MATHEMATICAL ANALYSIS AND SIMULATION FOR THE ELECTROSTATIC DISCHARGE (ESD) ACCORDING TO THE EN 61000-4-2

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ABSTRACT

This paper is a study for the better understanding of the Electrostatic Discharge (ESD). Electrostatic discharge can change the electrical characteristics of a semiconductor device, degrading or destroying it. Both a mathematical analysis and a simulation process of the ESD generator's circuit of the European Standard 61000-4-2 are presented. In the mathematical aspect of the problem the differential equations for different types of EUT (Equipment Under Test) are solved. In the second part of this paper a computer simulation of these two types of circuits is shown, using both PSPICE and Matlab programs. A comparison between theoretical results of the mathematical analysis and the two simulation processes is obtained. Finally, conclusions for the ESD phenomenon are presented.

Keywords: Electrostatic Discharge (ESD), EN 61000-4-2, Computer simulation, ESD generator, Differential Equations, Human Body model (HBM)

1. INTRODUCTION

The age of electronics brought with it new problems associated with static electricity and electrostatic discharge (ESD) and as electronic devices became faster and smaller, their sensitivity to ESD increased. ESD phenomena in industrial environments occur frequently and when problems do occur, solutions are randomly applied until the symptoms go away. Designers are often not sure whether the problem has really been solved, because they do not understand how to perform accurate verification tests [1, 2].

ESD is defined as the sudden transfer of charge between objects at different electrostatic potentials [3]. Electrostatic charge is most commonly created by the contact and separation of two materials. Creating electrostatic charge by contact and separation of materials is known as "triboelectric charging." It involves the transfer of electrons between materials. The atoms of a material with no static charge have an equal number of positive (+) protons in their nucleus and negative (-) electrons orbiting the nucleus. For example, people walking across the floor generate static electricity as shoe soles contact and then separate from the floor surface.

Electrostatic discharge can change the electrical characteristics of a semiconductor device, degrading or destroying it. It may also upset the normal operation of an electronic system, causing equipment malfunction or failure. The very fast pulses, especially the initial peak can cause serious problems for high-speed digital systems. ESD may cause damage by its currents and by its field. Many researchers have made considerable studies on ESD current waveforms and it has been shown that the amplitudes and risetimes vary with the charging voltages, approach speeds, electrode types and humidity [4-9].

ESD has become an independent field of EMC due to its numerous research fields and aspects [10]. One of these aspects is the immunity study against ESD. Most of the ESD procedures are according to the latest European Standard 61000-4-2 [11]. The object of this standard is to establish a common and reproducible basis for evaluating the performance of electrical and electronic equipment when subjected to electrostatic discharges. This standard is based on the Human Body Model, which simulates the electrostatic discharge of the human body on electrical or electronic equipment.

In this paper an analytical mathematical approach of the circuit of the ESD generator of the European Standard 61000-4-2 taking into consideration the inductance is presented. Also, simulation processes using both PSPICE and Matlab programs for the circuit mentioned is presented for the elements of the ESD generator circuit that the Standard defines.

2. THE EUROPEAN STANDARD 61000-4-2

The European standard 61000-4-2 relates to equipment, systems, sub-systems and peripherals which may be involved in static electricity discharges owing to environmental and installation conditions, such as low relative humidity, use of low conductivity carpets etc., which may exist in allocations classified in standards relevant to electrical and electronic equipment [11]. According to the standard the electrostatic discharges can occur either as contact discharges or as air discharges. Contact discharge is the preferred test method and air discharges shall be used in cases, where contact discharges cannot be applied. The range of the test level voltages for the contact discharges is 2 to 8 kV and for the air discharges is 2 to 15 kV.

