An extension of the Georgiou-Smith example: Boundedness and attractivity in the presence of unmodelled dynamics via nonlinear PI control

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Abstract

In this paper, a nonlinear extension of the Georgiou-Smith system is considered and robustness results are proved for a nonlinear PI controller with respect to fast parasitic first-order dynamics. More specifically, for a perturbed nonlinear system with sector bounded nonlinearity and unknown control direction, sufficient conditions for global boundedness and attractivity have been derived. It is shown that the closed loop system is globally bounded and attractive if (i) the unmodelled dynamics are sufficiently fast and (ii) the PI control gain has the Nussbaum function property. For the case of nominally unstable systems, the Nussbaum property of the control gain appears to be crucial. A simulation study confirms the theoretical results.

1 Introduction

The unknown control direction problem has attracted significant research interest over the last three decades. Nussbaum gains [1], [2] have become the main theoretical tool for the problem. Nussbaum functions (NFs) are continuous functions $N(\cdot)$ with the property

$$\limsup_{\zeta\to\pm\infty}\frac{1}{\zeta}\int_0^{\zeta} N(s)ds = +\infty \tag{1}$$

$$\liminf_{\zeta\to\pm\infty}\frac{1}{\zeta}\int_0^{\zeta} N(s)ds = -\infty. \tag{2}$$

Examples of NFs are $\zeta^2\cos(\zeta)$, $\exp(\zeta^2)\sin(\zeta)$ among others.

For the simple integrator $\dot{y} = bu$ with b a nonzero constant of *unknown sign*, standard analysis [1] shows that the Nussbaum control law

$$\begin{aligned} u &= \zeta^2\cos(\zeta)y \\ \dot{\zeta} &= y^2 \end{aligned} \tag{3}$$

ensures convergence of the output y to the origin and boundedness of the Nussbaum parameter ζ. However, Georgiou and Smith demonstrated in [3] that the proposed controller is nonrobust to fast parasitic unmodelled dynamics. Particularly, they considered the system

$$\begin{aligned} \dot{x} &= bu \\ \dot{y} &= M(x-y) \end{aligned} \tag{4}$$

and showed divergence for $M > 1$ when the controller (3) is used.

An alternative nonlinear PI methodology was proposed by Ortega, Astolfi and Barabanov in [4] to address the unknown control direction problem. For the simple integrator case, their controller takes the form

$$u = z\cos(z)y \tag{5}$$

with z a PI square error defined by $z = (1/2)y^2 + \lambda\int_0^t y^2(s)ds$. The main difference between the two controllers (3), (5) is the existence of the proportional term in the control gain of (5) (see also p. 166 of [5]). It was hinted in [4],[5] that such a controller is robust to fast parasitic first-order perturbations and therefore can stabilize the Georgiou-Smith example system if $\lambda < M$. Their argument, however, was based on the fact that the related transfer function is positive real and cannot be carried over to the case of an unstable unforced linear system or even a nonlinear system. In fact, the introduction of a simple destabilizing pole in the system

$$\begin{aligned} \dot{x} &= \alpha x + bu \\ \dot{y} &= M(x-y) \end{aligned} \tag{6}$$

($\alpha > 0$) may result in instability of the closed-loop system with the controller (5) even if $\alpha + \lambda < M$ (see Section 3).

It remains therefore an open problem to design a nonlinear PI controller robust to fast parasitic dynamics when the plant to be controlled is originally unstable and nonlinear. To this end, we consider an extension of the Georgiou-Smith system. Particularly, we examine the overall stability of the nonlinear system with first-order unmodelled dynamics given by

$$\begin{aligned} \dot{x} &= f(x) + bu \\ \dot{y} &= M(x - y) \end{aligned} \tag{7}$$

with a nonlinear PI control law u designed for the unperturbed system

$$\dot{y} = f(y) + bu. \tag{8}$$

Function $f(\cdot)$ is assumed a sector-bounded nonlinearity, i.e. $f(0) = 0$, $f(y) = y\alpha(y)$ and there exist some constants $\alpha_1, \alpha_2 \in \mathbb{R}$ such that $\alpha_1 \leq \alpha(y) \leq \alpha_2$ $\forall y \in \mathbb{R}$. A controllability assumption is also imposed, that is $b \neq 0$.

