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ASTRONOMICAL AZIMUTH DETERMINATION BY THE HOUR ANGLE OF POLARIS USING ORDINARY TOTAL STATIONS

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ABSTRACT

The determination of the astronomical azimuth of a line is not a difficult task for surveyors any more. The aim of this paper is to analyze the theoretical details and errors in order to propose the use of ordinary total stations, for an easy, efficient and accurate determination of the astronomical azimuth of a line by the hour angle method via Polaris sightings. As many modern total stations have a built – in quartz clock they can register automatically the UTC time as well as the angle measurements (horizontal and zenith) of each observation. The total fieldwork time needed is about 10 minutes and the accuracy that may be achieved is about ± 2″. This procedure will be proven to be easier than the determination of the geodetic azimuth of the same line. The calculation is independent and the result is free of the errors that the coordinates of a survey mark may contain, because they are not used. However good positional data is required from other sources. Astronomical azimuths are an alternative solution for the surveyors in order to check or orient their field surveys and arbitrary networks independent of the GPS system.

KEYWORDS: Astronomical azimuth. Hour angle. Total station. Polaris. Quartz clock.

INTRODUCTION

The determination of astronomical azimuths is not very popular. This is due to the rumour about the difficulty of the calculations and the need for night observations. Supposedly, special skills are needed in order to operate the appropriate instruments for the observations. Fortunately, this is not the case. The former myth, about the level of difficulty, is no longer true. Today the revolution in geodetic instrumentation plays a major role in making the total process easier and quicker. Additionally, it would be very convenient for the surveyors to have quick, easy and accurate astronomical azimuth determination to orient or check a field survey or network. However, clear skies are required for astronomical observations.

This determination is also independent of a national geodetic reference system. So it is not necessary to know any points or pillar coordinates and therefore it is free of the uncertainties that these coordinates may contain.

Surveyors in the Northern hemisphere have a great advantage in determining the astronomical azimuth of a line, especially between latitude 15° and 60° due to Polaris. Polaris is the star that helps anyone find his orientation towards the north. It makes a small circle, about 1° radius, around the North Astronomical Pole and its apparent motion is very slow. It is a bright star with a magnitude of 2.1, which makes it visible at night and easily recognised, as it is the last star at the tail of the Ursa Minoris (Little Bear) constellation. Besides that, the star constellations of Ursa Majoris (Great Bear) and Cassiopeia help to find this valuable star in the celestial sphere.

At latitudes less than 15°, it is not so convenient to observe Polaris as it is located very low on the celestial sphere near the local horizon. At those latitudes the errors caused by astronomical refraction are magnified. At latitudes larger than 60° the method of Polaris observations is affected by the error of the value of the astronomical

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latitude Φ of the station point, which is needed in the calculations. In addition, the geometry of the spherical triangle is very weak and special equipment, for example, an astronomical diagonal eyepiece is indispensable for the observation. In such a case other methods, such as the observation of star pairs near elongation or Black's method, are used [**7**],[**8**].

In the Southern hemisphere a corresponding star is σ Octantis. This is located about 45′ from the South Celestial Pole and has a magnitude of 5.5. It is difficult to recognize on the celestial sphere and for this reason it is necessary to know the approximate azimuth of the reference meridian.

In general, the precision of the astronomical azimuth determination depends on the quality of the instrument used, the method of determination, the number of measurements, the successful removal of systematic errors, the experience of the observer and the latitude of the instrument station. [**8**]

A basic parameter in most observations in geodetic astronomy, as in the astronomical azimuth determination, is the registration of the time of each observation. Many of the modern total stations, even those manufactured for common surveying fieldwork, seem to solve this problem, as they have built in crystal quartz clocks.

Some uses of the astronomical azimuth are:

- Orientation of airport radar and satellite or telecommunication antennas.
- Navigation
- Orientation of army systems.
- Control of underground surveying work in caves and mines from above ground to the start or the end of the traverses.

In addition, a common use of this determination is for the correct orientation of a land survey or an arbitrary network. In most surveying fieldwork a precision of ±2″ to ±5″ is sufficient.

ASTRONOMICAL AZIMUTH DETERMINATION

The astronomical azimuth A_{AB} of a line AB, defined between two points on the earth's surface, may be determined by measuring the horizontal angle between the given direction and the vertical plane of a celestial body and applying this angle to the astronomical azimuth of the celestial body. The following method is the most convenient for determining astronomical azimuths of stars for geodetic purposes, as the best precision will be achieved.

