Multi Parametric Evaluation of Back Propagation Artificial Neural Network in Determination of Geoid Undulations

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Abstract

The increased use of GNSS for technical constructions and infrastructure works makes necessary the calculation of local accurate geoid undulation's models for the determination of accurate orthometric heights. Apart from the conventional methods, artificial neural networks (ANN) are also used for this purpose. Specially, back propagation artificial neural networks (BPANN) are widely applied for engineering practice. It is well known that the training of ANNs consists a "black box", as the user cannot interlope in the procedure. The aim of this work is to investigate the impact of some crucial parameters to the accuracy, of geoid undulations determination, when BPANN is used. Parameters as the different types of input data (ellipsoidal or Cartesian coordinates, ellipsoidal or orthometric heights etc.), the allocation of the known points at a concrete area, the number of neurons and the number of hidden layers are examined. The results are evaluated by means of the achieved RMSEs, they are compared to each other and to the initial approximation by a polynomial interpolation using GNSS/leveling data. For the trial run an urban region of Athens city was used, where 37 points of known geoid undulations are located at 12Km^2 area. Useful diagrams illustrate the evaluation of the 176 BPANNs that were tested. It is concluded that the number of the known points is not as crucial as their regular and equidistant allocation. Also the increment of neurons in the hidden layers, optimize the results. Moreover simple input data set with two or three neurons including the coordinates of the known points is adequate as it provides better assessments.

Keywords: back propagation artificial neural networks, local geoid model, urban region, Cartesian coordinates.

1. Introduction

Almost fifty years ago, Bomford (1971) states that the accurate determination of geoid is one of the main subjects of Geodesy. Nowadays the evolution of technology on geodetic instrumentation and mainly on GNSS receivers provides to surveyors the possibility for quick and accurate coordinates' determination. As GPS provides easily ellipsoidal heights h, the accurate value of the geoid undulation N for every point on the earth is the most valued for surveying engineers worldwide. By performing the simple and well known equation H= h-N (Bomford 1971, Heiskanen 1981, Torge 2001), the orthometric heights H can be calculated. Many efforts and hundreds of studies were carried out in order to reach an accurate calculation of the geoid undulation N (Schneider et al 1997, Balmino 1978). Gravimetric methods are widely used as there are available data worldwide (Arabelos et al 1998, Moritz 1980, Tscherning et al 2001, Grebenitcharsky et al 2005). By using mainly these data several global geoid models are produced (Pavlis et al 2008, Pavlis et al 2012, Gilardoni et al 2016). Also other more accurate methods as astrogeodetic leveling (Lambrou 2003, Lambrou 2015) and spirit leveling, have minor use as they are cumbersome and time consuming (Hofmann-Wellenhof et al 2006, Featherstone 2001, Torge 2001,Yilmaz et al 2008). Usually local geoid models, were developed by using least-squares collocation (LSC), polynomial surface fitting and finite-element methods (FEM) utilizing the acquired data from GNSS and spirit leveling methods (Balmino 1978, Kotsakis et al 1999, Zhong 1997, Yanalak et al 2001, Yang et al 1999, Gullu et al 2011, Zhan et al 1999, Sansò et al 2013). Almost twenty years ago the Artificial Neural Networks (ANN) was brought forward in this field in order to provide better approximations of the geoid undulation N (Ambrozic et al 1999, Hu et al 2002). The ANNs were applied in many scientific fields (finances, physics, politics, socials) having a large number of patterns and algorithms.

