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A NEW METHOD TO DESIGN A CIRCUIT FOR THE ELECTROSTATIC DISCHARGE CURRENT

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Summary: The aim of this paper is to give an alternative and more accurate way of designing a circuit that will produce the electrostatic discharge (ESD) current. An equation, which is very accurate, is been used as the equation of the ESD current. Furthermore, the Prony method is applied on this equation to obtain an approximation of the impulse response of a circuit that produces it. Then, a theoretical realization is proposed using operational amplifiers.

***Keywords*:** Circuit design, electrostatic discharge (ESD) current, Prony method.

1. Introduction

The problem of the aberration between the circuit proposed by the IEC 61000-4-2 [1] and the typical ESD current waveform included in the Standard is approached in this paper. The goal is to create a circuit based on an accurate equation of ESD current, which is useful for future simulations and applications.

The Prony method [2-5], that was used here, approximates a function by a sum of exponentials. The Prony method has been used in many applications such signal processing and filtering [2], power system protection [3], communications [4], and medical science [5].

The Prony method was applied on the electrostatic discharge (ESD) current equation, and then some of its disadvantages were revealed. It failed to give accurate results when applied on Heidler's equation which is one with a very sophisticated form [6-8]. Heidler's equation [8] is the most accurate equation that describes the ESD current. In [9], this equation has been improved, since Genetic Algorithm was used to extract new values for its parameters.

In this paper the Prony method had to be modified. There have been two modifications of the original method. The aim is always the minimization of the relative error between Heidler's equation and the approximation of the impulse response obtained.

2. The ESD current's equation

The typical waveform of the ESD current, defined by the Standard [1, 8], is demonstrated in Figure 1.

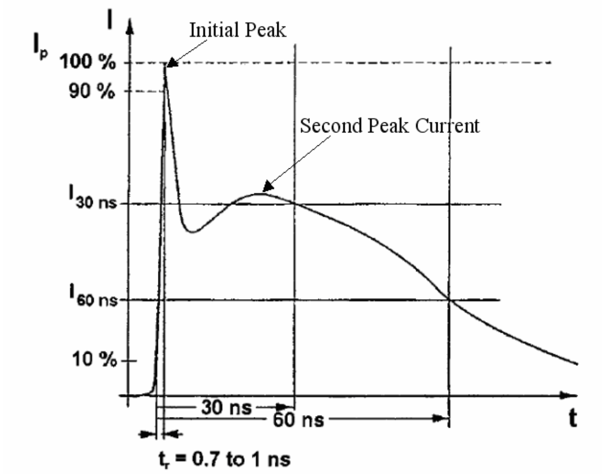

Fig. 1. Typical waveform of the output current of the ESD generator [1].

The Heidler's equation as it was in [8] is demonstrated hereunder in Eq. 1.

$$i(t)=\frac{i_1}{k_1}\cdot\frac{\left(\frac{t}{t_1}\right)^n}{1+\left(\frac{t}{t_1}\right)^n}\cdot exp\left(\frac{-t}{t_2}\right)+\frac{i_2}{k_2}\cdot\frac{\left(\frac{t}{t_3}\right)^n}{1+\left(\frac{t}{t_3}\right)^n}\cdot exp\left(\frac{-t}{t_4}\right) \tag{1}$$

where,

$$k_1 = exp\left(-\frac{t_1}{t_2}\cdot\left(\frac{nt_2}{t_1}\right)^{1/n}\right) \tag{2}$$

$$k_2 = exp\left(-\frac{t_3}{t_4}\cdot\left(\frac{nt_4}{t_3}\right)^{1/n}\right) \tag{3}$$

For a charging voltage of +4kV, the values of the parameters are presented in Tab. 1 (case 1). This equation has a rather satisfactory shape. The crucial values of the parameters I_p, t_r, I_{30ns}, I_{60ns}, are shown in Tab. 2.

New parameters corresponding to the Heidler's equation were found in [9] using the Genetic Algorithm, and for a charging voltage of +4kV, the values of the parameters are presented in Tab. 1 (case 2). The Heidler's equation now, seems to have a finely satisfactory shape, and the crucial values of the parameters I_p, t_r, I_{30ns}, I_{60ns}, are shown in Tab. 2.

