Optimal multidimensional solitary wave steering

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Received September 14, 2004; revised manuscript received December 8, 2004; accepted December 24, 2004

Solitary wave steering under interaction with continuous waves is studied. An analytical approach based on two conserved quantities of the wave evolution is used to study transverse wave number variations. The method is applicable to any number of transverse dimensions and any kind of nonlinearity. The analytical results provide useful information for the investigation of optimal parameters of interaction for efficient multidimensional steering. Numerical simulations for specific cases are used to confirm the analytical results.

OCIS codes: 190.4420, 190.5530, 200.4770, 230.4320.

1. INTRODUCTION

The n-dimensional nonlinear Schrödinger equation (ND NLSE),

$$iu_z + \Delta_\perp u + F(|u|^2)u = 0, \qquad (1)$$

models nonlinear wave evolution in a variety of branches of physics. It describes the slowly varying complex envelope of a wave packet in conservative, dispersive systems. Such systems appear in hydrodynamics,[1,2] plasma physics,[3,4] and nonlinear optics.[5,6] Also, the one-dimensional (1D) equation describes the propagation of the Davydov solitons on an α-helix protein,[7] while the three-dimensional equation applies to the description of Bose–Einstein condensates.[8]

In the context of nonlinear optics, this equation describes the evolution of the complex envelope $u(z,\mathbf{r}_\perp)$ of an electric field within the paraxial model of self-focusing.[6] The z coordinate measures the propagation distance and the transverse coordinates $(\mathbf{r}_\perp)$ might be spatial and–or temporal with the corresponding terms of the linear operator $\Delta_\perp$ describing diffraction and–or (anomalous) dispersion, respectively. Thus the 1D equation describes a light pulse or a self-focusing beam in a planar waveguide, while the two-dimensional (2D) equation describes a self-focusing beam in a bulk medium or a localized wave packet in both spatial and temporal dimensions. Finally, the three-dimensional equation models the evolution of spatiotemporal localized structures known as "light-bullets."

In the subsequent analysis we refer to the transverse coordinates as being spatial, although the arguments also hold for the spatiotemporal case. The nonlinear function $F(I)$ models the intensity-dependent refractive index of the medium and has the form $F(I)=I/(1+s^2I)$, where $I=|u|^2$ and $0\leqslant s\leqslant 1$ is the saturation parameter of the nonlinear medium. For $s=0$, we have a cubic (Kerr) nonlinearity and for $s\ll 1$ the function approximates the competing cubic–quintic nonlinearity $F(I)=I-s^2I^2$.

The 1D equation with $s=0$ is known to be completely integrable in terms of the inverse scattering transform; it admits soliton solutions and has an infinite number of conserved quantities that are related to symmetries.[9] Although the NLSE is not known to be completely integrable in higher dimensions, the balance between diffraction (and–or dispersion) and nonlinearity can result in the formation of localized structures. The stability of these structures has been a subject of major interest in the past decade, and it has been shown that for $s=0$, they can diffract or collapse in a finite distance of propagation, depending on their initial power.[9,10] Collapse-arresting mechanisms, such as high saturation of the refractive index,[11] the effect of nonparaxiality,[12,13] and wave self-rectification,[14] have been proposed, while collapse-free nonlinearities such as quadratic nonlinearities can be used to achieve stable solitary waves.[15–17] However, for a variety of applications, including all-optical switching devices, collapsing may be irrelevant, provided that the power of the beam and the length of propagation are chosen within appropriate value ranges.[18,19]

Among the properties of solitary wave propagation in nonlinear media, one of the most promising for applications is beam steering under interaction with other waves. This feature of the interactions is very desirable in designing all-optical and dynamically reconfigurable switching devices for potential applications in signal processing and telecommunications, and a beam steering technique based on the use of the "walking soliton" concept for non-Galilean invariant systems has been studied for quadratic media.[20–23]

In this work we investigate interactions of continuous waves (CWs) with solitary beams. These interactions are capable of affecting certain parameters of the solitary beams, the most important being their transverse velocity. Thus the intentional injection of an appropriate CW can be used as a control mechanism for changing a beam's transverse velocity, resulting in the capability of multidimensional beam steering. In fact a variety of beam-steering techniques have been investigated for the ND NLSE with n=1 and 2 transverse dimensions,[24–27] and the propagation of 1D solitons lying on a CW background has been studied both analytically and numerically.[28,29]

