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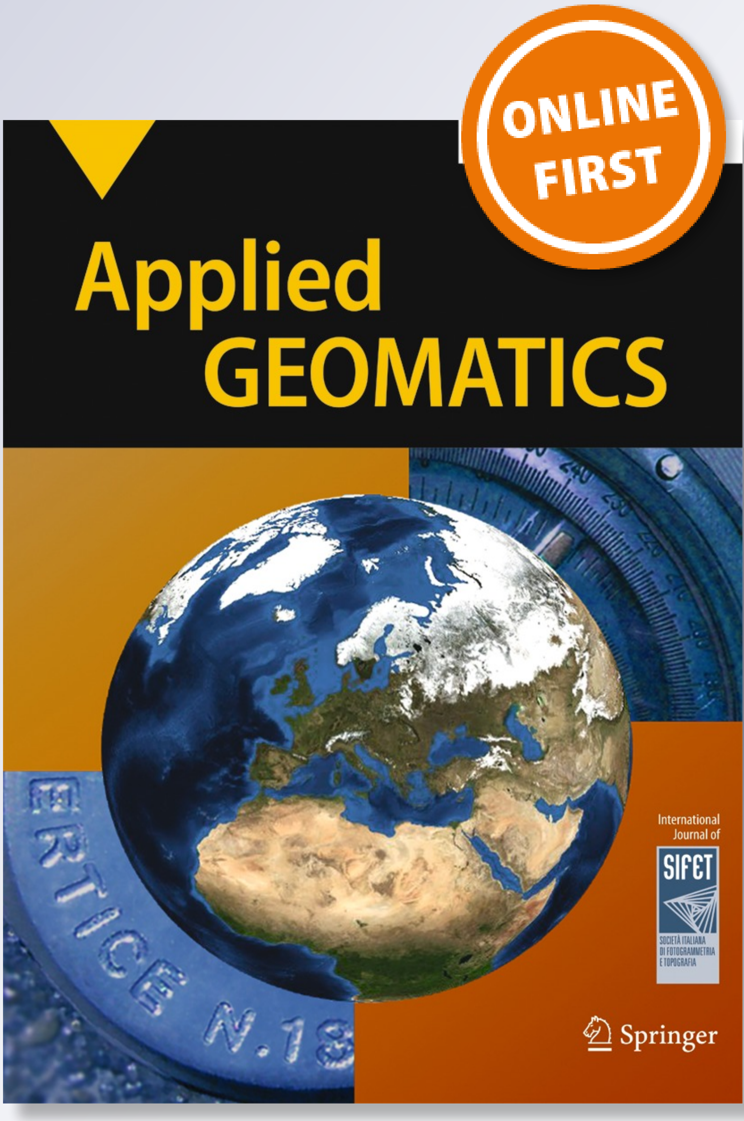

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ORIGINAL PAPER

A comparative evaluation of various models for prediction of displacements

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Abstract One of the main subjects of Geodesy is the monitoring of position changes of artificial structures (buildings, dams, bridges etc.). Such position changes can be caused by a variety of reasons such as vehicles for cable bridges and earthquakes. Various mathematical models have been developed in order to monitor and to analyze this phenomenon. This study presents the main models which are used by geodesists for the description of points' displacements. These are the descriptive models (which are separated into the congruence and the kinematic ones) and the cause-response models (which are separated into the static and the dynamic). Moreover, several models, which are based on time series analysis and are used mainly for the prediction of financial parameters, are referred in parallel. These are the smoothing models, the time series decomposition models, and the ARIMA models. All the abovementioned models are discussed and compared in order to emerge their advantages, disadvantages, and limitations. The goal of this study is to substantiate which of these models could be used with reliability for prediction of displacements. A case study using the most appropriate models is carried out. The experiment deals with the prediction of displacements of a set of permanent GNSS stations. The results proved that the linear kinematic models have the best performance, in comparison with the other examined models.

Keywords Geodesy · Prediction of displacements · Steps of prediction · GNSS permanent stations · Forecasting models

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Introduction

Nowadays, forecasting is one of the most important and growing areas in most sciences (such as economics and medicine), attracting the attention of many researchers for more extensive study, as Steyerberg et al. (2010) and Dhar (2011). Also, geodesists have a special interest to the monitoring of displacements of artificial structures in order to modeling the structure behavior. (Eichhorn 2007; van der Meij 2008; Dermanis 2011; Moschas and Stiros 2011).

In Geodesy, various models were used for the monitoring of the phenomenon rather than for its prediction. In general, the entire process of prediction presents several difficulties so, it is crucial to follow some basic steps, before choosing the appropriate model.

Therefore, the aim of this study is to present, compare and use both conventional models according to Welsch and Heunecke (2001), as well as predictive statistical models which are used in other scientific fields. These models are based on the theory of time series.

Besides the abovementioned models, artificial intelligence (i.e., artificial neural networks) has been also developed in order to produce predictions (Xie et al. 2006; Shuo et al. 2012; Pantazis and Alevizakou 2013). However, these techniques are not purposely included in this study which is focused on the conventional mathematical models.

