Cooperative Control of Multiple Agents with Unknown High-frequency Gain Signs under Unbalanced and Switching Topologies

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***Abstract*—Existing results on cooperative control of multi-agent systems with unknown control directions require that the underlying topology is either fixed with a strongly connected graph or switching between different strongly connected graphs. Furthermore, in most cases the graph is assumed to be balanced. This paper proposes a new class of nonlinear PI based algorithms to relax these requirements and allow for unbalanced and switching topologies having a jointly strongly connected basis. This is made possible for single-integrator (SI) and double-integrator (DI) agents with non-identical unknown control directions by a suitable selection of the distributed nonlinear PI functions. Moreover, as a special case, the proposed algorithms are applied to strongly connected and fixed graphs. Finally, simulation examples are given to show the validity of our theoretical results.**

***Index Terms*—Unknown control directions, multi-agent systems, consensus, nonlinear PI, switching topologies.**

I. Introduction

Multi-agent coordination has attracted intensive research interest over the past decade [1]–[4]. Consensus as a fundamental topic [5]–[7], aims to design algorithms that guarantee collective behaviors by using local neighborhood information. Applications include formation control of unmanned air vehicles (UAVs), clusters of satellites, self-organization problems and congestion control in communication networks.

In some control problems such as, the course-keeping controller design of ships [8] or uncalibrated-visual servoing [9], the control direction might not be always available a priori. In order to handle the unknown control directions, the Nussbaum gain technique was first proposed in [10]. To date, the Nussbaum gain approach has been extensively employed in various control schemes [11]–[16]. An alternative approach to the problem, the so called nonlinear PI based method, was later proposed in [17]. Results in [18]–[21] indicate that the nonlinear PI based method has better robustness properties for certain types of unmodelled dynamics.

Recently, a few efforts appeared in the literature on cooperative control of multi-agent systems with unknown control directions. In [22], the consensus of first-order and second-order agents with unknown identical control directions was considered using a novel Nussbaum function. Nussbaum functions were also employed in [23]–[26] for cooperative output regulation, in [27], [28] for SI agents, in [29], [30] for high-order agents, and in [31], [32] for nonlinear systems. It is observed that in most cases with the exception of [25]–[27] the Nussbaum gain approach requires that all the unknown control directions should be the same. Also, in [33] an adaptive approach was proposed to relax such a requirement and allow non-identical control directions in which partial control directions should be known.

In a recent paper [34], using the nonlinear PI based method, the consensus problem for SI and DI agents with non-identical unknown control directions was investigated for the first time under switching topologies. It was shown in [34] that if the switching graphs are balanced and strongly connected, then asymptotic consensus among the agents is ensured with the proposed control laws. It is worth noting that the consensus of agents with non-identical unknown control directions under unbalanced and switching topologies which are not strongly connected is to the best of our knowledge an open problem.

With the above motivations, in this work, we consider the consensus problem of SI and DI agents with non-identical unknown control directions and propose a new class of nonlinear PI based algorithms to allow for unbalanced and switching topologies which are not strongly connected. As a special case, we apply the nonlinear PI based algorithms for SI and DI agents under a strongly connected and fixed graph.

The main contributions of this paper are the following. First, we remove the balanced graph assumption of the work [34]. We also introduce a new class of switching topologies, namely those having a jointly strongly connected basis, generalizing the jointly connected property [38], [39] to digraphs. Thus, in this work the graphs of switching topologies do not need to be balanced or strongly connected but only to have a jointly strongly connected basis. These extensions are not trivial and have not been considered in the related literature [22]–[34]. Our results are obtained with the introduction of suitable novel nonlinear PI terms and a new technical analysis (Section III and Lemma 5). Second, with the proposed algorithms, the consensus problem of agents with non-identical unknown

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control directions under a strongly connected and fixed graph is also tackled as a special case.

The rest of this paper is organized as follows. In Section II, some preliminaries are given and basic lemmas and definitions are presented. Also, the problem under study is formulated. In Section III, a new class of nonlinear PI based algorithms is proposed and the main results of the paper (Theorems 1-2) are proved. Two examples are considered to verify the obtained results in Section IV. Section V concludes this paper with some remarks.

Notations: $\mathcal{L}_\infty$ and $\mathcal{L}_2$ are the spaces of bounded signals and square integrable signals, respectively. For $x \in \mathbb{R}$ we denote by $\lfloor x \rfloor$ the largest integer smaller than or equal to x.

II. Problem Formulation and Preliminaries

A. Preliminaries

Let $\{t_j\}_{j\in I}$ be the finite or infinite sequence of discontinuity points of a piecewise continuous function with index set $I = \{1, 2, \ldots\} \subseteq \mathbb{N}_+$ and denote $t_{n_0+1} = +\infty$ if I has finite cardinality $\mathrm{card}(I) = n_0$.

Definition 1: Consider a real-valued piecewise right continuous function $f : [0, \infty) \to \mathbb{R}$ and let $\{t_j\}_{j\in I}$ be the sequence of discontinuity points. The function $f(\cdot)$ is said to be uniformly piecewise right continuous if for any $\epsilon > 0$ there exists $\delta(\epsilon) > 0$ such that

$$\left| f\left(\bar{t}_2\right) - f\left(\bar{t}_1\right) \right| \leq \epsilon$$

for $\bar{t}_1,\ \bar{t}_2 \in [t_j, t_{j+1})\,,\ j \in I$, with $\left|\bar{t}_2 - \bar{t}_1\right| \leq \delta(\epsilon)$.

Two useful lemmas from [34] are introduced as follows:

Lemma 1: Consider a piecewise right continuous differentiable function $\phi : [0, \infty) \to \mathbb{R}$, and let $\{t_j\}_{j\in I}$ be the sequence of discontinuity points with $I \subseteq \mathbb{N}$. Suppose that ϕ has a bounded derivative except at the points t_j $(j \in I)$ and $\lim_{t\to\infty} \int_0^t \phi(s)ds$ exists and is finite. If there exists $\tau > 0$ such that $t_{j+1} - t_j > \tau$ for $j \in I$ then $\lim_{t\to\infty} \phi(t) = 0$.

Lemma 2: Let $M : [0, t_f) \to \mathbb{R}$ be a piecewise right-continuous function, and $S : [0, t_f) \to \mathbb{R}$ is a continuous, piecewise differentiable function such that

$$\dot{S}(t) = \left[\alpha_1 + \alpha_2 S(t) \cos(S(t))\right] M(t)$$

where α_1 and α_2 are two constants. If $\alpha_2 \neq 0$, then $|S(t) - S(0)| \leq 2\left(\pi + |\alpha_1/\alpha_2|\right)$ for $t \in [0, t_f)$.

The following Lemma will also be important for the subsequent analysis.

Lemma 3: Consider a real-valued continuous function $f : [0, \infty) \to \mathbb{R}$ with a uniformly piecewise right continuous derivative and let $\{t_j\}_{j\in I}$ be the sequence of discontinuity points of $\dot{f}$. If there exists $\tau > 0$ such that $t_{j+1} - t_j > \tau$ for $j \in I$ and $\lim_{t\to\infty}[f(t)\dot{f}(t)] = 0$ then $\lim_{t\to\infty} \dot{f}(t) = 0$.

