4.Seismic Ground Response

or

soil "amplification" of seismic ground motions

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Professor N.T.U.A..

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Suggested Reading:

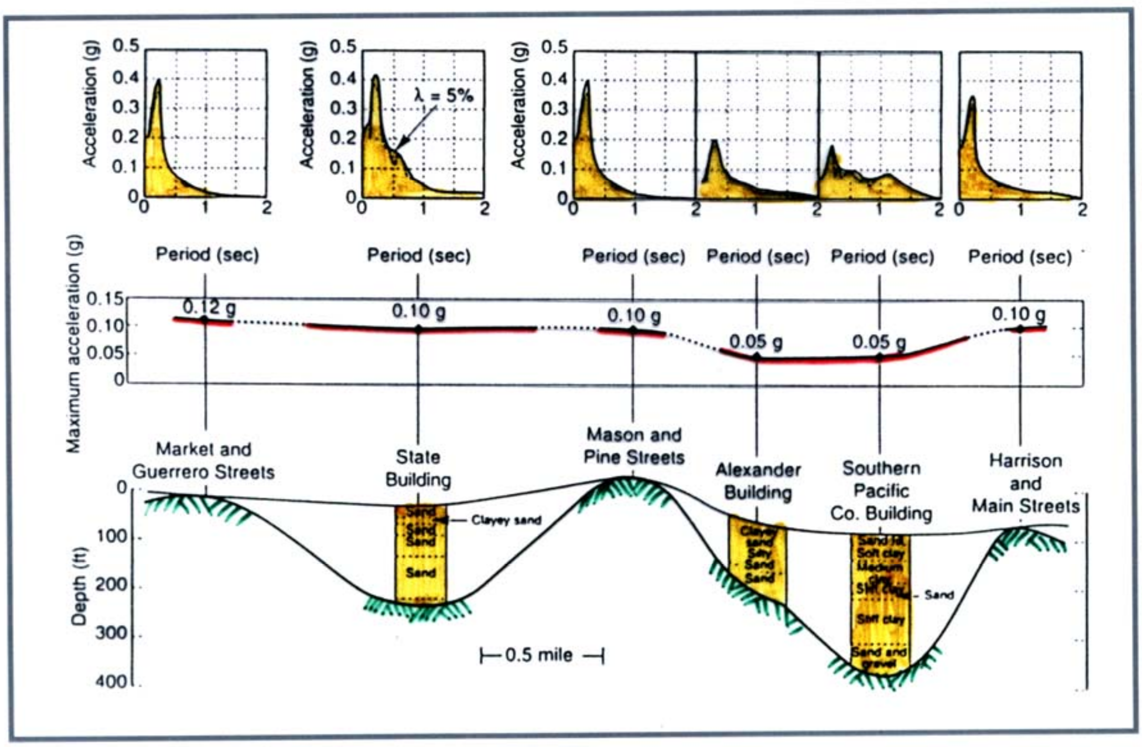
- **Steven Kramer:** Chapter 7 (7.1 & 7.2)
- **George Gazetas:** Chapter 4 (4.2)
- **SHAKE '92** – Users' Manual

4.1 EXAMPLES FROM (REAL) CASE HISTORIES

The examples which follow come from real seismic events and may help fro a first qualitative as well as quantitative evaluation of ***soil effects on recorded seismic ground motions****. Thus, keep notes with regard to the:*

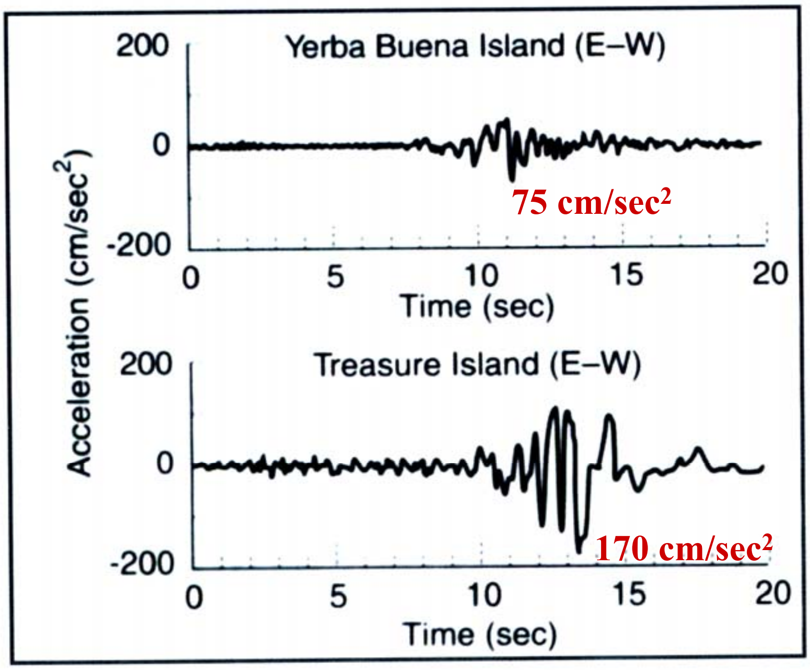
A. Soil "amplification" coefficient
B. Important soil and seismic motion parameters
C. Frequency dependence of soil amplification effects

San Francisco 1957 . . . .



Variation of peak ground acceleration and elastic response spectra in connection with local soil conditions at recording stations . . .

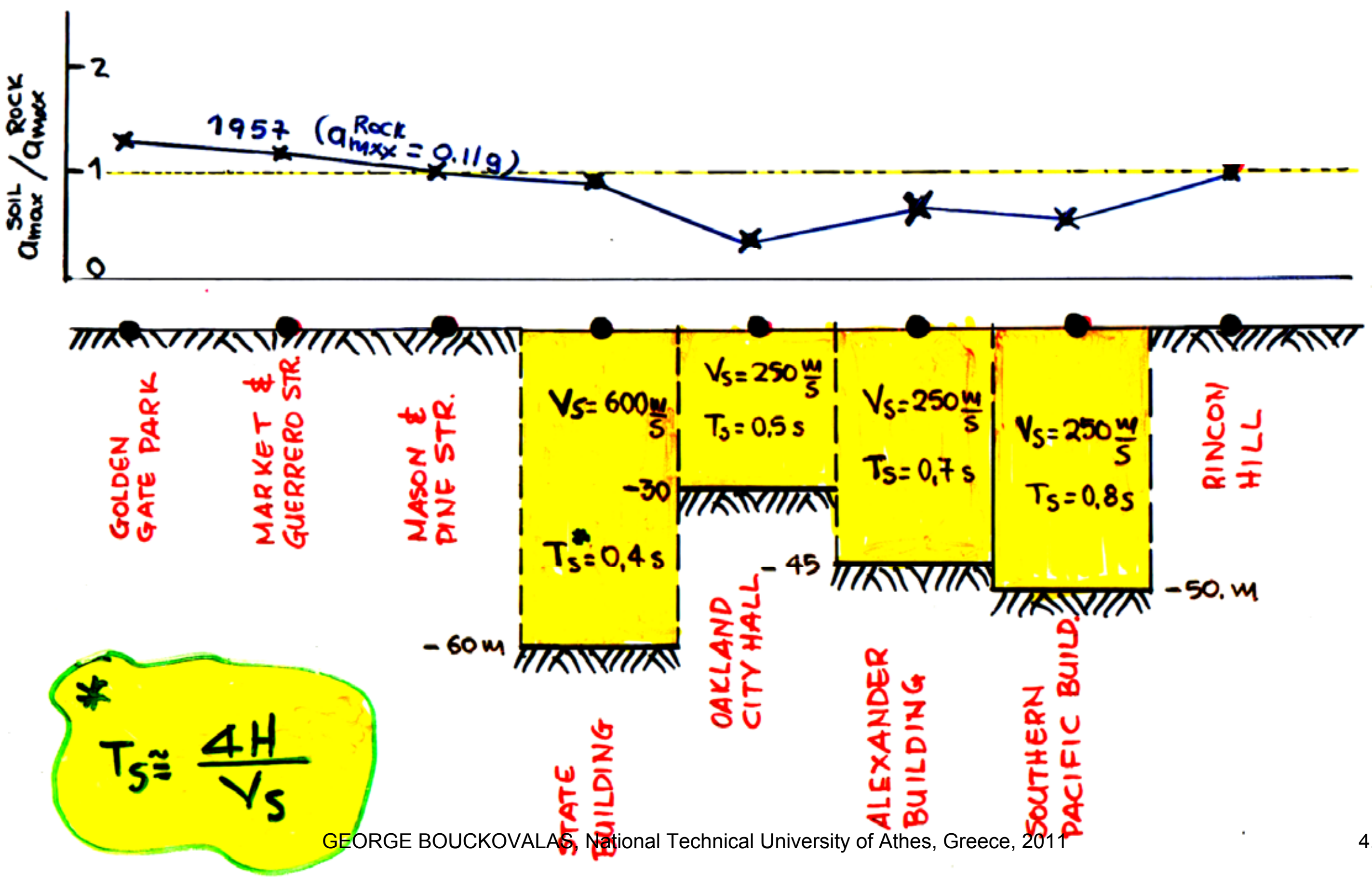
San Francisco 1989 . . . .