Main steps of prediction

The first step of the forecasting procedure is the *definition of the problem*. The aim of the prediction and the use of the results need to be clear. Also, it is useful to define the timescale (one-step-ahead prediction or multi-step-ahead prediction, short-term or long term) as well as the accuracy of the

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desired prediction. In addition, it is crucial to explore some external factors, such as the cost of the model, which will depend on the requirements of the process and perhaps the special equipment that might be required, as well as its effortlessness or complexity (Agiakoglou and Economou 2004).

The second step is that of the *data acquisition*. This step deals with the collection of past data that will form the data pool for the prediction. This data will be analyzed by various ways in order to predict the phenomenon at a particular future time.

A mathematical statistical analysis is carried out for finding a particular pattern that the data might follow. However, the word "data" does not only refer to the numerical data but also on any other kind of knowledge that the researcher has. Therefore, the experience and expertise of the scientist play a significant role.

The third step is the *exploratory analysis* where various statistical indicators are calculated. These are the central tendency, the standard deviation, the minimum, the maximum, and the linear trend. These factors will assist in choosing the appropriate model. Also, the outliers in the data, which should be removed, must be also detected and eliminated.

It is necessary to note that the above statistical factors assume that the population follows the normal distribution. If not, non-descriptive statistics, e.g., quartiles and median, should be used.

The final step is the *choosing and the evaluation* of the prediction. The model, which generates the most accurate prediction, is decided after the mathematical analysis in order to satisfy the proper indicators-criteria, which are particularly important.

Indicators-criteria for the evaluation of a model

In order to evaluate a prediction model, the produced results need to be compared with their real known values. The usual assessment process for multi-step-ahead predictions is the segregation of the data into "estimation-training" data and into "evaluation" data. Empirically, the 80% of the data is usually used for the model running and the 20% for the evaluation.

Some of the following mathematical indicators can be used (Smith and Cormick 1978; Charnes et al. 1985; Mayer and Glauber 1994; Schroeder et al. 2009; Erdogan 2010; Yilmaz and Gullu 2014). For multi-step-ahead predictions, all these indicators are calculated for the 20% of the evaluation data, as already mentioned. However, in the case of a one step-ahead prediction, these criteria can be also used.

In Table 1, is the real value, is the predicted value in a time t and N is the amount of the 20% of the actual values and predictions available (multi-step-ahead prediction).

The final step of the evaluation is to perform a statistical test (e.g., z-statistic test, *t* statistical test). The importance of each factor is controlled for (1-a) % confidence level, where "a" is the level of significance.

Table 1 Indicators-criteria for the evaluation of a model

Error	$e_t = D_t - Y_t$
Mean error	$(\mathrm{ME}) = \frac{1}{N} \cdot \sum_{t=1}^{N} e_t$
Mean absolute error	$(\mathrm{MAE}) = \frac{1}{N} \cdot \sum_{t=1}^{N} \lvert e_t \rvert$
Mean squared error	$(\mathrm{MSE}) = \frac{1}{N} \cdot \sum_{t=1}^{N} e_t^2$
Root mean squared error	$(\mathrm{RMSE}) = \sqrt{\frac{\sum_{t=1}^{N} e_t^2}{N}}$
Average percentage error	$(\mathrm{MPE}) = \frac{1}{N} \cdot \sum_{t=1}^{N} \frac{e_t}{D_t} \cdot 100(\%)$
Mean absolute percentage error	$(\mathrm{MAPE}) = \frac{1}{N} \cdot \sum_{t=1}^{N} \left\lvert \frac{e_t}{D_t} \right\rvert \cdot 100(\%)$

Traditional models for determination of displacements in Geodesy

In order to describe the displacement of a point, several models are being developed. At the same time, these models are used to confirm their functionality and also for research purposes. Lastly, there is the possibility to use these models in order to make a prediction of a phenomenon.

According to Welsch and Heunecke (2001), the models, depending on whether or not they include the sense of time and the reason or the forces causing the changes, are split in the two following categories, with their corresponding subcategories: *Descriptive Models* and *Cause-Response Models*.

Descriptive models

The descriptive models are the most conventional models for depicting a displacement and they are the ones primarily used. An object is represented by a number of points and the forces causing the displacement are not taken into consideration. They are distinguished as

- *Congruence models*

They evaluate the identity models or the correlation of an object between two or more time periods (Welsch and Heunecke 2001). A comparison of the geometry of the object in some time is made using some of its characteristics (Neumann and Kutterer 2006).

- *Kinematic models*

These models do not take into account the causes which provoke the displacements, as done in congruence models. They use functions in order to calculate the kinematic parameters. A distinction of these models can be made in simple linear kinematic model, the kinematic model of simple polynomials (Telioni 2003 and 2004) and kinematic geometric models of surface's speed (Arnoud de Bruijne et al. 2001; Mualla and Temel 2005; Acar et al. 2008). The most widespread are the linear kinematic models and the kinematic models using polynomials (Ehigiator-Irigue et al. 2013).