To study this kind of interaction in higher dimensions, one may apply standard variational methods to a perturbed NLSE, which is obtained if we treat the CW as an effective external potential. However, this method results in a nonautonomous, multidimensional dynamical system for the amplitude, width, phase, center position, and transverse velocity of the beam whose complex dynamical features prevent simple, intuitive understanding of the important interaction features. Instead a much simpler and more intuitive analytical perturbational approach based on the quasi-particle picture of the wave interaction[30] and utilizing two conserved quantities of the NLSE, namely the "mass" and the "momentum," has been adopted. This approach applies for any dimension and nonlinearity function. More important, it results in simple formulas for "mass" and "momentum" variation that provide useful guidelines for optimal parameter selection for efficient beam steering.

As shown in the following the presence of a CW affects both the mass and the momentum of the solitary beam. The mass-dependent, self-focusing instability of the higher-dimensional NLSE makes it necessary to select the interaction parameters so that the resulting mass of the beam does not lead to self-focusing. The latter is quite undesirable because, for the Kerr-type nonlinearity, it leads to beam collapse, while even for a saturable nonlinearity where there is no collapse, self-focusing and the corresponding increase in beam amplitude reduce the steering efficiency of the interaction. Direct numerical simulations are used to confirm the analytically obtained estimations. In Section 2 the analytical approach is applied and formulas for the mass and momentum variation of the beam are given. In Section 3 results of numerical simulations combined with estimations obtained by the analytical approach are discussed. The main conclusions are given in Section 4.

2. ANALYTICAL APPROACH

The mass and momentum of a solution of Eq. (1) are defined, respectively, as

$$P = \int |u|^2 \mathrm{d}S, \tag{2}$$

$$\mathbf{M} = i \int (u^* \nabla_\perp u - u \nabla_\perp u^*) \mathrm{d}S, \tag{3}$$

where $\mathrm{d}S$ is the area element normal to z, which elements are conserved when u evolves under Eq. (1). Since we are interested in interactions between solitary beams and CWs, we express u as a sum of a solitary part u_{s} and a CW part u_{cw} according to the quasi-particle approach:

$$u = u_{\mathrm{s}} + u_{\mathrm{cw}}, \tag{4}$$

where $u_{\mathrm{s}}=U(\mathbf{r}_\perp;\lambda)\exp(i\lambda z)$ is a standing-wave (SW) solution of Eq. (1) [or any traveling wave solution obtained as a Galilean transformation of the solution

$$u_{\mathrm{s}} = U(\mathbf{k}_\perp^{\mathrm{s}} z - \mathbf{r}_\perp;\lambda)\exp(i\mathbf{k}_\perp^{\mathrm{s}} \cdot \mathbf{r}_\perp/2 - i|\mathbf{k}_\perp^{\mathrm{s}}|^2 z/4 + i\lambda z)$$

and

$$u_{\mathrm{cw}} = \alpha \exp(-i\mathbf{k}_\perp^{\mathrm{cw}} \cdot \mathbf{r}_\perp + ik_z^{\mathrm{cw}} z + i\phi)$$

with $k_z^{\mathrm{cw}}=-(1/2)|\mathbf{k}_\perp^{\mathrm{cw}}|^2$ as obtained by the dispersion relation of the linearized Eq. (1) for small CW amplitudes α. The Eq. (4) form of the solution represents the actual launched transverse profile at $z=0$ for the initial condition problem of propagation and also has a physical meaning for $z>0$, since the SW retains its localized character during propagation, and the solution can be written as the sum of a (localized) solitary wave and a remaining wave background, each having its mass and momentum. Substituting Eq. (4) into Eq. (2) we obtain

$$P = P_{\mathrm{s}} + P_{\mathrm{cw}} + \Delta P_{\mathrm{s}},$$

$$\Delta P_{\mathrm{s}} = \int (u_{\mathrm{s}} u_{\mathrm{cw}}^* + u_{\mathrm{s}}^* u_{\mathrm{cw}}) \mathrm{d}S, \tag{5}$$

where $P_{\mathrm{s}}+\int|u_{\mathrm{s}}|^2\mathrm{d}S$ and $P_{\mathrm{cw}}=\int_{S_0}|u_{\mathrm{cw}}|^2\mathrm{d}S$ with S_0 being an area much larger than the characteristic width of the beam, but finite, so that the CW has finite mass according to the usual meaning of mass for solitary waves with nonzero background.[31] Similarly, substitution of Eq. (4) into Eq. (3) results in