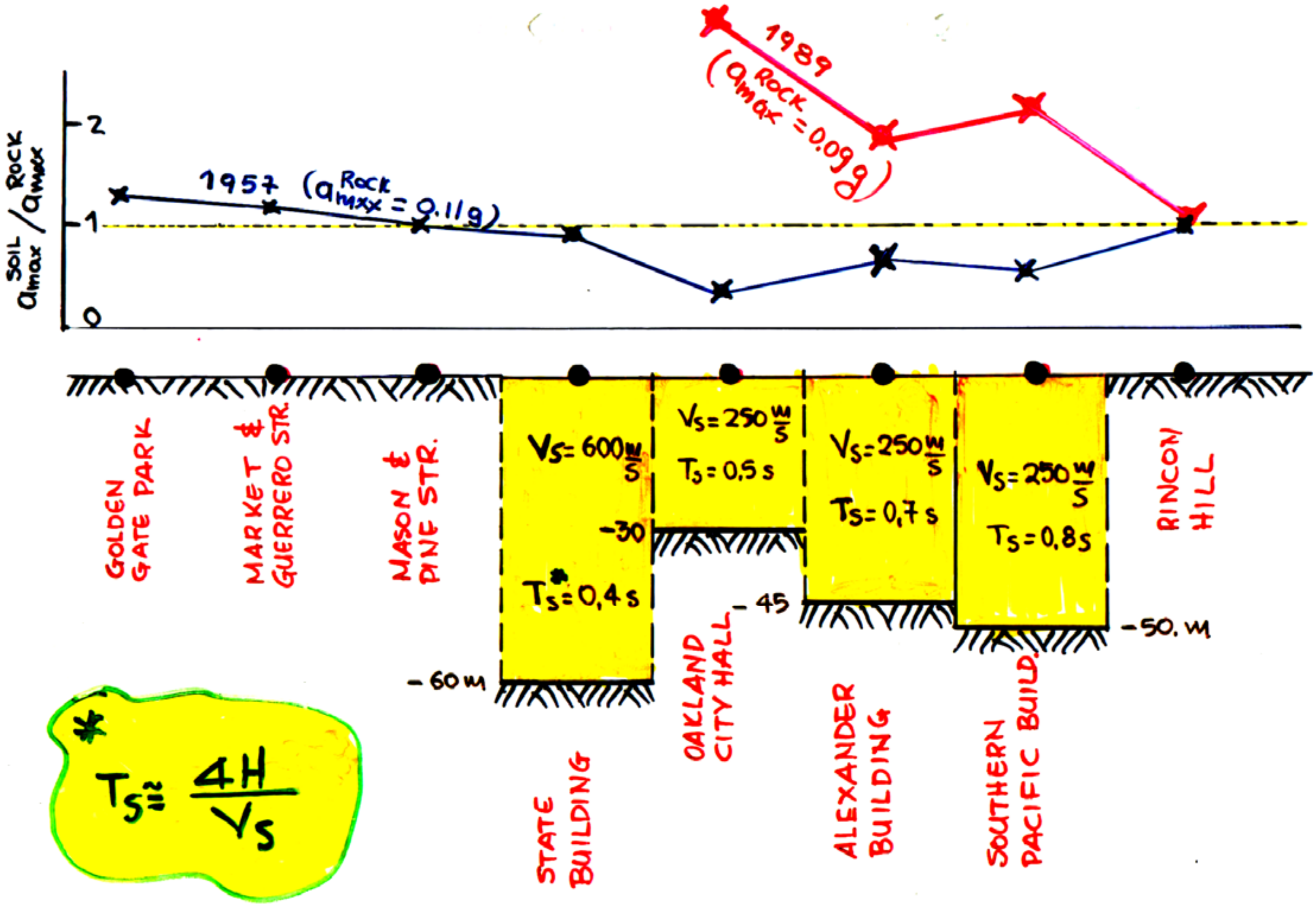
Recording at
outcropping ROCK

Recording at the surface of **earth fill on Bay mud**

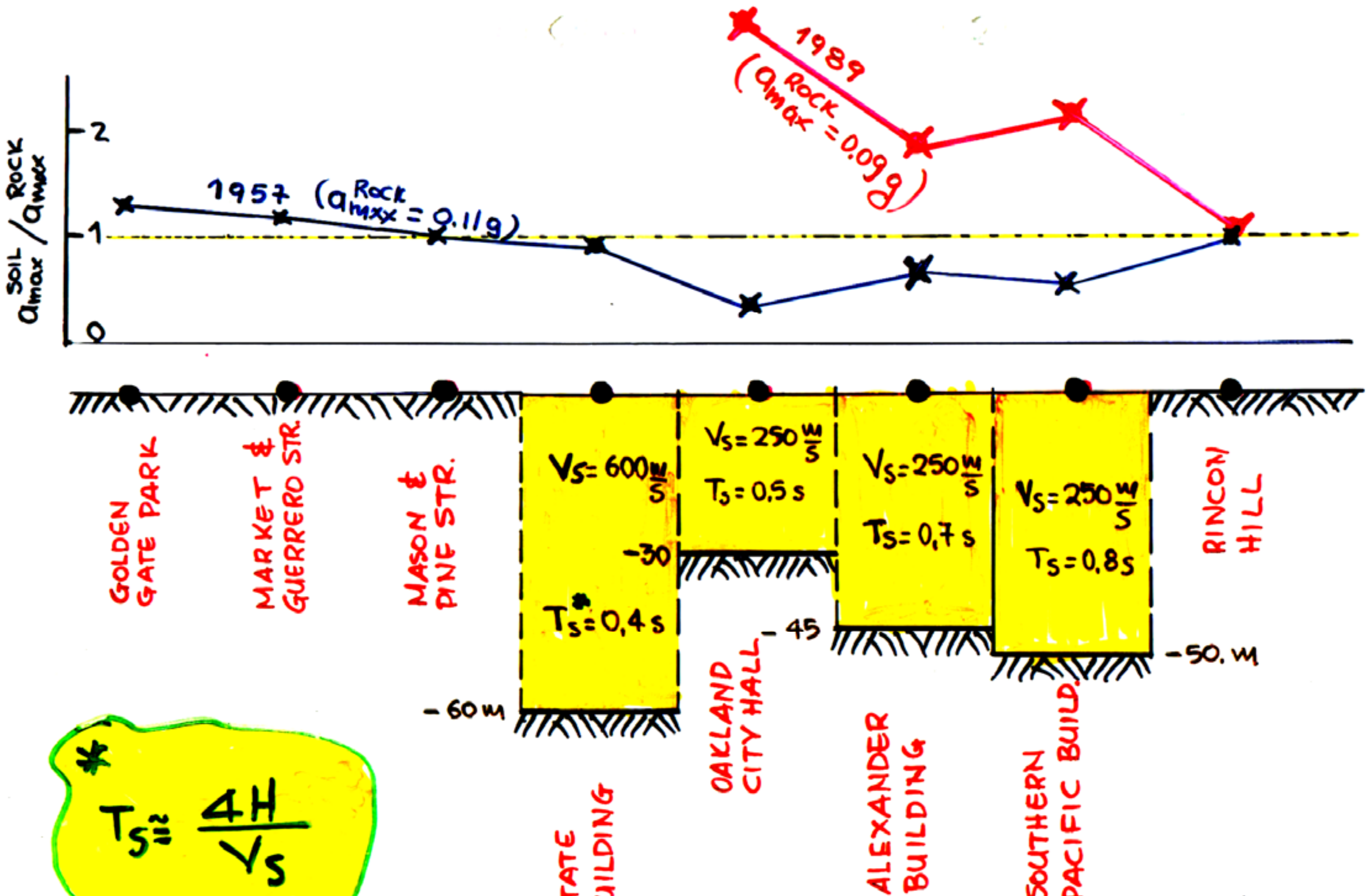
Soil "amplification" or "de-amplification" ?