Specifically, the linear kinematic model is represented by the following equation:

$$x_i^{t_v} = V_i \cdot (t_v - t_0) + x_i^{t_0} = V_i \cdot \Delta t + x_i^{t_0} \tag{1}$$

and respectively the kinematic model of simple polynomials by the Eq. (2):

$$\begin{aligned} x_i^{t_v} &= x_i^{t_0} + \frac{dx}{dt} \cdot (t_v - t_0) + \frac{1}{2} \cdot \frac{d^2x}{dt^2} \cdot (t_v - t_0)^2 + \ldots = \\ &= x_i^{t_0} + V_i \cdot \Delta t + \frac{1}{2} \cdot \gamma_i \cdot \Delta t^2 + \ldots \end{aligned} \tag{2}$$

where $x_i^{t_v}$ is the position of a point i in time period with the initial time (the moment of the first series of measurements). The unknown parameters, which must be calculated for the creation of the model, are the position $x_i^{t_0}$ of the point in the time , the rate of change of the position (displacement speed) and the rate of change of speed (acceleration) (in the case of the kinematic model of simple polynomials).

Cause-response models

These models differ from the previous ones as they do not focus only on the position change, but also embody the reasons which cause these changes. They consider that the displacement is the result (output) of a dynamic process. The two basic categories are the dynamic models and the static ones. But beyond this differentiation, they can also be distinguished in parametric and non-parametric models (Welsch and Heunecke 2001).

- *Dynamic models*

The majority of dynamic models consists of *non-parametric models*, without excluding dynamic models which can be *parametric*. In parametric models, the relation of input and output is known and can be modeled, as cannot be modeled in a non-parametric way. Hence, the displacement is a function both of weight and time, considering that the object is constantly moving.

In addition, the dynamic models can differentiate depending on the number of the input quantities (e.g., causes of deformation) and the number of the output quantities (e.g., deformation) in SISO (single input-single output), MISO (multiple input-single output), and MIMO (multiple input-single output).

The fundamental equation of a *parametric dynamic model* is the following:

$$|\mathbf{K}\ \ \mathbf{D}\ \ \mathbf{M}| \begin{vmatrix} x(t) \\ \frac{dx}{dt} \\ \frac{d^2x}{dt^2} \end{vmatrix} = y(t) \tag{3}$$

where the matrices**K**, **D**, and **M** in the case of a building displacements are the parameters of rigidity, damping and mass (Welsch and Heunecke 2001).

The more common case of a non-parametric model is that of a SISO model which is represented by an ordinary differential equation (Welsch 1996; Welsch and Heunecke 2000):

$$\begin{aligned} &a_q \cdot \frac{d^q x_i}{dt^q} + a_{q-1} \cdot \frac{d^{q-1} x_i}{dt^{q-1}} + \ldots + a_1 \cdot \frac{dx_i}{dt} + a_0 \cdot x_i = \\ &b_p \cdot \frac{d^p y_i}{dt^p} + b_{p-1} \cdot \frac{d^{p-1} y_i}{dt^{p-1}} + \ldots + b_1 \cdot \frac{dy_i}{dt} + b_0 \cdot y_i \end{aligned} \tag{4}$$

- *Static models*

The characteristic of these models is the description of the relation between stress and strain. The stress is caused by loads or forces which acting on the object and cause its geometric change. The static models can be regarded as a subcategory of the dynamic models and are expressed with the following equation (Welsch and Heunecke 2001):

$$\mathbf{K} \cdot x(t) = y(t) \tag{5}$$

The *main characteristics of all the abovementioned models* are presented briefly on the following table. Specifically, with ✗ is declared the lack of the corresponding characteristic and with ✔ its existence (Table 2).

General statistical forecasting models based on time series analysis

Many models, whose main goal is the prediction of a phenomenon, are based on the analysis and theory of time series. Especially in the last few years, the evolution of computers

Table 2 Brief presentation of models in Geodesy

MODEL		Time modeling	Force/charges modeling
Cause-response	Static	✗	✔
	Dynamic	✔	✔
Descriptive	Kinematic	✔	✗
	Congruence	✗	✗

and respective software made them one of the most basic tools of researchers. Depending on each occasion the suitability of each model should be tested, using the criteria analyzed before (Smith and Cormick 1978; Charnes et al. 1985; Mayer and Glauber 1994; Schroeder et al. 2009; Erdogan 2010; Yilmaz and Gullu 2014).

It should be mentioned that these models belong to broader category of the quantitative forecasting techniques. There are also the qualitative or judgmental forecasting techniques, in which the experience of the researcher is taken into consideration and the technological forecasting techniques. These two will not be analyzed in this essay. They are used mainly in cases where the available data is insufficient. On the contrary, quantitative techniques are "impartial" and demand a series of data of the examined phenomenon for their mathematic modeling. According to Vaidanis (2005), a quantitative prediction can be based on the following:

- time series models, in which obviously the information is in a time series of data and on
- Casual models, in which the variable to be predicted depends on one or more parameters.

These two categories can be combined. Therefore, according to Agiakoglou and Economou (2004), predicting a variable through the analysis of time series can occur by means of three categories of predictions: smoothing models ("Smoothing models" section), ARIMA analysis ("ARIMA analysis" section), and time series decomposition ("Time series decomposition" section).

Smoothing models

Smoothing models, such as Simple mean, Simple moving average, and Simple exponential, are well suited for one-step-ahead forecasting. To obtain a two-step-ahead prediction or any multi-step-ahead prediction, the forecasted value can be added to the end of the time series.

The other smoothing models can produce directly prediction in the desired time (h).