Observing a close circumpolar star at any hour angle (hour angle method).

By this method the UTC time of each observation to the star must be measured. Then the hour angle h of the star is computed by the equation [**8**]:

$$\mathrm{h} = \theta + \Lambda - a \tag{1}$$

where

a = Right ascension of the star

Λ = Astronomical longitude of the instrument station

and the Greenwich sidereal time θ can be calculated as follows [**8**]:

$$\theta = \theta^{0hUT} + (UTC + DUT) \cdot f \tag{2}$$

where

UTC = Coordinated Universal Time

θ^{0hUT} = Greenwich sidereal time at 0^hUT

DUT = Correction at the Coordinated Universal Time (UTC)

f = Ratio of the duration of the mean solar day to the mean sidereal day
= 1.00273790935

The astronomical azimuth A_s of the star is calculated by the formula from the basic rules of spherical trigonometry applied to the astronomical triangle [**8**]:

$$\tan A_s = \frac{-\sin h}{\cos \Phi \cdot \tan \delta - \sin \Phi \cdot \cos h} \qquad (3)$$

where

δ = Declination of the star

Φ = Astronomical latitude of the instrument station

Polaris is the most suitable star for the application of this method. The precision achieved will be analyzed in the following paragraphs.

PROPOSED METHODOLOGY

An ordinary total station that is to be used for astronomical observations must be equipped with a built - in crystal quartz clock, of one second time resolution, a dual – axis level sensor, illumination of the cross hairs and, for convenience, a diagonal eyepiece. In these total stations the levelling errors are largely eliminated as they have digital levelling sensors. Also for instruments with dual axis level sensors, small levelling deviations are corrected instantly and automatically, so that the instruments display the ″correct″ angle value in each measurement. It must be checked that the level sensors and the angle correction system are switched on during the observations. The reading and estimation errors of the observer, which are the booking errors, are eliminated since the measurements are registered automatically. Consequently the only remaining error is the observer’s sighting error that may be reduced by repeated pointings to the target. Thus a high angle measuring precision may be achieved. These instruments are easy to operate and the surveyors use them for simple construction, land surveys and setting out tasks in general.

For the determination of the astronomical azimuth of a line by the hour angle method mentioned above, the time of each observation must be recorded. This can be achieved by using a total station equipped with a built – in crystal quartz clock, which permits the measurement and the registration of the time of each observation, to one second resolution.

The main criterion, which distinguishes a good timekeeper from a poor one, is the stability of its oscillator. Modern technology provides frequency standards with standard deviation of no more than 1μs [**11**]. However, the point is to sychronize the instrument clock with the *UTC* time with a sufficient accuracy. The *UTC* time results by adding or subtracting the integral number of hours of the zone time and the Daylight Saving Time to or from the civil time at every place. The sychronization may be done by:

- The conventional method where the observer hears the broadcast radio time signals and sets the correct time in the instrument clock, as it was done with the old portable chronometer used in geodetic astronomy. This synchronization creates an error of the order of ± 0.5 seconds, which mainly depends on the skill and the experience of the observer.
- The connection of a GPS receiver with the total station [**1**], [**3**], [**4**]. Some GPS receivers can provide accurate time information in the form of 1pps (pulse per second) output, which is synchronised with the *UTC* time to an accuracy of few microseconds [**9**]. This accurate procedure is not easy to apply, as it needs the appropriate software and instrumentation [**3**].

After Polaris has been observed, its astronomical azimuth at the time of each observation is calculated by equation (3). The celestial coordinates of Polaris a, δ, which are required as well as the θ^{0hUT} and DUT values are available for each day of the year on an appropriate website with the digital astronomical ephemeris [**10**].

Instead of the astronomical coordinates Φ, Λ of the instrument station, the geodetic coordinates φ, λ may be used in the equations (1) and (3). A map or a small hand – held GPS receiver may provide these coordinates.

Therefore, one sets up the total station at point A and sights to the desired point B. Repeatedly sight to Polaris and register the measured horizontal angle b (Fig.1) and the UTC time. After these measurements in face left (FL) of the instrument, repeated measurements to Polaris and one measurement to point B follow in face right (FR).