1.1 Nonlinear PI for the unperturbed system

For system (8), consider a nonlinear PI controller of the form

$$u = \kappa(z) y \tag{9}$$

$$z = (1/2)y^2 + \lambda \int_0^t y^2(s) ds \tag{10}$$

with $\lambda > 0$ and PI gain $\kappa(z) = z^2 \sin z$. A more general approach to arbitrary NFs is also possible but is omitted due to space limitations. We can now analyze the closed-loop system (8)-(10) similarly to [4], [5]. Let us define $\xi := \int_0^t y^2(s) ds$, the augmented state vector $x_{ag} := [y, \xi]^T$ and the dynamical system $\dot{x}_{ag} = \tilde{f}(x_{ag}) := [f(y) + b\kappa(y^2/2 + \lambda\xi)y, y^2]^T$. Function $\tilde{f}$ is locally Lipschitz w.r.t. x_{ag} and therefore a unique maximal solution exists for $\dot{x}_{ag} = \tilde{f}(x_{ag})$ within a time interval $[0, T)$ for some $T > 0$ [6], [7]. From (8)-(10) we obtain

$$\dot{z} \leq \big[\max\{|\alpha_1|, |\alpha_2|\} + \lambda + b\kappa(z)\big] y^2.$$

Note that whenever $z(t) = z_k$ with $z_k := (\pi/2)[4k + 2 + \mathrm{sgn}(b)]$ we have

$$\dot{z}(t) \leq \big[\max\{|\alpha_1|, |\alpha_2|\} + \lambda - 4|b|k^2\pi^2\big] y^2(t). \tag{11}$$

Thus, $\dot{z}(t) \leq 0$ whenever $z(t) = z_k$ for every $k \geq k' := \lceil (\max\{|\alpha_1|, |\alpha_2|\} + \lambda)^{1/2}/(2\pi\sqrt{|b|})\rceil$ ($\lceil x \rceil$ denotes the smallest integer greater or equal than x) which in turn implies that z is upper bounded by $z(t) \leq z_{k_0}$ for all $t \in [0, T)$ where $k_0 := \max\{k', \lceil y^2(0)/4\pi \rceil\}$. Hence x_{ag} is bounded within the compact set $W := \{x_{ag} | \xi \geq 0$ and $(1/2)y^2 + \lambda\xi \leq z_{k_0}\}$ for all $t \in [0, T)$. Using a contradiction argument, the boundedness property of x_{ag} ensures that $T = \infty$ i.e. the solution can be extended up to infinity (see Theorem 3.3 of [7] or Proposition 8.5 in [6]). Thus $z \in \mathcal{L}_\infty$ which in turn implies $y \in \mathcal{L}_\infty \cap \mathcal{L}_2$ and $u \in \mathcal{L}_\infty$ from (10) and (9) respectively. Also, from (8) we have $\dot{y} \in \mathcal{L}_\infty$. Barbalat's lemma can now be recalled to show that $\lim_{t\to\infty} y(t) = 0$.

Assume now the existence of parasitic first order unmodelled dynamics in the form of (7). Sufficient conditions are given in the next section for *global boundedness and attractivity* for the closed-loop system comprised from (7) and the nonlinear PI controller (9) and (10).

2 Extended Georgiou-Smith system with sector nonlinearity

In this section we consider system (7) with a sector-bounded nonlinearity

$$f(x) = \alpha(x)x\,, \quad \alpha(x) \in [\alpha_1, \alpha_2] \qquad \forall x \in \mathbb{R} \tag{12}$$

for some $\alpha_1, \alpha_2 \in \mathbb{R}$. Note that α_1, α_2 can also take positive values rendering the unforced system unstable. We have established the following theorem.

Theorem 1. *Let the perturbed system described by*

$$\begin{aligned} \dot{x} &= f(x) + bu \\ \dot{y} &= M(x - y) \end{aligned} \tag{13}$$

with sector-bounded nonlinearity given by (12) *and control law*

$$u = \kappa(z)y \tag{14}$$

$$z = (1/2)y^2 + \lambda \int_0^t y^2(s)ds. \tag{15}$$

If

(i) $0 < \lambda < M - \max\{0, \alpha_2\}$

(ii) *$\kappa(z)$ has the Nussbaum property* (1),(2)

(iii) $\frac{2\lambda}{\sqrt{M-\lambda}}\left[\sqrt{M-(\lambda+\alpha_1)}+\sqrt{M-(\lambda+\alpha_2)}\right] \geq \alpha_2 - \alpha_1$

then, all closed-loop signals are bounded and $\lim_{t\to\infty} y(t) = \lim_{t\to\infty} x(t) = 0$.