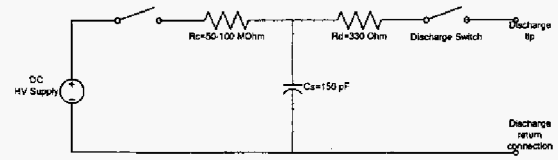

Figure 1: Simplified diagram of the ESD generator

Figure 1 shows a simplified diagram of the ESD generator. According to the standard it consists of the charging resistor R_c (50-100 MΩ), the energy-storage capacitor C_S (150pF ± 10%), the discharge resistor R_d (330Ω ± 10%) and the EUT (Equipment Under Test). The values mentioned have been selected according to the Human Body Model (HBM). The value of the energy-storage capacitor C_S is representative of the value of the human body, while the resistance of 330 Ω has been chosen to represent the source resistance of a human body holding a metallic object such as a key or tool. Also, in figure 1 two switches are depicted. When the first switch is closed, the second is opened in order the capacitor to be charged. After the capacitor's charge, the first switch opens and the second closes and so the electrostatic discharge on the EUT occurs. These two switches are known as relays and they are the only triggering device known today, which is able to produce repeatable and fast rising discharge currents.

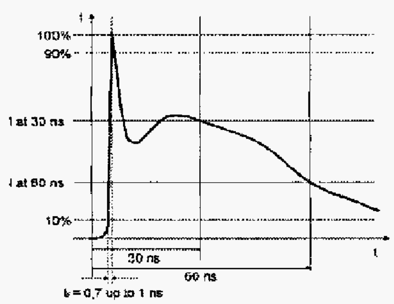

Figure 2: Typical waveform of the output current of the ESD generator [4]

The HBM pulse that the ESD generator must produce is shown in figure 2. This pulse is divided in two parts: A first peak called as "Initial Peak", caused by a discharge of the hand and a second peak, which is caused by a discharge of the body. The rise time of the Initial Peak is between 0.7 ns and 1 ns and its amplitude depends on the charging voltage of the ESD simulator.

3. SOLVING THE DIFFERENTIAL EQUATIONS OF THE ESD GENERATOR'S CIRCUIT.

At this point it is fundamental to analyse the circuit of the ESD generator [11], which is proposed by the European Standard. This analysis is divided into two parts. In the first part the inductance is ignored, while in the second part we take into consideration the effect of the inductance that the equipment on which the Electrostatic Discharge takes place may have. In both cases the equation of the current on the EUT is calculated, solving the differential equations of the circuit. In general for the RLC circuit it is considered that the EUT has an inductance and a resistance ($Z_{EUT}=R_{EUT}+jX_{EUT}$) as it can be seen in figure 3.

RC circuit analysis

For the RC circuit the inductance of the EUT is ignored and it assumed that the EUT is only an ohmic resistance ($Z_{EUT}=R_{EUT}$). The initial conditions of the circuit as it is shown in figure 3 for $L_{EUT}=0$, are $I(0)=0$ and $U_C(0)=V_0$, (V_0 is the initial voltage of the capacitor) and the switches 1 and 2 are opened and closed respectively. Solving the differential equation of the circuit the capacitor's voltage is calculated from equation (1):

$$U_R+U_C=0 \Rightarrow RC\frac{dU_c(t)}{dt}+U_c(t)=0$$

$$\Rightarrow \frac{U'_c(t)}{U_c(t)}=-\frac{1}{RC} \Rightarrow \left.\begin{array}{l} U_c(t)=K\cdot e^{-\frac{t}{RC}} \\ U_c(0)=V_0 \end{array}\right\} \Rightarrow$$

$$U_C(t)=V_0\cdot e^{-\frac{t}{\tau}},\ t\geq 0 \qquad (1)$$

where τ (τ=RC, $R=R_{EUT}+R_d$) is the time constant

Consequently the current can be obtain as:

$$i(t)=\frac{V_0}{R}\cdot e^{-\frac{t}{\tau}},\ t\geq 0^+ \qquad (2)$$

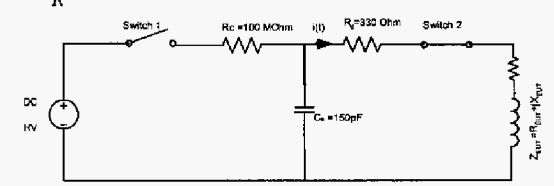

Figure 3: The circuit of the ESD during the discharge taking into consideration the impedance of the EUT

RLC circuit analysis

Taking into consideration the inductance of the EUT ($Z_{EUT}=R_{EUT}+jX_{EUT}$) the initial conditions of the circuit as it is shown in figure 3 are also $I(0)=0$ and $U_C(0)=V_0$.