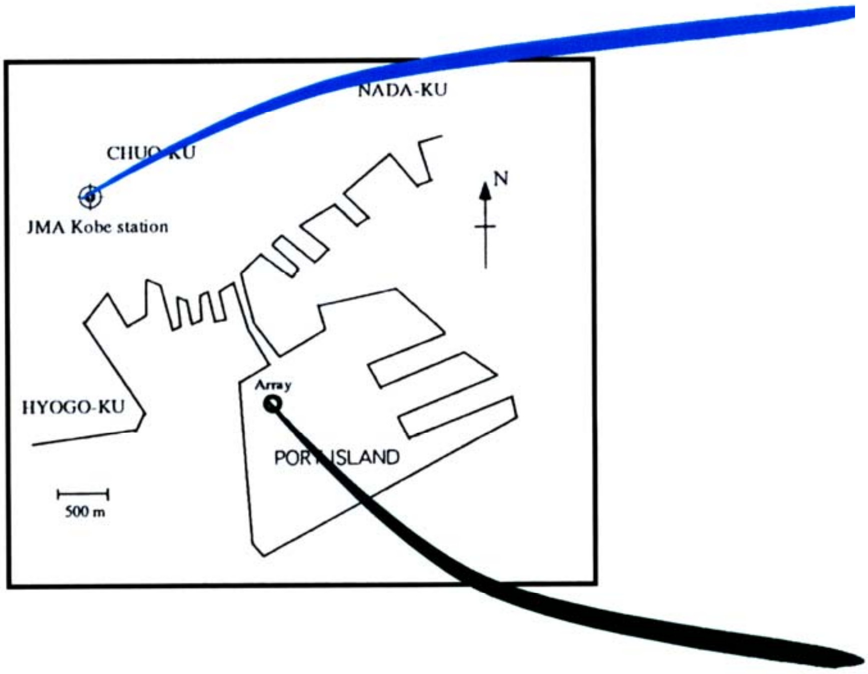
Soil “amplification” or “de-amplification” ?



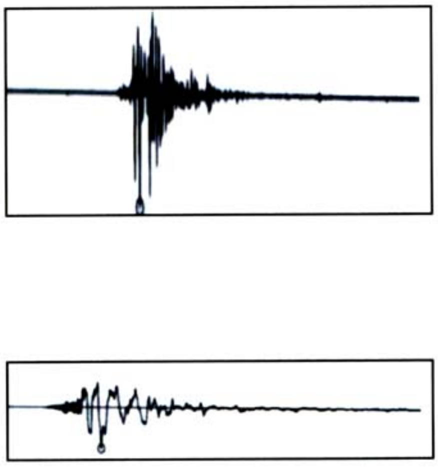
Is this a pure soil effect?



Kobe, Japan 1995

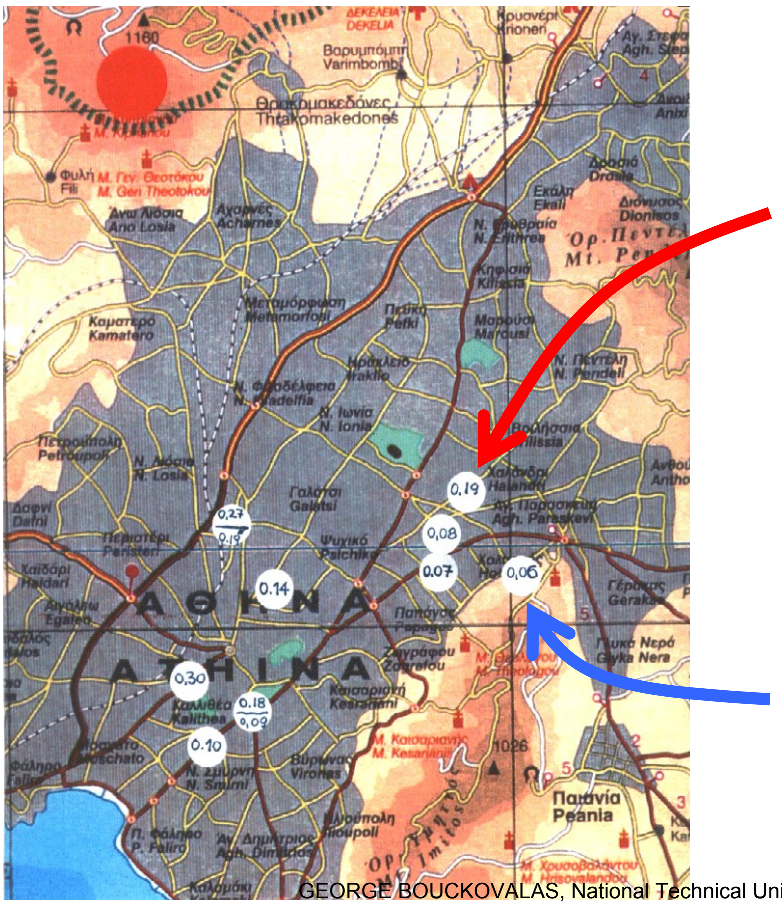


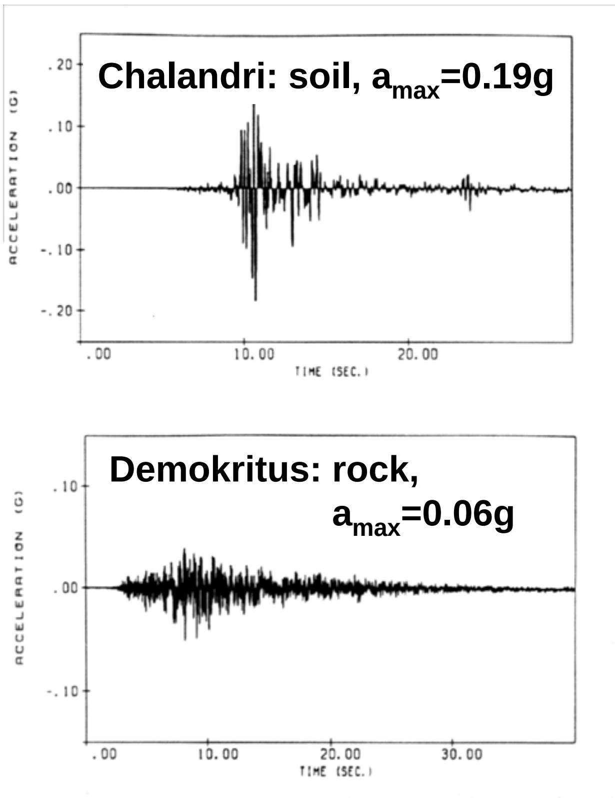
stiff soil, a_{max}=0.82g



loose fill,
a_{max}=0.32g

Athens, Greece 1999 . . . .