Proof. Define now $\xi := \int_0^t y^2(s)ds$, the augmented state vector $x_{p,ag} := [x, y, \xi]^T$ and the dynamical system $\dot{x}_{p,ag} = \tilde{f}_p(x_{p,ag}) := [f(x) + b\kappa(y^2/2 + \lambda\xi)y, M(x-y), y^2]^T$. Function $\tilde{f}_p$ is locally Lipschitz w.r.t. $x_{p,ag}$ and therefore a unique maximal solution exists for $\dot{x}_{p,ag} = \tilde{f}_p(x_{p,ag})$ within a time interval $[0, t_f)$ for some $t_f > 0$ [6], [7]. From the definition of the PI error z in (15) and (13) we have that

$$\dot{z} = Mxy - (M-\lambda)y^2. \tag{16}$$

Consider now the function S given by

$$S := \frac{\lambda}{2}x^2 + \frac{1}{2}(M-\lambda)(x-y)^2 + \frac{c}{M}z - b\int_0^z \kappa(s)ds \tag{17}$$

with $c \in \mathbb{R}$ to be defined. Substituting from (13), (14), (15), (16) we have for its time derivative that

$$\begin{aligned}\dot{S}(t) &= \lambda x\dot{x} + (M-\lambda)(x-y)(\dot{x}-\dot{y}) + \frac{c}{M}\dot{z} - b\kappa(z)\dot{z} \\ &= [\lambda x + (M-\lambda)(x-y)][\alpha(x)x + b\kappa(z)y] - M(M-\lambda)(x-y)^2 \\ &\quad + \frac{1}{M}\left(c - bM\kappa(z)\right)\left[Mxy - (M-\lambda)y^2\right], \quad \forall t \in [0, t_f).\end{aligned} \tag{18}$$

In (18) the terms involving $\kappa(z)$ can be cancelled and we result in

$$\begin{aligned}\dot{S}(t) &= \lambda\alpha(x)x^2 - M(M-\lambda)(x-y)^2 \\ &\quad + (M-\lambda)\alpha(x)x(x-y) + cxy - \frac{c}{M}(M-\lambda)y^2\,, \ \forall t \in [0, t_f).\end{aligned} \tag{19}$$

To simplify notation we define the constant $\epsilon := 1/M$. Eq. (19) can then be written in matrix form as

$$\dot{S}(t) = -M \begin{bmatrix} x(t) & y(t) \end{bmatrix} \Lambda(x(t)) \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}, \ \forall t \in [0, t_f) \tag{20}$$

where $\Lambda(x)$ is defined by

$$\Lambda(x) := \begin{bmatrix} M-(\lambda+\alpha(x)) & -\frac{1}{2}[c\epsilon+(M-\lambda)(2-\epsilon\alpha(x))] \\ * & (M-\lambda)(c\epsilon^2+1) \end{bmatrix} \tag{21}$$

and $*$ denotes a symmetric w.r.t. the main diagonal element of $\Lambda(x)$. We claim that there is some constant $c \in \mathbb{R}$ such that $\Lambda(x)$ is positive definite for all $x \in \mathbb{R}$. Equivalently, we can prove that, for some $c \in \mathbb{R}$, the two principal minors of $\Lambda(x)$ given by

$$\begin{aligned}
\Delta_1(x) &:= M-(\lambda+\alpha(x)) \\
\Delta_2(x) &:= [M-(\lambda+\alpha(x))](M-\lambda)(c\epsilon^2+1) - \frac{1}{4}[c\epsilon+(M-\lambda)(2-\epsilon\alpha(x))]^2
\end{aligned}$$

are positive $\forall x \in \mathbb{R}$. From assumption (i) of Theorem 1, it is obvious that $\Delta_1(x) > 0 \ \forall x \in \mathbb{R}$. For $\Delta_2(x)$ we have that

$$\Delta_2(x) = -(1/4)\big[Ac^2 + B(x)c + \Gamma(x)\big] \tag{22}$$

with $A = \epsilon^2 > 0$,

$$\begin{aligned}
B(x) &= 2\epsilon(M-\lambda)(2-\epsilon\alpha(x)) - 4\epsilon^2(M-\lambda)[M-(\lambda+\alpha(x))] \\
&= 2\epsilon^2(M-\lambda)[\alpha(x)+2\lambda]
\end{aligned} \tag{23}$$

and

$$\begin{aligned}
\Gamma(x) &= (M-\lambda)^2(2-\epsilon\alpha(x))^2 - 4(M-\lambda)[M-(\lambda+\alpha(x))] \\
&= \epsilon(M-\lambda)\alpha(x)[4\lambda+\epsilon\alpha(x)(M-\lambda)].
\end{aligned} \tag{24}$$