Solving the differential equation of the circuit the capacitor's voltage is calculated from equation (3):

$$\left.\begin{array}{l} U_C(t)+L\cdot\frac{di(t)}{dt}+R\cdot i(t)=0 \\ i(t)=C\cdot\frac{dU_c(t)}{dt} \\ i(0)=0, U_c(0)=V_0 \end{array}\right\} \Rightarrow$$

$$L\cdot C\cdot\frac{d^2 i}{dt^2}+R\cdot C\cdot\frac{di}{dt}+i(t)=0 \qquad (3)$$

or equivalently

$$L\cdot C\cdot\frac{d^2U_c}{dt^2}+R\cdot C\cdot\frac{dU_c}{dt}+U_c(t)=0 \qquad (4)$$

The analytical solution of equation (2) is presented in the Appendix. Synoptically the current i(t) is given by equation (5):

$$i(t)=\begin{cases} C\cdot\dfrac{\lambda_1\cdot\lambda_2}{\lambda_1-\lambda_2}\cdot V_0\cdot\left(e^{\lambda_2\cdot t}-e^{\lambda_1\cdot t}\right),\ if\ R>2\sqrt{\dfrac{L}{C}} \\ \dfrac{C\cdot R}{2\cdot L}\cdot V_0\cdot\left(\dfrac{R}{2\cdot L}\cdot t-2\right)\cdot e^{-\frac{R}{2L}\cdot t},\ if\ R=2\sqrt{\dfrac{L}{C}} \\ C\cdot V_0\cdot e^{a\cdot t}\left(-\dfrac{\alpha^2}{\beta}\cdot\sin(\beta t)-\beta\cdot\sin(\beta t)\right),\ if\ R<2\sqrt{\dfrac{L}{C}} \end{cases} \quad (5)$$

where λ_1, λ_2, α, β are coefficients defined in the Appendix.

4. PSPICE SIMULATION FOR THE CIRCUIT OF THE EUROPEAN STANDARD 61000-4-2

In order to confirm the equations' solutions as they described in the previous paragraph and the waveform that it has to be produced by the ESD generator, the PSPICE program is used for various types of EUT [8]. The waveform of the current on the EUT is presented. Firstly, the PSPICE simulation was made for both the RC and the RLC circuit for a DC voltage of 2 KV. For the RC circuit it is considered that the EUT is only an ohmic resistance with values 10 Ω, 120 Ω and 1 KΩ. The program's results for the current can be seen in figure 4.

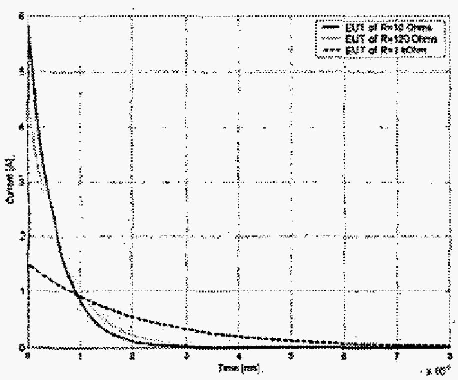

Figure 4: Current waveforms from the PSPICE program for the RC circuit with ohmic EUT.

Three sets of values for the resistance R_{EUT} and the inductance L_{EUT} of the EUT are examined below:

- R_{EUT}=10 Ω and L_{EUT}=1 μH ($R_{tot} > 2\sqrt{\frac{L}{C}}$)
- R_{EUT}=10 Ω and L_{EUT}=4,335 μH ($R_{tot} = 2\sqrt{\frac{L}{C}}$)
- R_{EUT}=10 Ω and L_{EUT}=20 μH ($R_{tot} < 2\sqrt{\frac{L}{C}}$)

where R_{tot} is the total resistance for the RLC circuit (R_{tot}=R_d + R_{EUT}) when the switches 1 and 2 are open and closed respectively. The circuits and the respective waveforms of the program are shown in figure 5.

