# A STANDARD NETWORK TO CONTROL GPS RECEIVERS

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## ABSTRACT

**This work deals with the establishment of a close–range standard 3-dimensional geodetic network in order to certify the GPS receivers' proper function. The standard network is located at the area of the University Campus of NTUA at Zografos Athens, Greece.**
**The first aim is the determination of the X and Y coordinates and the orthometric height H of the network's points as well as the determination of their uncertainties by accurate terrestrial measurements.**
**The final target is to check any pair of GPS receivers.**
**The above standard network consists of five points, marked by special pillars, which ensure the stability of their place and the single setting of both a Total Station and a GPS receiver.**
**The terrestrial observations consist of horizontal and vertical angles and distance measurements between the points. The measurements were carried out by using a digital total station Leica TCA 1800 and the indispensable and appropriate accessories such as tribraches, prisms, targets, heavy forced centering bases, prism holders and adapters, which lead to the maximum possible measuring accuracy.**
**The adjustment of the terrestrial measured data was carried out in the Greek Geodetic Reference System 1987 (GGRS ′87) by the least square method.**
**The standard network was used in order to check a pair of GPS receivers Trimble 4600LS. Ten baselines were measured and determined the coordinates of the five points and the corresponding uncertainties in the same reference system.**
**The comparison of the measured elements such as distances, horizontal angles and height differences as well as the calculated coordinates of both solutions, leads with credibility to the certification of the proper or improper function of GPS receivers and to the checking of their industrial prescribed accuracy.**

## 1. INTRODUCTION

**The rapid evolution of the GPS system and its systematic use for the measurement of 3d geodetic networks and for other common surveying works, surface the need of checking the accuracy and the proper function of the GPS receivers.**
**This control refers to both the initially measured networks' elements as azimuths, slope**

distances and height differences and to the determined coordinates in a reference system.
Following a procedure to control the GPS receiver is analyzed which is based on the establishment, the measurement and the adjustment of a 3d high accuracy geodetic network.
This network will be the reference for checking the proper function of any pair of GPS receivers and the succeeded accuracy in the positioning.

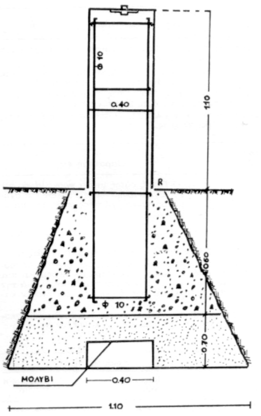
## 2. THE NETWORK

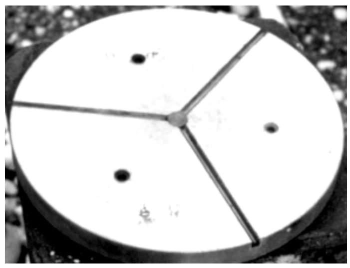
A 3d geodetic network defined as a set of points on the earth's surface which are connected by on site measurements, in order to determine, by common procedure, the coordinates X, Y and Z of each point in a Reference System (geocentric or topocentric).

A standard 3d geodetic network was established at the University Campus of NTUA and consists of 5 points (Piniotis et al., 2000). The choice of each point was made by using the following criteria:

- The insurance of permanence and stability of its position.
- The creation, as best as possible, of a regular geometric shape by the network's sites.

The marking of the network points was done by using special constructions as pillars (fot. 1), which insures the unique definition of the point and the easy setting of the instruments or the receivers of the GPS system. Figure 1 illustrates the geometric shape of the network.





