FAST AND ACCURATE MEASUREMENT OF DIRECTIONS BY USING DIGITAL THEODOLITES

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Abstract

The mechanical or precise old theodolites have graduated glass circles. In order to least the errors of a random wrong graduation of the instrument's circle, one should adopt an observation program of several sets of measurements on both telescope's face positions, by the method of rounds or by the method of directions or by the method of angles measurement. The starting point of measurement in each set must change according to the formula $200^g/n$, where n is the number of sets, in order to use the entire circle surface in the measurements. The following questions arise immediately. Is it indispensable nowadays to carry out sets of measurements? Is it necessary when using digital theodolites to change the initial reading point on the digital circle while measuring?

An experiment was carried out in order to lead at a new proposal for fast and accurate measurements of directions by using digital theodolites without the traditional procedure of the sets. A network was established in the laboratory. Special and distinct targets marked the points of the network. All the directions between the main pillar and the network's points were measured by using the mechanical theodolite WILD T2, on both telescope faces and in four sets by the method of the closing rounds. The same angles were also measured by using the digital theodolites Leica TDM 5000, TCA 1800, TCR303 and Trimble DR^+ 5605 with repeatable measurements on each target at the same time without following the sets procedure.

The results, the comparison of the values of the calculated angles and the achieved accuracy lead to useful conclusions.

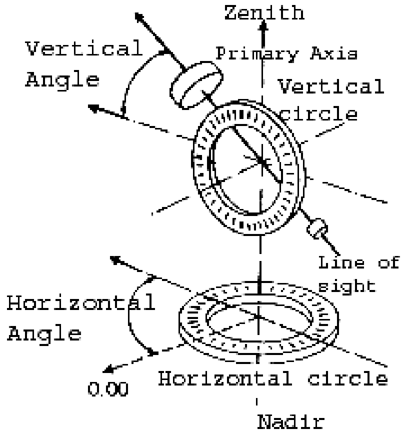
1. Introduction

The evolution of the digital technology, in the geodetic instruments has influenced the procedure of the angle and direction measurements. The modern digital theodolites are embodied in the digital total stations and have replaced today the old mechanical theodolites. This change has brought important improvement in their use, for instance:

- The setting out of the instruments (centering by laser beam and digital leveling)
- The digital reading and the automatic registration of the measured values.

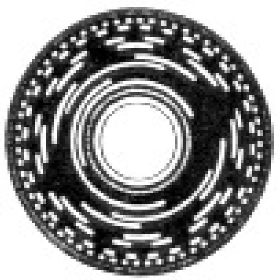
Initially, the mechanical theodolites have reading circles (horizontal and vertical), (fot. 1) which were metallic, and their graduation was carried out by mechanical tools. Later on, the metallic circles were replaced by glass ones, which are nicked with a mechanical way. At the modern mechanical theodolites, the glass-circles are graduated by photographic methods. The glass-circles are uniform and they let the light-rays pass through them. Thus, the graduation accuracy has increased in comparison to the initial model.

The diameter of the circles fluctuates from 6 to 25cm, according to the providing accuracy by the instrument. The graduation is in grads from 0^g to 399^g and the readings increase clockwise.



Fot. 1. Circles of the mechanical theodolite

The digital theodolites differ from the mechanical ones at the graduation manner. They have code-circles of readings, which have printed on them the codes that correspond to the reading values. The circumference of the circle is divided in equal intervals (parts) of 10^c. Each part allows a light-sign to enter or not (Gray-code). Fot. 2 illustrate the code circle that was used in the digital theodolites (Balodimos, Stathas, Arabatzi, 2000).



Fot.2. Code-circle of the digital theodolites

2. A new procedure for direction measurements

When using of mechanical theodolites for the measurement of directions, in order to eliminate the systematic or random errors caused by the bad circle graduation or by the circle eccentricity or by the observer's estimation of the readings, it was indispensable to carry out sets of measurements, using all the surface of the graduated circle. In each set the starting point of the measurements must change according to the formula $200^g/n$, where n is the number of sets.

Today by using the digital theodolites as there are neither graduation errors of the digital circles nor reading errors, there is no need to perform the above mentioned measurement methodology.

An application of a new measurement methodology is carried out. According to this methodology, it is sufficient to have repeatable sightings on each target for ***n*** indispensable measurements at the same time. This application investigates the accuracy and the correctness of the results compared to the results of the previous adopted methodology.