Proof. The proof is given in Appendix A. ■

In what follows, we revisit basic definitions on graph theory. A directed graph is denoted by $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, where $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$ represents the finite and nonempty set of nodes, and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges. $\mathcal{A} = [a_{ik}] \in \mathbb{R}^{N\times N}$ is the adjacency matrix, where a_{ik} represents the coupling strength of edge (k, i) with $a_{ik} > 0$ if (k, i) belongs to $\mathcal{G}$ and $a_{ik} = 0$ otherwise. The union $\mathcal{G}_1 \cup \mathcal{G}_2$ of two graphs $\mathcal{G}_1$, $\mathcal{G}_2$ with $\mathcal{G}_p = (\mathcal{V}_p, \mathcal{E}_p, \mathcal{A}_p)$ $(p = 1, 2)$ is defined as a new graph with vertices $\mathcal{V}_1 \cup \mathcal{V}_2$ and edges $\mathcal{E}_1 \cup \mathcal{E}_2$. A union adjacency matrix can also be defined but we leave this definition out since it is not needed in our analysis. Denote by $N_i = \{k \in \mathcal{V} : (k, i) \in \mathcal{E}\}$ the set of node i's neighbors. Let $d_i = \sum_{k=1}^N a_{ik}$ be the in-degree of vertex i, and denote by $D = \mathrm{diag}\{d_1, \ldots, d_N\}$ the in-degree matrix. Then the Laplacian matrix is defined as $L = D - \mathcal{A}$. The directed path with length l is defined with a sequence of edges in the form $((i_1, i_2), (i_2, i_3), \ldots, (i_l, i_{l+1}))$ where $(i_m, i_{m+1}) \in \mathcal{E}$ for $m = 1, \ldots, l$ and $i_m \neq i_n$ for $m, n = 1, \ldots, l$ and $m \neq n$. If there exists a directed path between any two distinct nodes in a directed graph $\mathcal{G}$, the graph is said to be strongly connected.

Definition 2: [35] A *basis bicomponent* of a directed graph $\mathcal{G}$ is a strongly connected subgraph of $\mathcal{G}$ with no incoming links from other nodes of $\mathcal{G}$.

Remark 1: The concept of basis bicomponent is an important one in graph theory. In [36] and [37], the number of basis bicomponents d is shown to be equal to the out-forest complexity of a directed graph and $rank(L) = N - d$. Moreover, Chebotarev and Agaev proved in [36] (Corollary 1) that the standard consensus protocol $\dot{x} = -Lx$ ensures that all vertices in a basic bicomponent reach consensus for any Laplacian matrix L.

Lemma 4: Consider the directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ which has a basis bicomponent $\mathcal{G}_b = (\mathcal{V}_b, \mathcal{E}_b, \mathcal{A}_b)$ with $\mathcal{V}_b := \{i_1, i_2, \cdots, i_r\} \subset \mathcal{V}$. Denote by L_r the reduced matrix which is obtained by deleting all columns and rows of the original Laplacian matrix L that correspond to nodes not included in the subgraph. Then, L_r is a Laplacian matrix for the strongly connected graph defined by $\{i_1, i_2, \cdots, i_r\}$. Denote by $\omega_r := [\omega_1, \omega_2, \ldots, \omega_r]^T$ the left eigenvector of L_r associated with the zero eigenvalue. Then, it holds true that

$$\sum_{m=1}^{r}\sum_{n=1}^{r} \omega_m a_{i_m,i_n} \zeta_m \left(\zeta_m - \zeta_n\right) = \frac{1}{2}\sum_{m=1}^{r}\sum_{n=1}^{r} \omega_m a_{i_m,i_n} \left(\zeta_m - \zeta_n\right)^2$$

for all $\zeta_m \in \mathbb{R}$ with $\omega_m > 0$ for every $m \in \{1, \ldots, r\}$ and $\sum_{m=1}^r \omega_m = 1$.

Proof. The proof is given in Appendix B. ■

We define now the new property of jointly strongly connected basis.

Definition 3: Consider a group of graphs $\mathcal{G}_\ell = (\mathcal{V}, \mathcal{E}_\ell, \mathcal{A}_\ell)$ $(\ell = 1, \cdots, M)$ defined over the same set of vertices $\mathcal{V}$ with each graph $\mathcal{G}_\ell$ having the basis bicomponents $\mathcal{G}_{b,\ell 1}, \cdots, \mathcal{G}_{b,\ell d_\ell}$ $(\ell = 1, \cdots, M)$. The graph group is said to have a *jointly strongly connected basis* if the union $\cup_{\ell=1}^M \cup_{j=1}^{d_\ell} \mathcal{G}_{b,\ell j}$ of all the graph basis bicomponents forms a strongly connected graph having as vertices all elements of $\mathcal{V}$.

The above definition is illustrated in Fig. 1. Note that even though graph $\mathcal{G}_2$ has a strongly connected subgraph defined by agents 1 and 2, a basis bicomponent does not exist due to the incoming link to agent 2 from agent 4. From the union of the respective basis bicomponents one can deduce that the group of graphs $\mathcal{G}_1, \mathcal{G}_2, \mathcal{G}_3$ has a jointly strongly connected basis.

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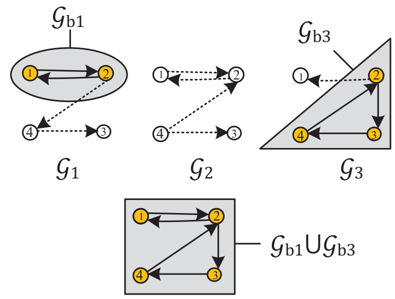

Fig. 1. Graphs $\mathcal{G}_1, \mathcal{G}_2, \mathcal{G}_3$, their basis bicomponents $\mathcal{G}_{b1}, \mathcal{G}_{b3}$ and the strongly connected graph $\mathcal{G}_{b1} \cup \mathcal{G}_{b3}$.

Remark 2: The jointly strongly connected basis property can be regarded as a generalization for directed graphs of the widely known concept of jointly connected topologies [38], [39]. Note that for undirected topologies, each group of connected agents forms a basis bicomponent since there are no incoming/outcoming links among separate groups.

Consider now a graph that switches between different elements of a jointly strongly connected basis as described in the following assumption:

Assumption 1: Let $\{t_j\}_{j=1}^{\infty}$ be a sequence of switching times and consider a set of network topologies which are given by the Laplacian matrices $\{L_\ell\}_{\ell=1}^{M}$ and a mapping $\bar{n} : \mathbb{N}_+ \to \{1, 2, \cdots, M\}$ such that $L(t) = L_{\bar{n}(j)}$ for all $t \in [t_j, t_{j+1})$, $j \in \mathbb{N}_+$. Assume that the switching topologies $\{L_\ell\}_{\ell=1}^{M}$ have a jointly strongly connected basis and there exists some unknown constant τ_{min} such that $t_{j+1} - t_j > \tau_{min}$ for all $j \in \mathbb{N}_+$. Also, for each topology L_ℓ there exist sequences of activation and deactivation times $\{T_{\ell\nu}^a\}_{\nu=1}^{\infty}, \{T_{\ell\nu}^d\}_{\nu=1}^{\infty} \subset \{t_j\}_{j=1}^{\infty}$ and constant $\tau_{\max} > 0$ such that $L(t) = L_\ell$ for $t \in [T_{\ell\nu}^a, T_{\ell\nu}^d)$ $(\nu \in \mathbb{N}_+)$ and

$$T_{\ell,\nu+1}^a - T_{\ell\nu}^d \le \tau_{max}$$

for all $\ell = 1, \cdots, M, \nu \in \mathbb{N}_+$.

Remark 3: Assumption 1 states that every topology L_ℓ is reactivated (at $T_{\ell,\nu+1}^a$) within time less than or equal to $\tau_{\max}$ from its previous deactivation time $T_{\ell\nu}^d$ and the time between two consecutive switchings is greater than or equal to $\tau_{\min}$.