**Fot.1: The marking of the points of the 3d standard geodetic network.**

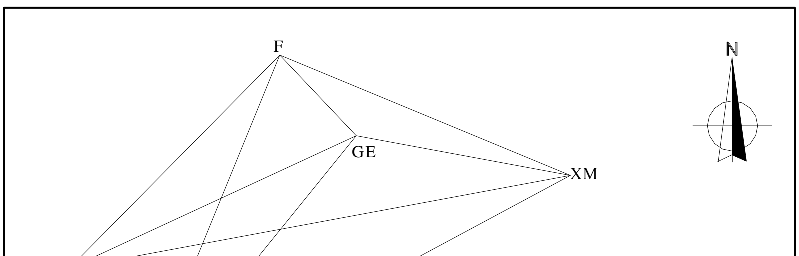
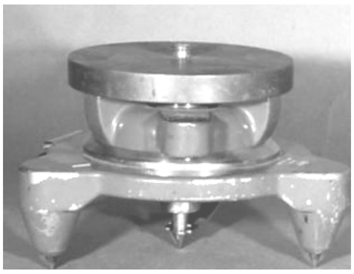



**Figure 1: The geodetic network.**

# 3. TERRESTRIAL MEASUREMENTS

**Terrestrial measurements of horizontal and vertical angles and distances were carried out by using a total station Leica TCA 1800 which provides accuracy of ±3$^{cc}$ in angle measurements and ±1mm ± 2ppm in distance measurements. Special accessories are used in order to succeed high accuracy in the field – measurements correspondent to the provided total station. These accessories are:**

- **Very heavy special bases for forced instrument centering, which insure a centering error ±0.1mm (fot. 2).**



**Fot. 2 : Base for forced centering.**

- **Appropriate prism - holders and targets with their corresponding tribraches.**
- **Digital thermometer – barometer for temperature and pressure measurements.**
- **Digital level and bar – code staff for the accurate determination of the instrument's height.**

**Initially all the tribraches are leveled by another checked instrument for accurate leveling procedure. Horizontal angles are measured by the direction sighting method. All the horizontal and vertical angles are measured for both position of the telescope and for four sets as well as the slope distances between all network points. 9 distances, 14 horizontal angles and 13 vertical angles are in total measured.**
**The unique centering is insured by mutual alternation between the total station and the targets on the fixed tribraches during the measurements. Special care is given to the instrument's height measurement. The following method is applied for the determination of the instrument's height by accuracy ± 0.25mm.**

**On a marked point A, close to the instrument station point B, a rod was placed. The total station was used as level, by setting the line of sight perpendicular to the plumb line (vertical angle =100$^g$) and sighting the rod. The reading e on the rod was measured twice on the first and second (300$^g$) telescope position (fig. 2a).**

**When the total station got off point B, a rod was placed on point B, and a digital level was placed in the middle – perpendicular of the distance between A and B (fig. 2b). Two measurements, Backsight (to point B) and Foresight (to point A), on the rod were taken and the height difference $\Delta H_{AB}$ was calculated as the difference between the two readings Backsight – Foresight.**
**The instrument height i was calculated by using the formula:**

$$i = e + \Delta H_{AB}$$

**The accuracy of this method is about ±0.25 mm, considering that:**

- **Readings on rods have an error of ±0.01mm, by using a digital level and a barcode rod.**
- **The reading of the observer on the rod by using the total station as level has an error of ±0.25mm.**