3. Application

The application of the methodology was performed at a network, established in a laboratory room. There were pillars where the instruments were put onto a forced centering base in order to eliminate the centering errors. The pillar B1 was chosen. The points of the network were marked by special targets. Among four different targets for industrial geodesy applications, one is chosen, as the sightings were very close, about 10m, and high pointing accuracy is requested. The chosen target insures:

- The ability of unique and accurate pointing from any direction as it has the appropriate shape and size.
- The stability and permanence of its position in order to reuse it.

The network consists of five points placed all around the pillar B1, at such places to cover the entire horizontal circle. Fig. 1 presents the laboratory room and the positions of the network points and fig.2 illustrates the target.

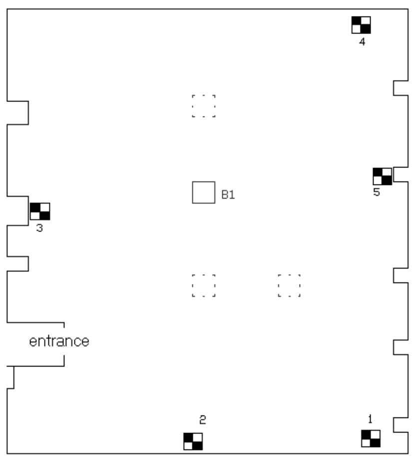


Figure 1. The laboratory room

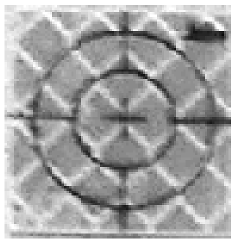


Figure 2. The chosen target.

The measurements were carried out by using:

- The mechanical precise theodolite Wild T2. According to the classical proceedings, four sets of measurements (n=4) were carried out in both telescope faces by the method of the closing rounds. In each set there was a different starting point of the indications according to the formula $200^g/n$.
- The digital total stations Leica TDM 5000, TCA 1800, TCR303 and Trimble DR^+ 5605. There were performed four (4) indispensable repeatable measurements, at the same time, both on I and II telescope faces, to the five targets and the results were the final values of the five measured directions, by each instrument. All the used instruments have the ability of digital leveling and digital automatic registration of the measurements.

A special heavy base for forced instrument centering, which insure a centering error of ±0.1mm is used in order to achieve high accuracy in the measurements corresponding to the provided accuracy by the total stations.

3.1 Measurements with the theodolite Wild T2

The mechanical precise theodolite Wild T2 provides readings of 1^{cc} and has an optical micrometer for the accurate estimation of the indicated values (Wild Heerbrugg Ltd).
The mean values of the five horizontal angles, which were formed between the five targets, of the network and their standard deviation were calculated. The results appear in table 1.

From	To	FACE I	FACE II	*Mean Value*	*Mean referring Value*	*General Mean Value*	*Standard Deviation(cc)*	*Angle Value(g)*	*Standard Deviation (cc)*
B1	1	0.0002	199.9973	399.9984	0.0000				
	2	40.8029	240.7999	40.8014	40.8032				
	3	129.3926	329.3910	129.3918	129.3936				
	4	250.5058	50.5022	250.5040	250.5058				
	5	304.6518	104.6558	304.6538	304.6556				
	1	0.0004	199.9964	399.9980					
B1	1	49.9983	249.9976	49.9980	0.0000				
	2	90.7970	290.7996	90.7983	40.8000				
	3	179.3932	379.3907	179.3919	129.3936				
	4	300.5062	100.5024	300.5043	250.5060				
	5	354.6512	154.6528	354.6520	304.6537				
	1	49.9996	249.9978	49.9987					
B1	1	100.0149	300.0114	100.0132	0.0000				
	2	140.8158	340.8148	140.8153	40.8023				
	3	229.4075	29.4053	229.4064	129.3932				
	4	350.5209	150.5219	350.5214	250.5084				
	5	4.6696	204.6668	4.6682	304.6552				
	1	100.0142	300.0114	100.0128					
B1	1	150.0027	350.0001	150.0014	0.0000	0.0000	±6.1		
	2	190.8043	390.8011	190.8027	40.8012	*40.8017*	*±5.8*	***40.8017***	***±8.4***
	3	279.3913	79.3949	279.3931	129.3916	*129.3930*	*±4.8*	***88.5913***	***±7.5***
	4	0.5116	200.5102	0.5109	250.5094	*250.5074*	*±8.9*	***121.1144***	***±10.1***
	5	54.6548	254.6568	54.6558	304.6543	*304.6547*	*±3.1*	***54.1473***	***±9.4***
	1	150.0028	350.0003	150.0016				***95.3453***	***±6.8***

Table 1. Measurements with the theodolite Wild T2.