A great number of researches were carried out by using ANNs procedures as encouraging results were achieved (James et al 2001, Miima et al 2001, Seager et al 1999, Kutoglu 2006, Stopar et al 2006, Kavzoglu et al 2005). For engineering applications and especially for geoid undulations determination the Back – Propagation algorithm (BPANN) was used in the majority of researches (Lin 2007). Briefly the architecture of a BPANN, is structured by three types of layers, the input, hidden and output layers (Abdalla et al 2010). Additionally, a pre-input and a post-output transformation layers are needed for the alteration of the input and output values according to the sigmoid activation function f(x) in the range from 0 to 1 and vice-versa. The training goal is to minimize the root of mean square error of the differences between the network's outputs and the target (known) outputs. The MATLAB toolbox includes a BPANN user friendly environment for data simulations. This software allows the changing of all the crucial parameters according to the user's desire. Also, it provides statistical data and charts helping the results' perception (Demuth et al 2002). During the training procedure the choice of the involved parameters is critical as influence the ability of the network to learn and to provide an accurate solution. The number and size of hidden layer, the number of neurons (Kavzoglu et al 2005, Stopar et al 2006, Yilmaz 2013), the range of the initial weights, the learning rate, the number of training iterations (Kavzoglu et al 1999, Kavzoglu 2001, Garson 1998) and the number of used points (Yilmaz et al 2011) are some of them. Additionally it is worth to mention that usually two criteria are used in order to evaluate and to compare the performance of different BP ANNs. The RMSE of the training set is the main criterion. However the combination that provides, the smallest RMSE and the biggest correlation coefficient (ρ), is the best (Gourine et al 2012, Yilmaz et al 2011, Kavzoglu et al 2005, Gullu et al 2011). Hence, the feed-forward back propagation algorithm via MATLAB toolbox will be used in this study in order to investigate the influence of some crucial parameters, as the type of input data, the number of hidden layers, the number of neurons in each layer and the number of the known points in a given area,for the calculation of geoid undulations.

2. Investigation Scenarios

According to the above mentioned technique the scope of this research is to investigate whether different type of input data or BPANN's main architecture may change the accuracy of the simulation namely the outcome RMSE. Thus the geoid undulation N is the only neuron in the output layer.

The following scenarios are examined:

- The impact of the number and the type of neurons at the input layer. Thus i=1 to 8 input data set are formed:
  $(set_1) = \varphi\text{-}\lambda$, (set_2) = X-Y-Z, $(set_3) = \varphi\text{-}\lambda\text{-}\xi\text{-}\eta$, $(set_4) = \varphi\text{-}\lambda\text{-}H$,
  $(set_5) = \varphi\text{-}\lambda\text{-}\xi\text{-}\eta\text{-}H$, (set_6) = X-Y-Z-H, $(set_7) = \varphi\text{-}\lambda\text{-}h$, $(set_8) = \varphi\text{-}\lambda\text{-}\xi\text{-}\eta\text{-}h$.
  Where φ = latitude, λ=longitude, H=orthometric height, h = ellipsoidal height, X, Y, Z = geocentric coordinates, ξ , η = the components of the deflection of the vertical
- The number of hidden layers and the included neurons in each one. Thus eleven different structures of BPANNs are tested (j =1 to 11) as following:
  One hidden layer including 20, 40, 60 neurons consecutively (str_1:20:1, str_2:40:1, str_3:60:1). Two hidden layers including 20, 40, 60 neurons by turns (str_4:20:20:1, str_5:20:40:1, str_6:20:60:1, str_7:40:20:1, str_8:40:40:1, str_9:40:60:1, str_{10}:60:20:1, str_{11}:60:40:1)
- The impact of the number and the distribution of the known points in a given area. Thus two scenarios are tried on. The first scenario uses all the available points as the second one uses the half of the known points which have a regular allocation at the area. So 88 different combinations are come up, with architecture from 2:20:1 to 5:40:60:1, which are tested twice. Hence 176 different BPANNs are formed for the same simulation.