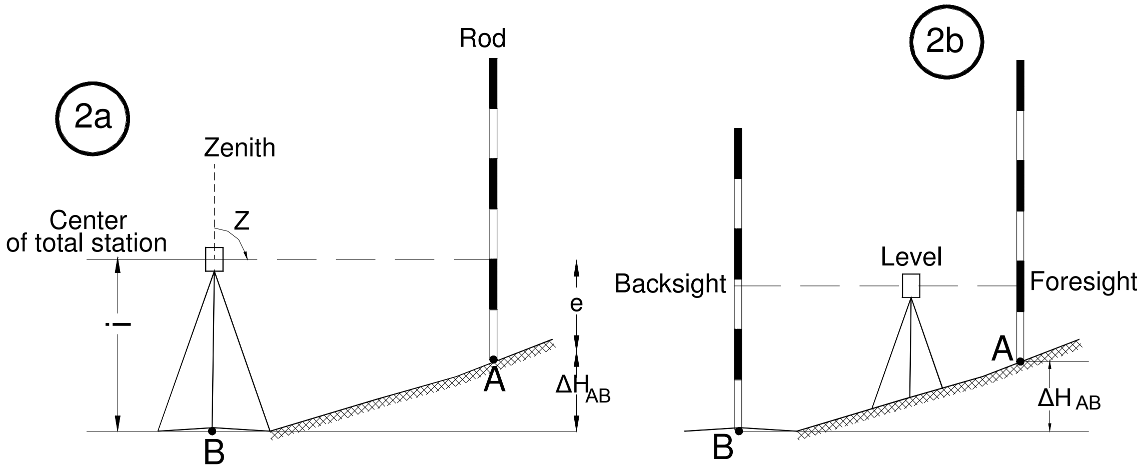



**Figure 2. Accurate determination of the instrument's height.**

**The height of the target from the tribrache is equal to the instrument's height according the industrial specifications of the special accessories construction.**

**The data adjustment was carried out in the Greek Geodetic Reference System GGRS 87. The orthometric height H is used as the third coordinate.**
**The appropriate corrections were applied to the measured elements before the adjustment. These are:**

- **Correction to the vertical angles due to the atmospheric refraction.**
- **Correction to the distances due to the cyclic error of the total station and their reduction to mean sea level.**
- **Correction of distances due to the cartographic projection of the Greek Geodetic Reference System (GGRS 87) which change the measured distances according the following equation:**

$$S_{GGRS\ 87} = S_m \cdot \mathrm{K}$$

**where K is the scale factor of the map projection, which is calculated by the formula**

$$\mathrm{K} = \mathrm{K}_o + 0.012311 \cdot (\mathrm{X} - \mathrm{X}_o)^2 \qquad (1)$$

**where X = mean X coordinate of the location in Mm**
**$K_o$ = scale factor at the central meridian (λ=24°) for the GGRS 87, equal to 0.9996**
**$X_o$ = X coordinate of the central meridian arbitrarily chosen equal to 0.5 Mm.**

**The network adjustment was carried out by the least square method. Three kind of observation equations were used, that arise from the derivation of the following equations:**

- **Horizontal angle**

$$\arctan\frac{\widehat{X}_k-\widehat{X}_i}{\widehat{Y}_k-\widehat{Y}_i}-\arctan\frac{\widehat{X}_j-\widehat{X}_i}{\widehat{Y}_j-\widehat{Y}_i}=l+u \tag{2}$$

- **Slope distance**

$$\sqrt{(\widehat{X}_j-\widehat{X}_i)^2+(\widehat{Y}_j-\widehat{Y}_i)^2+(\widehat{H}_j-\widehat{H}_i)^2}=l+u \tag{3}$$

- **Vertical angle**

$$\arccos\frac{1-\sqrt{1-\frac{2\cdot(1-K)}{R}\cdot\left[(\widehat{H}_j-\widehat{H}_i)^2-(h_i-h_t)-(1-K)\frac{(\widehat{X}_j-\widehat{X}_i)^2+(\widehat{Y}_j-\widehat{Y}_i)^2+(\widehat{H}_j-\widehat{H}_i)^2}{2\cdot R}\right]}}{(1-K)\cdot\frac{\sqrt{(\widehat{X}_j-\widehat{X}_i)^2+(\widehat{Y}_j-\widehat{Y}_i)^2+(\widehat{H}_j-\widehat{H}_i)^2}}{R}}=l+u \tag{4}$$

**where** $\widehat{X}_i$, $\widehat{Y}_i$, $\widehat{H}_i$, **temporary coordinates of the instrument station i**
$\widehat{X}_j$, $\widehat{Y}_j$, $\widehat{H}_j$ **and** $\widehat{X}_k$, $\widehat{Y}_k$, $\widehat{H}_k$, **temporary coordinates of the sighting points j and k**
**$h_i$ instrument height**
**$h_t$ target height**
***K* atmospheric refraction coefficient (equal about 0.16)**
***R* earth radius.**
$l$ **measured value**
$u$ **difference between the calculated and measured value of each element.**

**The coordinates of point LAMP and the azimuth of the direction between the points LAMP and F were considered fixed in the adjustment. The final points coordinates and their uncertainties are calculated and illustrate in table 1.**