- *Simple mean*

In this case, the prediction is made through calculating the average value of the data.

$$F_{t+1} = \frac{\sum_{i=1}^{t} X_i}{n} \tag{6}$$

where is the prediction for the next time step t + 1, X_i is the known series value for time period i and n is the multitude of values of variable of the time series.

- *Simple moving average* (*MA*)

The average value is calculated using only the data of a certain "window" of the recent past. Hence, every time a new observation is entered, the new average of the sample is calculated, discarding the oldest observation. Thus, there is always the same number of observations, albeit updated.

$$F_{t+1} = \frac{X_t + X_{t+1} + \ldots + X_{t-n+1}}{n} = \frac{1}{n} \cdot \left(\sum_{i=t-n+1}^{t} X_i \right) \tag{7}$$

By the addition of a new observation and hence the discarding of the oldest one, the Eq. 7 becomes as:

$$F_{t+1} = F_t + \frac{X_t}{n} - \frac{X_{t-n}}{n} \tag{8}$$

- *Simple* (*or single*) *exponential smoothing* (SES)

The new observations are considered with greater weight in the average calculation than the older observations. Thus, the following equation is used (Ostertagova and Ostertag 2011):

$$F_{t+1} = \frac{1}{n} \cdot X_t + \left(1 - \frac{1}{n}\right) F_t = \alpha \cdot X_t + (1 - \alpha) \cdot F_t \tag{9}$$

where α (alpha) is a measure of weight of the most recent real value in relation to the most recent prediction (Vaidanis 2005) and is named smoothing constant, taking values from 0 to 1. The choice of alpha is made by tests, when MSE is the minimum. Generally, if data show large randomness, it can be used small α values (e.g., 0.01<α<0.3) as if it shows pattern larger values of α can be used.

- *Double moving average*

This works much like simple smoothing except that two components must be updated each period—level and trend. The equation of this method is the following one (http://people.duke.edu/~rnau/411avg.htm):

$$F_{t+h} = \alpha_t + h \cdot b_t \tag{10}$$

where F_{t+h} is the desired prediction in the time h. And so, there is a possibility of predicting the next time period or even more future periods.

For the above association, the simple moving average from the following association and then the double moving average have to be calculated:

$$M_{t+1} = \frac{1}{n} \cdot \sum_{j=1}^{n} X_{t-j+1} \quad M'_{t+1} = \frac{1}{n} \cdot \sum_{j=1}^{n} M_{t-j+1} \tag{11}$$

$$\alpha_t = 2 \cdot M_t - M'_t \quad b_t = \frac{2}{n-1} \cdot \left(M_t - M'_t \right) \tag{12}$$

- *Double exponential smoothing* (*one parameter*) *or Brown's approach* (Brown 1956)

The same procedure as previously is followed as has the same presuppositions, albeit smoothing the values of the original time series. The equation of this method for the calculation of the prediction F_{t+h} in a future time h is the same with the previous one (Eq. 10). In this case, the original observation needs to be smoothed with the method of simple exponential smoothing:

$$A_t = \alpha \cdot X_t + (1-\alpha) \cdot A_{t-1} \tag{13}$$

where α is the smoothing constant (its choice is made after tests), A_t are the values after the smoothing for $t = 2,3,..,n$ and for $t = 1$ the initial condition $A_1 = X_1$ is set (Agiakoglou and Economou 2004). Following that, second smoothing needs to be done:

$$A'_t = \alpha \cdot A_t + (1-\alpha) \cdot A'_{t-1} \tag{14}$$

$$\alpha_t = 2 \cdot A_t - A'_t \quad b_t = \frac{\alpha}{1-\alpha} \cdot \left(A_t - A'_t \right) \tag{15}$$

- *Double exponential smoothing* (*two parameter*) *or Holt's approach* (Holt 1957)

In this case, there are two smoothing parameters, the smoothing constant for the level α and the smoothing constant for the trend β, so the prediction comes out from the equation:

$$F_{t+h} = A_t + h \cdot T_t \tag{16}$$

for h = 1,2,3,.. and

$$A_t = \alpha \cdot X_t + (1-\alpha) \cdot (A_{t-1} + T_{t-1}) \tag{17}$$

where $A_t$are the value after the smoothing for $t = 2,3,..,n$ and for $t = 1$ the initial condition $A_1 = X_1$ is set and

$$T_t = \beta \cdot (A_t - A_{t-1}) + (1-\beta) \cdot T_{t-1} \tag{18}$$

where T_t is the values after the trend smoothing for $t = 2,3,..,n$. There are several methods to choose the initial values for T_t.

In this essay for $t = 1$ the initial condition $T_1 = X_2 - X_1$ is set. T_1 can be also set as, according to Kalekar P. (Kalekar 2004): $T_1 = (X_n - X_1)/(n-1)$ or $T_1 = [(X_2 - X_1) + (X_3 - X_2) + (X_4 - X_3)]/3$.

The constants α ($0 < \alpha < 1$) and β ($0 < \beta < 1$) are chosen after testing, when the MSE is minimized.