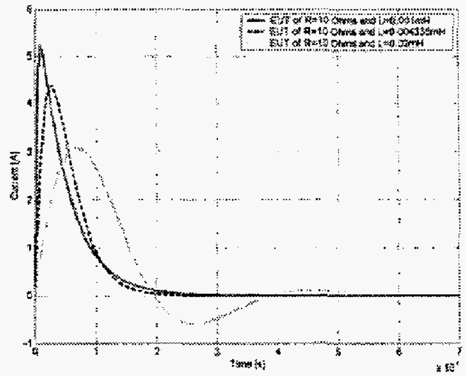

Figure 5: Current waveforms from the PSPICE program for the RLC circuit for EUT with resistance and inductance.

5. MATLAB SIMULATION FOR THE CIRCUIT OF THE EUROPEAN STANDARD 61000-4-2

In order to simulate the ESD generator of the European Standard 61000-4-2, the Matlab program has been used, as well with the PSPICE program described in the previous paragraph. The simulation was made only for the RC circuit for a DC voltage of 2 KV. The simulation for the RLC circuit could not be made because in Matlab the modelling of the non linear elements has been done with current sources. Consequently, the inductance cannot be in series with the current source. This is a disadvantage of Matlab in comparison with PSPICE.

The circuit as it was designed in Matlab can be seen in figure 6. The routine of arithmetic integration ode23t (mod.stiff/Trapezoidal) was used. For the RC circuit it is considered that the EUT has the same values of 10 Ω, 120 Ω and 1 KΩ mentioned in paragraph 4 for the PSPICE simulation. The program results for the voltage and the current can be seen in figure 7.

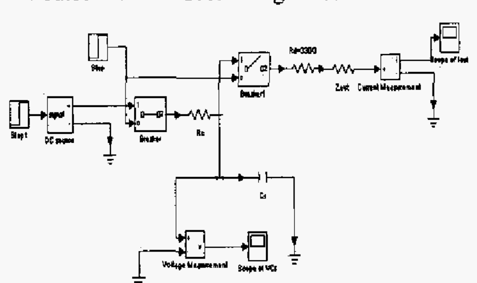

Figure 6: Circuit of the ESD generator as it was designed in Matlab.

In figure 8 it is shown a very good congruity between PSPICE and Matlab for ohmic EUT. These results is a proof of the reliability of these two programs and that anyone wanting to work in circuits analysis should choose taking into consideration the program that fits better to his needs.

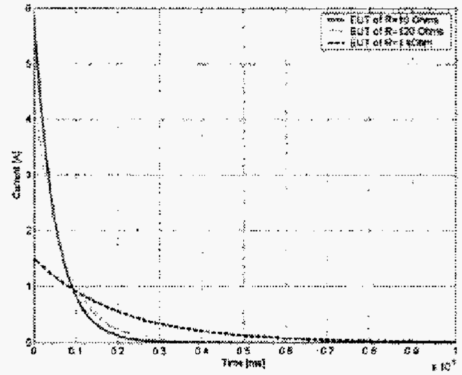

Figure 7: Current waveforms from the Matlab program for the RC circuit with ohmic EUT.

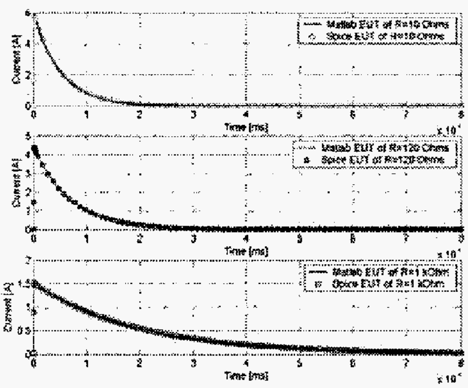

Figure 8: PSPICE and Matlab comparison for the current waveforms for the EUT mentioned.