**Table 1. The coordinates and their uncertainties in GGRS 87.**

| Point | X(m) | $\sigma_X$(mm) | Y(m) | $\sigma_Y$(mm) | H(m) | $\sigma_H$(mm) |
|---|---|---|---|---|---|---|
| LAMP | 480547.715 | | 4202792.111 | | 210.860 | |
| F | 480757.358 | 0.7 | 4203004.499 | 0.8 | 203.256 | 2.2 |
| GE | 480832.887 | 1.1 | 4202923.733 | 1.3 | 216.102 | 2.2 |
| XM | 481047.547 | 1.4 | 4202885.334 | 2.4 | 222.321 | 3.0 |
| FE | 480612.952 | 1.2 | 4202652.057 | 1.1 | 237.316 | 2.2 |

**The absolute and relative ellipses of all points as well as the errors in each distance and angle measurement are calculated for confidence level 99%.**

## 4. GPS MEASUREMENTS

**The above-described network is used for checking the proper function of a pair of receivers Trimble 4600LS. These receivers measure in L1 frequency and C/A code. The network was measured by the method of relative static positioning by using the correspondent accessories that insure the unique centering and the right leveling of the receivers. Also the antennas were directed towards the North during the measurements**

**in order to efface the error of the position of the antenna's phase center, which isn't constant but is angle dependent. 10 bases were measured. The results appear in table 2.**

**Table 2: Baselines' ratios & reference variances of GPS measurements.**

| From Station | To Station | Ratio | Reference Variance |
|---|---|---|---|
| FE | F | 52.9 | 0.197 |
| FE | XM | 110.4 | 0.185 |
| GE | FE | 32.3 | 0.516 |
| GE | F | 45.5 | 0.524 |
| GE | LAMP | 65.0 | 0.280 |
| GE | XM | 46.4 | 0.266 |
| LAMP | FE | 34.3 | 0.438 |
| LAMP | F | 29.5 | 0.393 |
| XM | F | 68.9 | 0.186 |
| XM | LAMP | 57.8 | 0.150 |

**The GPS data adjustment was carried out in the GGRS 87 by using the Trimble GPSurvey programme (Trimnet plus,1992). The point LAMP was considered fixed in the adjustment. The small size of the network (maximum site length about 500m) and the smooth relief of the area permit the supposition that the geoid undulation N remains constant. Thus as the orthometric height H, of the fixed point LAMP, is given in the adjustment, the orthometric heights of the network points are calculated. Table 3 illustrates the coordinates X and Y and the orthometric height H of each network point and their uncertainties.**

**Table 3. Coordinates of network points and errors from GPS measurements.**

| Point | X(m) | $\sigma_X$(mm) | Y(m) | $\sigma_Y$(mm) | H(m) | $\sigma_H$(mm) |
|---|---|---|---|---|---|---|
| LAMP | 480547.715 | | 4202792.111 | | 210.860 | |
| F | 480757.357 | 0.7 | 4203004.496 | 0.9 | 203.263 | 2.2 |
| GE | 480832.884 | 0.8 | 4202923.731 | 0.9 | 216.105 | 2.5 |
| XM | 481047.541 | 0.6 | 4202885.330 | 0.8 | 222.327 | 2.0 |
| FE | 480612.950 | 0.7 | 4202652.059 | 0.8 | 237.324 | 2.2 |

## 5. COMPARISON

**To certify the proper function of the GPS receiver, one must compare measured and calculated elements of the network as a result of the two measuring campains and adjustments.**

**The compared elements are horizontal angles, slope distances, height differences and the point coordinates as they were calculated by the two adjustments.**

**When the difference in the value of an element between the GPS and Terrestrial measurements is smaller than its calculated error at the confidence level 99%, this difference is expected as a measurement error and is also accepted.**