Table 1. Values for the 6 parameters of Eq. 1 at +4 kV

	Case 1 [8]	**Case 2 [9]**
i_1 [A]	17.5	17.46
i_2 [A]	9	7.81
t_1 [ns]	1.3	0.75
t_2 [ns]	2	0.82
t_3 [ns]	12	3.43
t_4 [ns]	37.8	67.8
n	1.8	3

Table 2. Parameters' values of Heidler's equation of ESD current at +4kV.

	I_p (A)	**t_r (ns)**	**I_{30ns} (A)**	**I_{60ns} (A)**
IEC 61000-4-2 [8]	15±10%	0.7÷1	8±30%	4±30%
Case 1 [8]	15.14	0.88	7.83	3.98
Case 2 [9]	14.99	1.00	7.83	4.00

In the present paper the Heidler's equation as defined in case 2 [9], will be used in order to extract the transfer function of a circuit that produces it as an output.

3. The Prony method

3.1. Introduction to the Prony Method

Let $g_d(t)$ be a continuous function. Let us consider the values of $g_d(t)$ at a set of equally spaced points $t=kT$, $k=0,1,\ldots$ and form the matrix:

$$P(N,M)=\begin{bmatrix} g_d(0) & g_d(T) & . & . & . & g_d[(M-1)T] \\ g_d(T) & g_d(2T) & . & . & . & g_d(MT) \\ . & . & & & & . \\ . & . & & & & . \\ . & . & & & & . \\ g_d[N-1)T] & g_d(NT) & . & . & . & g_d[(N+M-2)T] \end{bmatrix} \quad (4)$$

If $g_d(t)$ is the impulse response of a linear, time invariant system of finite order n, it can be written:

$$g_d(kT)=\sum_{i=1}^{n} A_i \exp(s_i kT)=\sum_{i=1}^{n} A_i z_i^k . \quad (5)$$

where,

$$z_i = \exp(s_i T) \quad (6)$$

Let us define the polynomial in z having roots z_i, $i=1,2,\ldots n$, as follows.

$$y(z)=\prod_{i=1}^{n}(z-z_i)=z^n+\sum_{m=0}^{n-1} b_m z^m \quad (7)$$

Then, for N, M greater than n

$$rankP(N,M)=n \quad (8)$$

This is equivalent to saying that any n+1 columns of the matrix $P(N,M)$ are linearly dependent and for its first n+1 columns the following relation holds:

$$\begin{bmatrix} g_d(0) & g_d(T) & . & . & . & g_d(nT-T) \\ g_d(T) & g_d(2T) & . & . & . & g_d(nT) \\ . & . & & & & . \\ . & . & & & & . \\ . & . & & & & . \\ g_d[N-1)T] & g_d(NT) & . & . & . & g_d[(N+n-2)T] \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ . \\ . \\ . \\ b_{n-1} \end{bmatrix} = - \begin{bmatrix} g_d(nT) \\ g_d(nT+T) \\ . \\ . \\ . \\ g_d[(N+n-1)T] \end{bmatrix} \quad (9)$$

where b_i are the coefficients of $\psi(z)$, as in (7). Furthermore, the system of equations:

$$\begin{bmatrix} 1 & 1 & . & . & . & 1 \\ z_1 & z_2 & . & . & . & z_n \\ . & . & & & & . \\ . & . & & & & . \\ . & . & & & & . \\ z_1^{N-1} & z_2^{N-1} & . & . & . & z_n^{N-1} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ . \\ . \\ . \\ A_n \end{bmatrix} = \begin{bmatrix} g_d(0) \\ g_d(T) \\ . \\ . \\ . \\ g_d(NT-T) \end{bmatrix} \quad (10)$$

is consistent and has a solution.

The function $g(t)$ is a interpolation of order n, when

$$g_d(kT)=g(kT), k=0,1,2,..,2n-1 \quad (11)$$

To determine the order n of the approximation the rank of the matrix $P(N,M)$ has to be found. This can be obtained by examining the determinant of the submatrix $P(k,k)$ consisted by the first k rows and the first k columns of $P(N,M)$ for k=1, 2,… and n is the dimension of the last non zero minor. However since $g_d(t)$ does not correspond to a linear time invariant system, the order n is considered as the minimum n so that:

$$\left|\det P(n+1,n+1)\right| < e_q \quad (12)$$

This is equivalent to saying that e_q is treated as an equivalent zero for the approximation and n columns of the submatrix $P(n,n)$ are linearly independent.