$\Delta_2(x)$ is therefore a quadratic polynomial with respect to c that is positive definite for all x iff

(a) $\Delta(x) := B^2(x) - 4A\,\Gamma(x) > 0$

(b) there exists some constant $c \in \big(c_1(\alpha(x)), c_2(\alpha(x))\big)$ for all $x \in \mathbb{R}$ where $c_1(\alpha(x))$, $c_2(\alpha(x))$ are the two roots of $\Delta_2(x)$ given by

$$\begin{aligned}
c_1(\alpha(x)) &:= -(M-\lambda)(2\lambda+\alpha(x)) - 2\lambda(M-\lambda)^{1/2}\big[M-(\lambda+\alpha(x))\big]^{1/2} \\
c_2(\alpha(x)) &:= -(M-\lambda)(2\lambda+\alpha(x)) + 2\lambda(M-\lambda)^{1/2}\big[M-(\lambda+\alpha(x))\big]^{1/2}.
\end{aligned}$$

If we carry out the calculations we have that

$$\Delta(x) = 16\epsilon^3\lambda^2(M-\lambda)[1-\epsilon(\lambda+\alpha(x))]$$

and therefore the condition $\Delta(x) > 0$ is satisfied if $\lambda + \max\{0, \alpha_2\} < M$. For condition (b) to be true, as $\alpha(x)$ varies in $[\alpha_1, \alpha_2]$, there must be some $c \in \mathbb{R}$ such that $c \in [c_1(\alpha), c_2(\alpha)]$ for all $\alpha \in [\alpha_1, \alpha_2]$. This holds true iff $\max_{\alpha\in[\alpha_1,\alpha_2]} c_1(\alpha) < \min_{\alpha\in[\alpha_1,\alpha_2]} c_2(\alpha)$. Function $c_2(\cdot)$ is obviously decreasing w.r.t. $\alpha(x)$ with minimum value $c_2(\alpha_2) > c_2(M-\lambda) = \lambda^2 - M^2$. Function $c_1(\cdot)$ on the other hand is decreasing up to some point $\alpha_0 = M(M-2\lambda)/(M-\lambda)$ and then increasing up to $M-\lambda$ with value $c_1(M-\lambda) = \lambda^2 - M^2 < c_2(\alpha_2)$. Thus, (b) holds true if $c_1(\alpha_1) < c_2(\alpha_2)$ which is exactly assumption (iii) of the theorem. Selecting now $c := \epsilon_0 c_1(\alpha_1) + (1-\epsilon_0)c_2(\alpha_2)$ for any $\epsilon_0 \in (0,1)$ we have $\dot{S}(t) \le 0\ \forall t \in [0, t_f)$. Integrating over $[0,t]$ we have that $S(t) \le S(0)$ $\forall t \in [0, t_f)$ or equivalently

$$\frac{\lambda}{2}x^2(t) + \frac{1}{2}(M-\lambda)(x(t)-y(t))^2 < S(0) - \frac{c}{M}z(t) + b\int_0^{z(t)} \kappa(s)ds \quad (25)$$

for all $t \in [0, t_f)$. The above inequality and the Nussbaum property of $\kappa(\cdot)$ ensure the boundedness of z. To prove this, let us assume the contrary. From the Nussbaum property (ii) of $\kappa(z)$ there exists a strictly increasing sequence $\{z_k\}_{k=1}^{\infty}$ such that $\lim_{k\to\infty} z_k = +\infty$ and

$$\lim_{k\to\infty} \frac{1}{z_k}\int_0^{z_k} b\kappa(w)dw = -\infty. \quad (26)$$