6. CONCLUSIONS

Comparing the PSPICE program with Matlab it is obvious that the PSPICE program is better. The weakness of Matlab is that it is unable to make the simulation for an RLC circuit, because the inductance cannot be in series with the current source. This happens because in Matlab the modelling of the non linear elements has been done with current sources.

From the previously PSPICE simulation it is obvious that for both circuits types (RC and RLC) the smaller the impedance Z_{EUT} ($Z_{EUT} = \sqrt{R_{EUT}^2 + L_{EUT}^2}$) the shorter the rise time and the greater the peak current ($I = \frac{U}{Z}$). Specifically, from figure 5 it can be seen that the presence of the inductance decreases the peak current and increases the rise time. This difference is more distinct as the value of L_{EUT} increases. The conclusions are the same for the Matlab simulation for the RC circuits as it can be seen from figure 8. This result is really helpful for the better understanding of ESD and specifically on how should be the current waveforms that should be expected for various EUT, when ESD occurs on them during laboratory tests.

This observation can be helpful to the better understanding of the ESD phenomenon. The influence of the inductance to the effect of ESD damage can not be neglected, since it affects two important factors of the ESD phenomenon, the rise time and the peak current. Certainly, the reliability is the same for the two programs, despite the weakness of Matlab.

7. REFERENCES

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APPENDIX

The characteristic equation of the 2nd class homogeneous differential equation:

$$L\cdot C\cdot\frac{d^2U_c}{dt^2}+R\cdot C\cdot\frac{dU_c}{dt}+U_c(t)=0 \quad (A.1)$$

is: $L\cdot C\cdot\lambda^2+R\cdot C\cdot\lambda+1=0$ (A.2)

where $\Delta=(RC)^2-4LC$

Taking cases for Δ the below cases are examined:

- If $\Delta>0\Rightarrow R>2\sqrt{\frac{L}{C}}$

then the equation A.1 has real roots λ_1,λ_2. Consequently the solution of A.1 is:

$$U_C(t)=C_1e^{\lambda_1 t}+C_2e^{\lambda_2 t} \quad (A.3)$$

where C_1, C_2 are coefficients and:

$$\lambda_{1,2}=-\frac{R}{2L}\pm\sqrt{\left(\frac{R}{2L}\right)^2-\frac{1}{LC}} \quad (A.4)$$

Placing:

$$\alpha=\frac{R}{2L} \quad (A.5)$$

and $\beta=\sqrt{\left(\frac{R}{2L}\right)^2-\frac{1}{LC}}$, then (A.6)

$$\lambda_{1,2}=-\alpha\pm\beta \quad (A.7)$$

In order the coefficients C_1, C_2 to be calculated the below system of equations A.8 and A.9 is being solved:

$$\left.\begin{array}{l} i(0)=0\Rightarrow C\frac{dU_C(0)}{dt}=0\Rightarrow\frac{dU_C(0)}{dt}=0 \\ U_C'(t)=C_1\lambda_1e^{\lambda_1 t}+C_2\lambda_2e^{\lambda_2 t}\end{array}\right\}\Rightarrow$$

$$C_1\lambda_1+C_2\lambda_2=0 \quad (A.8)$$

and from A.3 $U_c(0)=V_0\Rightarrow C_1+C_2=0$ (A.9)

The solution of the above system is:

$$U_c(t)=\frac{\lambda_2V_0}{\lambda_2-\lambda_1}e^{\lambda_1 t}+\frac{\lambda_1V_0}{\lambda_1-\lambda_2}e^{\lambda_2 t} \quad (A.10)$$

where λ_1, λ_2 are given by the equations A.4 to A.7. Consequently the current i(t) is:

$$i(t)=C\cdot\frac{dU_C(t)}{dt}\Rightarrow\quad i(t)=C\cdot\frac{\lambda_1\cdot\lambda_2}{\lambda_1-\lambda_2}\cdot V_0\cdot\left(e^{\lambda_2\cdot t}-e^{\lambda_1\cdot t}\right) \quad (A.11)$$

The coefficient $\alpha=\frac{R}{2L}$ is known as damping constant and it is measured in sec^{-1}. The coefficient ω_0 is known as resonance frequency of the RLC circuit in series and it is measured in rad/sec.