- *Exponential smoothing adjusted for trend and seasonality*

It is used when a specific trend appears in the tested time series along with a specific seasonality (L). The outcome for the prediction for n periods is given from the form:

$$F_{t+n} = (S_t + b_t \cdot n) \cdot I_{t-L+n} \tag{19}$$

where the exponentially smoothing series S_t, the assessment of seasonality I_t, and the trend estimator b_t need to be updated, respectively:

$$S_t = \alpha \cdot \frac{X_t}{I_{t-L}} + (1-\alpha) \cdot (S_{t-1} + b_{t-1}) \tag{20}$$

$$I_t = \beta \cdot \frac{X_t}{S_t} + (1-\beta) \cdot I_{t-L} \tag{21}$$

$$b_t = \nu \cdot (S_t + b_t \cdot n) \cdot I_{t-L+n} \tag{22}$$

ARIMA analysis

Autoregressive Integrated-Moving Average models (ARIMA) are stochastic mathematical models which are mainly used to describe the evolution of a random process. These models are also called Box-Jenkins Models (Reinsel et al. 1977). ARIMA combines auto-regression, which fits the current data point to a linear function (usually) of some prior data points. Also, it includes moving averages, adding together several consecutive data points and getting their mean, and then using that to compute estimations of the next value. A non-seasonal ARIMA model is classified as an "ARIMA (p,d,q)" model, where p is the number of autoregressive terms, d is the number of non-seasonal differences needed for stationarity, and q is the number of lagged forecast errors in the prediction equation. Generally, a p-order ARIMA model defined as follow:

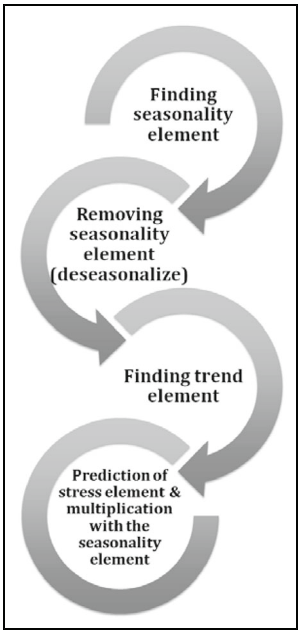

Fig. 1 Process of prediction with time series decomposition

$$Y_t = c + \varphi_1 \cdot Y_{t-1} + \varphi_2 \cdot Y_{t-2} + \ldots + \varphi_p \cdot Y_{t-p} + e_t \quad (23)$$

Time series decomposition

The time series decomposition is based on finding the key characteristics of a time series (i.e., trend, cyclicality, seasonality and randomness) and then to distinguish them. The process of the prediction with the analysis-division of a timetable aims to find any trend occurs and to adjust it according to the seasonality and circularity indicators, which have been set from the analysis of the time series according to the figure below (Fig. 1).

Limitations and advantages of general prediction models

As far as it concerns, the smoothing models can be separated according to the timescale of the predictions. Specifically, simple mean, simple moving average and simple exponential are well suited for one-step-ahead forecasting as the other, are suitable for making multi step predictions.

The simple mean and the simple moving average can be easily understood and computed. They also provide sure forecasts, when the time series has no trend, but ignore complex relationships in data.

The single exponential method should only be used, when the data set contains no seasonality and is suitable for no trend series. Both moving average and Single exponential are appropriate for stationary series and depend on a single parameter. However, the first requires all past data points to compute new forecast while the other only requires last forecast and last observation of "demand" to continue.

When using the double moving average, the researcher must have observed whether the time series values present an upward or downward trend.

The double exponential smoothing is used, when the data shows also a trend and should not be applied when seasonality is present. Brown's approach and Holt's approach are generally suitable for linear trend series.

The exponential smoothing adjusted for trend and seasonality. It is used when a specific trend appears in the tested time series along with a specific seasonality. As far as the time series decomposition model is concerned, it is not used as a prediction tool in general. It is used as a tool before selecting another model for better understanding of the examined time series.

The ARIMA models advantage is that, with enough elements regressed and averaged, almost any time series can be fitted. The major disadvantage of them is that they are difficult to understand and usually computational expensive. Furthermore, the underlying theoretical model and structural relationships are not distinct as in some simple forecasts models (such as simple exponential smoothing). Finally, the ARIMA models are essentially "backward looking." Such that the long-term forecast eventually goes to be straight line. [https://libres.uncg.edu/ir/uncw/f/zhai2005-2.pdf].

Case study

An experimental application by using geodetic data (time series of geocentric coordinates X, Y and Z) is carried out. As regards the predictions, it is not possible to use all the previously mentioned models. The chosen ones must model time and simultaneously must not model the forces which cause the displacement. An additional criterion for choosing the appropriate models is to reduce the time and cost of their implementation.

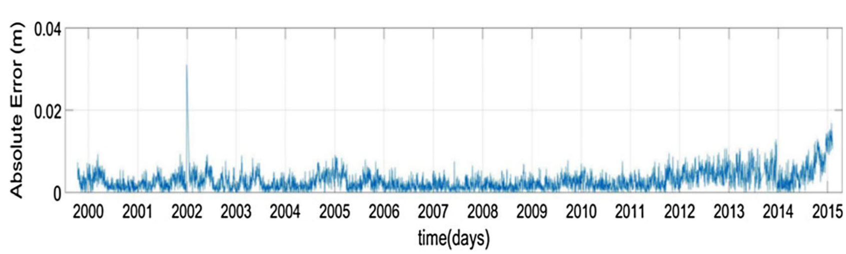

Fig. 2 Graphical representation of the differences for the detection of the outliers (Coordinate X time series)