3.2 Measurements with the Leica TDM 5000

The modern digital total station Leica TDM 5000 (Leica, 1997), is today (2004) one of the most accurate total stations all over the world. It provides readings of 0.1^{cc} and has accuracy by the DIN ′18723, of $\pm1.5^{cc}$.
The instrument has the ability to correct automatically the measured angular values, if there's a small leveling deviation. It has also endless snails without restrainers.
The mean values of the measured horizontal angles appear in table 2.

From	To	FACE I	FACE II	*Mean Value*	*General Mean Value (g)*	*Mean referring Value (g)*	*Angle Value(g)*	*Standard Deviation (cc)*
B1	1	0.00007	200.00073	0.00040	0.00044			
		0.00077	200.00016	0.00046				
		0.00007	200.00067	0.00037				
		0.0057	200.00050	0.00054				
B1	2	40.80250	240.80266	40.80258	40.80248	40.80204	**40.80204**	**± 1.4**
		40.80240	240.80320	40.80280				
		40.80220	240.80260	40.80240				
		40.80160	240.80270	40.80215				
B1	3	129.39321	329.39335	129.39328	129.39318	129.39274	**88.5907**	**± 1.5**
		129.39324	329.39320	129.39322				
		129.39320	329.39312	129.39316				
		129.39312	329.39300	129.39306				
B1	4	250.50840	50.50820	250.50830	250.50820	250.50776	**121.11502**	**± 0.6**
		250.50816	50.50820	250.50818				
		250.50802	50.50826	250.50814				
		250.50816	50.50820	250.50818				
B1	5	304.65526	104.65534	304.65530	304.65535	304.65491	**54.14715**	**± 0.5**
		304.65528	104.65556	304.65542				
		304.65509	104.65577	304.65543				
		304.65500	104.6552	304.65526				
							95.34509	**± 0.5**

Table 2. Measurements with the total station Leica TDM 5000.

3.3 Measurements with the Leica TCA 1800

The digital total station Leica TCA 1800, is one of the more accurate instruments, gives readings of 1^{cc} and has an accuracy by the DIN 18723, of $\pm 3^{cc}$ (Leica, 1997).
Table 3 illustrates the measurements, the final mean values of the horizontal angles and their determined standard deviations.

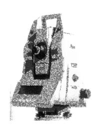
From	To	FACE I	FACE II	*Mean Value*	*General Mean Value (g)*	*Mean referring Value (g)*	*Angle Value(g)*	*Standard Deviation (cc)*
B1	1	0.0002	200.0007	0.0004	0.0005			
		0.0009	200.0005	0.0007				
		0.0004	199.9998	0.0003				
		0.0008	200.0006	0.0007				
B1	2	40.8024	240.8032	40.8028	40.8023	40.8018	**40.8018**	**± 2.0**
		40.8028	240.8022	40.8025				
		40.8022	240.8018	40.8020				
		40.8025	240.8014	40.8019				
B1	3	129.3939	329.3936	129.3938	129.3934	129.3929	**88.5911**	**± 2.6**
		129.3930	329.3936	129.3933				
		129.3932	329.3930	129.3931				
		129.3933	329.3937	129.3935				
B1	4	250.5078	50.5082	250.5080	250.5080	250.5075	**121.1146**	**± 2.1**
		250.5084	50.5088	250.5086				
		250.5085	50.5075	250.5080				
		250.5068	50.5080	250.5074				
B1	5	304.6564	104.6556	304.6560	304.6553	304.6548	**54.1473**	**± 2.4**
		304.6550	104.6552	304.6551				
		304.6540	104.6556	304.6548				
		304.6550	104.6555	304.6552				
							95.3452	**± 2.4**

Table 3. Measurements with the total station Leica TCA 1800.

3.4 Measurements with the Leica TCR 303

The total station Leica TCR 303, gives readings of 5^{cc} and has accuracy, according the DIN ′18723 of $\pm 9^{cc}$ (Leica, 2000). It has also the capability to measure distances with or without reflector up to 100m. TCR 303 is widely used in common geodetic fieldworks. In table 4 appear the results of the measurements with this instrument.