It is: $\omega_0=2\pi f_0$ and: $\beta=\sqrt{\alpha^2-\omega_0^2}$

Also the quality factor Q of the RLC circuit is defined as:

$$Q=\frac{\omega_0}{2\alpha}=\frac{L\omega_0}{R}=\frac{1}{RC\omega_0}=\frac{1}{R}\sqrt{\frac{L}{C}} \quad (A.12)$$

This case where $R>2\sqrt{\frac{L}{C}}$ or $\alpha>\omega_0$ or $Q<\frac{1}{2}$ is known as over damped oscillation.

- If $\Delta=0\Rightarrow R=2\sqrt{\frac{L}{C}}$ or $\alpha=\omega_0$ or $Q=\frac{1}{2}$

then the equation A1 has two double real roots λ_1,λ_2. Consequently the solution of A.1

is: $\lambda_1=\lambda_2=\lambda=-\frac{R}{2L}=-\alpha$ (A.13)

Consequently the solution is:

$$U_C(t)=C_1\cdot e^{\lambda t}+C_2\cdot t\cdot e^{\lambda t} \quad (A.14)$$

C_1, C_2 are calculated from the system below:

$$\begin{cases}C_1=V_0\\ C_1\lambda+C_2=0\end{cases}\Rightarrow\begin{cases}C_1=V_0\\ C_2=-V_0\lambda\end{cases}$$

Consequently:

$$U_C(t)=V_0\left[1-\frac{R}{2L}t\right]e^{-\frac{R}{2L}t} \quad (A.15)$$

The current i(t) is:

$$i(t)=\frac{C\cdot R}{2\cdot L}\cdot V_0\cdot\left(\frac{R}{2\cdot L}\cdot t-2\right)\cdot e^{-\frac{R}{2L}t} \quad (A.16)$$

This case where $R=2\sqrt{\frac{L}{C}}$ or $\alpha=\omega_0$ or $Q=\frac{1}{2}$ is known as damped oscillation.

- If $\Delta<0\Rightarrow R<2\sqrt{\frac{L}{C}}$ or $\alpha<\omega_0$ or $Q>\frac{1}{2}$ then the equation A1 has two complex roots λ_1,λ_2:

$$\lambda_{1,2}=\frac{-RC\pm i\sqrt{4LC-(RC)^2}}{2LC}=-\frac{R}{2L}\pm i\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}\right)^2} \quad (A.17)$$

or $\lambda_{1,2}=-\alpha\pm i\beta$ (A.18)

Consequently the solution is:

$$U_C(t)=e^{\alpha t}\cdot\left[C_1\cos(\beta t)+C_2\sin(\beta t)\right] \quad (A.19)$$

C_1, C_2 are calculated from the system:

$$\left.\begin{array}{l} i(0)=0\Rightarrow\alpha C_1+\beta C_2=0\\ U_C(0)=V_0\Rightarrow C_1=V_0\end{array}\right\}\Rightarrow\begin{cases}C_1=V_0\\ C_2=-\frac{\alpha V_0}{\beta}\end{cases}$$

Consequently:

$$U_c(t)=e^{\alpha t}\cdot V_0\left[\cos(\beta\cdot t)-\frac{\alpha}{\beta}\sin(\beta\cdot t)\right] \quad (A.20)$$

The current i(t) is:

$$i(t)=C\cdot V_0\cdot e^{\alpha t}\left(-\frac{\alpha^2}{\beta}\cdot\sin(\beta\cdot t)-\beta\cdot\sin(\beta\cdot t)\right) \quad (A.21)$$

where α and β are calculated from A.5, A.6.

This case where $R<2\sqrt{\frac{L}{C}}$ is known as under damped oscillation.

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