Due to continuity of z, if z grows unbounded, then it will eventually pass from every element $z_k \ge z(0)$ of $\{z_i\}_{i=1}^{\infty}$ over $[0, t_f)$. Thus, there exist times $t_k \in [0, t_f)$ at which $z(t_k) = z_k$ such that from (25) we have

$$\frac{\lambda}{2}x^2(t_k) + \frac{1}{2}M(1-\epsilon\lambda)(x(t_k)-y(t_k))^2 < S(0) - \epsilon c z_k + b\int_0^{z_k} \kappa(w)dw. \quad (27)$$

Due to (26) the r.h.s. of (27) tends to $-\infty$ as $k \to \infty$. Since the l.h.s. of (27) is nonnegative, z must be bounded from $z(t) \le z_{k_1}$ with $k_1 := \{k \in \mathbb{N} | S(0) - \epsilon c z_k + b\int_0^{z_k} \kappa(w)dw < 0 \text{ and } z_k \ge z(0)\}$ for all $t \in [0, t_f)$. From (25) we have

$$x^2(t) \le \nu := \frac{2}{\lambda}S(0) + \frac{2}{\lambda}\max_{z\in[0,z_{k_1}]}\Big\{-\frac{c}{M}z + b\int_0^z \kappa(w)dw\Big\} \quad \forall t \in [0, t_f) \quad (28)$$

and therefore the augmented state vector $x_{p,ag}$ is bounded within the compact set $W_0 := \{x_{p,ag} | \xi \geq 0,\ (1/2)y^2 + \lambda\xi \leq z_{k_1} \text{ and } x^2 \leq \nu\}$. Using now Theorem 3.3 of [7] we have that $t_f = \infty$. Thus, $z, x \in \mathcal{L}_\infty$ and therefore $y \in \mathcal{L}_\infty \cap \mathcal{L}_2$, $u \in \mathcal{L}_\infty$. Then, from (13) $\dot{x}, \dot{y} \in \mathcal{L}_\infty$ and Barbalat's lemma yields the desired property $\lim_{t\to\infty} x(t) = \lim_{t\to\infty} y(t) = 0$. □

Remark 1. *For a linear system $f(x) = \alpha x$, condition (iii) of Th. 1 is no longer needed and (i) reduces to $\lambda < M$ if $\alpha \leq 0$ (nominally stable system) and $\alpha + \lambda < M$ if $\alpha > 0$ (nominally unstable). Note that in the latter case the necessary condition for stabilization by simple output feedback is $\alpha < M$.*

Remark 2. *If $c_2(\alpha_2) \geq 0$ then the constant c in the definition of S can be nonnegative. This means that in (25) $-c\epsilon z(t) \leq 0$ and the Nussbaum condition (ii) for $\kappa(z)$ in Theorem 1 can be relaxed to*

$$\limsup_{z\to\infty} \int_0^z \kappa(s)ds = +\infty\,, \qquad \liminf_{z\to\infty} \int_0^z \kappa(s)ds = -\infty. \tag{29}$$

After calculations one can show that condition $c_2(\alpha_2) \geq 0$ holds true iff $\alpha_2 \leq 0$. Thus, if the unforced linear system is stable ($\alpha \leq 0$) and $\lambda < M$ then, the controller (5) results in bounded and attractive closed-loop behavior. This also provides a strict proof for the integrator example of [4], [5].

Remark 3. *Note that the l.h.s. of condition (iii) tends to 4λ in the limit $M \to \infty$. Thus, for some sector bounded nonlinearity (12), if we select $\lambda > (\alpha_2 - \alpha_1)/4$ then there exists some $M_0 > 0$ such that for all $M > M_0$ the closed-loop system (13), (14), (15) is globally bounded and attractive.*

Remark 4. *An alternative approach to the robust control problem under study could be pursued along the lines of universal adaptive control [8]-[11]. Using the results of [10], the Nussbaum gain $\zeta^2 \cos\zeta$ in (3) can be replaced by a function $(K \circ \beta)(\zeta)$ with $K(\cdot), \beta(\cdot)$ having properties A1-A3 in [10] ($q = 2$). Examples of such functions are $K(\beta) = \beta^{1/4} \sin\beta$ and $\beta(\zeta) = \sqrt{\ln(\zeta + 2)}$. This controller ensures asymptotic regulation for the perturbed system (13) but simulations indicate poor transient behavior especially for an unstable unforced nominal system. This is possibly due to the fact that the controller gain $K(\beta(\zeta(t)))$ slowly converges to a stabilizing gain allowing the output to take extremely large values in this time span.*