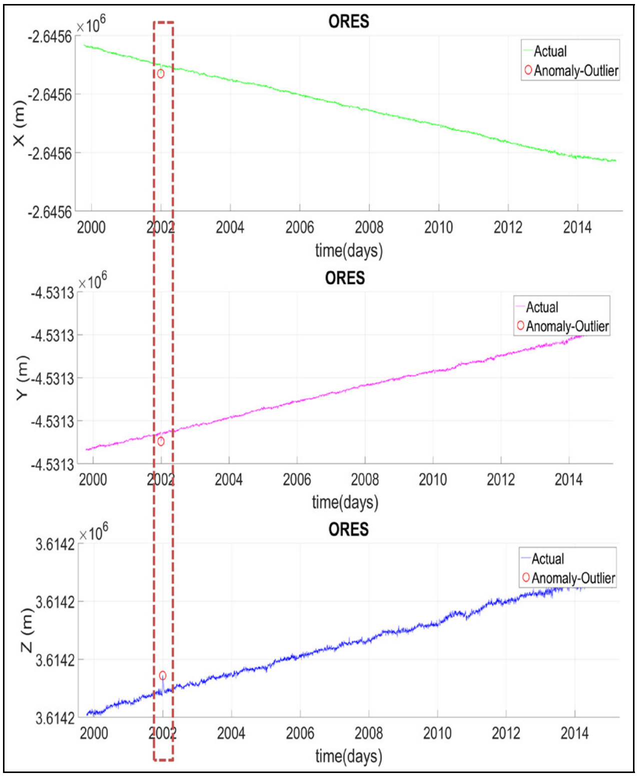

Fig. 3 Graphical representations of geocentric coordinates X, Y, and Z time series (October 1999–February 2015) and outliers detected

In this study, the problem is defined as the possibility of prediction of displacement (in the order of a few cm) of a permanent GNSS station. In the particular application, the models can produce the following:

- One-step-ahead prediction
- Multi-step-ahead prediction

A reliable forecasting model requires large number of data for each point. Hence, the chosen network is part of the Plate Boundary Observatory (PBO) (part from the Earthscope, http://www.earthscope.org), which consists of GNSS stations which are in operation from 1995. Specifically, a sub-network of 174 consecutive operation permanent GNSS stations was selected. The evaluation of the selected models was performed for all the GNSS stations.

Description and data preprocessing

The original data are the time series of the geocentric coordinates X, Y, and Z from the 174 GNSS stations in the Global Reference Frame IGS08. The data are registered from 1995 to 2015 (the oldest station, 6792 daily recordings) and from 2008 to 2015 (the newer station, 2557 daily recordings).

In most cases, GNSS time series present problems, like signal loss (i.e., changing of the antenna) or inaccurate data. So, techniques of preprocessing are applied to solve these problems. For this reason, an in-house built code (in the MATLAB®) software was composed. The code checks lack of data, double data and whether a recording is inaccurate (outlier).

Eventually, it was proved that some stations had inconsecutive recordings from the start day of its operation. After the stage of the preprocessing, the station with the largest number of recordings had 6648 continuous daily recordings and the one with the smallest had 93 continuous recordings. Thus, the station, which was chosen as a representative example (ORES latitude = 34° 44′ 20.76″, longitude = 239° 43′ 17.04″) had 5546 continuous recordings (from 13 October 1999 to 07 February 2015) and only one outlier detected.

This code also detects if a data is inaccurate and can be considered as outlier. After tests, it was found that the best and most proper way to locate these outliers is to fit a smoothing spline to the data. In this way, any large sudden spike will have large difference (absolute error) between the original data and the data from the spline (Fig. 2). The main issue is to define the threshold on which the value of the time series can be regarded as an outlier. This value depends on the application

Table 3 Results for the multi-step-ahead prediction of the dX time series ("ORES" station), test set 1109 observations

Criteria for the dX "ORES" station Multi-step-ahead prediction

MODELS	ME (mm)	MAE (mm)	Max MAE (mm)	Min MAE (mm)	MSE (mm^2)	RMSE (mm)
Linear Kinematic	0.55	4.63	18.84	0.01	34	5.86
Second degree Kinematic	−1.98	4.88	14.99	0.00	32	5.65
Double moving average (window 150)	5.81	6.50	27.10	0.01	86	9.29
Brown's (a = 0.05)	−6.67	6.73	21.35	0.04	60	7.72
Brown's (a = 0.10)	−4.61	5.06	18.79	0.00	36	6.04
Holt's (a = 0.1,b = 0.1)	−2.79	4.15	16.53	0.01	25	5.04
Holt's (a = 0.1,b = 0.01)	13.77	13.92	43.28	0.05	325	18.02
Holt's (a = 0.2,b = 0.01)	0.43	4.03	17.20	0.00	27	5.22

Table 4 Results for the multi-step-ahead prediction of the dY time series ("ORES" station), test set 1109 observations

Criteria for the dY "ORES" station Multi-step-ahead prediction

MODELS	ME (mm)	MAE (mm)	Max MAE (mm)	Min MAE (mm)	MSE (mm^2)	RMSE (mm)
Linear Kinematic	2.77	5.68	23.81	0.00	55	7.41
Second degree Kinematic	9.98	10.36	34.80	0.02	169	13.00
Double moving average (window 150)	−14.83	14.94	38.43	0.05	287	16.95
Brown's (a = 0.05)	−15.51	15.59	39.11	0.04	308	17.55
Brown's (a = 0.10)	6.35	7.83	30.90	0.02	109	10.43
Brown's (a = 0.15)	1.08	5.01	20.65	0.00	42	6.45
Holt's (a = 0.1,b = 0.01)	−14.32	14.41	37.73	0.02	266	16.31
Holt's (a = 0.2,b = 0.01)	−12.32	12.44	35.40	0.00	202	14.22

and on the available data, following a try-and-error process in order to find it.