$$\mathbf{M} = \mathbf{M}_{\mathrm{s}} + \mathbf{M}_{\mathrm{cw}} + \boldsymbol{\Delta}\mathbf{M}_{\mathrm{s}},$$

$$\boldsymbol{\Delta}\mathbf{M}_{\mathrm{s}} = i \int (u_{\mathrm{s}}^* \nabla_\perp u_{\mathrm{cw}} - u_{\mathrm{s}} \nabla_\perp u_{\mathrm{cw}}^*) \mathrm{d}S, \tag{6}$$

where $\mathbf{M}_{\mathrm{s}}$ and $\mathbf{M}_{\mathrm{cw}}$ are defined analogously. It can be easily shown that $\mathbf{M}_{\mathrm{s}}=P_{\mathrm{s}}\mathbf{k}_\perp^{\mathrm{s}}$, $\mathbf{M}_{\mathrm{cw}}=P_{\mathrm{cw}}\mathbf{k}_\perp^{\mathrm{cw}}$, and $\boldsymbol{\Delta}\mathbf{M}_{\mathrm{s}}=\Delta P_{\mathrm{s}}\mathbf{k}_\perp^{\mathrm{cw}}$.

Under the assumption of small amplitude CWs and an efficiently sort distance of propagation, the effect of modulational instability (MI) is negligible and the background (CW) remains practically unaffected. The MI is an effect with finite bandwidth, that is, it can only be excited under the influence of a wave possessing a spectral component whose transverse wave number difference Δk with respect to the CW falls into the spectral range of the instability. The latter is proportional to the amplitude of the CW, thus, the MI cannot be excited provided that $2\alpha^2 \leqslant \Delta k^2$.[32] On the other hand, the spectral content of a Gaussian SW falls in the range $(\Delta k_\perp - 1/(2a_{\mathrm{FWHM}}), \Delta k_\perp + 1/(2a_{\mathrm{FWHM}}))$, where $\Delta k_\perp$ is the SW carrier transverse wave number difference with the CW, and $a_{\mathrm{FWHM}} \simeq 1.665 a_r$ is the effective beam width with a_r being the width parameter of the Gaussian. Thus it is possible to prevent MI and the corresponding background deformation by appropriate choice of the carrier wave number difference, the beam width, and the CW amplitude. Moreover, even for the case where a spectral component of the SW falls into the wave number range of MI, the gain has a finite value, so that the effect of MI can still be small for a small length of propagation, which is the case under investigation, and practically suitable for applications in optical devices. In a rough estimate based on the bandwidth of MI for the case of optimal transverse wave number difference (which is shown in the following analysis to be equal to the inverse width parameter of the Gaussian SW a_r) and an ${a_r}^2=2$ (used in the numerical simulations that follow), a maximum value for the CW amplitude of $\alpha_{max} \simeq 0.35$ is obtained. For $\alpha < \alpha_{max}$ (applied in the numerical simulations) the undesirable effects of MI can thus be avoided. For such a case of practically constant CW, it is reasonable to assume that the variations ΔP and $\mathbf{\Delta M}$ should be considered as variations of the mass and the momentum of the beam and not the CW. The variation of the transverse wave number of the beam can be written in the following form

$$\mathbf{\Delta k}_{\perp}^{s} = \frac{\Delta P_s}{P_s}\mathbf{\Delta k}_{\perp}, \quad \mathbf{\Delta k}_{\perp} \equiv \mathbf{k}_{\perp}^{cw} - \mathbf{k}_{\perp}^{s}. \tag{7}$$

Thus the variation of the transverse wave number (velocity) of the beam ($\mathbf{\Delta k}_{\perp}^{s}$) has the same direction as the transverse wave number difference between the beam and the CW ($\mathbf{\Delta k}_{\perp}$). This property provides the capability of beam steering in any desired direction by appropriately choosing $\mathbf{\Delta k}_{\perp}$, as shown in Fig. 1 for the 2D case.