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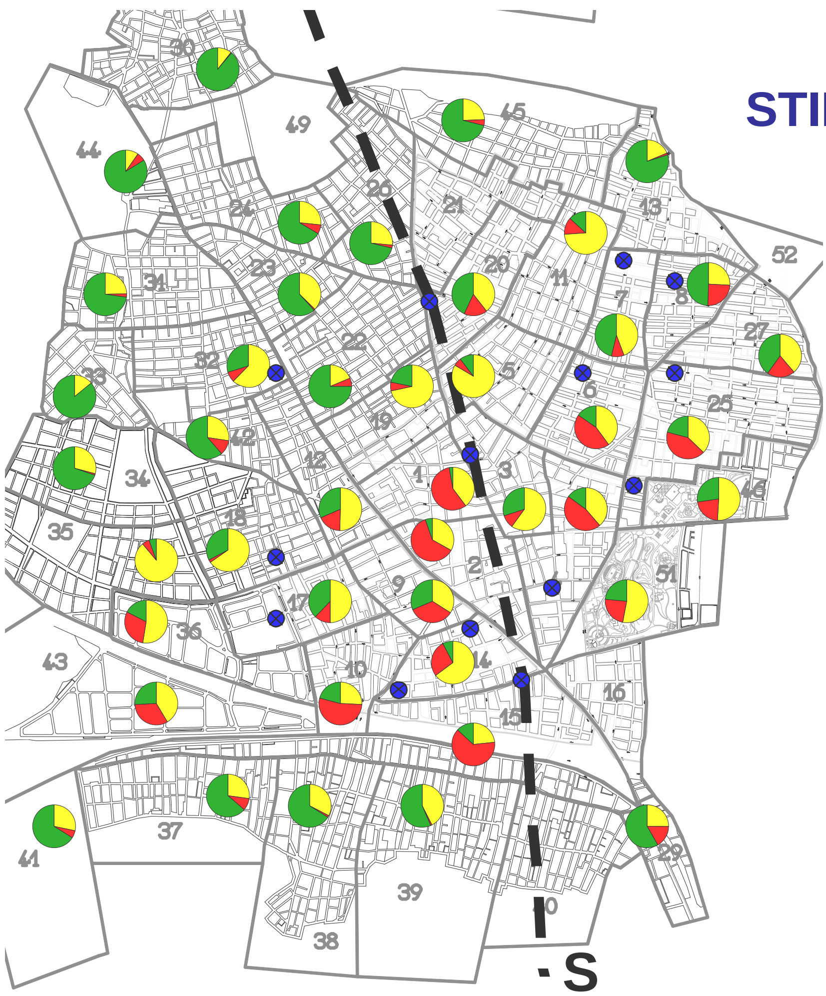
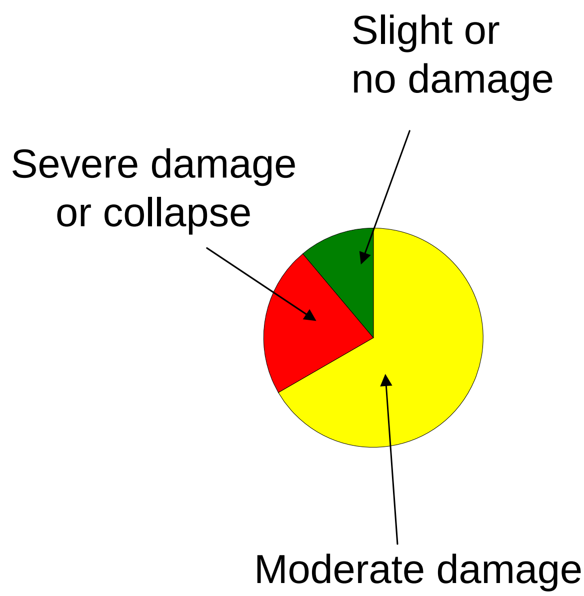
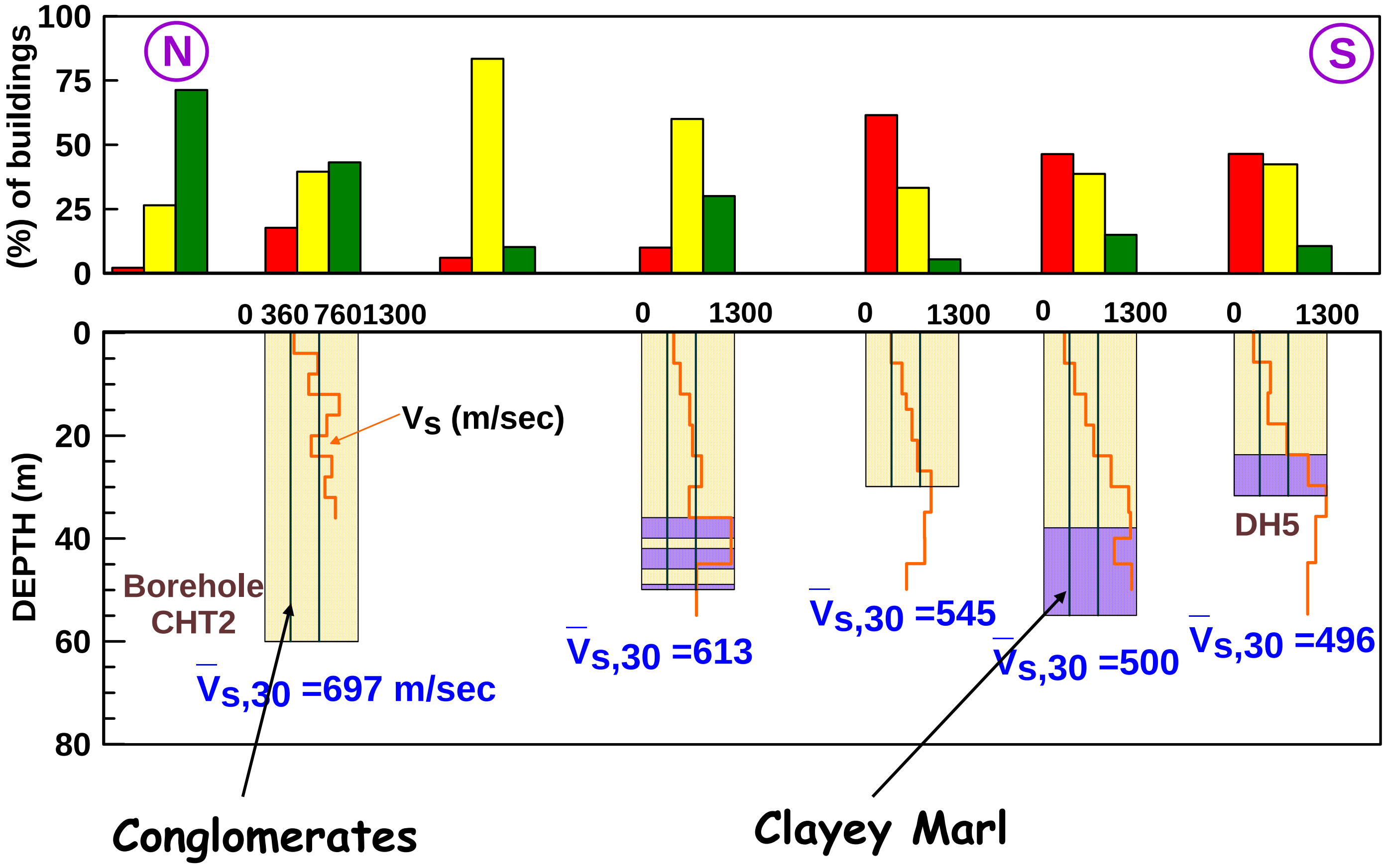


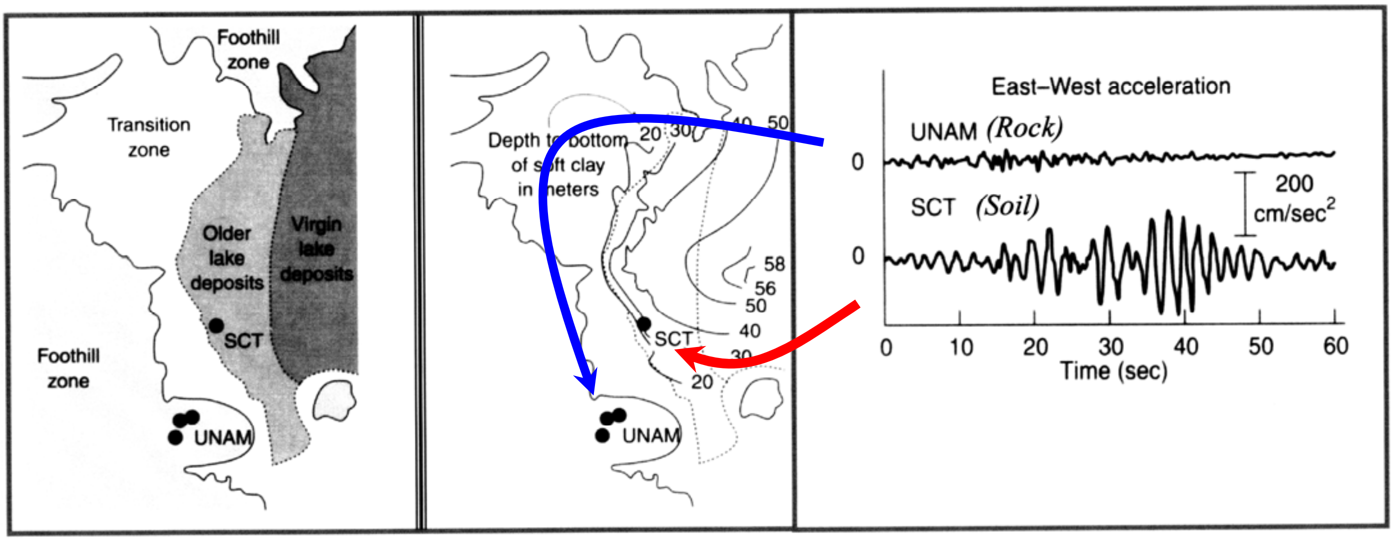
STIFF SOIL AMPLIFICATION at Ano Liosia municipality