To determine z_i's, the coefficients of $\psi(z)$ are determined from the relation:

$$\begin{bmatrix} g_d(0) & g_d(T) & . & . & . & g_d(nT-T) \\ g_d(T) & g_d(2T) & . & . & . & g_d(nT) \\ . & . & & & & . \\ . & . & & & & . \\ . & . & & & & . \\ g_d(nT-T) & g_d(nT) & . & . & . & g_d(2nT-2T) \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ . \\ . \\ . \\ b_{n-1} \end{bmatrix} = - \begin{bmatrix} g_d(nT) \\ g_d(nT+T) \\ . \\ . \\ . \\ g_d(2nT-T) \end{bmatrix} \quad (13)$$

Having obtained the roots of $\psi(z)$, the A_i's are determined from the relation:

$$\begin{bmatrix} 1 & 1 & . & . & . & 1 \\ z_1 & z_2 & . & . & . & z_n \\ . & . & & & & . \\ . & . & & & & . \\ . & . & & & & . \\ z_1^{n-1} & z_2^{n-1} & . & . & . & z_n^{n-1} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ . \\ . \\ . \\ A_n \end{bmatrix} = \begin{bmatrix} g_d(0) \\ g_d(T) \\ . \\ . \\ . \\ g_d(nT-T) \end{bmatrix} \tag{14}$$

3.2. The first modified Prony method

A more accurate method of tracking the transfer function will be demonstrated in what follows.

The aim is to use the maximum possible number and not only the first $2n$ samples in order to obtain the approximation. The matrix $P(N,M)$ is formed again and linearly independent columns in the submatrix $P(N,k)$ consisted by the first N rows and the first k columns of $P(N,M)$ are sought.

If the first n+1 columns are linearly dependent, the minimum singular value of the matrix $P(N,n+1)$ is equal to zero. Similarly to the standard method, since $g_d(t)$ does not correspond to a linear time invariant system, the order n of the approximation is defined as the minimum n such that:

$$\left| s_{\min} P(N, n+1) \right| < e_q \tag{15}$$

Again e_q is treated as an equivalent zero for the approximation. Equations (9) and (10) now do not posses solutions and best approximations of the solutions minimizing the euclidian norm of the errors are obtained using generalized inverses. So the vectors containing the parameters A_i and s_i are determined.

3.3. The second modified Prony method

An even more accurate method of tracking the transfer function will be demonstrated in what follows.

The aim, again, is to use the maximum possible number and not only the first $2n$ samples in order to obtain the approximation. The matrix $P_1(2N,N)$ is now formed and linearly independent columns in the submatrix $P_1(2N\text{-}k,k)$ consisted by the first $2N$-k rows and the first k columns of $P(N,M)$ are sought.

If the first n+1 columns are linearly dependent, the minimum singular value of the matrix $P(2N\text{-}n\text{-}1,n+1)$ is equal to zero. Similarly to the standard method, since $g_d(t)$ does not correspond to a linear time invariant system, the order n of the approximation is defined as the minimum n such that:

$$\left| s_{\min_d} P(2N-n-1, n+1) \right| < e_q \tag{16}$$

Again e_q is treated as an equivalent zero for the approximation. Again, equations (9) and (10) do not posses solutions and best approximations of the solutions minimizing the Euclidian norm of the errors are obtained using generalized inverses. So the vectors containing the parameters A_i and s_i are determined.

$$P_1 = \begin{bmatrix} g(0) & g(T) & g(2T) & . & g(NT-T) \\ g(T) & g(2T) & g(3T) & . & g(NT) \\ g(2T) & g(3T) & . & . & . \\ g(3T) & . & . & . & g(2NT-2T) \\ & . & . & . & 0 \\ . & . & . & & 0 \\ . & . & g(2NT-2T) & . & 0 \\ . & g(2NT-2T) & 0 & . & 0 \\ g(2NT-2T) & 0 & 0 & . & 0 \end{bmatrix} \tag{17}$$

4. Applying the Approximation Methods.

There is an inherent difficulty in applying the Prony method to approximate the Heidler's equation. This waveform has a very fast mode and a slow mode so if a small sampling period T is chosen, the slow mode is not taken into account while if a large period T is selected the fast mode is ignored. To overcome this difficulty the Prony method is applied separately on each summand of the Heidler's equation. Since the systems considered are linear, applying the rule of superposition the overall approximation is the sum of the two approximations.