It is remarkable that for the case of a sole transverse dimension and Kerr-type nonlinearity, i.e., the completely integrable case, Eqs. (5) and (7) coincide with those obtained by means of perturbation on the associated linear eigenvalue problem of the NLSE according to the inverse scattering transform method.[33] However, our approach extends the capability of estimating the variation of the mass and the momentum (or the transverse wave number/velocity $\mathbf{k}_{\perp}^{s}$) due to the presence of a CW background in two directions: higher-dimensional NLSE and more general nonlinearity functions can be studied if u_s can be found numerically or be approximated by a Gaussian (or super-Gaussian).[34] For the 2D NLSE, a circular solitary beam can be written in the form

$$u_s = A \exp\left(-\frac{x^2+y^2}{2a_r^2}\right)\exp(-ik_x^s x - ik_y^s y + i\sigma). \tag{8}$$

Using Eq. (5), we find the variation of the mass is

$$\Delta P^s = 4\pi\alpha A a_r{}^2 \cos\Delta\phi \exp(-a_r{}^2|\mathbf{\Delta k}_{\perp}|^2/2), \tag{9}$$

where $\Delta\phi \equiv \phi - \sigma$ is the initial phase difference and a_r is the width parameter of the beam. The variation of the transverse velocity (or wave number) of the beam $\mathbf{\Delta k}_{\perp}^{s}$ can be obtained directly from Eqs. (7) and (9).

As can be seen from Eqs. (7) and (9), variations of the mass and the transverse velocity depend critically on the amplitude of the CW background and the initial phase difference between the beam and the CW. As expected from the perturbative character of our approach, both variations are linearly dependent on the amplitude of the CW. Moreover, the initial phase difference is shown to be crucial to the capability of altering the mass or the transverse velocity of a beam under interaction with a CW.

The role of the wave number difference between the two waves is shown in Fig. 2. Considering the dependence of ΔP^s on the transverse wave number difference between the beam and the CW ($\mathbf{\Delta k}_{\perp}$), it is obvious that the maximum ΔP is attained for $\mathbf{\Delta k}_{\perp}=0$ [see Fig. 2(a)]. Deviations from $\mathbf{\Delta k}_{\perp}=0$ have effects that depend strongly on the characteristic size of the beam since ΔP^s is a Gaussian function of $|\mathbf{\Delta k}_{\perp}|$ whose width depends on a_r. The maximum velocity along a transverse direction can be achieved for $|\mathbf{\Delta k}_{\perp}|_{max}=a_r{}^{-1}$. For transverse wave number differences that are not comparable with the characteristic size of the beam, the CW background cannot affect significantly the transverse velocity of the beam [Fig. 2(b)]. According to Eqs. (7) and (9) the injection of a CW of appropriate transverse wave number difference for efficient beam steering leads to an increasing beam mass, that because of the self-focusing effect, is detrimental to the evolution of the beam width and amplitude. The latter determines strongly the propagation length in which the amplitude of the CW is large enough in comparison with the beam amplitude that the interaction mechanism actually works.

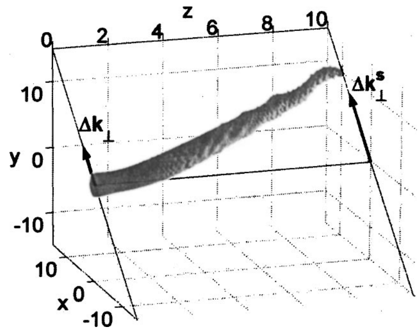


Fig. 1. Steering of a 2D circular Gaussian beam under interaction with a CW.

3. RESULTS AND DISCUSSION

It is well known that solitary wave propagation in nonlinear media governed by the NLSE is not stable in general.[9] For a Kerr-type nonlinearity the 2D case is critical for stable propagation. Catastrophic collapse or diffraction can occur depending on the initial mass of a beam: that is, there is a critical value for the mass P_{cr} above which the amplitude of the beam increases to infinity and its width decreases to zero after a finite propagation distance. Beams with mass values below P_{cr} continuously diffract and are also destroyed. The critical value of the mass, as well as the collapse distance, has been calculated analytically for the Gaussian approximation ($P_{cr}=4\pi$) and numerically for the exact stationary solution of the NLSE ($P'_{cr}=11.7$).[18] Beam destruction under the aforementioned instability can be easily avoided in practical applications by the choice of a medium length shorter than the collapse distance. On the other hand a saturable nonlinearity can be used as a collapse-arresting mechanism. In this case the beam mass needed for self-trapping increases with the saturation parameter.[34] Under interaction with a CW the self-focusing effect causes the beam characteristics to vary during propagation. However, beam amplitude, width, and transverse wave number evolution under propagation can be well understood and predicted by use of Eqs. (7) and (9) combined with stability considerations. This provides the capability of an appropriate selection of the interaction parameters for effective beam steering as shown in numerical simulations.