The following Lemma is a central result for consensus under switching topologies having a jointly strongly connected basis:

Lemma 5: Consider a group of switching topologies described by Assumption 1 and N continuously differentiable almost everywhere (except $\{t_j\}_{j=1}^{\infty}$) functions $y_i : [0, \infty) \to \mathbb{R}$. Define $\xi(t) = L(t)y(t)$ with $\xi = [\xi_1, \xi_2, \cdots, \xi_N]^T$ and $y = [y_1, y_2, \cdots, y_N]^T$. If $\lim_{t\to\infty} y_i(t)\xi_i(t) = 0$ and $\lim_{t\to\infty} \dot{y}_i(t) = 0$ then $\lim_{t\to\infty}(y_i(t) - y_k(t)) = 0$ for all $i, k \in \{1, 2, \cdots, N\}$.

Proof. The proof is given in Appendix C. ∎

B. Problem formulation

Consider either N SI agents with state $x_i \in \mathbb{R}$ and dynamics

$$\dot{x}_i = b_i u_i, \quad i = 1, 2, \ldots, N, \tag{1}$$

or N DI agents with position $x_i \in \mathbb{R}$, velocity $v_i \in \mathbb{R}$ and dynamics

$$\begin{cases} \dot{x}_i = v_i \\ \dot{v}_i = b_i u_i \end{cases}, \quad i = 1, 2, \ldots, N, \tag{2}$$

where $u_i \in \mathbb{R}$ is the control input and $b_i \in \mathbb{R}$ is the control gain with the unknown sign.

Assumption 2: The control gains b_i, $i = 1, 2, \ldots, N$ are unknown and nonzero constants.

Remark 4: The assumption $b_i \neq 0$ for all $i = 1, 2, \ldots, N$ is necessary for the controllability of each agent dynamics. The signs of the gains b_i may be different and their prior knowledge is no longer needed.

The design objective is to propose a new class of distributed control algorithms for agents (1) or (2) under Assumptions 1 and 2 such that either

$$\lim_{t\to\infty} (x_i(t) - x_k(t)) = 0 \tag{3}$$

for SI agents with $i, k \in \{1, 2, \ldots, N\}$, or

$$\begin{cases} \lim_{t\to\infty} (x_i(t) - x_k(t)) = 0 \\ \lim_{t\to\infty} (v_i(t) - v_k(t)) = 0 \end{cases} \tag{4}$$

for DI agents with $i, k \in \{1, 2, \ldots, N\}$.

III. Main results

A. Consensus algorithm design for SI agents

For SI agents (1), we define $e_i(t) := \sum_{k=1}^{N} a_{ik}(t) (x_i(t) - x_k(t))$. The main result is stated as follows:

Theorem 1: Consider a network of SI agents (1) satisfying Assumption 2 with switching topologies described by Assumption 1. The consensus problem (3) is solved if the distributed control algorithms are designed by

$$\begin{aligned} u_i(t) &= S_i(t) \cos(S_i(t)) e_i(t) \\ &\quad \times \left[\lambda_1 x_i(t) e_i(t) + \lambda_2 \int_0^t x_i(s) e_i(s) ds \right] \end{aligned} \tag{5}$$

with

$$S_i(t) = \frac{x_i^2(t)}{2} + \lambda_1 \int_0^t x_i^2(s) e_i^2(s) ds + \frac{\lambda_2}{2} \left[\int_0^t x_i(s) e_i(s) ds \right]^2 \tag{6}$$

where $\lambda_1 > 0$ and $\lambda_2 > 0$. Moreover, all x_i and u_i are bounded for $i = 1, 2, \ldots, N$.

Proof. The augmented state vector $x_{ag} := [x^T, z_1^T, z_2^T]^T$ with $x := [x_1, x_2, \ldots, x_N]^T$ and $z_j := [z_{j1}, z_{j2}, \ldots, z_{jN}]^T$, $(j = 1, 2)$ is defined where for each $i = 1, 2, \ldots, N$,

$$z_{1i} := \int_0^t x_i^2(s) e_i^2(s) ds, \tag{7}$$

$$z_{2i} := \int_0^t x_i(s) e_i(s) ds. \tag{8}$$

The closed-loop dynamics of SI agents (1) with (5), (6), (7) and (8) take the form

$$\begin{cases} \dot{x}_i = Q_i(x_{ag}) \left(\lambda_1 x^T \varepsilon_i \varepsilon_i^T L(t) x + \lambda_2 z_{2i} \right) \varepsilon_i^T L(t) x \\ \dot{z}_{1i} = \left(x^T \varepsilon_i \varepsilon_i^T L(t) x \right)^2 \\ \dot{z}_{2i} = x^T \varepsilon_i \varepsilon_i^T L(t) x \end{cases} \tag{9}$$

with

$$\left\{ \begin{array}{l} Q_i\left(x_{ag}\right) = b_i S_i \cos\left(S_i\right) \\ S_i = \frac{1}{2}x_i^2 + \lambda_1 z_{1i} + \frac{1}{2}\lambda_2 z_{2i}^2 \end{array} \right. \quad (10)$$

where ε_i is the i-th column of the identity matrix. It is seen from (9) that for the dynamical system $\dot{x}_{ag} = f\left(x_{ag}, t\right)$ the mapping f is piecewise continuous and locally Lipschitz wrt x_{ag}. Hence from section 8.5 in [40], a unique continuous solution $x_{ag}(\cdot)$ exists over some maximal interval $[0, t_f)$. In view of the control law (5), the time derivative of $S_i(t)$ is

$$\begin{aligned} \dot{S}_i(t) &= \left[1 + b_i S_i(t)\cos(S_i(t))\right] x_i(t) e_i(t) \\ &\quad \times \left[\lambda_1 x_i(t) e_i(t) + \lambda_2 \int_0^t x_i(s) e_i(s) ds\right] \end{aligned}$$

for all $t \in [0, t_f)$. Thus, according to Lemma 2, we have

$$|S_i(t) - S_i(0)| \leq 2\left[\pi + (1/|b_i|)\right]$$

which means that $S_i(t)$ is bounded in $[0, t_f)$. It is observed from (6) that $S_i(t) \geq 0$ for any $t \geq 0$. Therefore, boundedness of $S_i(t)$ with (7) and (8) yields boundedness of x_i, $z_{1i}(t)$ and $z_{2i}(t)$ in $[0, t_f)$. Thus, the whole state vector x_{ag} is bounded in $[0, t_f)$ and the solution can be extended up to $t_f = \infty$. Since the related bounds are independent from the final time t_f, they remain unchanged when the solution is extended to $t_f = \infty$, i.e. $x_i, z_{1i}, z_{2i}, S_i \in \mathcal{L}_\infty$. In addition, we have $x_i(t)e_i(t) \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ and from (1), it is obtained $d[x_i e_i]/dt = \dot{x}_i(t)e_i(t) + x_i(t)\dot{e}_i(t) \in \mathcal{L}_\infty$ except at the points of switching topology t_j ($j \in \mathbb{N}_+$). Therefore, using Lemma 1 for $\phi(t) = x_i^2(t)e_i^2(t)$, we have $\lim_{t\to\infty} x_i(t)e_i(t) = 0$.

We will further prove that $\lim_{t\to\infty} \dot{x}_i(t) = 0$. Multiplying both sides of (1) by x_i and using (5) we obtain the limit $\lim_{t\to\infty}[x_i(t)\dot{x}_i(t)] = \lim_{t\to\infty}[x_i(t)e_i(t)] = 0$. Also, from (9) and the boundedness of x_{ag} we have that $\dot{x}_{ag}$ is a bounded piecewise continuous function. Further differentiation at all times except the points t_j proves that $\dot{x}_i$ is a piecewise continuous function with bounded derivative. Thus, from Lemma 2 of [34], $\dot{x}_i$ is a uniformly piecewise right continuous function. A direct application now of Lemma 3 yields the desired $\lim_{t\to\infty} \dot{x}_i(t) = 0$. Since $\lim_{t\to\infty} \dot{x}_i(t) = 0$ and $\lim_{t\to\infty} x_i(t)e_i(t) = 0$, the consensus property (3) is derived from Lemma 5 by setting $\xi_i(t) = e_i(t)$ and $y_i(t) = x_i(t)$. ∎

For a strongly connected and fixed graph the following Corollary for SI agents (1) is obtained.