Figure 3 presents the time series of the X, Y, and Z geocentric coordinate as an example but mainly focus on the presentation of the outlier found which is highlighted in the red circle.

Also, a check was made using past data, to confirm that it was indeed a wrong recording and not an extreme phenomenon had happened (like an earthquake). For this check, some more information about the permanent stations were used as the file of the velocities of each station. In this file, a station may have more than one velocity with their respective time, for example if the station was near a large earthquake and was affected by post-seismic displacement. Consequently, the code compares the result of the time where the outlier was detected with the time where the earthquake occurred (start day of a different velocity) in order to ensure that it was an outlier indeed.

Lastly, the segregation of the data into "estimation-training" data and "evaluation" data is carried out. The data, which are not held out, are used to estimate the parameters of each model. The model is then tested on data in the evaluation period, if the results are satisfactory, and the forecasts are then generated beyond the end of the estimation and evaluation periods.

This segregation was done empirically and by following the bibliography, where usually the 80% is used for the model and 20% is used for the evaluation. Specifically, for the chosen station, from the 5546 recordings, the 4437 (80%) were used as "training set" and the 1109 (20%) as "evaluation set".

The station with the biggest size of data had 5318 recordings used as "training set" and respectively the one with the smallest had 74 recording. Holding data out for evaluation purposes is probably the most important diagnostic test, as it gives the best indication of the accuracy that can be expected when forecasting the future.

Evaluation of the examined models

The scope of this study did not allow the application of all models, taking into consideration their limitations. As far as traditional models of Geodesy are concerned, the only one applied was the *kinematic model* (linear and second degree polynomial).

Table 5 Results for the multi-step-ahead prediction of the dZ time series ("ORES" station), test set 1109 observations

Criteria for the dZ "ORES" station Multi-step-ahead prediction

MODELS	ME (mm)	MAE (mm)	Max MAE (mm)	Min MAE (mm)	MSE (mm^2)	RMSE (mm)
Linear Kinematic	−1.68	4.63	22.41	0.00	37	6.12
Second degree Kinematic	0.34	4.32	19.33	0.01	30	5.50
Double moving average (window 150)	4.23	5.00	22.17	0.00	37	6.10
Brown's (a = 0.05)	11.24	11.34	30.23	0.07	160	12.66
Brown's (a = 0.10)	−13.53	13.77	44.69	0.00	303	17.42
Brown's (a = 0.51)	−18.63	18.75	54.75	0.01	535	23.12
Holt's (a = 0.4,b = 0.01)	1.59	3.73	18.95	0.01	23	4.75
Holt's (a = 0.5,b = 0.01)	−0.94	3.78	18.85	0.02	25	5.01

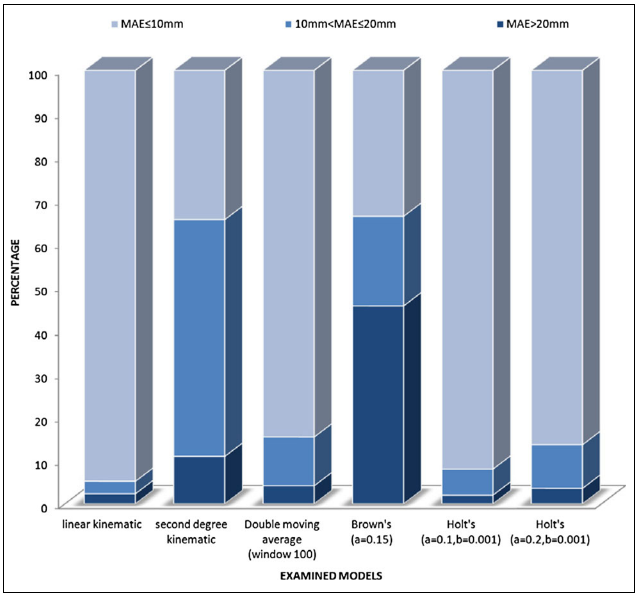

Fig. 4 Classification of MAE for the multi-step-ahead prediction of the dX

Also, the models *Simple mean, Simple moving average, Simple exponential smoothing, Double moving average*, and *Brown and Holt* were actualized. For all these models, the comparison was done using the evaluation criteria ("Indicators-criteria for the evaluation of a model" section).

As far as the one-step-ahead prediction is concerned, all the abovementioned models had very good results for the predictions, with values of MAE close to zero. The most complicated problem was to extend the predictions in the future.