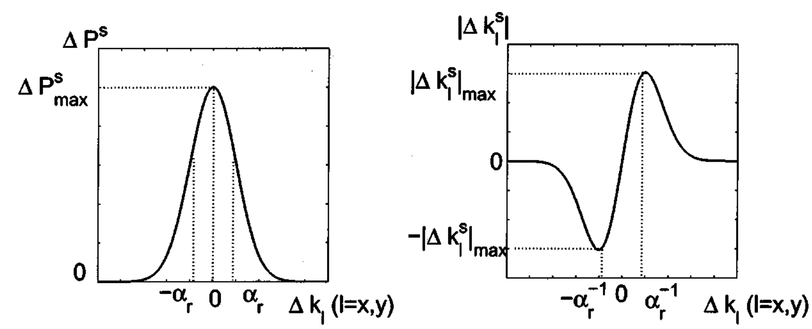
According to Fig. 2(b), for a Gaussian beam with $P=P_{cr}$ interacting with a CW having $\Delta\phi=0$, $\alpha=0.2$, and an optimal choice of $\Delta k_x=a_r^{-1}$, the mass increases in accordance with Eq. (9), resulting in beam collapse after a short propagation distance for a Kerr-type nonlinearity ($s=0$). To prevent beam collapse nonlinearities with nonzero saturation parameter can be used as shown in Figs. 3(a)–3(c). Since the mass needed for beam self-trapping increases with the saturation parameter, the following remark can be made: For a large s the beam mass increase due to the presence of a specific CW may not be sufficiently large for self-trapping, leading to the diffraction of the beam [Fig. 3(a)]; on the other hand, for a small s the beam self-focuses with increasing amplitude [Fig. 3(c)]. In the first case the amplitude of the beam becomes more comparable with the amplitude of the CW, and the transverse velocity of the beam can change significantly, in contraposition with the second case in which the interaction becomes weaker as the beam amplitude increases. However, intermediate values of s can prevent large amplitude variations of the beam and efficient beam steering [Fig. 3(b)]. On the other hand, beam collapse can be avoided if the initial "mass" of the beam is below P_{cr} even for a Kerr-type nonlinearity, which is the case considered in the following.

The strong dependence of beam evolution on the CW parameters is shown for a Gaussian beam having $P=P_{\mathrm{cr}}/2$ in the following figures obtained by direct numerical simulations of the NLSE. In Figs. 4(a)–4(c) the CW has been chosen so that $\Delta\phi=0$, $\Delta k_x=a_r^{-1}$ (optimal choice for effective beam steering), and $\alpha=0.1$, 0.2, 0.3, respectively. According to Eq. (9) the increase of beam mass is proportional to the CW amplitude α, so that, depending on α, the resulting beam mass can lead to diffraction [Fig. 4(a)], quasistable propagation [Fig. 4(b)], or self-focusing [Figure 4(c)]. In the third case it is shown that the increase in beam amplitude and the corresponding decrease in beam width reduces the efficiency of transversal steering after a short propagation distance.

The effect of a nonzero initial phase difference is shown in Figs. 5(a) and 5(b) for $\Delta\phi=\pi/2$ and π, respectively, while the rest of the parameters are the same as in the case of Fig. 4(b). For $\Delta\phi=\pi/2$, the mass of the beam does not increase significantly, in agreement with Eq. (9), so that the beam continuously diffracts, while the transverse wave number does not change significantly, in agreement with Eq. (7). A more radical evolution occurs for $\Delta\phi=\pi$ since, according to Eq. (9), the presence of the CW actually decreases the mass of the beam, resulting in drastic beam diffraction and the formation of a secondary beam that is fixed in the transverse dimension.