When plotted together, the output of each one of these methods (the impulse response), should ideally be the same as the Heidler's equation that is to be approximated.

4.1 Applying the Prony Method

If the Standard Prony method is applied on the Heidler's equation, the impulse response obtained is shown in Eq. 18 and presented below, in Figure 4.

$$G_P(s) = \frac{49.1851\left(\left(\frac{s}{10^9}\right)^2 + 0.4597\,\frac{s}{10^9} + 0.1229\right)}{\left(\left(\frac{s}{10^9}\right) + 0.3266\right)\left(\left(\frac{s}{10^9}\right) + 0.015\right)\left(\left(\frac{s}{10^9}\right)^2 + 2.293s + 1.956\right)} \tag{18}$$

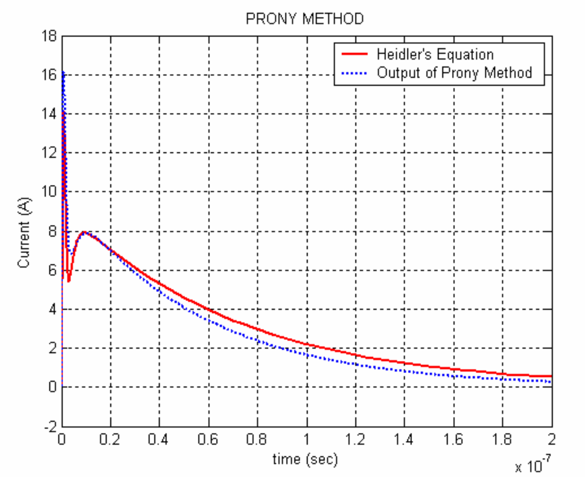

Fig. 4. Plot of the graph that occurs from the application of Prony method on the Heidler's equation, along with Heidler's equation.

4.2 Applying the first modified Prony Method

If the (first) modified Prony method is applied on the Heidler's equation, the impulse response obtained is shown in Eq. 19 and presented below, in Figure 5.

$$G_{1MP}=\frac{114.9985\left(\left(\frac{s}{10^{9}}\right)-2.463\right)\left(\left(\frac{s}{10^{9}}\right)+0.1656\right)\left(\left(\frac{s}{10^{9}}\right)^{2}+0.4118s+0.04784\right)}{\left(\left(\frac{s}{10^{9}}\right)+0.383\right)\left(\left(\frac{s}{10^{9}}\right)+0.1443\right)\left(\left(\frac{s}{10^{9}}\right)+0.0145\right)\left(\left(\frac{s}{10^{9}}\right)+1.311\right)}\cdot\frac{\left(\left(\frac{s}{10^{9}}\right)^{2}+6.655\left(\frac{s}{10^{9}}\right)+25.49\right)\left(\left(\frac{s}{10^{9}}\right)^{2}+9.526\left(\frac{s}{10^{9}}\right)+125.9\right)}{\left(\left(\frac{s}{10^{9}}\right)-2.419\right)\left(\left(\frac{s}{10^{9}}\right)^{2}+7.871s+32.53\right)\left(\left(\frac{s}{10^{9}}\right)^{2}+9.428\left(\frac{s}{10^{9}}\right)+124.8\right)} \quad (19)$$