Cross-section N-S



Mexico City 1985

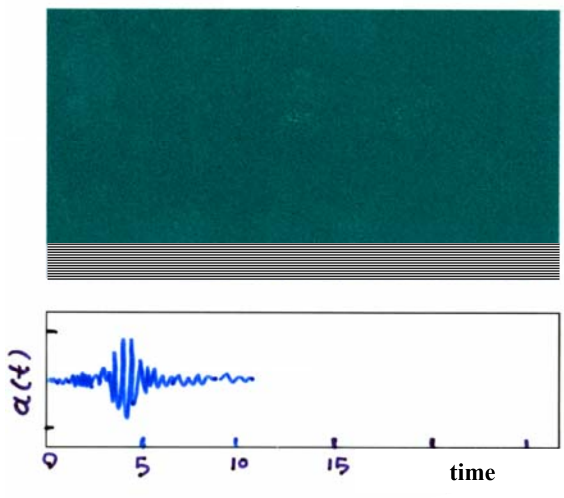


geology

thickness of soil deposits

Typical recordings on the surface of SOIL & ROCK

Simplified mechanism of SOIL effects



Initial Conclusions :

A. **Soil "amplification" coefficient:** (0.40-3.20) 0.6-2.0

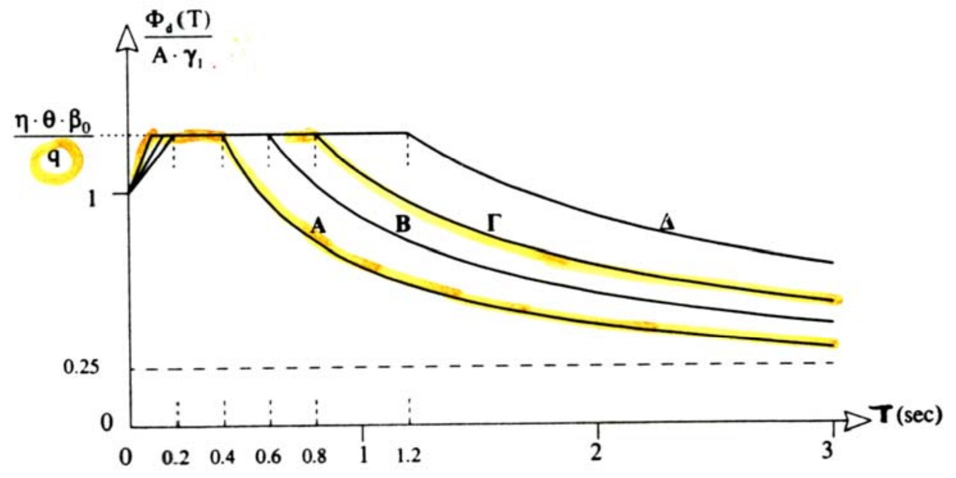
B. **Important soil and seismic motion parameters:**
soil & excitation characteristics

C. **Frequency dependence of soil amplification effects:**
a_{max} & Sa

Quite often,

the effect of a few (tens of) meters of soft soil
is larger and more significant than
than the effect a few kilometers of earth crust….

Seismic Codes: EAK-2000 ………

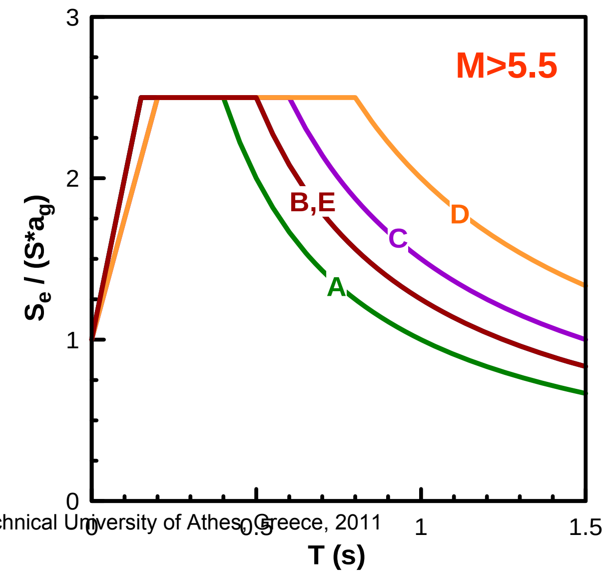


π.χ.
Κατηγορία Β: Εντόνως αποσαθρωμένα βραχώδη ή εδάφη που από μηχανική άποψη μπορούν να εξομοιωθούν με κοκκώδη. Στρώσεις κοκκώδους υλικού μέσης πυκνότητας (και) πάχους μεγαλύτερου των 5μ, ή μεγάλης πυκνότητας (και) πάχους μεγαλύτερου των 70μ.

Seismic Codes: EC-8 ………

	Description	Soil Parameters
A		
B		
C	Deep deposits of dense or medium-dense sand, gravel or stiff clay with thickness from several tens to many hundreds of m	$V_{s,30}$=180-360m/s N_{SPT}=15-50 C_u=70-250KPa
D		
E		

	S		T_B		T_C	
	M>5,5	M<5,5	M>5,5	M<5,5	M>5,5	M<5,5
A	1.0	1.0	0.15	0.05	0.4	0.25
B	1.2	1.35	0.15	0.05	0.5	0.25
C	1.15	1.5	0.20	0.10	0.6	0.25
D	1.35	1.8	0.20	0.10	0.8	0.30
E	1.4	1.6	0.15	0.05	0.5	0.25



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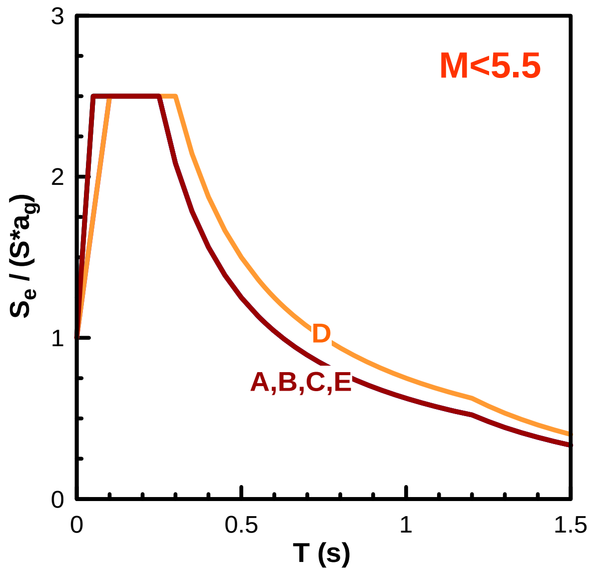


In principle, analysis-prediction of soil effects is possible with one of the three following methods (with greatly different complexity, as well as, accuracy):

accuracy ↓

I. Analytical (harmonic excitation, uniform & elastic soil, ..)

II. Empirical (based on geological and seismological input)

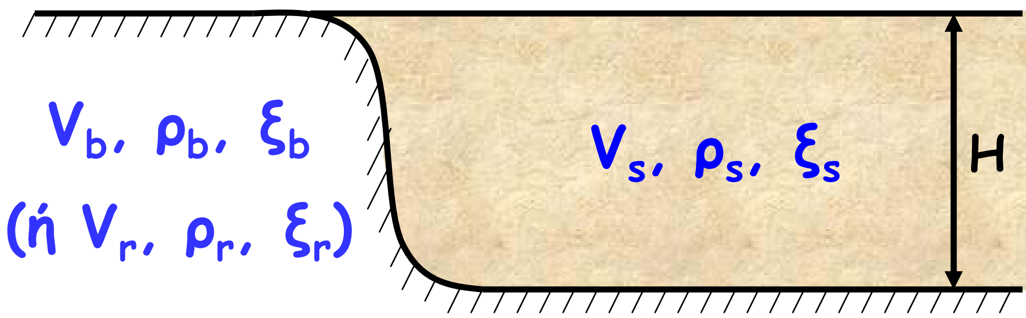
III. Numerical (SHAKE, DESRA, …)

simplicity ↑

4.3 ANALYTICAL METHODS

- Uniform elastic soil on rigid bedrock
- Uniform visco-elastic soil on rigid bedrock
- Uniform visco-elastic soil on flexible bedrock
- Non-uniform visco-elastic soil on flexible bedrock