The astronomical azimuth A_{AB} of the line AB will be derived, by the equation (4), by adding or subtracting (depending on the relative position of the points) the horizontal angle b between the direction AB and Polaris to or from the calculated astronomical azimuth of Polaris A_P via equation (3).

$$A_{AB} = 360^\circ - b + A_P \quad (4)$$

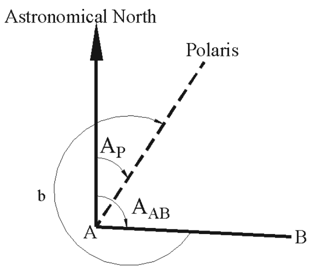

Fig. 1. The astronomical azimuth A_{AB} of the direction AB

From each sighting to Polaris a different azimuth of the star A_P will be calculated according to the observation time and a different horizontal angle b will be registered between the star and the earth's direction AB as the star is moving. So the sum A_{AB} of the two above values will be always the same. Ten to fifteen sightings are sufficient to eliminate the observer's sighting error.

ERROR ANALYSIS

The total error of the determination of the astronomical azimuth A_{AB} of a line AB contains:

- Error of the measured angle b, which includes:
  - Error of the sighting to the point B
  - Error of the sightings to the star
- Error of the declination δ of Polaris
- Error of the used value of the astronomical latitude Φ of the instrument station
- Error of the determination of the hour angle h

Each one of the above errors will be analyzed in the following paragraphs.

- Sighting error to point B.

  An experimental investigation of several types of targets, using several total stations that have nominal precision of ± 0.5″, ± 1″, ± 3″ and ± 7″, was carried out to determine this error.

A luminous target, a cone target and a single prism were used as targets. The single prism is the most commonly used target by the surveyors. The targets were put at distances of 300 m and 100 m and repeated sightings were carried out in four different measurement series.

The results proved that the mean value of the sighting error is of the order of half of the nominal direction measuring precision for each total station. The smallest sighting error was found in the luminous target at distance of 300 m.

- Error of the sightings to the star.

This can be reduced by repeated measurements. Ten to fifteen sightings to the star may be carried out within 5 to 10 minutes. The standard deviation of the single azimuth value determination is calculated by formula (5)

$$\sigma_x = \pm\sqrt{\frac{[\upsilon\upsilon]}{(n-1)}} \tag{5}$$

and correspondingly the standard deviation of the mean azimuth value is calculated by the following formula [**8**].

$$\sigma_{\bar{x}} = \pm\sqrt{\frac{[\upsilon\upsilon]}{n\cdot(n-1)}} \tag{6}$$

- The value of the declination δ of Polaris is given by the digital astronomical ephemeris with a precision of ±0″.01 [**10**]. The derivation of equation (3) gives the following equation (7), which presents the influence of the error in δ, to the error in the azimuth determination.

$$\sigma_{A_\delta} = \pm\frac{\cos\Phi\cdot\sin h}{(\sin z\cdot\cos A_s)^2\cdot(1+\sin^2 h)}\cdot\sigma_\delta \tag{7}$$

This error takes a maximum value of half the given precision of the coordinate δ, about ±0.005″.

- Error of the used value of astronomical latitude Φ.

In equation (3) the astronomical latitude Φ of the instrument station must be used. Since it is quite difficult to determine this the geodetic coordinate φ may be used instead. The difference of the two types of coordinates depends on the fitting of the used ellipsoid to the geoid at the station point. The value of the difference fluctuates between 5″ - 20″ in most areas.

The differentiation of equation (3) leads to equation (8) that calculates the error in the azimuth caused by the uncertainty of the Φ value.

$$\sigma_{A_\Phi} = \pm\sin A_s\cdot\cot z\cdot\sigma_\Phi \tag{8}$$

Figure 2 depicts the error σ_{A_Φ} in the calculated Polaris astronomical azimuth relative to the error σ_Φ in the used value of Φ, and to the astronomical latitude Φ of the instrument station. According to this diagram, an error of 15″ in the Φ value causes error of ±0.″2 in the determination of the astronomical azimuth if the observation point is located at the latitude of 40°.