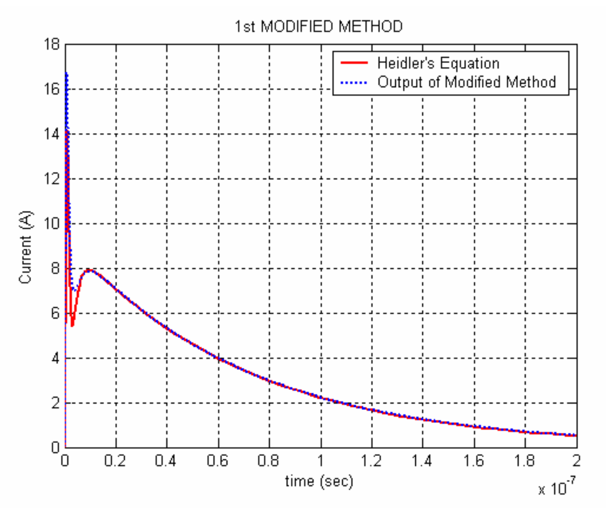

Fig. 5. Plot of the graph that occurs from the application of the first modified Prony method on the Heidler's equation, along with Heidler's equation.

4.3 Applying the second modified Prony Method

If the (second) modified Prony method is applied on the Heidler's equation, the impulse response obtained is shown in Eq. 20 and is presented below, in Figure 6.

$$G_{2MP}(s)=\frac{0.27614\cdot10^{9}+s}{0.23680\cdot10^{9}+s}\cdot\frac{3.27112\cdot10^{15}+s}{3.27753\cdot10^{9}+s}\cdot\frac{s}{1.21326\cdot10^{9}+s}\cdot\frac{251.86748\cdot10^{18}-17.04970\cdot10^{9}\cdot s+s^{2}}{2.15900\cdot10^{18}+2.57591\cdot10^{9}\cdot s+s^{2}}\cdot\frac{2.08342\cdot10^{18}+2.64707\cdot10^{9}\cdot s+s^{2}}{40.21040\cdot10^{18}+10.48931\cdot10^{9}\cdot s+s^{2}}\cdot\frac{4.55\cdot10^{-6}(0.20657\cdot10^{18}+0.46878\cdot10^{9}\cdot s+s^{2})}{0.00854\cdot10^{18}+0.60265\cdot10^{9}\cdot s+s^{2}} \quad (20)$$

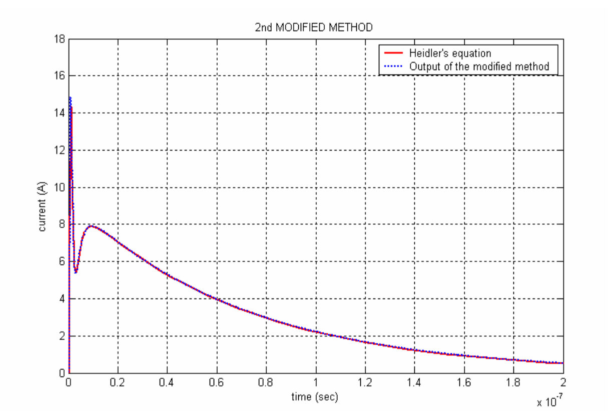

Fig. 6. Plot of the graph that occurs from the application of the second modified Prony method on the Heidler's equation, along with Heidler's equation.

4.4 Comparison of relative errors

The goal of making these successive modifications of the Prony method is the minimization of the relative error *RE* between the theoretical equation of Heidler (Eq. 1 as defined in [9]) and the impulse response obtained by the application of the Approximation methods. The relative error *RE* is given by the following equation:

$$RE(t_k)=\frac{|P(t_k)-H(t_k)|}{H(t_k)} \quad (21)$$

where $H(t_k)$ is the value of Heidler's equation on time:

$$t_k = kT\ ,\quad k=1, 2, .., \quad (22)$$

and $P(t_k)$ is the value of the impulse response extracted from the application of the (standard or modified) Prony methods.

With a sampling step of T=0.5nsec, the maximum and the average relative error norms of these values are displayed in Tab. 3. The errors have been calculated for t_k=0÷200nsec, as well as t_k=2÷60nsec, which is considered to be one of high interest.

Table 3. Relative error norms calculated for the application of each method.

Method	t_k	Relative error norm	
		maximum (%)	average (%)
Prony method	2–60 ns	65.8191	2.8207
	0–200 ns	65.8191	4.5290
1ST modified method	2–60 ns	49.2468	2.4532
	0–200 ns	49.2468	0.7334
2ND modified method	2–60 ns	11.3470	0.1624
	0–200 ns	11.3470	0.3792

5. The circuit

Evaluating the results of the methods applications', the second modified method is chosen to be applied on the Heidler's equation.