Corollary 1: Consider a network of SI agents (1) satisfying Assumption 2 with the strongly connected graph $\mathcal{G}$. The consensus problem (3) is solved if the distributed control algorithms (5) and (6) are selected. Furthermore, all x_i and u_i are bounded for $i = 1, 2, \ldots, N$.

Proof. The result is a direct consequence of Theorem 1, and therefore its proof is omitted. ∎

Remark 5: It is worth pointing out that the nonlinear PI functions for SI agents in Theorem 1 of [34] are still effective for the case of Corollary 1, where the fixed graph is strongly connected. The nonlinear PI functions of SI agents in [34] are

$$S_i(t) = \frac{1}{2}x_i^2(t) + \lambda \int_0^t x_i(s)e_i(s)ds \quad (11)$$

where $e_i(t) = \sum_{k=1}^N a_{ik}(t)\left(x_i(t) - x_k(t)\right)$ and $\lambda > 0$. By the similar analysis in the proof of Theorem 1 we obtain $S_i(t)$ is bounded. Since the fixed graph is strongly connected, it follows that $\sum_{i=1}^N \omega_i S_i(t)$ is bounded, where $\omega_i > 0$ are the elements of the left eigenvector of L associated with the zero eigenvalue. Then, we have

$$\begin{aligned} \sum_{i=1}^N \omega_i S_i(t) =& \frac{\lambda}{2}\int_0^t \sum_{i=1}^N \sum_{k=1}^N \omega_i a_{ik}\left(x_i(s) - x_k(s)\right)^2 ds \\ &+ \frac{1}{2}\sum_{i=1}^N \omega_i x_i^2(t). \end{aligned} \quad (12)$$

The desired consensus property follows from the boundedness of $\sum_{i=1}^N \omega_i S_i(t)$, (12) and the strong connectivity property of the graph. The details are omitted for brevity.

However, for the case of switching topologies having a jointly strongly connected basis, the above approach cannot be applied due to the fact that $\omega_i, (i = 1, 2, \ldots, N)$ are not fixed but change over different time intervals.

B. Consensus algorithm design for DI agents

For DI agents (2), we define $\zeta_i := \sum_{k=1}^N a_{ik}\left(v_i - v_k\right)$, $\eta_i := \sum_{k=1}^N a_{ik}\left(x_i - x_k\right)$, $q_i := v_i + \rho x_i$ and $r_i := \zeta_i + \rho\eta_i$ with $\rho > 0$. The main result for DI agents is the following:

Theorem 2: Consider a network of DI agents (2) satisfying Assumption 2 with switching topologies described by Assumption 1. The consensus problem (4) is solved if the distributed control algorithms are selected by

$$\begin{aligned} u_i(t) = R_i(t)\cos(R_i(t))\Bigg[&(\rho + 1)v_i(t) \\ &+ r_i(t)\left(\lambda_1 q_i(t) r_i(t) + \lambda_2 \int_0^t q_i(s) r_i(s) ds\right)\Bigg] \end{aligned} \quad (13)$$

with

$$\begin{aligned} R_i(t) = &\frac{1}{2}q_i^2(t) + \frac{\rho}{2}x_i^2(t) + \lambda_1 \int_0^t q_i^2(s) r_i^2(s) ds \\ &+ \int_0^t v_i^2(s) ds + \frac{\lambda_2}{2}\left[\int_0^t q_i(s) r_i(s) ds\right]^2 \end{aligned} \quad (14)$$

where $\lambda_1 > 0$ and $\lambda_2 > 0$. Moreover, all x_i, v_i and u_i are bounded for $i = 1, 2, \ldots, N$.

Proof. Consider the augmented state vector $\bar{x}_{ag} := \left[x^T, v^T, \bar{z}_1^T, \bar{z}_2^T\right]^T$ with $x := \left[x_1, x_2, \ldots, x_N\right]^T$, $v := \left[v_1, v_2, \ldots, v_N\right]^T$ and $\bar{z}_j := \left[\bar{z}_{j1}, \bar{z}_{j2}, \ldots, \bar{z}_{jN}\right]^T$, $(j = 1, 2)$ where for each $i = 1, 2, \ldots, N$,

$$\bar{z}_{1i} := \lambda_1 \int_0^t q_i^2(s) r_i^2(s) ds + \int_0^t v_i^2(s) ds, \quad (15)$$

$$\bar{z}_{2i} := \int_0^t q_i(s) r_i(s) ds. \quad (16)$$

The closed-loop dynamics of the DI agents (2) with (13) and (14) are

$$\left\{ \begin{array}{l} \dot{x}_i = v_i \\ \dot{v}_i = W_i\left(\bar{x}_{ag}\right)\Big[(\rho + 1)v_i + (v + \rho x)^T L^T(t)\varepsilon_i \\ \qquad \times \left(\lambda_1 (v + \rho x)^T \varepsilon_i \varepsilon_i^T L(t)(v + \rho x) + \lambda_2 \bar{z}_{2i}\right)\Big] \\ \dot{\bar{z}}_{1i} = \lambda_1 \left[(v + \rho x)^T \varepsilon_i \varepsilon_i^T L(t)(v + \rho x)\right]^2 + v_i^2 \\ \dot{\bar{z}}_{2i} = (v + \rho x)^T \varepsilon_i \varepsilon_i^T L(t)(v + \rho x) \end{array} \right. \quad (17)$$

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with

$$\begin{cases} W_i\left(\bar{x}_{ag}\right)=b_iR_i\cos\left(R_i\right) \\ R_i=\frac{1}{2}\left(v_i+\rho x_i\right)^2+\frac{\rho}{2}x_i^2+\bar{z}_{1i}+\lambda_2\bar{z}_{2i}^2 \end{cases} \tag{18}$$

where ε_i is the i-th column of the identity matrix. It is seen from (17) that the dynamical system $\dot{\bar{x}}_{ag}=\bar{f}\left(\bar{x}_{ag},t\right)$ has a piecewise continuous and locally Lipschitz mapping $\bar{f}$ wrt $\bar{x}_{ag}$. Thus, from section 8.5 in [40], a unique continuous solution $\bar{x}_{ag}\left(\cdot\right)$ exists over some maximal interval $[0,\bar{t}_f)$. In view of the control law (13), the time derivative of $R_i(t)$ is

$$\begin{aligned}\dot{R}_i(t) \;=\;& \left[1+b_iR_i(t)\cos(R_i(t))\right]q_i(s)\Bigg[(\rho+1)v_i(t) \\ &+r_i(t)\left(\lambda_1q_i(t)r_i(t)+\lambda_2\int_0^t q_i(s)r_i(s)ds\right)\Bigg]\end{aligned}$$