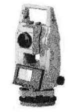
From	To	FACE I	FACE II	*Mean Value*	*General Mean Value (g)*	*Mean referring Value (g)*	*Angle Value(g)*	*Standard Deviation (cc)*
B1	1	0.0000	200.0005	0.0002	0.0003			
		0.0005	200.0005	0.0005				
		0.0005	200.0000	0.0002				
		0.0005	200.0000	0.0002				
B1	2	40.8015	240.8020	40.8018	40.8020	40.8017	**40.8017**	**± 1.3**
		40.8015	240.8020	40.8018				
		40.8015	240.8025	40.8020				
		40.8020	240.8025	40.8022				
B1	3	129.3930	329.3935	129.3932	129.3931	129.3928	**88.5911**	**± 1.2**
		129.3930	329.3930	129.3930				
		129.3925	329.3935	129.3930				
		129.3930	329.3935	129.3932				
B1	4	250.5085	50.5090	250.5088	250.5088	250.5085	**121.1157**	**± 1.2**
		250.5085	50.5095	250.5090				
		250.5080	50.5090	250.5085				
		250.5085	50.5090	250.5088				
B1	5	304.6555	104.6570	304.6562	304.6556	304.6553	**54.1468**	**± 2.3**
		304.6540	104.6570	304.6555				
		304.6545	104.6560	304.6552				
		304.6550	104.6560	304.6555				
							95.3447	**± 2.2**

Table 4. Measurements with the total station Leica TCR303.

3.5 Measurements with DR^+ 5605

The total station DR^+ 5605 is a direct reflex, servo driven instrument, provides readings of 1^{cc} and its accuracy according the DIN ′18723 is $\pm 15^{cc}$ (Trimble, 2002). Table 5 presents the results of the measurements with this instrument.

From	To	FACE I	FACE II	*Mean Value*	*General Mean Value (g)*	*Mean referring Value (g)*	*Angle Value(g)*	*Standard Deviation (cc)*
B1	1	0.0006	199.9985	399.9995	399.9998			
		0.0014	199.9987	0.0000				
		399.9995	200.0005	0.0000				
		0.0000	199.9987	399.9994				
B1	2	40.8027	240.8018	40.8022	40.8023	40.8025	**40.8025**	**± 1.9**
		40.8037	240.8010	40.8024				
		40.8025	240.8025	40.8025				
		40.8033	240.8010	40.8021				
B1	3	129.3934	329.3919	129.3927	129.3922	129.3924	**88.5899**	**± 2.6**
		129.3922	329.3910	129.3916				
		129.3933	329.3909	129.3921				
		129.3934	329.3915	129.3925				
B1	4	250.5037	50.5017	250.5027	250.5029	250.5031	**121.1107**	**± 2.9**
		250.5047	50.5011	250.5029				
		250.5038	50.5016	250.5027				
		250.5042	50.5026	250.5034				
B1	5	304.6521	104.6498	304.6509	304.6505	304.6507	**54.1476**	**± 2.9**
		304.6517	104.6480	304.6499				
		304.6532	104.6476	304.6504				
		304.6526	104.6490	304.6508				
							95.3493	**± 2.9**

Table 5. Measurements with the total station Trimble DR^+ 5605.

3.6 Comparison

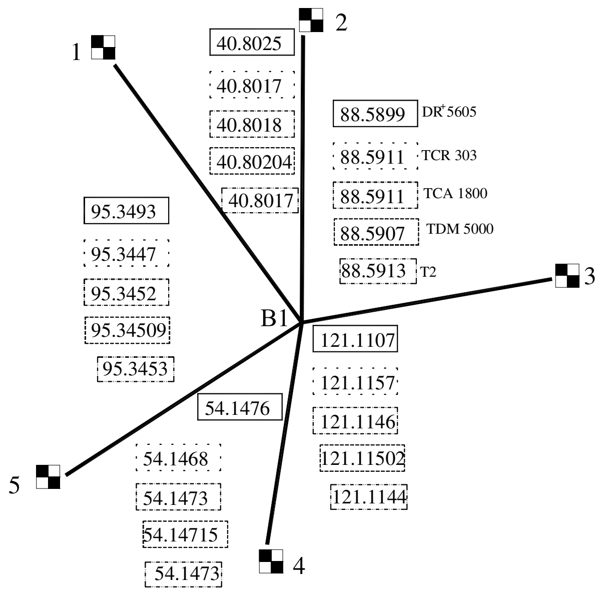


Figure 3. The horizontal angle values calculated by each instrument.