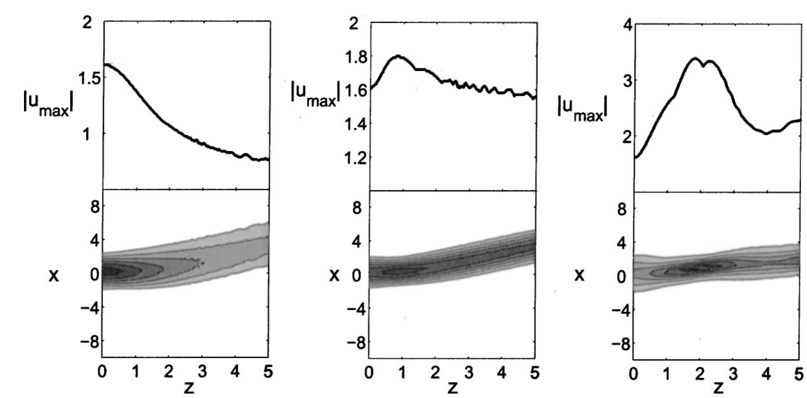
The dependence of the interaction of the beam with the CW on the tranverse wave number difference Δk_x is shown to be critical for both the stability and the capability of effective beam steering. In Figs. 6(a) and 6(b) the parameters of the beam and the CW are the same as with Fig. 4(b) except that $\Delta k_x=0$ and 1, respectively.

According to Fig. 2(a) the presence of a CW with the same transverse wave number as the beam is shown to



(a) (b)

Fig. 2. Gaussian beam's variation of (a) mass (ΔP^s) and (b) transverse wave number (Δk_l^s, $l=x,y$) due to the the presence of a CW with transverse wave number difference $\Delta k_l (l=x,y)$.



(a) (b) (c)

Fig. 3. Evolution of a Gaussian beam with $A=\sqrt{2}$, $a_r^2=2(P=P_{\mathrm{cr}})$ in the presence of a CW with $\alpha=0.2$, $\Delta k_y=0$, $\Delta k_x=a_r^{-1}$, and saturation parameter $s=$ (a), 1 (b), 0.5 (c) 0.25.

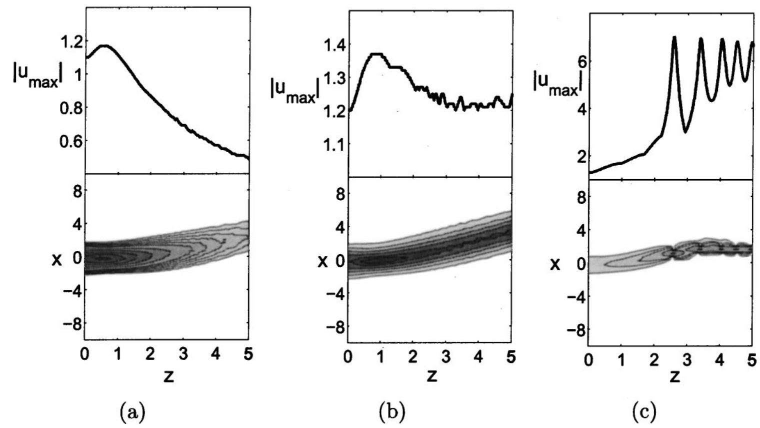


Fig. 4. Evolution of a Gaussian beam with $A=1$, $a_r^{\ 2}=2$ $(P=P_{\mathrm{cr}}/2)$ in the presence of a CW with $\Delta\phi=0$, $\Delta k_y=0$, $\Delta k_x=a_r^{\ -1}$ and $\alpha=$ (a), 0.1 (b), 0.2 (c) 0.3. The saturation parameter is $s=0$ (Kerr-type nonlinearity).

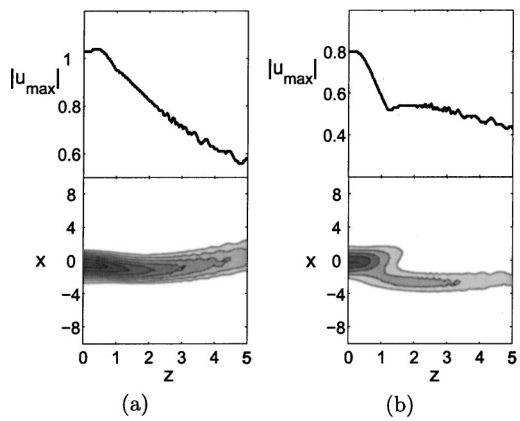


Fig. 5. Evolution of a Gaussian beam with $A=1$, $a_r^{\ 2}=2$ $(P=P_{\mathrm{cr}}/2)$ in the presence of a CW with $\alpha=0.2$, $\Delta k_y=0$, $\Delta k_x=a_r^{-1}$, and $\Delta\phi=$ (a), $\pi/2$ (b) π. The saturation parameter is $s=0$ (Kerr-type nonlinearity).