The same models were used for multi-step-ahead predictions. The evaluation shows that it is not possible for all of

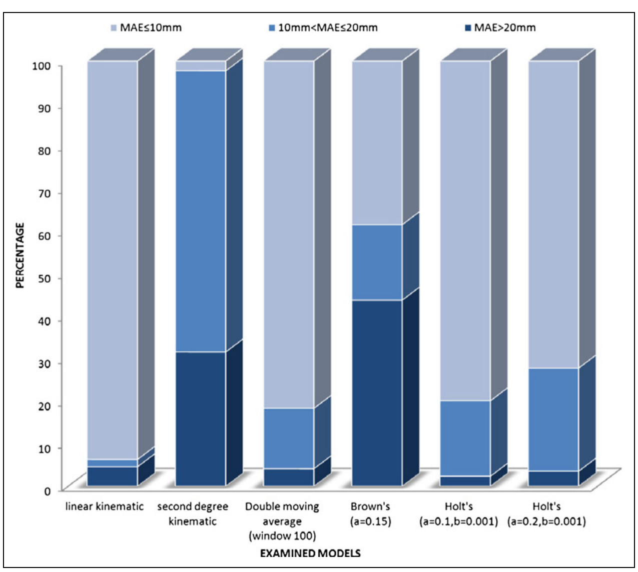

Fig. 5 Classification of MAE for the multi-step-ahead prediction of the dY

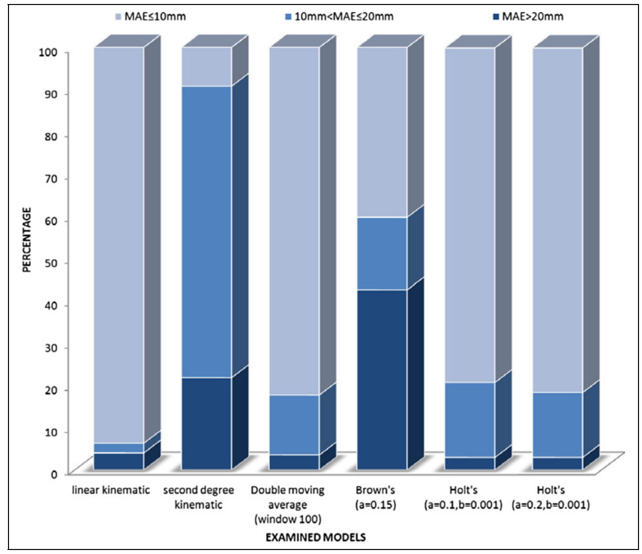

Fig. 6 Classification of MAE for the multi-step-ahead prediction of the dZ

them to be used for the predictions of displacement of the order of a few cm. Therefore, the results of the s*imple mean*, *simple moving average*, *and simple exponential smoothing* were rejected since they did a prediction with a μAE of the order of 25–30 cm.

All the criteria for the examined station are presented in the Tables 3, 4, 5.

Generally, the root mean square error (RMSE) and the mean absolute error (MAE) are the most widely used in the model evaluation studies. However, in this study, it was decided to select the MAE, as it leads to simpler mathematical results. The values of MAE of the 174 GNSS station were clustered for each model. Figures 4, 5, and 6 present the results values of the MAE error for dX, dY and dZ for all the stations in percentages. The clusters that were used are MAE ≤10 mm, 10 mm <MAE ≤ 20 mm, and MAE >20 mm.

It is obvious from the Figs. 4, 5, and 6 that the linear kinematic model has the best performance for the majority of the GNSS stations. The 95% of the stations produces predictions of dX, dY, and dZ with MAE smaller than 10 mm. Close to these results are those of the Holt's model (a = 0.2 and b = 0.001). The 86% of the stations produce predictions with MAE smaller than 10 mm for dX, 75% for dY, and 82% for dZ.

Concluding remarks

The purpose of this research was the presentation, test, and comparison of some mathematical techniques and of the traditional geodetic models of deformation in order to use them as a tool for prediction of displacement.

These mathematical methods are widely used by the scientific community in other applications but are rarely used for the prediction of displacements.

The traditional models in Geodesy and some key features that differentiate them from one another are presented. Specifically, the main classification characteristics are whether they model the causes of the displacement or the modeling of time. The latter option, the modeling of time was the main idea for the investigation of their usability in forecasting and not just for modeling such phenomena.

The aim of the present study is to further highlight the main forecasting methods based on time series analysis and to determine the possibility of using some of the models to forecast displacement. From the theoretical exposition of these methods, it is clear that it is not possible to use all of them, ultimately only those that are also capable of modeling time.

Before choosing the appropriate model, it is crucial to define the timescale of the prediction. All the mentioned models can be used in order to produce one-step-ahead prediction, with MAE of a few mm. The problem becomes more complicated when it comes to multi-step ahead predictions.

An experimental application by using geodetic data (time series of geocentric coordinates X, Y and Z from 174 GNSS stations), was carried out. In order to make multi-step predictions kinematic model, as well as the double moving average, double exponential smoothing (Brown's approach) and exponential smoothing adjusted for trend method (Holt's approach), were used. The other three models, simple mean, simple moving average, and simple exponential smoothing, could not be used for this reason as they presented a MAE of the order of 25–30 cm, even if the forecasted value were added to the end of the time series.

Finally, the results of the application showed that the linear kinematic model produced the best results, and close to this were the results of the Holt's model.

This work was the first step in a larger research and it is proposed to investigate further these models and others, using even more data.

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