The mean values of the five formed horizontal angles of the network, as measured by the five different instruments must be examined. Fig. 3 presents all the results.
Each value is accompanied by its standard error as calculated by the measurements' results. The results of the digital total stations (TDM5000, TCA 1800, TCR303, DR^+ 5605), which measured with the new methodology of the repeatable measurements, will be compared to the results of the mechanical theodolite Wild T2, which measured using the old methodology of the sets.
So the following equation must be true for each one of the five measured angles, for each total station:

$$t \cdot \sqrt{\left(\sigma^2_{T2angle_i} + \sigma^2_{TotalStation\,angle_i}\right)} \geq \left|\mathrm{G.M.V}_{\mathrm{Hz,T2}} - \mathrm{G.M.V}_{\mathrm{Hz,TotalStation}}\right| \qquad (1)$$

Where,
t : the value of the coefficient of the normal distribution for one dimension check and for confidence level 95%, equal to1.96.
$\sigma_{T2angle_i}$: The standard error of the i angle measured by the Wild T2.
$\sigma_{TotalStation\,angle_i}$: The standard error of the i angle measured by the total station.
$\mathrm{G.M.V}_{\mathrm{Hz,T2}}$: The general mean value of each horizontal angle, measured by the Wild T2.
$\mathrm{G.M.V}_{\mathrm{Hz,TotalStation}}$: The general mean value of each horizontal angle, measured by the total station.
The comparison of all the measured angle values and the valid of the equation (1) for each total station relative to the mechanical theodolite T2 are presented in the tables from 6 to 9.

Angle	Check (95%)
1-B1-2	$1.96 \cdot \sqrt{8.4^2 + 1.4^2} \geq \lvert 40.8017 - 40.80204 \rvert \Rightarrow 16.7^{cc} \geq 3.4^{cc}$
2-B1-3	$1.96 \cdot \sqrt{7.5^2 + 1.5^2} \geq \lvert 88.5913 - 88.5907 \rvert \Rightarrow 15.0^{cc} \geq 6.0^{cc}$
3-B1-4	$1.96 \cdot \sqrt{10.1^2 + 0.6^2} \geq \lvert 121.1144 - 121.11502 \rvert \Rightarrow 19.8^{cc} \geq 6.2^{cc}$
4-B1-5	$1.96 \cdot \sqrt{9.4^2 + 0.5^2} \geq \lvert 54.1473 - 54.14715 \rvert \Rightarrow 18.5^{cc} \geq 1.5^{cc}$
5-B1-1	$1.96 \cdot \sqrt{6.8^2 + 0.5^2} \geq \lvert 95.3453 - 95.34509 \rvert \Rightarrow 13.4^{cc} \geq 2.1^{cc}$

Table 6. Comparison of the horizontal angles values between the Wild T2 and the Leica TDM 5000

Angle	Check (95%)
1-B1-2	$1.96 \cdot \sqrt{8.4^2 + 2.0^2} \geq \lvert 40.8017 - 40.8018 \rvert \Rightarrow 16.9^{cc} \geq 1.0^{cc}$
2-B1-3	$1.96 \cdot \sqrt{7.5^2 + 2.6^2} \geq \lvert 88.5913 - 88.5911 \rvert \Rightarrow 15.6^{cc} \geq 2.0^{cc}$
3-B1-4	$1.96 \cdot \sqrt{10.1^2 + 2.1^2} \geq \lvert 121.1144 - 121.1146 \rvert \Rightarrow 20.2^{cc} \geq 2.0^{cc}$
4-B1-5	$1.96 \cdot \sqrt{9.4^2 + 2.4^2} \geq \lvert 54.1473 - 54.1473 \rvert \Rightarrow 19.0^{cc} \geq 0.0^{cc}$
5-B1-1	$1.96 \cdot \sqrt{6.8^2 + 2.4^2} \geq \lvert 95.3453 - 95.3452 \rvert \Rightarrow 14.1^{cc} \geq 1.0^{cc}$

Table 7. Comparison of the horizontal angles values between the Wild T2 and the Leica TCA 1800.