Elastic soil no RIGID bedrock

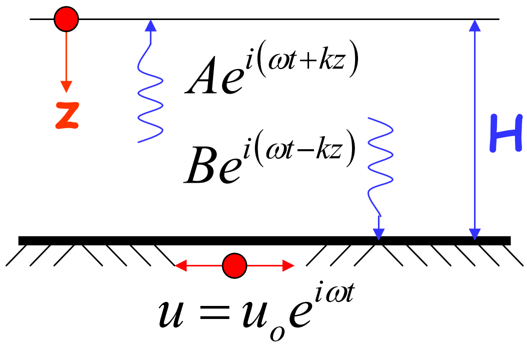


Definitions :

$$V = \sqrt{\frac{G}{\rho}}$$

$$T_s = \frac{4H}{V_s}, T_b \text{ ή } T_r = \frac{4H}{V_b}$$

$$k = \frac{\omega}{V} \quad (wave\, number)$$



$$u = Ae^{i(\omega t+kz)} + Be^{i(\omega t-kz)}$$

Boundary condition : $\tau_{z=0} = 0$

$$G\left.\frac{\partial u}{\partial z}\right|_{z=0} = 0 \Rightarrow Gik\left[Ae^{i(\omega t+kz)} + Be^{i(\omega t-kz)}\right] = 0$$

$$\text{for } z=0: \quad Gik(A-B)e^{i\omega t} = 0 \Rightarrow A = B$$

$$\text{thus}: u = 2Ae^{i\omega t}\frac{e^{ikz}+e^{-ikz}}{2} = 2Ae^{i\omega t}\cos kz$$

Boundary condition : $u_{z=H} = u_o e^{i\omega t}$

$$\text{thus}: \quad u_o = 2A\cos kH \;\; \acute{\eta} \;\; A = \frac{u_o}{2\cos kH}$$

stationary wave with amplitude

$$\overline{u}_o = 2A\cos kz = u_o\frac{\cos kz}{\cos kH}$$

Amplification factor:

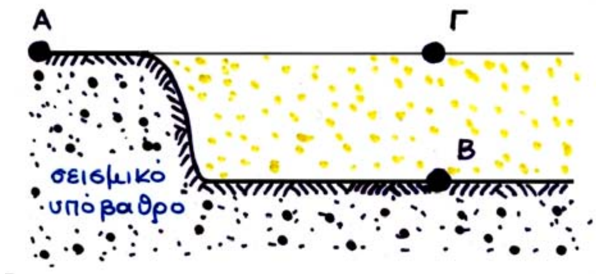
$$F_1(\omega) = \frac{\kappa\acute{\iota}\nu\eta\sigma\eta\ \Gamma}{\kappa\acute{\iota}\nu\eta\sigma\eta\ \mathrm{B}} = \frac{1}{\cos kH} = \cdots \frac{1}{\cos\left(\dfrac{\pi}{2}\dfrac{T_{soil}}{T_{exc.}}\right)}$$

Resonance:

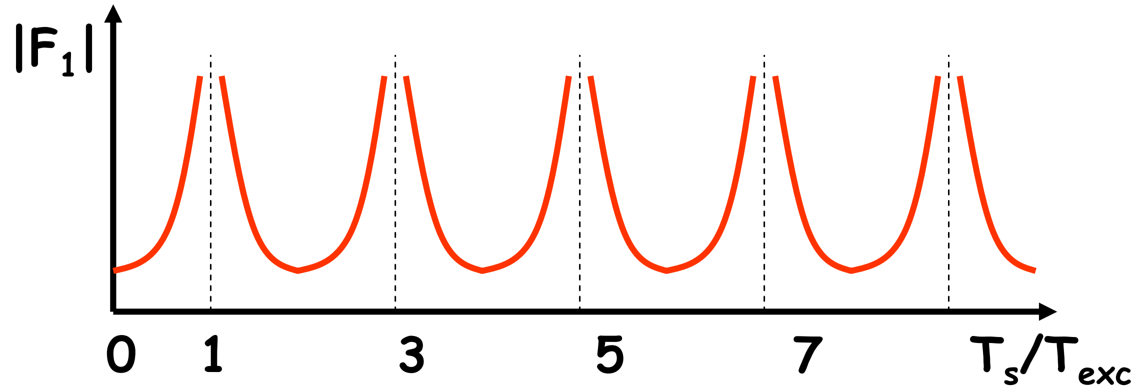
$$T_{soil}/T_{exc} = 2n-1, \quad n = 1,2,\ldots$$

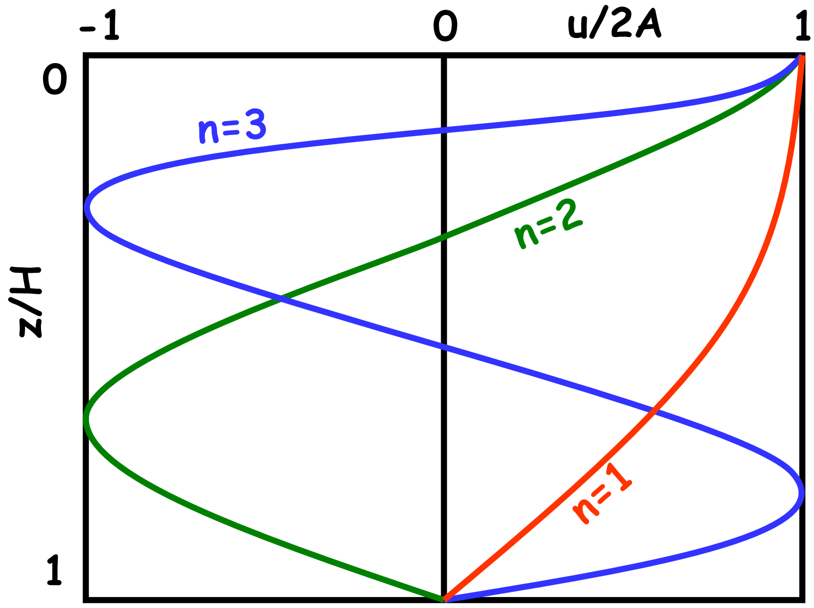
Modal shapes:

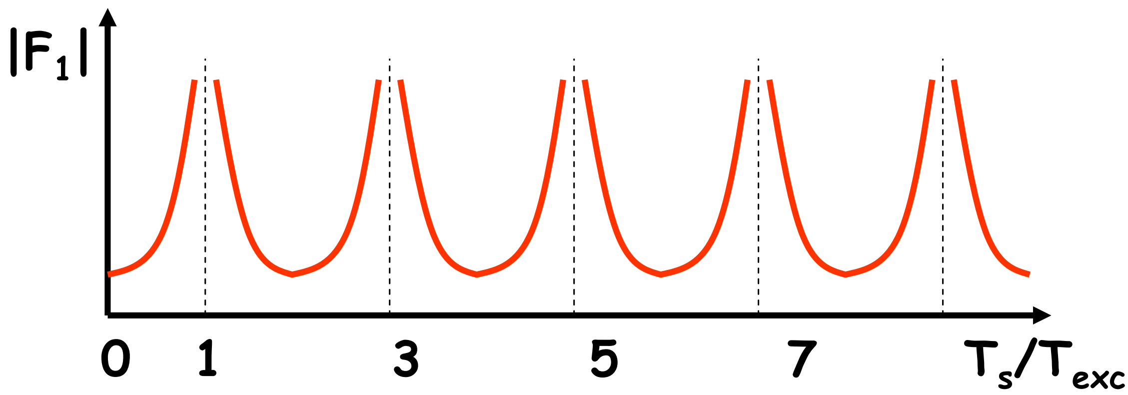
$$\frac{u}{2A} = \cos\left[\frac{\pi}{2}(2n-1)\frac{z}{H}\right]$$