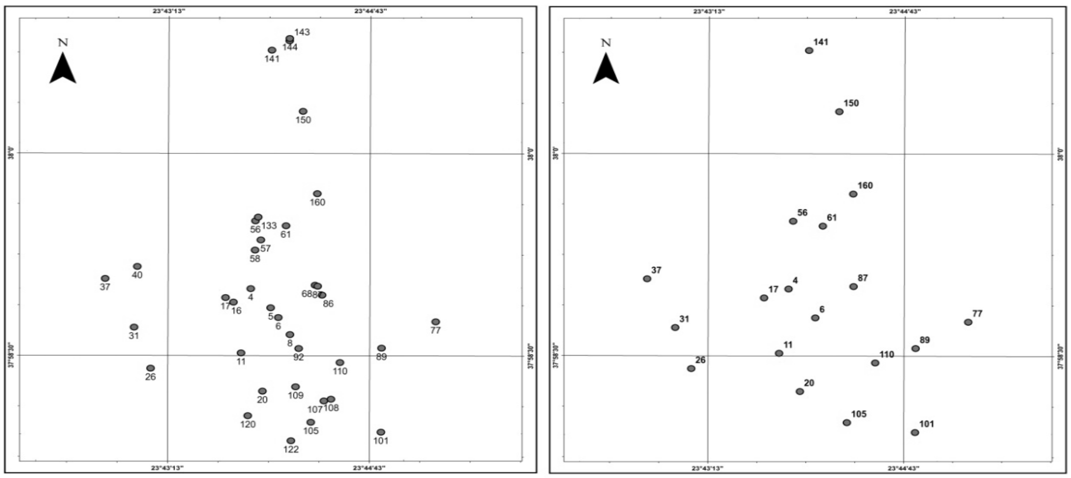
3. Data procedure

The study area includes 37 distributed points at an urban region of about 12 km^2 (figure 1a). For these points both the geocentric coordinates X, Y, Z and the geodetic ones φ, λ, h, were determined by using the GNSS static positioning method as their orthometric heights were determined by spirit leveling (Alevizakou 2010, Boubakis, 2011). Then the total accuracy of the geoid undulation determination for each point is estimated to be less than ±1cm. The components ξ and η of the deviation of the vertical for these points are determined by using EGM2008 global geopotential model (Pavlis et al. 2008). The variation of N (ΔN) for the examined area is about 20cm. For this area a first order polynomial adaptation was applied, (equation 1) which provides RMSE equal to ±5cm.

$$N_i(m) = 38.4937 + 252.7577 \cdot (\varphi_i - \varphi_0) + 152.5094 \cdot (\lambda_i - \lambda_0) \quad (1)$$

Where φ_i, λ_i are the coordinates, latitude and longitude, of each point i and φ_o, λ_o are the coordinates of the central point of the region, both in WGS ′84. Although the area is small, it is obvious that the achieved RMSE is not satisfied. So it is decided to test if a neural network could approximate better the geoid in this area. Initially the 37 points are used (figure 1a), thereinafter 19 points, out of the 37 are selected (figure 1b), which show more regular allocation. The Levenberg-Maquart (trainlm), which is a type of the back propagation algorithm, is used as training algorithm. Moreover the sigmoid function (tansig) is used for the activation of the hidden neurons.

Figure 1: The maps of 37 points (a) and 19 points(b)



(a) (b)

Tables 1 and 2 present the results by means of the RMSE of the training data set, for all the combinations, by using 37 and 19 points correspondingly.

Table 1. RMSE (in cm) of the training sets for 37 points

set_i \ str_j	*20:1*	*40:1*	*60:1*	*20:20:1*	*20:40:1*	*20:60:1*	*40:20:1*	*40:40:1*	*40:60:1*	*60:20:1*	*60:40:1*
φ-λ	4.5	5.5	~~3.2~~	5.3	~~4.0~~	4.3	4.6	4.5	~~3.9~~	~~2.0~~	1.9
X-Y-Z	~~4.2~~	~~4.1~~	4.9	~~5.1~~	~~4.0~~	~~4.0~~	~~3.5~~	~~3.9~~	4.0	2.1	~~1.5~~
φ-λ-ξ-η	4.6	5.6	5.3	7.3	4.9	4.7	5.1	~~8.4~~	4.7	5.5	4.5
φ-λ-H	~~5.3~~	4.2	7.1	8.1	5.5	3.7	4.7	4.0	4.9	3.9	3.4
φ-λ-ξ-η-H	4.9	4.8	5.7	8.7	5.0	5.1	7.1	8.3	6.2	~~4.9~~	~~4.7~~
X-Y-Z-H	3.5	5.0	6.1	8.1	6.0	4.8	3.6	4.3	4.0	3.6	3.4
φ-λ-h	4.6	5.2	~~7.8~~	~~8.8~~	4.5	4.2	4.4	8.1	~~6.7~~	4.5	2.9
φ-λ-ξ-η-h	4.8	~~6.6~~	5.9	7.5	~~6.1~~	~~5.8~~	~~7.3~~	7.5	6.1	4.3	4.4