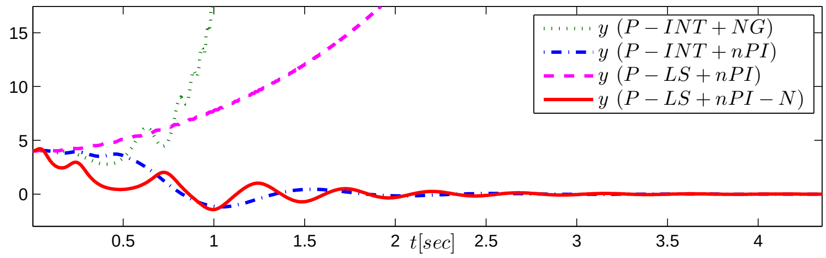

Figure 1: Time responses of $y(t)$ for the cases of a perturbed integrator (P-INT) and a perturbed linear system (P-LS) for the three controllers NG, nPI, nPI-N.

3 Simulation results

A simulation study was performed for the perturbed integrator (P-INT) and the perturbed linear system (P-LS) described by (4), (6) respectively with parameters $\alpha = 1$, $b = 1/2$, $\lambda = 2.5$, $M = 4$ and initial conditions $x(0) = y(0) = 4$. For the specific parameters, condition (i) of Theorem 1 holds true. We tested the case of a Nussbaum gain based (NG) controller (3) and a nonlinear PI controller (5) with gains $\kappa(z) = z\cos(z)$ (not a Nussbaum function) denoted as nPI and $\kappa(z) = z^2\cos(z)$ (Nussbaum function) denoted as nPI-N. The output response y shown in Fig. 1 verifies the theoretical results. Particularly, for the P-INT system with the NG controller, y is divergent as shown in [3]. If the nPI controller is used then y remains bounded and converges to zero [4], [5]. However, the nPI control for the P-LS also yields unbounded solutions. Convergent solutions are obtained for the P-LS only when the nPI-N is employed.

Consider now the perturbed nonlinear system (13) with $f(x) = 3[1 + \sin^2(x)]x$ where $\alpha_1 = 3$, $\alpha_2 = 6$ and $b = 1$. Selecting $\lambda = 2.5$, we have that both conditions (i) and (iii) are satisfied for every $M > \alpha_2 + \lambda = 8.5$ (see Remark 3). For the control law (14), (15), $\kappa(z) = z^2\sin(z)$ simulation results are shown in Fig. 2 with $M = 10$ and initial conditions $x(0) = y(0) = 4$. As expected, all x, y, u are bounded and converge to the origin as time passes.

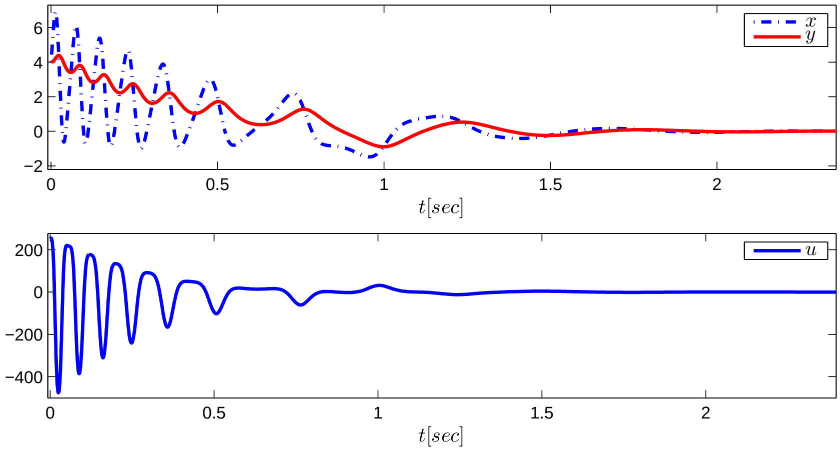

Figure 2: Time responses of system states x, y and control input u for the perturbed nonlinear system with controller nPI-n.

4 Conclusions

Sufficient conditions are derived for global boundedness and attractivity of a perturbed nonlinear system with sector-bounded nonlinearity under nonlinear PI control. The results further demonstrate improved robustness of the nonlinear PI controls compared to Nussbaum gain based schemes.

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