for all $t\in[0,\bar{t}_f)$. Hence, according to Lemma 2, we have

$$|R_i(t)-R_i(0)|\le 2\left[\pi+(1/\left|b_i\right|)\right]$$

which means $R_i(t)$ is bounded in $[0,\bar{t}_f)$. Since $R_i(t)\ge 0$ for any $t\ge 0$, boundedness of $R_i(t)$ with (14) yields boundedness of q_i,v_i,x_i,r_i $\bar{z}_{1i}$ and $\bar{z}_{2i}$ in $[0,\bar{t}_f)$. Thus, the whole state vector $\bar{x}_{ag}$ is bounded and therefore the solution holds up to $\bar{t}_f=\infty$. Since the related bounds are independent from $\bar{t}_f$, they remain unchanged for $\bar{t}_f=\infty$, i.e. x_i, v_i, q_i, r_i, $R_i\in\mathcal{L}_\infty$ and $q_ir_i\in\mathcal{L}_2$. Moreover, (13) yields that $u_i\in\mathcal{L}_\infty$. Combining these properties we obtain $d[q_ir_i]/dt=\dot{q}_ir_i+q_i\dot{r}_i\in\mathcal{L}_\infty$. From Lemma 1, we now have $\lim_{t\to\infty}q_i(t)r_i(t)=0$. Note that since $v_i\in\mathcal{L}_\infty\cap\mathcal{L}_2$ and $\dot{v}_i=b_iu_i\in\mathcal{L}_\infty$ we have from Lemma 1 that $\lim_{t\to\infty}v_i(t)=0$. We will also prove that $\lim_{t\to\infty}\dot{q}_i(t)=0$. Using (13) we obtain $\lim_{t\to\infty}[q_i(t)\dot{q}_i(t)]=\lim_{t\to\infty}[q_i(t)\dot{v}_i(t)]+\rho\lim_{t\to\infty}[q_i(t)v_i(t)]=0$. From (17) and the boundedness of $\bar{x}_{ag}$ we have that $\dot{\bar{x}}_{ag}\in\mathcal{L}_\infty$ and therefore $\dot{q}_i$ is a bounded piecewise continuous vector function. Further differentiation at all times except the points t_j proves that $\dot{q}_i$ is a piecewise continuous function with bounded derivative. Thus, from Lemma 2 of [34], $\dot{q}_i$ is a uniformly piecewise right continuous function. A direct application now of Lemma 3 yields the desired $\lim_{t\to\infty}\dot{q}_i(t)=0$.

Since $\lim_{t\to\infty}q_i(t)r_i(t)=0$ and $\lim_{t\to\infty}\dot{q}_i(t)=0$, we obtain $\lim_{t\to\infty}(q_i(t)-q_k(t))=0$ from Lemma 5 with $\xi_i(t)=r_i(t)$ and $y_i(t)=q_i(t)$. Also since $\lim_{t\to\infty}v_i(t)=0$, we have

$$\lim_{t\to\infty}\left(v_i(t)-v_k(t)\right)=\lim_{t\to\infty}v_i(t)-\lim_{t\to\infty}v_k(t)=0$$

for all $i,k\in\{1,2,\cdots,N\}$. In view of the definition of q_i,

$$\begin{aligned}\lim_{t\to\infty}\left(x_i(t)-x_k(t)\right) \;=\;& (1/\rho)\lim_{t\to\infty}\left(q_i(t)-q_k(t)\right) \\ &-(1/\rho)\lim_{t\to\infty}\left(v_i(t)-v_k(t)\right)=0\end{aligned}$$

for all $i,k\in\{1,2,\cdots,N\}$ which completes the proof. ■

For a strongly connected and fixed graph the following Corollary for DI agents (2) is obtained.

Corollary 2: Consider a network of DI agents (2) satisfying Assumption 2 with the strongly connected graph $\mathcal{G}$. The consensus problem (4) is solved if the distributed control algorithms are selected as (13) and (14). Furthermore, all x_i, v_i and u_i are bounded for $i=1,2,\ldots,N$.

Proof. The result is a direct consequence of Theorem 2, and we omit its proof. ■

Remark 6: Using a similar analysis to that in Remark 5 we can prove that the controller in Theorem 2 of [34] ensures consensus for DI agents with a fixed, strongly connected graph but the method cannot be generalized for DI agents having a jointly strongly connected basis. However, extensive simulations for both SI and DI agents have not contradicted the claim that the controllers of [34] can still achieve consensus for the case of switching topologies having a jointly strongly connected basis. Theoretically proving this claim is to the best of our knowledge a challenging open problem.

IV. Simulation Examples

In this section, a group of four agents with SI dynamics (Case 1) or DI dynamics (Case 2) is considered under unbalanced and switching topologies having a jointly strongly connected basis shown in Fig. 2. An infinite sequence of switchings occurs in a periodic manner with transitions $\mathcal{G}_1\to\mathcal{G}_2\to\mathcal{G}_3\to\mathcal{G}_1\to\cdots$ with activated topology at time t

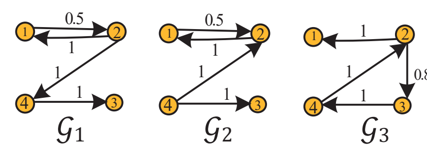

Fig. 2. The switching topologies $\mathcal{G}_1,\mathcal{G}_2,\mathcal{G}_3$.

$$\mathcal{G}(t)=\begin{cases}\mathcal{G}_1, & \text{if } t \mod 2\in[0,0.5) \\ \mathcal{G}_2, & \text{if } t \mod 2\in[0.5,1) \\ \mathcal{G}_3, & \text{if } t \mod 2\in[1,2)\end{cases}.$$

For both cases let initial states $x(0)=[-1,1.2,-3,1.5]^T$ and non-identical unknown control gains $b_1=1$, $b_2=-4$, $b_3=-3$, $b_4=6$. For Case 2 let the initial condition $v(0)=[-0.2,-1,0.2,1]^T$. The proposed control laws (5), (6) and (13), (14) are employed with parameters $\lambda_1=0.4$, $\lambda_2=0.2$, $\rho=0.55$. Simulation results are shown in Fig. 3-4 for Case 1, and in Fig. 5-7 for Case 2, respectively. It is clear that for both cases asymptotic consensus is achieved and all the signals x_i, v_i and u_i are bounded.

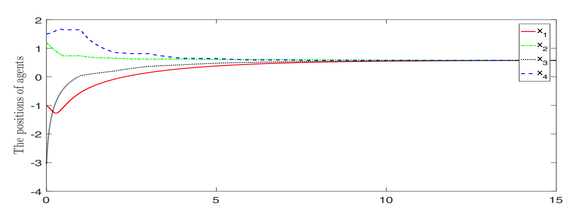

Fig. 3. The positions x_i for SI agents ($i=1,\ldots,4$)

V. Conclusion

We present a method to solve the consensus problem for agents with non-identical unknown control directions under unbalanced and switching topologies. A new class of nonlinear PI based algorithms are constructed by a suitable selection of

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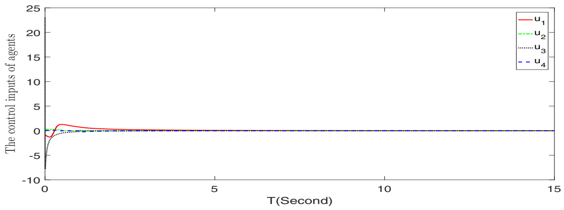

Fig. 4. The control inputs u_i for SI agents $(i = 1, \ldots, 4)$

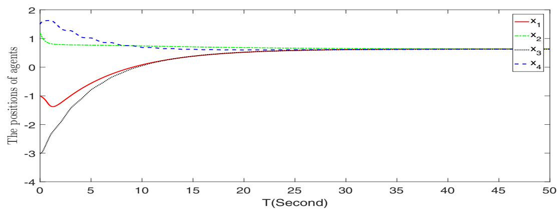

Fig. 5. The positions x_i for DI agents $(i = 1, \ldots, 4)$

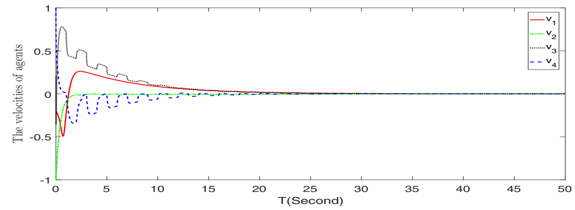

Fig. 6. The velocities v_i for DI agents $(i = 1, \ldots, 4)$

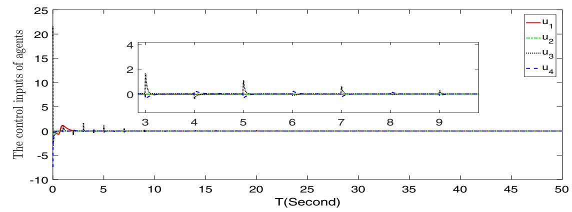

Fig. 7. The control inputs u_i for DI agents $(i = 1, \ldots, 4)$

the distributed nonlinear PI functions. It has been rigorously proven that the consensus of SI and DI agents with non-identical unknown control directions can be achieved under switching topologies having a jointly strongly connected basis or a strongly connected and fixed graph.