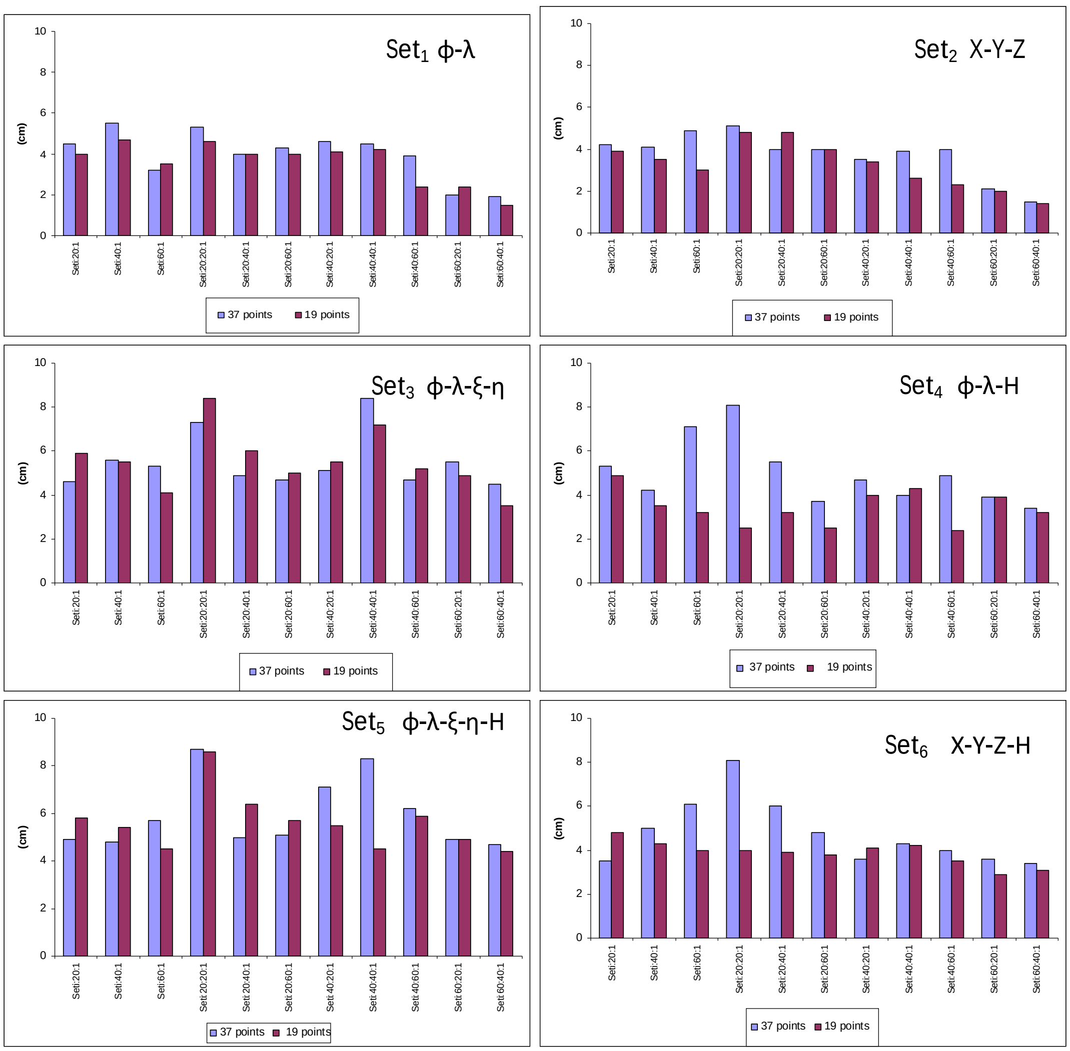
Table2. RMSE (in cm) of the training sets for 19 points