**For example if $S_{12(t)}$ is the distance between points 1 and 2, measured by terrestrial measurement, and $S_{12(GPS)}$ is the distance measured by GPS measurement then $\Delta S_{12} = S_{12(t)} - S_{12(GPS)}$. Therefore**

$$-\sigma_{\Delta S_{12}} \cdot Z_{99\%} \leq |\Delta S_{12}| \leq +\sigma_{\Delta S_{12}} \cdot Z_{99\%}$$

**where $\sigma_{\Delta S_{12}}$ is the standard error in $\Delta S_{12}$, calculated by the following equation:**

$$\sigma_{\Delta S_{12}} = \pm\sqrt{\sigma^2_{S_{12(t)}} + \sigma^2_{S_{12(GPS)}}}$$

**where $\sigma_{S_{12(t)}}$ = error in $S_{12}$, determined by the terrestrial measurements**

**$\sigma_{S_{12(GPS)}}$ = error in $S_{12}$, determined by the GPS measurements**

**$Z_{99\%}$ = factor of the normal distribution for confindence level 99%.**

## 5.1 Comparison of measured elements.

**The measured elements to be compared between the terrestrial and the GPS measurements are:**
- **Slope distances**
- **Horizontal angles (as azimuth differences from GPS)**
- **Height differences between points**

**⇨ In the comparison of the slope distances the following are used:**

- **The terrestrial measured distances as they were calculated after the application of all the above mentioned reductions and corrections.**
- **The GPS distances as they result from the solution of the bases.**

**In table 4 appear the values of the measured distances by the two methods and in addition the calculated differences between them and the accepted difference value in the last column.**

**Table 4: Comparison of terrestrial and GPS measured slope distances.**

| From Station | To Station | Distance from Total Station (m) | Distance from GPS (m) | Difference ΔS (mm) | $\sigma_{\Delta S} \cdot Z_{99\%}$ (mm) |
|---|---|---|---|---|---|
| FE | F | 382.564 | 382.560 | 3.6 | 5.2 |
| FE | XM | 493.685 | 493.681 | 3.6 | 5.2 |
| GE | FE | 350.335 | 350.332 | 2.7 | 5.3 |
| GE | F | 111.372 | 111.371 | 0.6 | 5.3 |
| GE | LAMP | 314.259 | 314.257 | 2.4 | 5.2 |
| GE | XM | 218.249 | 218.248 | 0.6 | 5.2 |
| LAMP | FE | 156.814 | 156.817 | -3.4 | 5.2 |
| LAMP | F | 298.653 | 298.651 | 1.8 | 5.2 |
| XM | LAMP | 508.796 | 508.797 | -0.7 | 5.3 |

**It's noticed that the differences are all smaller than the expected measurements error.**

⇨ **In the comparison of height differences the following are used:**

- **The terrestrial height differences calculated by trigonometric heighting by using the measurements of vertical angles, distances, instrument heights and target heights.**
- **The GPS calculated height differences between the points from the solution of bases. These height differences (dh) as mentioned above, may be compared to the orthometric height differences (dH), as ΔN ≈ 0m at the network's area.**

**Table 5 illustrates the measured height differences between the points, the calculated differences of the measured height differences and the accepted difference values in the last column.**

**Table 5: Comparison of height differences between terrestrial and GPS measurements.**

| From Station | To Station | Height differences From GPS (m) | Height differences From terrestrial (m) | ΔDH Difference (mm) | $\sigma_{\Delta DH} \cdot Z_{99\%}$ (mm) |
|---|---|---|---|---|---|
| FE | F | -34.061 | -34.053 | -8.0 | 15.6 |
| FE | XM | -14.939 | -14.940 | 1.0 | 15.6 |
| GE | FE | 21.191 | 21.195 | -4.0 | 15.7 |
| GE | F | -12.873 | -12.860 | -13.0 | 15.7 |
| GE | LAMP | -5.286 | -5.280 | -6.0 | 15.9 |
| GE | XM | 6.242 | 6.232 | 10.0 | 15.7 |
| LAMP | FE | 26.462 | 26.465 | -3.0 | 15.6 |
| LAMP | F | -7.599 | -7.596 | -3.0 | 15.7 |
| XM | LAMP | -11.520 | -11.522 | -2.0 | 15.7 |