The above transfer function is realized as a cascade connection of biquads. The pole-zero pairing and the place of each biquad were selected using standard rules. For each one of the first three factors, a circuit of the form of Figure 7 was considered.

For creating a circuit that corresponds to the three last factors of the expression of the transfer function G(s), the Tow-Thomas circuit, depicted in Figure 8, was used.

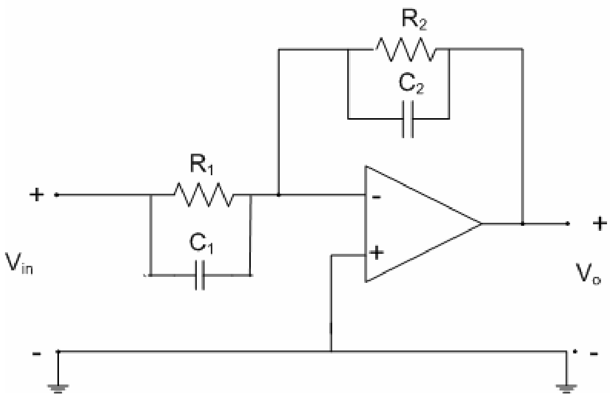

Fig. 7. Form of the inverting connection used to design circuit that corresponds to factors 1 to 7 of the transfer function.

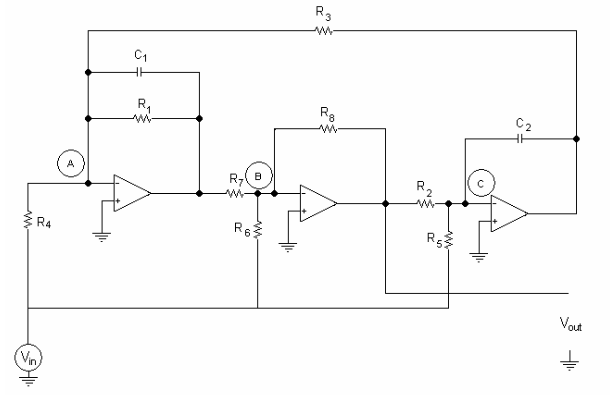

Fig. 8. Form of the connection used to design circuit that corresponds to the last factor of the transfer function, the Tow-Thomas circuit.

The simulation of the circuit in Fig. 9 with ideal operational amplifiers, in PSpice gave the output, shown in Fig 10. In the same figure Heidler's equation is presented in order to evaluate the behavior of the circuit.

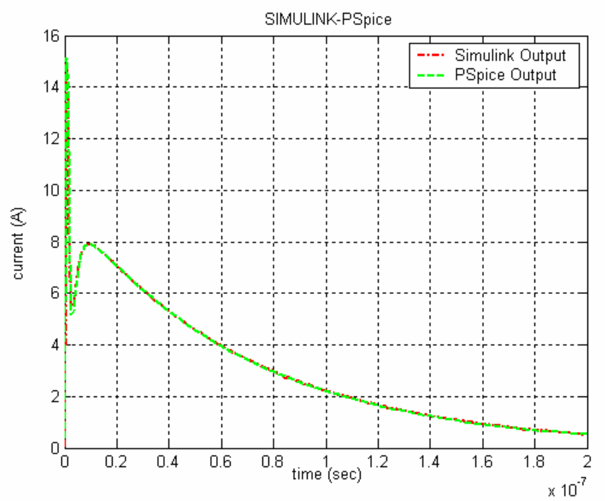

Fig. 10. Plot of the simulation of the circuit, along with the Heidler's equation, as defined in [9].