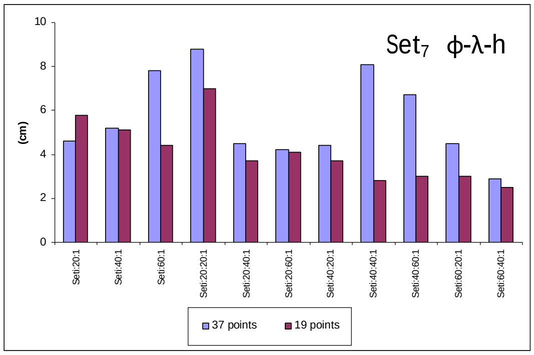
set_i \ str_j	*20:1*	*40:1*	*60:1*	*20:20:1*	*20:40:1*	*20:60:1*	*40:20:1*	*40:40:1*	*40:60:1*	*60:20:1*	*60:40:1*
φ-λ	4.0	4.7	3.5	4.6	4.0	4.0	4.1	4.2	2.4	2.4	1.5
X–Y-Z	~~3.9~~	~~3.5~~	~~3.0~~	4.8	4.8	4.0	~~3.4~~	~~2.6~~	~~2.3~~	~~2.0~~	~~1.4~~
φ-λ-ξ-η	~~5.9~~	5.5	4.1	8.4	6.0	5.0	5.5	~~7.2~~	5.2	4.9	3.5
φ-λ-H	4.9	3.5	3.2	~~2.5~~	~~3.2~~	~~2.5~~	4.0	4.3	2.4	3.9	3.2
φ-λ-ξ-η-H	5.8	5.4	~~4.5~~	~~8.6~~	~~6.1~~	5.7	5.5	4.5	~~5.9~~	~~4.9~~	~~4.4~~
X-Y-Z-H	4.8	4.3	4.0	4.0	3.9	3.8	4.1	4.2	3.5	2.9	3.1
φ-λ-h	5.8	5.1	4.4	7.0	3.7	4.1	3.7	2.8	3.0	3.0	2.5
φ-λ-ξ-η-h	5.3	~~6.0~~	4.3	7.1	5.3	~~5.3~~	~~6.8~~	5.6	5.3	4.2	4.0

For each row of the table, the light gray box presents the max RMSE as the dark gray one presents the min RMSE.

Also for each column of the table the box with the single stripe presents the min RMSE as the double striped box presents the max RMSE. When 37 points are used the RMSE fluctuates from 1.5cm to 8.8cm as the correlation coefficient ρ fluctuate from 0.80 to 0.99. When 19 points are used, the RMSE fluctuates from 1.4cm to 8.6cm, as the correlation coefficient ρ fluctuate from 0.46 to 0.99. Moreover in order to highlight the influence of the points' number and allocation, figure 2 illustrates the RMSE for each one of the eight different input data sets and the eleven BPANN's structures by using 37 and 19 points. According to figure 2, the 90% of the BPANN's structure combinations have smaller RMSE, when 19 points are used. That means that the regular allocation of the points plays significant role. In addition table 3 presents, the minimum, the maximum RMSE for all the input data sets (3a) and for all BPANN structures (3b) both, when 37 and 19 points are used. Moreover figure 3 illustrates how many times a BPANN structure achieves min or max RMSE by using all the input data sets, as figure 4 presents how many times an input data set achieves min or max RMSE by using all the BPANN structures.

Figure 2: RMSE for each one of the eight input data set by using 37 or 19 points.





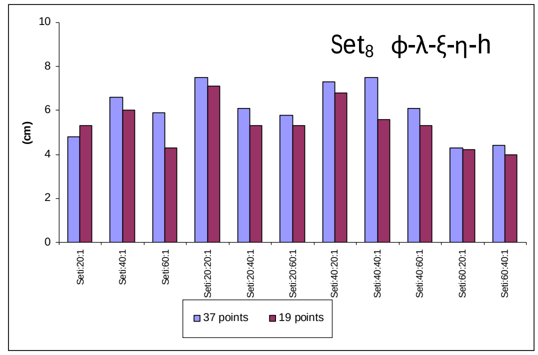


Table 3: The min and max RMSE for every input data set (a) and for every BPANN's structure (b) for both the 37 and 19 points.

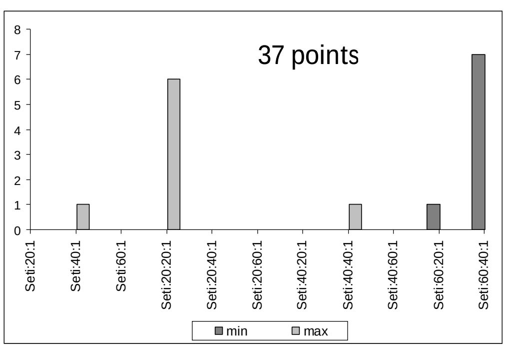
	37 points		*19 points*	
set_i	*RMSE min(cm)*	*RMSE max(cm)*	*RMSE min(cm)*	*RMSE max(cm)*
φ-λ	1.9	5.5	1.5	4.7
X-Y-Z	1.5	5.1	1.4	4.8
φ-λ-ξ-η	4.5	8.4	3.5	8.4
φ-λ-H	3.4	8.1	2.4	4.9
φ-λ-ξ-η-H	4.7	8.7	4.4	8.6
X-Y-Z-H	3.4	8.1	3.1	4.8
φ-λ-h	2.9	8.8	2.5	7.0
φ-λ-ξ-η-H	4.3	7.5	4.0	7.1