⇨ **In the comparison of the horizontal angles the following are used:**

- **The terrestrial measured angle values calculated as differences between the corresponded sighting directions.**
- **The values of the angles calculated as azimuth differences from the GPS bases solution.**

**Table 6: Comparison of horizontal angles calculated by terrestrial and GPS measurements.**

| Target – Station – Target | Angles from terrestrial measurements (grad) | Angles from GPS measurements (grad) | Difference (cc) | $\sigma \cdot Z_{99\%}$ (cc) |
|---|---|---|---|---|
| F – LAMP- GE | 22.8851 | 22.8853 | -2 | 13.8 |
| GE – LAMP – XM | 15.7895 | 15.7902 | -7 | 13.8 |
| XM – LAMP – FE | 83.9876 | 83.9873 | 3 | 13.8 |
| GE – F – FE | 72.6231 | 72.6240 | -9 | 13.8 |
| FE – F – LAMP | 24.8294 | 24.8295 | -1 | 13.8 |
| LAMP – GE – F | 79.6612 | 79.6601 | 11 | 13.8 |
| F – GE – XM | 159.1362 | 159.1368 | -6 | 13.8 |
| XM – GE – FE | 132.0558 | 132.0550 | 8 | 13.8 |
| FE – GE – LAMP | 29.1468 | 29.1472 | -4 | 13.8 |
| FE – XM – LAMP | 19.6231 | 19.6228 | 3 | 13.8 |
| LAMP – XM – GE | 23.0069 | 23.0075 | -6 | 13.8 |
| LAMP – FE – F | 52.5069 | 52.5076 | -7 | 13.8 |
| F – FE – GE | 18.5682 | 18.5676 | 6 | 13.8 |

| | | | | |
|---|---|---|---|---|
| GE – FE - XM | 25.3143 | 25.3146 | -3 | 13.8 |

The angles of the network calculated by the terrestrial and the GPS measurements and their differences appear in table 6. Also in the last column the accepted difference value for each one is registered.

## 5.2 Comparison of the coordinates.

Comparing the calculated coordinates X and Y and the orthometric heights H of the standard network points (table 1) with those calculated by the GPS measurements (table 3) the following differences were registered in table 7. The results shown in table 7 are of the differences in the coordinates X and Y, which fluctuate between 1 to 4mm, and in the coordinate H, which fluctuate between 3 and 8 mm.

Table 7: Comparison of the coordinates.

| Point | ΔX(mm) | $\sigma_{\Delta X} \cdot Z_{99\%}$ (mm) | ΔY (mm) | $\sigma_{\Delta Y} \cdot Z_{99\%}$ (mm) | ΔH (mm) | $\sigma_{\Delta H} \cdot Z_{99\%}$ (mm) |
|---|---|---|---|---|---|---|
| LAMP | - | - | - | - | - | - |
| F | 1 | 3 | 3 | 3 | 7 | 8 |
| GE | 3 | 5 | 2 | 4 | 3 | 9 |
| XM | 4 | 4 | 4 | 7 | 6 | 9 |
| FE | 2 | 4 | 2 | 4 | 8 | 9 |

# 6. CONCLUSIONS

- A standard control network was established and measured by using a high accuracy total station and specific accessories by an accuracy of about ± 2mm.
- This fundamental work, which lasted about 10 hours, can be successfully used for checking any pair of GPS receivers.
- The terrestrial measured values of horizontal and vertical angles and slope distances as well as the calculated values of the standard network points' coordinates may be compared to the corresponding measurements achieved by any pair of GPS receivers.
- If the differences of all the corresponding elements of the network, between terrestrial and GPS measurements, are smaller than their determination error for confidence level 99%, this signifies the proper function of the utilized GPS receivers and hence they measure by the prescribed accuracy.
- The above-analyzed application certifies the proper function of the utilized GPS receivers since all the calculated differences of the compared elements were within the limit of the accepted difference values. So the two receivers passed the test successfully.

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