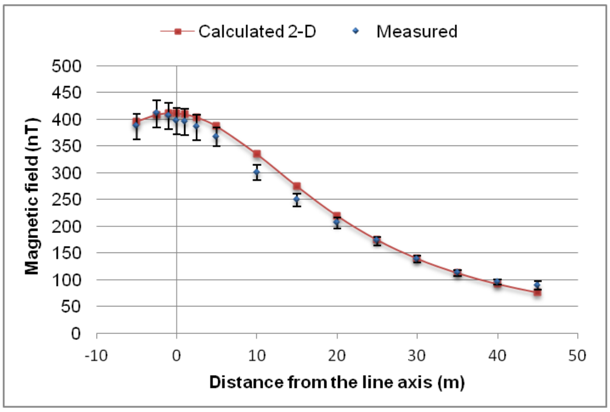


Figure 5: Lateral profile of the magnetic field (Case A)

3.2.2 Case B Both 2-D and 3-D calculation methods are applied at the mid-span, with balanced currents of 118 A and 70 A in each circuit. The transmission line parameters are presented in Table 3.

Table 3: Transmission line parameters (Case B)

Case B		
Span length (m)	200	
Ground clearance (m)	19	
Sag (m)	1	
Load current (A)	1st circuit	2st circuit
	116,8-120,3	69,3-70,9
Line configuration	Conventional	
Type of pylons	1st tower	2st tower
	S4	S4

The lateral profile of the magnetic flux density at height of 1m above the ground at the maximal sag position is presented in Figure 6.

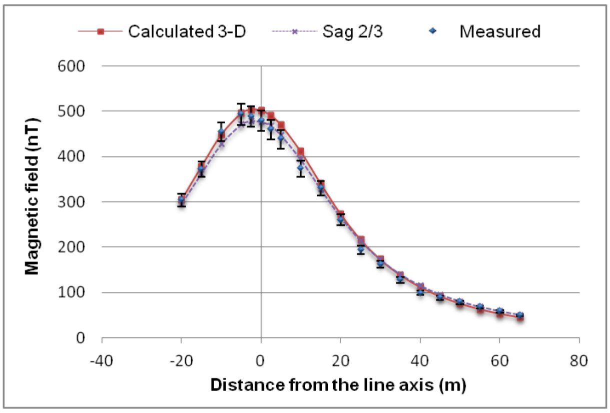


Figure 6: Lateral profile of the magnetic field (Case B)

Figure 6 show the differences among the results calculated for the sagged conductor and the ones obtained for the approximations of the sagged conductors with the straight ones placed at the height of 2/3 maximum conductor sagging. Applying the 3-D method it is observed that at the most measurement points the deviation varies from 3% to 9%. A similar agreement is obtained with 2-D calculation results where the difference with the measurements was less than 5% except from two points where the difference exceeds 10%.

3.2.3 Case C The 3-D method was applied because the tower heights were unequal as the type of towers were different and the first tower was placed on a hill of 3,5m, and the line of calculation was not at mid-span. In particular, the measurements of magnetic fields were performed along two lines perpendicular to the overhead line, which cut the x axis at the distances x_1= -35m (Case C1) and x_2= 60m (Case C2) away from midspan.

The measured and calculated magnetic field density produced by the double circuit transmission line, are shown in Figures 7 and 8. The calculated results obtained from the software package CDEGS was also used. The results are given for the overhead power line with balanced current of 167,5 A and 161 A in each circuit. Table 4 provides the transmission line parameters.

Table 4: Transmission line parameters (Case C)

Case C		
Span length (m)	330	
Ground clearance (m)	19	
Sag (m)	1	
Load current (A)	1st circuit	2st circuit
	166,4-170,8	159,3-162,6
Line configuration	Conventional	
Type of pylons	1st tower	2st tower
	R4	T4

In Figure 7 the computed values obtained from 3-D method are slightly smaller than the measured ones. The agreement in the centre of the line was about 2,5 % but it falls to 20 % for distances larger than 30m away from the power line axis. In the opposite, from Figure 7 it is clear that the agreement of CDEGS software with the measured values is better than the 3-D method. The variation with the measurements is less than 3,5 % at the central points but reaches 16% at the edges.

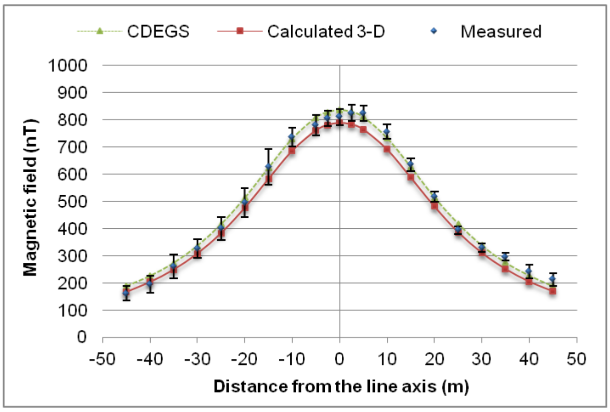


Figure 7: Lateral profile of the magnetic field (Case C1)

Additionally, Figure 8 shows the comparison of measurement results with the 3-D method at the second measurement line. Applying this method it is observed that the difference varies from 6% to 10 %. In the same figure are also shown that the consistency between the measured and calculated fields with the CDEGS software is very good as the deviation is smaller than 3% at the most points.

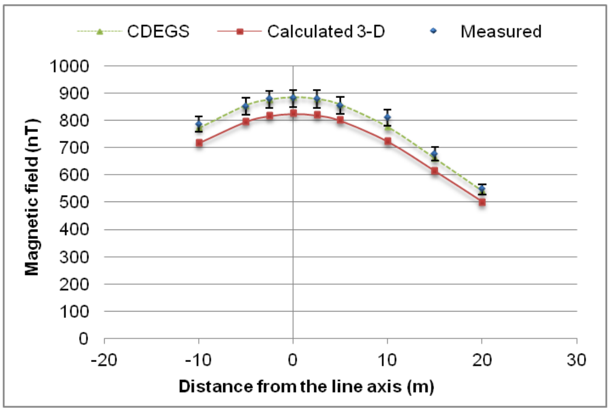


Figure 8: Lateral profile of the magnetic field (Case C2)

4 CONCLUSION

In this paper 2-D and 3-D methods for calculating of magnetic field produced by overhead power lines are presented. These methods are based on the Biot-Savart law and applied to three 150 kV double circuit power lines. Comparisons between the measurements results of three case studies and the corresponding calculation results were performed emphasizing the degree of approximation. A satisfying level of agreement between the two methods and the measurements was observed. Deviations between the results were caused by changes of the line load and the assumptions that had been made for both 2-D and 3-D in this paper. Further work should be considered improving the author's software in order to achieve more accurate computational results of the power frequency magnetic fields in the vicinity of power lines.

5 REFERENCES

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