(a)

	37 points		*19 points*	
str_j	*RMSE min(cm)*	*RMSE max(cm)*	*RMSE min(cm)*	*RMSE max(cm)*
20:1	4.2	5.3	3.9	5.9
40:1	4.1	6.6	3.5	6.0
60:1	3.2	7.8	3.0	4.5
20:20:1	5.1	8.8	2.5	8.6
20:40:1	4.0	6.1	3.2	6.1
20:60:1	4.0	5.8	2.5	5.3
40:20:1	3.5	7.3	3.4	6.8
40:40:1	3.9	8.4	2.6	7.2
40:60:1	3.9	6.7	2.3	5.9
60:20:1	2.0	4.9	2.0	4.9
60:40:1	1.5	4.7	1.4	4.4

(b)

Figure 3: The frequency of the min and max RMSE for every BPANN structure by using all the input data combinations



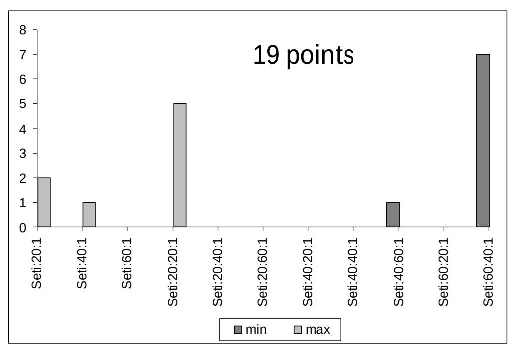
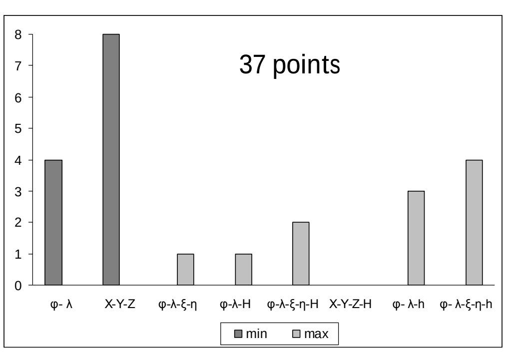
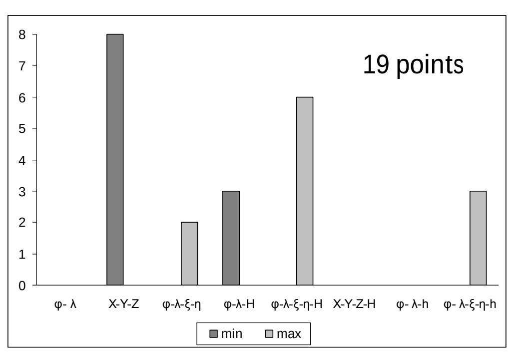


Figure 4: The frequency of the min and max RMSE for every input data combination by using all BPANNs structures





4 Evaluation of the results

According to table 3a it is obvious that by using only the points' coordinates, as input data, better results are achieved. The input data set_1 and set_2 give similar and compatible results for any BPANN structure with both 37 and 19 used points. When 19 points are used, the approximation is better.

Also the min RMSE fluctuates from 1.5cm to 4.7cm, when 37 points were used and from 1.4cm to 4.4cm, when 19 points were used. The input data set_2 = X-Y-Z provides better results (min RMSE$\leq$ 1.5cm) either for the 37 and 19 points, while the correlation coefficient ρ fluctuates from 0.934 to 0.999. When the components of the deviation of the vertical ξ and η are used as input data the simulation gets worst. This is observed in any combination (sets 3, 5 and 8). Also sets which have the H or h give bigger RMSEs. It is significant that when more input neurons are used with heterogeneous data the ANN provides worst approximation, as the RMSE increases. Thus the more input neurons the more confused than helpful.