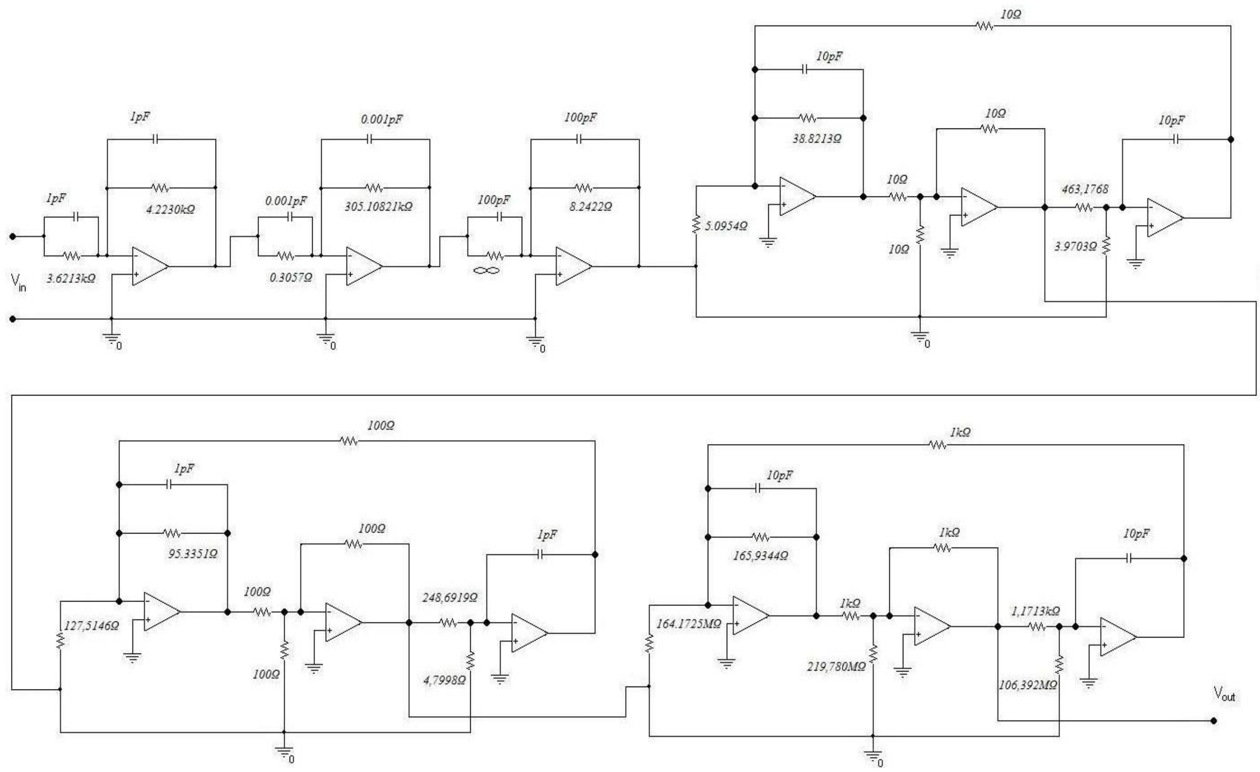

Fig. 9. The circuit that produces the ESD current at +4kV.

6. Conclusions

The problem of the aberration between the circuit proposed by the IEC 61000-4-2 [1] and the typical ESD current waveform included in the Standard is approached in this paper.

Prony method is used to obtain an approximation of the transfer function of a circuit with a known output. Prony method had to be re-checked and modified twice in order to be able to give results when applied on

Heidler's equation, which is one with very sophisticated composition. This improvement makes the method more efficient in other applications of similar difficulty.

The transfer function acquired in the above way was used to design the components of a circuit that produces the current of Electrostatic Discharge as it is described with Heidler's equation.

The improvement of the method, gave very accurate results for the ESD current waveshape. The approximation of the Heidler's equation is one with a very small error.

What has been done here is a realization of the circuit with ideal components. The most difficult aspect we had to deal with, is that operational amplifiers of vast Gain Bandwidth Product are needed, in order to design the active R C circuit producing the current of ESD.

Speaking with numbers, a plain observation of Eq. 20, can easily show that opamps with G.BW.P. at least 1000GHz are needed in order to have a satisfactory operation of the circuit.

In the present time no such opamps exist. In recent papers an effort to built very fast opamps is being made, as in [10] but the values of GBWP sought, are yet to be achieved. The experience in the progress of semi-conducting materials though, gives us well-grounded hopes soon to come across opamps like the ones we need.

7. Acknowledgement

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