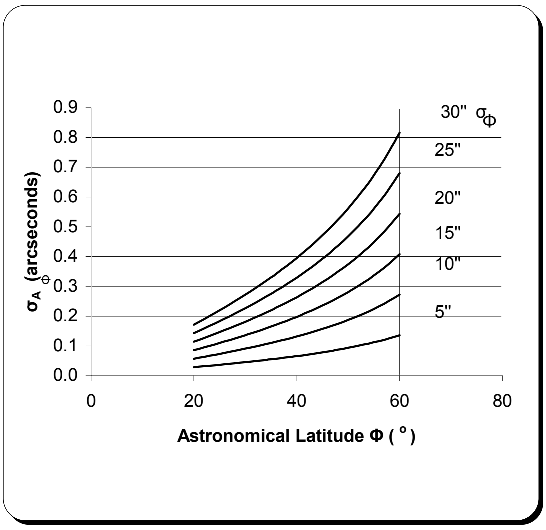

Fig. 2. Diagram of error σ_{A_Φ}

- Error in the determination of the hour angle.
  Another differentiation of equation (3) gives equation (9) that determines the error in the azimuth caused by the total uncertainty in the h value.

$$\sigma_{A_h} = \pm \cos \Phi \cdot (\tan \Phi - \cos A_s \cdot \cot z) \cdot \sigma_h \qquad (9)$$

The hour angle h is determined by equation (1). Therefore, the total error in the hour angle value, is calculated by equation (10):

$$\sigma_h = \pm \sqrt{\sigma_\theta^2 + \sigma_\Lambda^2 + \sigma_\alpha^2} \qquad (10)$$

σ_θ depends on the error of the measured *UTC* time. The measurement of the *UTC* time includes:

- *The synchronization error* of the instrument's clock to the *UTC* time, which is about ±0.5 s. If the *UTC* time information is provided by a GPS receiver connected to a high accuracy total station then the accuracy of the time information is of the order of ±1 ms.
- *The observer delay.* The personal delay of the observer is the time interval between the moment that the observer sights the star in the centre of the reticule's cross-hairs to the moment that he pushes the key of the instrument display to record the measurement. This time delay is different for each observer and depends mainly on his physical characteristics and his reflexes. The magnitude of this delay must be subtracted from the *UTC* time registered at each measurement. It can be determined only experimentally as follows.

A GPS receiver, which can provide accurate *UTC* time information in the form of 1pps output (e.g. Trimble 4000 DL), is connected to a total station (e.g. Leica TDM 5000) by the appropriate hardware and software (Fig.3) [**3**]. Each time the observer takes a measurement, the *UTC* time can be registered by the instrument with 1ms precision. Simultaneously the GPS receiver is connected by the appropriate cable with a light emitting diode (LED) that flashes every integral second as the GPS receiver emits a pulse per second. The observer sees the LED on the instrument's reticule and takes measurements when the LED is flashing.

As the LED flashes exactly every *UTC* integral second and the instrument registers the time that the observer pushes the key to take the measurement, the difference between the two times is the personal delay of the observer.

This time interval was calculated for different observers, according to the above experiment and was found to be about 0.3 s. This time error causes an almost insignificant error in the azimuth determination as illustrated in the following diagram (Fig.4) and may be ignored in Polaris observations. It may be mentioned that most instruments record the values of angles and time simultaneously. If an electronic delay of some μs occurs, it is similarly insignificant.

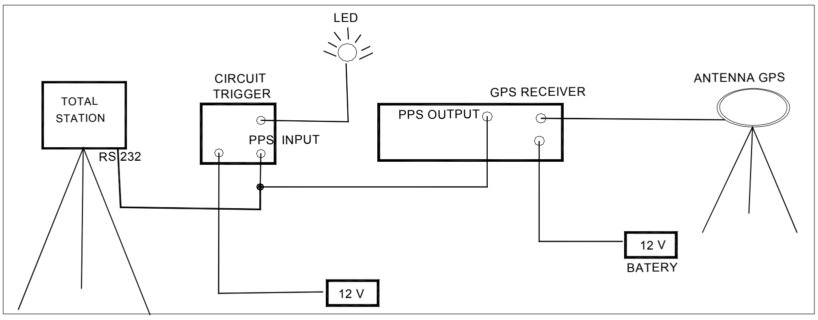

Fig. 3. The instrument's connection for the determination of the observer's delay [**3**]

The error σ_Λ in the astronomical longitude Λ is about ±5″ to ±20″. This is because the geodetic value of λ, is used in lieu of the astronomical value of Λ. σ_α is the error in the right ascension a of Polaris that is provided with a precision of ±0″.01 [**10**].