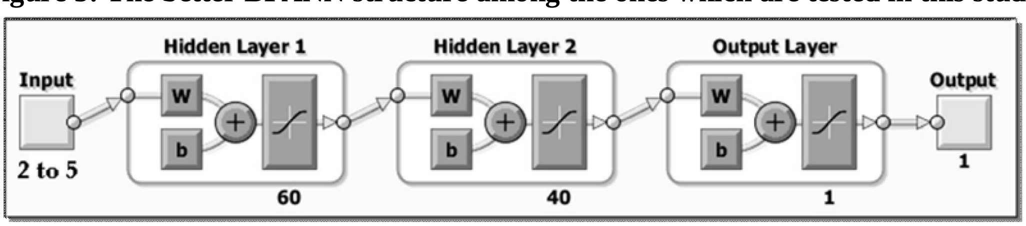
Table 3b presents that simple BPANN structure give similar results as the first order polynomial approximation. A substantive improvement of the results appears, when two layers are used and one of them contains at least 60 neurons. The RMSE is eliminated as the number of neurons is increased for all sets of the input data. The last more complicated structure 60:40:1 succeed min RMSEs for 7 out of 8 input data sets with both the 37 and 19 given points.

Moreover as fig 3 illustrates str_{11} = 60:40:1 gives constantly the min RMSE regardless the used input data set. On the contrary str_4 provides the max RMSE (5 times) with different input data sets. According to fig 4 the input data set_2 achieves 8 times the min RMSE with different structures. That means that it is responded very well to the simulation. Also set_5 and set_8 provides always the max RMSEs as ξ and η components are involved. It was found that input sets, which have similar input data as set_1 and set_2, set_4 and set_7 as well as set_5 and set_8 have similar approximations. A BPANN manages to approximate better the geoid undulation in this area than a first order polynomial as provides three times smaller RMSE.

Also as figure 2 illustrates the simulations with 19 points give better approximation (smaller RMSE) for the 90% of the combinations. Thus the uniform allocation of the known points over the examined area seems to be a crucial parameter for the function approximation. In the case of 37 points an over fitting may be occurred due to the dense allocation of the points or the slight differences of the points' coordinates as well as the minor alteration of N. On the other hand it seems that19 points are adequate to present the geoid undulation of such an area.

Finally tables 1 and 2 show that, for the same input data set and the same BPANN structure, the differences between the two spatial distributions of the points are small of the order of 1cm for the 90% of the combinations. It is proved by the tables 1 and 3b that the most complicated BPANN, which consisted of two hidden layers which are formed by 60 and 40 neurons (figure 5) provides the more accurate N values regardless the input data set and the number of the used points. This structure provides RMSE smaller than 4cm for the 60% of the combinations with 37 points and for the 90% of the combinations with 19 points.

Figure 5: The better BPANN structure among the ones which are tested in this study



5. Conclusions

This paper investigates the influence of the input data type, of the different ANN's structure and of the known points allocation, when an ANN is used for geoid undulation determination at an urban area of 12 km^2. The BPANN algorithm via Math lab tool box is used. According the results analysis it is concluded that the best achieved RMSE is ±1.4cm much better than the first order polynomial adaptation, which was ±5cm.

The regular allocation of points at the examined area plays significant role. The 90% of the scenarios provide smaller RMSE, when 19 regular and almost equidistant allocated points were used. Not regular distribution and high density of the known points may lead to over fitting phenomena which provide bigger RMSE values namely worst approximations. It is proved also that the spatial coordinates φ and λ or X, Y and Z are adequate as input data for the geoid undulation determination. Additionally set_2 = X-Y-Z as input data presents a constant behavior for all the BPANN structure. The fact that the geocentric coordinates X, Y, Z are accurately determined by using GNSS may enables these results.

Moreover the most complicated BPANN, which consisted of two hidden layers with 60 and 40 neurons provides the more accurate geoid undulation determination regardless of the input data set and the number of the used points (achieves 7 out of 8 min RMSEs). The RMSEs of this structure fluctuates from 1.5cm to 4.7 cm for all the input data sets. Also this structure provides the minimum RMSE for the 90% of the input data combination.

Thus it is proved that better approximations of the geoid undulation N can be achieved by using more complicated ANNs structures, simple input data set as the geocentric coordinates and equidistant allocation of the known points.

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