Appendix A
Proof of Lemma 3

Proof. Assume the opposite. Then, for some sufficiently small $\epsilon > 0$ there exists a sequence of times $\{T_\sigma\}_{\sigma=1}^{\infty}$ with $\lim_{\sigma\to\infty} T_\sigma = +\infty$ such that $|\dot{f}(T_\sigma)| > \epsilon$ for all $\sigma \in \mathbb{N}$. Obviously $T_\sigma \in [t_{j^*(\sigma)}, t_{j^*(\sigma)+1})$ for some $j^*(\sigma) \in I$ and since $t_{j^*(\sigma)+1} - t_{j^*(\sigma)} > \tau$ we have that either $[T_\sigma, T_\sigma + \tau/2] \subseteq [t_{j^*(\sigma)}, t_{j^*(\sigma)+1})$ or $[T_\sigma - \tau/2, T_\sigma] \subseteq [t_{j^*(\sigma)}, t_{j^*(\sigma)+1})$. Without loss of generality we assume that $[T_\sigma, T_\sigma + \tau/2] \subseteq [t_{j^*(\sigma)}, t_{j^*(\sigma)+1})$. Since $\dot{f}(t)$ is uniformly piecewise right continuous there exists some $\delta(\epsilon) > 0$ such that

$$|\dot{f}(t) - \dot{f}(T_\sigma)| \le \epsilon/2$$

for all $t \in [T_\sigma, T_\sigma + \delta(\epsilon)] \subset [t_{j^*(\sigma)}, t_{j^*(\sigma)+1})$. Hence,

$$|\dot{f}(t)| \ge |\dot{f}(T_\sigma)| - |\dot{f}(t) - \dot{f}(T_\sigma)| > \epsilon - \epsilon/2 = \epsilon/2, \quad (19)$$

for all $t \in [T_\sigma, T_\sigma + \delta(\epsilon)]$. Due to $\lim_{t\to\infty}[f(t)\dot{f}(t)] = 0$ there exist $\bar{\sigma} \in \mathbb{N}_+$ such that

$$|f(t)\dot{f}(t)| \le \frac{\delta(\epsilon)\epsilon^2}{8} \quad \forall t \in [T_\sigma, T_\sigma + \delta(\epsilon)],\ \forall \sigma \ge \bar{\sigma}. \quad (20)$$

From (19) and (20) we obtain

$$|f(t)| \le \frac{\delta(\epsilon)\epsilon^2}{8|\dot{f}(t)|} < \frac{\delta(\epsilon)\epsilon}{4} \quad \forall t \in [T_\sigma, T_\sigma + \delta(\epsilon)],\ \forall \sigma \ge \bar{\sigma}. \quad (21)$$

Using now the mean value theorem we have

$$f(T_\sigma + \delta(\epsilon)) = f(T_\sigma) + \delta(\epsilon)\dot{f}(T_\sigma + \theta\delta(\epsilon))\big|_{\theta\in(0,1)}. \quad (22)$$

Combining (20), (21) and (22) we result in

$$\begin{aligned} |f(T_\sigma + \delta(\epsilon))| &> \delta(\epsilon)\left|\dot{f}(T_\sigma + \theta\delta(\epsilon))\big|_{\theta\in(0,1)}\right| - |f(T_\sigma)| \\ &> \frac{\delta(\epsilon)\epsilon}{2} - \frac{\delta(\epsilon)\epsilon}{4} = \frac{\delta(\epsilon)\epsilon}{4} \end{aligned}$$

which is a contradiction to (21). This completes the proof. ■

Appendix B
Proof of Lemma 4

Proof. Since the nodes of the basis bicomponent $\mathcal{G}_b$ do not receive information from the remaining graph nodes, we have that $a_{i_m,k} = 0$ for all $k \ne i_1, i_2, \cdots, i_r$, $m = 1, \cdots, r$. Thus, the diagonal elements of the Laplacian L at the i_m rows are $\sum_{n=1}^{r} a_{i_m,i_n}$. Due to this property, deleting all other rows and columns we obtain the matrix L_r which is the Laplacian matrix that corresponds to the strongly connected subgraph with vertices $i_1, \cdots, i_r$. The rest of the proof follows from Lemma 7.7 in [41]. ■

Appendix C
Proof of Lemma 5

Proof. For every time interval wherein the topology L_ℓ is activated with a basis bicomponent that involves the agents $i_1, \cdots, i_r$ it holds true that $\xi_{i_m} = \sum_{n=1}^{r} a_{i_m i_n}(y_{i_m} - y_{i_n})$ and

$$\sum_{m=1}^{r} \omega_m y_{i_m}(t)\xi_{i_m}(t) = \frac{1}{2}\sum_{m=1}^{r}\sum_{n=1}^{r} \omega_m a_{i_m i_n}(y_{i_m} - y_{i_n})^2 \quad (23)$$

from Lemma 4. Since $\lim_{t\to\infty} y_i(t)\xi_i(t) = 0$ and $\omega_m > 0$, for a direct edge from i_m to i_n in the basis bicomponent $(a_{i_m i_n} > 0)$, there exists some integer $N_{i_m i_n} \in \mathbb{N}_+$ such that

$$|y_{i_m}(t) - y_{i_n}(t)| \le \epsilon/2(r-1), \quad \forall t \in [T_{\ell\nu}^a, T_{\ell\nu}^d] \quad (24)$$

for all integers $\nu \ge N_{i_m i_n}$. Due to strong connectivity of the subgraph defined by all those nodes $i_1, \cdots, i_r$ (basis bicomponent) there is a direct path from each i_m to each i_n that involves up to $r-1$ links. Thus, for sufficiently large $\nu \ge \max_{1\le m,n\le r} N_{i_m i_n}$ we then have from (24) and the triangle inequality that

$$|y_{i_m}(t) - y_{i_n}(t)| \le \epsilon/2 \quad \forall t \in [T_{\ell\nu}^a, T_{\ell\nu}^d] \quad (25)$$