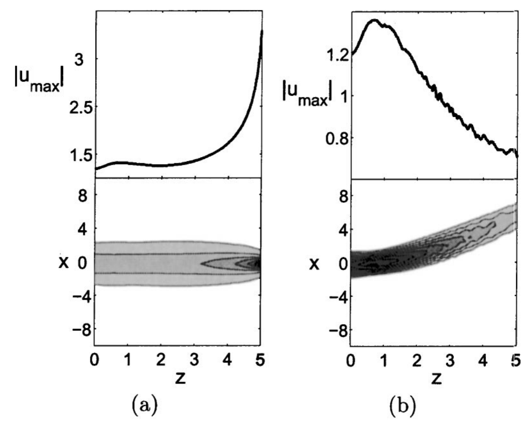


Fig. 6. Evolution of a Gaussian beam with $A=1$, $a_r^{\ 2}=2$ $(P=P_{\mathrm{cr}}/2)$ in the presence of a CW with $\alpha=0.2$, $\Delta\phi=0$, $\Delta k_y=0$, and $\Delta k_x=$ (a), 0 (b) 1. The saturation parameter is $s=0$ (Kerr-type nonlinearity).

result in a significant increase in the beam's mass and in continuous self-focusing, while the transverse wave number of the beam remains constant according to Fig. 2(b). Additionally a transverse wave number difference that is larger than the inverse characteristic initial size of the beam results in beam diffraction, since the presence of the CW does not increase significantly the mass of the beam [in accordance with Fig. 2(a)] so that it can be self-focused. However, as the beam diffracts the transverse wave number can change significantly. Comparing Figs. 6(a) and 6(b) with Fig. 4(b) we can conclude that effective beam steering with necessary stability of the beam can be achieved for a transverse wave number difference in the vicinity of the maximum shown in Fig. 2(b).

4. CONCLUSIONS

Interactions of a solitary wave of the multidimensional NLSE with continuous waves were investigated. The dependence of these interactions on specific parameters such as the mass of the SW, the amplitude of the CW, and the initial phase and wave number difference between the two waves was studied. Among the features of the interactions, the variation of the transverse wave number of the SW was studied for optimal parameter selection to achieve effective beam steering without significant deterioration of beam shape due to diffractive decay or self-focusing collapse for finite propagation distances that are interesting from the point of view of potential applications in optical devices. Moreover, apart from considering the CW as an intentionally injected control signal, several situations are for see able in which a residual CW due to a previous stage of optical signal transmission or processing may occur in realistic situations.

An analytical approach based only on two conserved quantities of the wave, namely, mass and momentum, has been applied. It was shown that an initial difference between the transverse wave number of the beam and the CW can lead to a variation of the transverse wave number (momentum) of the solitary beam along the same direction as the aforementioned difference, while the mass of the wave also changes due to the presence of the CW. Although the approach applies to any number of transverse dimensions and type of nonlinearity, the case of Gaussian beams in a 2D bulk medium with Kerr-type or saturable nonlinearity was studied. More important, the critical dependence of the beam's evolution on the characteristics of the CW was predicted in terms of analytical relations. The latter, combined with widely known stability considerations of the 2D NLSE, are shown to be capable of explaining and predicting the evolution of a radially symmetric beam as obtained by direct simulations. In the context of nonlinear optics, the capability of 2D spatial beam steering with the injection of an appropriate CW is very promising for potential applications in all-optical signal control. Moreover, considering the two transverse dimensions as spatial and temporal the aforementioned results can also be applied to controlled space and time steering (and corresponding frequency conversion) in planar geometries. Furthermore, in the three-dimensional case, a "light bullet" can be controlled in both 2D space and time. Finally, since the multidimensional NLSE is a universal model for wave propagation in the presence of nonlinearity and dispersion and-or diffraction, interesting applications in other branches of physics are expected. Extensions to elliptic Gaussian beams as well as to more general beam profiles can also be considered. This is a subject of current and future investigation.

ACKNOWLEDGMENTS

This work is supported by the grants "Herakleitos" and "Pythagoras" of the Hellenic Ministry of Education as well as by the Hellenic Association for Controlled Thermonuclear Fusion within Euratom.

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