Angle	Check (95%)
1-B1-2	$1.96 \cdot \sqrt{8.4^2 + 1.3^2} \geq \lvert 40.8017 - 40.8017 \rvert \Rightarrow 16.7^{cc} \geq 0.0^{cc}$
2-B1-3	$1.96 \cdot \sqrt{7.5^2 + 1.2^2} \geq \lvert 88.5913 - 88.5911 \rvert \Rightarrow 14.9^{cc} \geq 2.0^{cc}$
3-B1-4	$1.96 \cdot \sqrt{10.1^2 + 9^2} \geq \lvert 121.1144 - 121.1157 \rvert \Rightarrow 18.6^{cc} \geq 13.0^{cc}$
4-B1-5	$1.96 \cdot \sqrt{9.4^2 + 2.3^2} \geq \lvert 54.1473 - 54.1468 \rvert \Rightarrow 19.0^{cc} \geq 5.0^{cc}$
5-B1-1	$1.96 \cdot \sqrt{6.8^2 + 2.2^2} \geq \lvert 95.3453 - 95.3447 \rvert \Rightarrow 14.0^{cc} \geq 6.0^{cc}$

Table 8. Comparison of the horizontal angles values between the Wild T2 and the Leica TCR303.

Angle	Check (95%)
1-B1-2	$1.96 \cdot \sqrt{8.4^2 + 1.9^2} \geq \lvert 40.8017 - 40.8025 \rvert \Rightarrow 16.9^{cc} \geq 8.0^{cc}$
2-B1-3	$1.96 \cdot \sqrt{7.5^2 + 2.6^2} \geq \lvert 88.5913 - 88.5899 \rvert \Rightarrow 15.6^{cc} \geq 14.0^{cc}$
3-B1-4	$1.96 \cdot \sqrt{10.1^2 + 2.9^2} \geq \lvert 121.1144 - 121.1107 \rvert \Rightarrow 20.6^{cc} \leq 37.0^{cc}$
4-B1-5	$1.96 \cdot \sqrt{9.4^2 + 2.9^2} \geq \lvert 54.1473 - 54.1476 \rvert \Rightarrow 19.3^{cc} \geq 5.0^{cc}$
5-B1-1	$1.96 \cdot \sqrt{6.8^2 + 2.9^2} \geq \lvert 95.3453 - 95.3493 \rvert \Rightarrow 14.5^{cc} \leq 38.0^{cc}$

Table 9. Comparison of the horizontal angles values between the Wild T2 and the Trimble DR^{+} 5605.

4. Conclusions

- The modern digital total stations are released from the graduation or the eccentricity of the measuring circles errors, that the old mechanical theodolites used to have. Also small leveling deviations are today corrected automatically in some of them.
- The reading and estimation error of the observer were totally erased, as well as the registration error of each measurement, as it is registered automatically.
- The achieved accuracy of each horizontal angle measurement is according to the DIN ′18723 for each instrument. In some cases the calculated standard deviation of an angle was smaller than the expected according to the instrument manufacturer. This probably happens due to the very careful use of the instruments and the strict keeping of all the required conditions during the measurements.
- The results of the total stations' measurements are acceptable, according to the previous comparison. So, it isn't indispensable to carry out sets of measurements with the old methodology by changing the starting point of the indications in each set. It is sufficient to

perform some pointings, on each target the same time in order to reduce the observer's sighting error.

- By the presented methodology the observation time is eliminated, as the observer doesn't lose the target from the sighting field of the instrument and sights it again without searching for it for a long time.
- The application proved that it is sufficient to perform four (4) indispensable sightings in order to measure the right angle value.
- The time needed to perform four sets of the horizontal direction measurements at the above-described network is about one hour by using the WILD T2 theodolite, while by the methodology of the repeatable pointings is about 15 minutes by using each one of the digital total stations.
- The methodology of the repeatable pointings on the target was proved:
  - easy and convenient
  - quick
  - accurate
  - correct and acceptable

  for directions and horizontal angles measurements in networks or other geodetic applications.

References

1. Balodimos, D-D, Stathas, D., Arabatzi, O., *Geodesy: Networks-Detailing Procedures-Setting out procedures*, Athens, 2000, N.T.U.A. Edition.
2. Leica Heerbrugg AG, *Users manual for TCA 1800*, Switzerland, 1997.
3. Leica Heerbrugg AG, *Users manual for TM 5000/ TDM 5000 system*, V2.2, Switzerland, 1997.
4. Leica Heerbrugg AG, *Users manual for TCR303*, 2000.
5. Trimble Navigation, *Users manual for 5600*, 2002.
6. Wild Heerburgg Ltd, Wild *T2, Universal Theodolite with automatic index, Instruction for Use,* Switzerland

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