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for all $m,n\in\{1,2\cdots,r\}$. During the interim time intervals there exists a finite number of switching times which is less than or equal to $\varrho:=\lfloor\tau_{\max}/\tau_{\min}\rfloor-1$. Denote by $\tau_1^{\ell\nu}$, $\tau_2^{\ell\nu}$, $\cdots$, $\tau_{\gamma_{\ell\nu}}^{\ell\nu}\in\{t_j\}_{j=1}^{\infty}$ these switching times i.e., $T_{\ell\nu}^d<\tau_1^{\ell\nu}<\cdots<\tau_{\gamma_{\ell\nu}}^{\ell\nu}<T_{\ell,\nu+1}^a$ with $\gamma_{\ell\nu}\in\mathbb{N}_+$ such that $\gamma_{\ell\nu}\leq\varrho$. From the triangle inequality, for $t\in[T_{\ell\nu}^d,T_{\ell,\nu+1}^a]$ we have

$$\begin{aligned}|y_{i_\varsigma}(t)-y_{i_\varsigma}\left(T_{\ell\nu}^d\right)|\leq&|y_{i_\varsigma}(t)-y_{i_\varsigma}(\tau_{\varpi}^{\ell\nu})|\\&+\sum_{k=1}^{\varpi-1}|y_{i_\varsigma}(\tau_{k+1}^{\ell\nu})-y_{i_\varsigma}(\tau_k^{\ell\nu})|\\&+|y_{i_\varsigma}(\tau_1^{\ell\nu})-y_{i_\varsigma}\left(T_{\ell\nu}^d\right)|\end{aligned}\qquad(26)$$

with $\tau_{\varpi}^{\ell\nu}=\max\{\tau_\beta^{\ell\nu}:\tau_\beta^{\ell\nu}\leq t\ ,\ \beta=1,2,\cdots,\gamma_{\ell\nu}\}$ for all $\varsigma=1,2,\cdots,r$. Note that in each of the open intervals $(T_{\ell\nu}^d,\tau_1^{\ell\nu})$, $(\tau_i^{\ell\nu},\tau_{i+1}^{\ell\nu})$, $(\tau_{\gamma_{\ell\nu}}^{\ell\nu},T_{\ell,\nu+1}^a)$ x_i is continuously differentiable. Thus, from the mean value theorem we have that

$$|y_{i_\varsigma}(t)-y_{i_\varsigma}(\tau_{\varpi}^{\ell\nu})|\leq\sup_{\tau_{\varpi}^{\ell\nu}<s<t}|\dot y_{i_\varsigma}(s)|\Delta T_{\ell,\nu}\qquad(27)$$

$$|y_{i_\varsigma}(\tau_{k+1}^{\ell\nu})-y_{i_\varsigma}(\tau_k^{\ell\nu})|\leq\sup_{\tau_k^{\ell\nu}<s<\tau_{k+1}^{\ell\nu}}|\dot y_{i_\varsigma}(s)|\Delta T_{\ell,\nu},\quad(k=1,2,\cdots,\varpi-1)\qquad(28)$$

$$|y_{i_\varsigma}(\tau_1^{\ell\nu})-y_{i_\varsigma}\left(T_{\ell\nu}^d\right)|\leq\sup_{T_{\ell\nu}^d<s<\tau_1^{\ell\nu}}|\dot y_{i_\varsigma}(s)|\Delta T_{\ell,\nu}\qquad(29)$$

for all $t\in[T_{\ell\nu}^d,T_{\ell,\nu+1}^a]$, $\varsigma\in\{1,2\cdots,r\}$ with $\Delta T_{\ell,\nu}:=T_{\ell,\nu+1}^a-T_{\ell\nu}^d$. Since $\lim_{t\to\infty}\dot y_i(t)=0$ there exists time $T_{i_\varsigma}(\epsilon)>0$ such that $|\dot y_{i_\varsigma}(t)|\leq\epsilon/(4(\varrho+1)\tau_{\max})$ for all $t\geq T_{i_\varsigma}(\epsilon)$ with $t\neq t_j$ for all $j\in\mathbb{N}_+$, $\varsigma=1,2,\cdots,r$. Then, from (26)-(29) we obtain

$$|y_{i_\varsigma}(t)-y_{i_\varsigma}(T_{\ell\nu}^d)|\leq\frac{(\varpi+1)\epsilon}{4(\varrho+1)\tau_{\max}}(T_{\ell,\nu+1}^a-T_{\ell\nu}^d)\leq\frac{\epsilon}{4}\qquad(30)$$

for all $\nu\in\mathbb{N}_+$ such that $T_{\ell\nu}^d\geq T_{i_\varsigma}(\epsilon)$, $\varsigma\in\{1,2\cdots,r\}$. Using now the triangle inequality with (25) and (30) for $\varsigma=m,n$ we result in

$$\begin{aligned}|y_{i_m}(t)-y_{i_n}(t)|\leq&|y_{i_m}(t)-y_{i_m}(T_{\ell\nu}^d)|\\&+|y_{i_n}(t)-y_{i_n}(T_{\ell\nu}^d)|\\&+|y_{i_m}(T_{\ell\nu}^d)-y_{i_n}(T_{\ell\nu}^d)|\leq\epsilon\end{aligned}\qquad(31)$$

for all $t\in[T_{\ell\nu}^d,T_{\ell,\nu+1}^a]$ with sufficiently large ν ($\nu\geq\max_{1\leq m,n\leq r}N_{i_m i_n}$, $T_{\ell\nu}^d\geq\max\{T_{i_m}(\epsilon),T_{i_n}(\epsilon)\}$). Combining (25) and (31) we arrive at

$$\lim_{t\to\infty}(y_{i_m}(t)-y_{i_n}(t))=0\qquad(32)$$

for every set of agents i_m,i_n which belong to a basis bicomponent of some topology. Since the switching topologies have a jointly strongly connected basis, for every $i,k\in\{1,2,\cdots,N\}$ there exist agents $\sigma_1,\cdots,\sigma_\kappa$ such that the pairs (i,σ_1), (σ_1,σ_2), $\cdots$, $(\sigma_{\kappa-1},\sigma_\kappa)$, (σ_κ,k) have edges over basis bicomponents of the switching topologies. Thus, from (32) we have that $\lim_{t\to\infty}(y_i(t)-y_{\sigma_1}(t))=0$, $\lim_{t\to\infty}(y_{\sigma_w}(t)-y_{\sigma_{w+1}}(t))=0$, $(1\leq w\leq\kappa-1)$ and $\lim_{t\to\infty}(y_{\sigma_\kappa}(t)-y_k(t))=0$. If we define $\sigma_0:=i$, $\sigma_{\kappa+1}:=k$ then

$$\lim_{t\to\infty}(y_i(t)-y_k(t))=\sum_{w=0}^{\kappa}\lim_{t\to\infty}(y_{\sigma_w}(t)-y_{\sigma_{w+1}}(t))=0$$

for all $i,k\in\{1,2,\cdots,N\}$ and the proof is completed. ∎

REFERENCES

[1] R. Olfati-Saber, J. Fax, and R. Murray, "Consensus and cooperation in networked multi-agent systems," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 215 – 233, 2007.

[2] W. Ren and R. Beard, "Consensus seeking in multiagent systems under dynamically changing interaction topologies," *IEEE Trans. Autom. Control*, vol. 50, no. 5, pp. 655 – 661, 2005.

[3] X. Wang, Y. Hong, J. Huang, and Z. Jiang, "A distributed control approach to a robust output regulation problem for multi-agent linear systems," *IEEE Trans. Autom. Control*, vol. 55, no. 12, pp. 2891 – 2895, 2010.

[4] J. Qin, H. Gao, and C. Yu, "On discrete-time convergence for general linear multi-agent systems under dynamic topology," *IEEE Trans. Autom. Control*, vol. 59, no. 4, pp. 1054 – 1059, 2014.

[5] S. Liu, L. Xie, and H. Zhang, "Distributed consensus for multi-agent systems with delays and noises in transmission channels," *Automatica*, vol. 47, no. 5, pp. 920 – 934, 2011.

[6] W. Yu, G. Chen, and M. Cao, "Some necessary and sufficient conditions for second-order consensus in multi-agent dynamical systems," *Automatica*, vol. 46, no. 6, pp. 1089 – 1095, 2010.