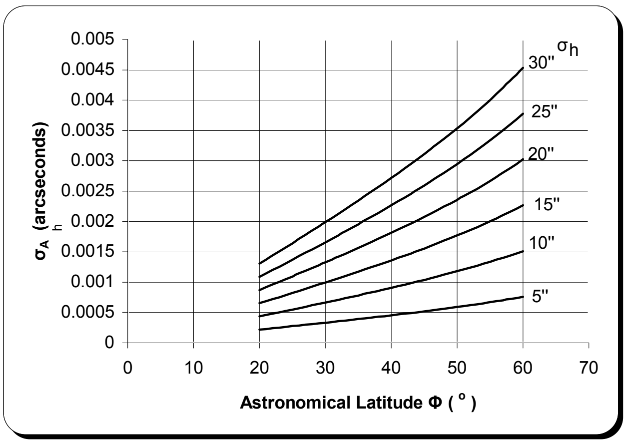

Fig. 4. Diagram of error σ_{Λ_h}

Figure 4 illustrates, according equation (9), the error σ_{A_h} in the determined Polaris astronomical azimuth, relative to the total error of the calculated hour angle according to equation (1) and the astronomical latitude Φ of the observation place. According to equation (10) and taking $\sigma_\theta = \pm 0.8$ s = ±12″, $\sigma_\Lambda = \pm 15''$ and $\sigma_\alpha = \pm 0.01''$ the total error in the determination of the hour angle is $\sigma_h = \pm 1.3$ s = ±20″. At latitude of 40°, a negligible error of ±0″.0018 is caused in the determined Polaris astronomical azimuth.

EXPERIMENT

A practical test was carried out on a pillar of the test field of the National Technical University of Athens. The astronomical coordinates Φ, Λ of the pillar are known. Two different ordinary total stations (Leica TCR303 and Leica TC307) were used for the determination of the astronomical azimuth of the line connecting the pillar to an illuminated target at a distance of 300 m. These instruments have all the required features that were defined previously and are widely used in surveying fieldwork. An observation set consisted of one sighting to the target (FL), 15 sightings to Polaris (FL), 15 sightings to Polaris (FR) and one sighting to the target (FR). The total stations were selected for use after having performed all tests that check their proper function. Each measurement set with one total station lasted about 10 minutes. Table 1 illustrates the results.

Table 1. *Test results*

Total Station	Date	Nominal Direction measuring precision	Time measuring precision	Time synchronization by:	Mean value of Astronomical Azimuth	Standard deviation of the Mean value of the Astronomical Azimuth
Leica TCR303	25/7/2005	3″	1 s	The observer	49° 27′ 01.5″	±2.0″
Leica TC307	28/7/2005	7″	1 s	The observer	49° 26′ 58.3″	±3.9″

DISCUSSION

1. The determination of the astronomical azimuth of a line today, is quick and easy to perform by using ordinary total stations.
2. Observing a close circumpolar star, as Polaris, at any hour angle is very convenient. Polaris, is an easily recognisable star on the celestial sphere of the Earth's Northern hemisphere.
3. Ten-minutes of fieldwork is sufficient for this determination.
4. The built – in quartz clock in most modern total stations helps with automatic registration of the *UTC* time of each observation.
5. The synchonization of the instrument built – in quartz clock to the *UTC* time is not a difficult task as an accuracy of ±1 s in the synchronization is easily achieved.
6. For the astronomical azimuth determination by the hour angle method using Polaris observations the need for high time accuracy is not essential as it causes very small errors in the final calculated azimuth value. Even a total error of four seconds in the

hour angle used causes a maximum error of ±0″.01 in the finally determined astronomical azimuth value.

7. The precision that may be achieved mainly depends on the total station that will be used. Typically, it is half of the nominal direction measuring precision of the instrument.

CONCLUSIONS

The use of modern ordinary total stations leads to fast, easy and accurate determination of the astronomical azimuth A of a given line. This is achieved by:

- Calculating the astronomical azimuth of the star by a simple formula without any computer programming needed.
- Reducing the random error of the observer by repeated sightings to the star.
- Registering directly the *UTC* time, as the built – in total station clock may be synchronized to it.
- Recording automatically and digitally all measured values (angle and time) eliminates reading errors and decreases the data acquisition time needed.

Using an ordinary total station, by sighting to Polaris via the hour angle method, the astronomical azimuth of a given line can be determined in ten minutes. This determination is independent of any national geodetic reference system and the uncertainties or biases that published pillar coordinates usually have.

The previously mentioned advantages, the easy operation, the general availability of modern digital total stations and the simple calculations make the proposed methodology efficient, easy and convenient to use by all surveyors in many land survey applications.

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