[7] G. Wen, Z. Duan, W. Yu, and G. Chen, "Consensus in multi-agent systems with communication constraints," *Int. J. Robust and Nonlinear Control*, vol. 22, no. 2, pp. 170 – 182, 2012.

[8] J. Du, C. Guo, S. Yu, and Y. Zhao, "Adaptive autopilot design of time-varying uncertain ships with completely unknown control coefficient," *IEEE Journal of Oceanic Engineering*, vol. 32, no. 2, pp. 346 – 352, 2007.

[9] A. Astolfi, L. Hsu, M. Netto, and R. Ortega, "Two solutions to the adaptive visual servoing problem," *IEEE Trans. Robotics and Automation*, vol. 18, no. 3, pp. 387 – 392, 2002.

[10] R. Nussbaum, "Some remarks on a conjecture in parameter adaptive control," *Systems & Control Letters*, vol. 3, no. 5, pp. 243 – 246, 1983.

[11] X. Ye, "Decentralized adaptive regulation with unknown high-frequency-gain signs," *IEEE Trans. Autom. Control*, vol. 44, no. 11, pp. 2072 – 2076, 1999.

[12] Z. Ding, "Global adaptive output feedback stabilization of nonlinear systems of any relative degree with unknown high-frequency gains," *IEEE Trans. Autom. Control*, vol. 43, no. 10, pp. 1442 – 1446, 1998.

[13] Y. Zhang, C. Wen, and Y. Soh, "Adaptive backstepping control design for systems with unknown high-frequency gain," *IEEE Trans. Autom. Control*, vol. 45, no. 12, pp. 2350 – 2354, 2000.

[14] S. Ge and J. Wang, "Robust adaptive tracking for time-varying uncertain nonlinear systems with unknown control coefficients," *IEEE Trans. Autom. Control*, vol. 48, no. 8, pp. 1463 – 1469, 2003.

[15] L. Liu and J. Huang, "Global robust output regulation of output feedback systems with unknown high-frequency gain sign," *IEEE Trans. Autom. Control*, vol. 51, no. 4, pp. 625 – 631, 2006.

[16] X. Yan and Y. Liu, "Global practical tracking for high-order uncertain nonlinear systems with unknown control directions," *SIAM J. Control Optim.*, vol. 48, no. 7, pp. 4453 – 4473, 2010.

[17] R. Ortega, A. Astolfi, and N. Barabanov, "Nonlinear PI control of uncertain systems: An alternative to parameter adaptation," *Systems & Control Letters*, vol. 47, no. 3, pp. 259 – 278, 2002.

[18] A. Astolfi, D. Karagiannis, and R. Ortega, *Nonlinear and adaptive control with applications*. Springer, 2007.

[19] H.E. Psillakis, "An extension of the Georgiou–Smith example: Boundedness and attractivity in the presence of unmodelled dynamics via nonlinear PI control," *Systems & Control Letters*, vol. 92, pp. 1 – 4, 2016.

[20] H.E. Psillakis, "Further results on robustness of the nonlinear PI control method: The ignored actuator dynamics case," in *Proc. the 24th Mediterranean Conference on Control and Automation*, Athens, Greece, 2016, pp. 77 – 81.

[21] H.E. Psillakis, "Integrator backstepping with the nonlinear PI method: An integral equation approach," *European Journal of Control*, vol. 28, pp. 49 – 55, 2016.

[22] W. Chen, X. Li, W. Ren, and C. Wen, "Adaptive consensus of multi-agent systems with unknown identical control directions based on a novel Nussbaum-type function," *IEEE Trans. Autom. Control*, vol. 59, no. 7, pp. 1887 – 1892, 2014.

[23] Z. Ding, "Adaptive consensus output regulation of a class of nonlinear systems with unknown high-frequency gain," *Automatica*, vol. 51, pp. 348 – 355, 2015.

[24] L. Liu, "Adaptive cooperative output regulation for a class of nonlinear multi-agent systems," *IEEE Trans. Autom. Control*, vol. 60, no. 6, pp. 1677 – 1682, 2015.

[25] Y. Su, "Cooperative global output regulation of second-order nonlinear multi-agent systems with unknown control direction," *IEEE Trans. Autom. Control*, vol. 60, no. 12, pp. 3275 – 3280, 2015.

[26] M. Guo, D. Xu, and L. Liu, "Cooperative output regulation of heterogeneous nonlinear multi-agent systems with unknown control directions," *IEEE Trans. Autom. Control*, vol. 62, no. 6, pp. 3039 – 3045, 2017.

[27] J. Peng and X. Ye, "Cooperative control of multiple heterogeneous agents with unknown high-frequency-gain signs," *Systems & Control Letters*, vol. 68, pp. 51 – 56, 2014.

[28] M. Radenkovic and M. Tadi, "Multi-agent adaptive consensus of networked systems on directed graphs," *International Journal of Adaptive Control and Signal Processing*, vol. 30, no. 1, pp. 46 – 59, 2016.

[29] Q. Ma, "Cooperative control of multi-agent systems with unknown control directions," *Applied Mathematics and Computation*, vol. 292, pp. 240 – 252, 2017.

[30] P. Shi and Q. Shen, "Cooperative control of multi-agent systems with unknown state-dependent controlling effects," *IEEE Trans. Autom. Science and Engineering*, vol. 12, no. 3, pp. 827 – 834, 2015.

[31] W. Wang, D. Wang, Z. Peng, and T. Li, "Prescribed performance consensus of uncertain nonlinear strict-feedback systems with unknown control directions," *IEEE Trans. Systems, Man and Cybernetics: Systems*, vol. 46, no. 9, pp. 1279 – 1286, 2016.

[32] G. Wang, C. Wang, L. Li, and Z. Zhang, "Designing distributed consensus protocols for second-order nonlinear multi-agents with unknown control directions under directed graphs," *Journal of the Franklin Institute*, vol. 354, no. 1, pp. 571 – 592, 2017.

[33] C. Chen, C. Wen, Z. Liu, K. Xie, Y. Zhang, and C. Chen, "Adaptive consensus of nonlinear multi-agent systems with non-identical partially unknown control directions and bounded modelling errors," *IEEE Trans. Autom. Control*, vol. 62, no. 9, pp. 4654 – 4659, 2017.

[34] H. Psillakis, "Consensus in networks of agents with unknown high-frequency gain signs and switching topology," *IEEE Trans. Autom. Control*, vol. 62, no. 8, pp. 3993 – 3998, 2017.

[35] A. Stanoev and D. Smilkov, *Consensus and synchronization in complex networks*. Springer, 2013.

[36] P. Chebotarev and R. Agaev, "The forest consensus theorem," *IEEE Trans. Autom. Control*, vol. 59, no. 9, pp. 2475 – 2479, 2014.

[37] R. Agaev and P. Chebotarev, "The matrix of maximum out forests of a digraph and its applications," *Automation and Remote Control*, vol. 61, no. 9, pp. 1424 – 1450, 2000.

[38] H. Yu and X. Xia, "Adaptive consensus of multi-agents in networks with jointly connected topologies," *Automatica*, vol. 48, no. 8, pp. 1783 – 1790, 2012.

[39] Z. Meng, T. Yang, G. Li, W. Ren, and D. Wu, "Synchronization of coupled dynamical systems: Tolerance to weak connectivity and arbitrarily bounded time-varying delays," *IEEE Trans. Autom. Control*, DOI: 10.1109/TAC.2017.2754219, 2017.

[40] M. Hirsch and S. Smale, *Differential equations, dynamical systems and linear algebra*. Academic Press, 1973.

[41] F. Lewis, H. Zhang, K. Hengster-Movric, and A. Das, *Cooperative control of multi-agent systems: optimal and adaptive design approaches*. Springer, 2014.