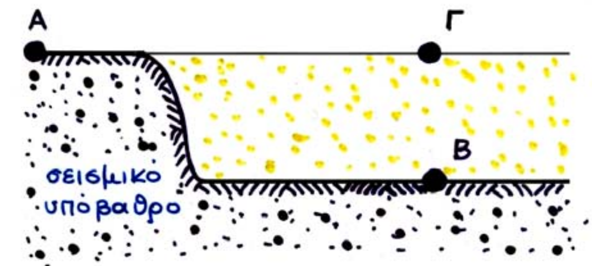


$$\frac{u}{2A} = \begin{cases} \cos\dfrac{\pi}{2}\dfrac{z}{H}, & n=1 \\ \cos\dfrac{3\pi}{2}\dfrac{z}{H}, & n=2 \\ \cos\dfrac{5\pi}{2}\dfrac{z}{H}, & n=3 \end{cases}$$





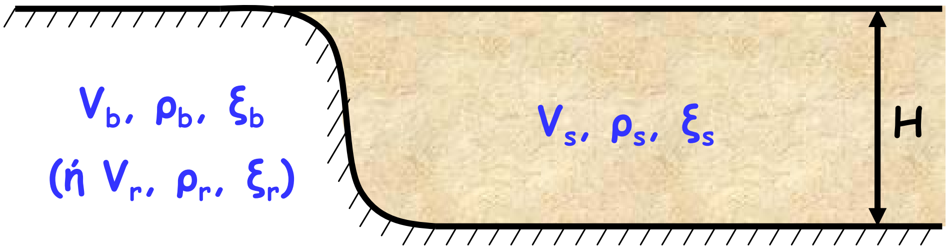




Due to the RIGID bedrock assumption
(motion A) = (motion B)
And consequently

$$F_1(\omega)=\frac{motion\quad \Gamma}{κίνηση\quad B}=\frac{motion\quad \Gamma}{motion\quad A}$$

Visco-elastic soil on RIGID bedrock



Definitions for elastic soil (ξ=0)

$$V = \sqrt{\frac{G}{\rho}}$$

$$T_s = \frac{4H}{V_s}, T_b \ ή \ T_r = \frac{4H}{V_b}$$

$$k = \frac{\omega}{V} \quad (wave\,number)$$

Definitions for visco-elastic soil (ξ≠0)

$$G^* = G(1+2i\xi)$$

$$V^* = \sqrt{\frac{G^*}{\rho}} \approx V(1+i\xi) \quad \gamma\iota\alpha\ \xi \le 0,30$$

$$T^* = \frac{4H}{V^*} \approx \frac{4H}{V}(1-i\xi) = T(1-i\xi)$$

$$k^* = \frac{\omega}{V^*} \approx \frac{\omega}{V}(1-i\xi) = k(1-i\xi)$$

According to the "correspondence principle", you may use the solution for elastic soil, with the real soil variables replaced by the corresponding complex variables. Thus,

Amplification factor

$$F_2(\omega) = \frac{1}{\cos k^* H} \cong \frac{1}{\cos[kH(1-i\xi)]} = \frac{1}{\cos(kH - i\xi kH)}$$

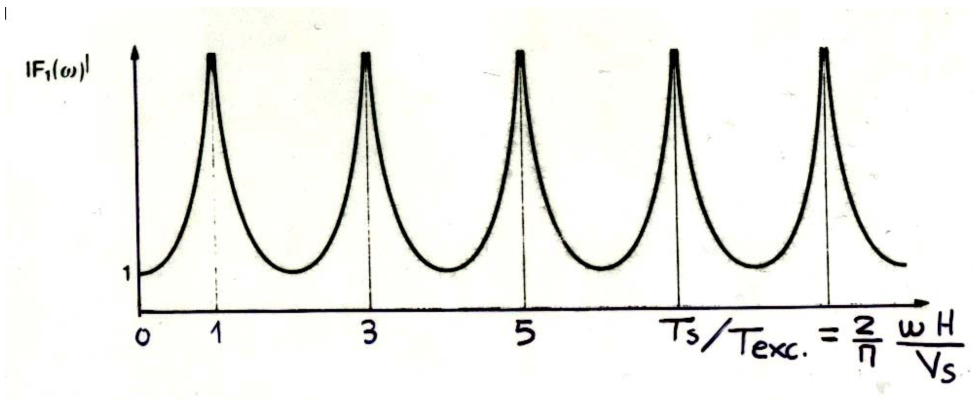
or, given that: $\left|\cos(x \pm iy)\right| = \sqrt{\cos^2 x + \sinh^2 y} \approx \sqrt{\cos^2 x + y^2} \quad (y \to 0)$

$$\left|F_2(\omega)\right| \approx \frac{1}{\sqrt{\cos^2 kH + (kH\xi)^2}} \quad where\ \kappa = \omega / V_s$$

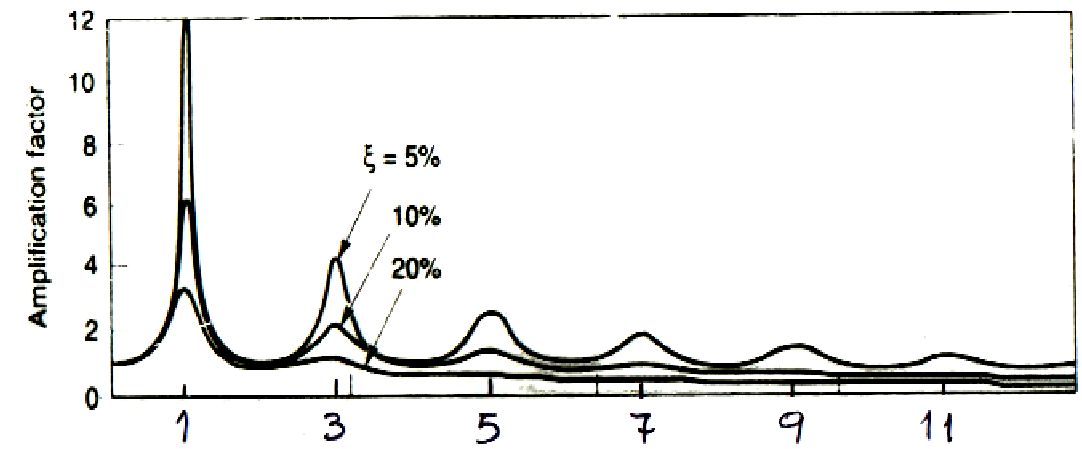
$$\max\left|F_2(\omega)\right| = \frac{2}{(2n-1)\pi}\frac{1}{\xi}$$

when

$$kH \approx (2n-1)\frac{\pi}{2} \quad or \quad \frac{T_{soil}}{T_{exc}} = 2n-1$$

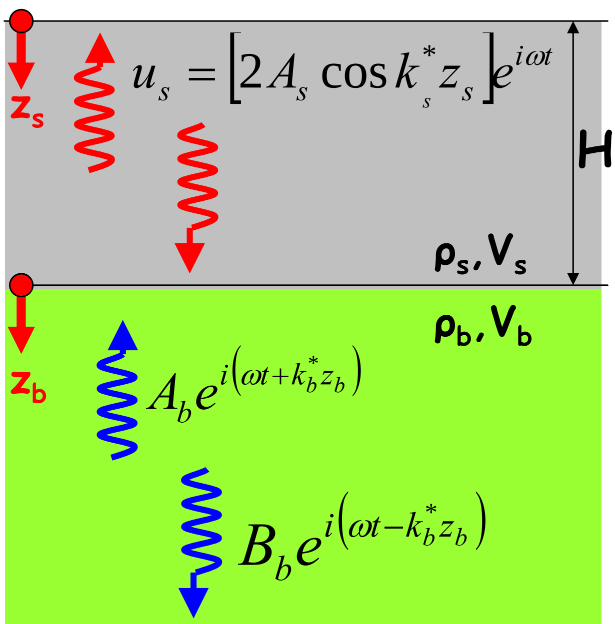


Elastic soil on RIGID bedrock



Visco-elastic soil on RIGID bedrock

Uniform visco-elastic soil on FLEXIBLE bedrock



$$u_s = \left[2A_s \cos k_s^* z_s\right] e^{i\omega t}$$

$$A_b e^{i\left(\omega t + k_b^* z_b\right)}$$

$$B_b e^{i\left(\omega t - k_b^* z_b\right)}$$

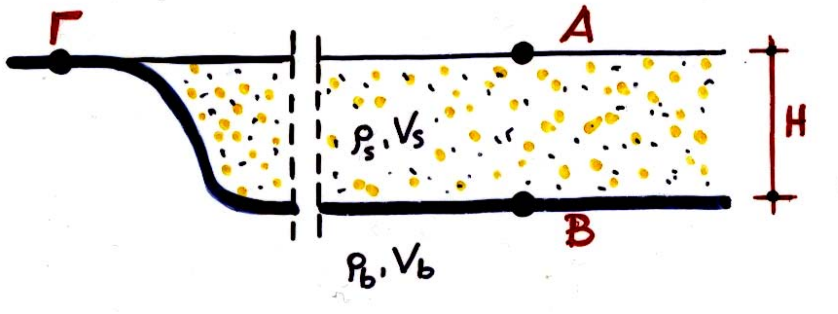
Boundary conditions:

$$z_s = 0 : \tau = G_s \frac{\partial u_s}{\partial z_s} = 0 \rightarrow \cdots u_s = \left(2A_s \cos k_s^* z_s\right) e^{i\omega t}$$

$$z_b = 0,\ z_s = H : u_s(H) = u_b(0) \ \&\ G_s \frac{\partial u_s}{\partial z_s} = G_b \frac{\partial u_b}{\partial z_b}$$

$$\Rightarrow \begin{cases} A_b = \frac{1}{2} A_s \left[\left(1 + a^*\right) e^{ik_s^* H} + \left(1 - a^*\right) e^{-ik_s^* H}\right] \\ B_b = \frac{1}{2} A_s \left[\left(1 - a^*\right) e^{ik_s^* H} + \left(1 + a^*\right) e^{-ik_s^* H}\right] \end{cases}$$

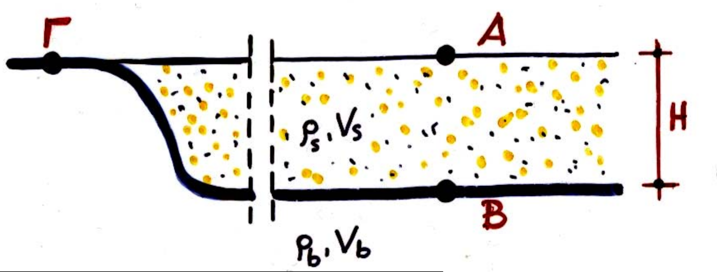
$$a^* = \frac{\rho_s V_s^*}{\rho_b V_b^*} = \frac{\rho_s V_s}{\rho_b V_b} \frac{1 + i\xi_s}{1 + i\xi_b} = a \frac{1 + i\xi_s}{1 + i\xi_b}$$



$$F_3(\omega) = \frac{u_A}{u_B} = \frac{u_s(0)}{u_b(0)} = \frac{2A_s}{A_b + B_b} \Rightarrow \cdots$$

$$\left|F_3(\omega)\right| = \frac{1}{\left|\cos\left(k_s^* H\right)\right|} \approx \frac{1}{\left[\cos^2\left(k_s H\right) + \left(\xi_s k_s H\right)^2\right]^{1/2}} = \left|F_2(\omega)\right|$$

However, in practice it is more important to know the ratio U_A/U_Γ:



$$F_4(\omega) = \frac{u_A}{u_\Gamma} = \frac{2A_s}{2A_b} \Rightarrow \cdots$$

$$\left|F_4(\omega)\right| = \frac{1}{\left|\cos\left(k_s^* H\right) + i a_s^* \sin\left(k_s^* H\right)\right|} \approx \frac{1}{\left[\cos^2\left(k_s H\right) + \left(\bar{\xi}_s k_s H\right)^2\right]^{1/2}}$$

όπου

$$\bar{\xi}_s = \xi_s + \frac{2}{\pi} a_s \frac{T_{exc.}}{T_s}$$

$$a_s = \frac{\rho_s V_s}{\rho_b V_b}$$

για άκαμπτο βράχο, $V_b \rightarrow \infty$, $a_s \rightarrow 0$, $\bar{\xi}_s \rightarrow \xi_s$, έτσι:

$$\left|F_4(\omega)\right| = \frac{1}{\left[\cos^2\left(k_s H\right) + \left(\bar{\xi}_s k_s H\right)^2\right]^{1/2}} = \left|F_3(\omega)\right|$$

Important soil and excitation parameters affecting soil amplification . . .

$$|F_4(\omega)| = \frac{1}{\left|cos(k_s^* H) + i a_s^* \, sin(k_s^* H)\right|} \approx \frac{1}{\left[cos^2(k_s H) + (\bar{\xi}_s k_s H)^2\right]^{1/2}}$$

where :

$$\bar{\xi}_s = \xi_s + \frac{2}{\pi} a_s \frac{T_{exc.}}{T_s}, \qquad a_s = \frac{\rho_s V_s}{\rho_b V_b}$$

Problem parameters:

$$k_s H = \frac{\omega H}{V_s} = \frac{\pi}{2} \frac{T_s}{T_{exc}}$$

$$\bar{\xi}_s = \xi_s + \frac{2}{\pi} a_s \left(\frac{T_s}{T_{exc}}\right)^{-1}$$

T_s / T_{exc}

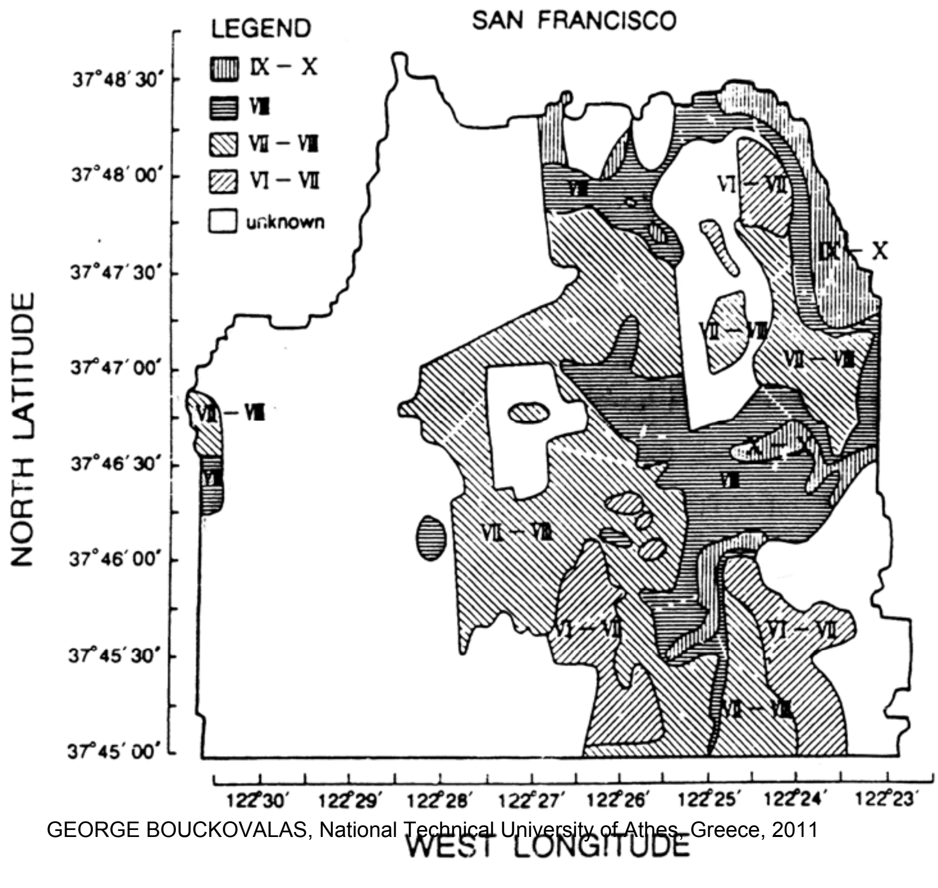
ξ_s

$$a_s = \frac{\rho_s V_s}{\rho_b V_b} \approx \frac{T_b}{T_S} \quad (ή \quad \frac{T_b}{T_{exc}})$$

4.4 EMPIRICAL METHODS

- ~~Analysis of damage from previous earthquakes~~
- ~~Correlation to (surface) geology~~
- ~~Attenuation relationships for a_{max}, V_{max}, ….~~
- ~~Correlation to shear wave velocity V_S~~
- Seismic Codes
- Semi-analytical relationships

Damage analysis from previous earthquakes:
San Francisco iso-seismal map
From the 1906 earthquake



Correlations to (surface) geology:
Factors of average spectral amplification

Geological unit	Relative amplification factor
Borcherdt and Gibbs (1976)	
Bay mud	11.2
Alluvium	3.9
Santa Clara Formation	2.7
Great Valley sequence	2.3
Franciscan Formation	1.6
Granite	1.0
Shima(1978)	
Peat	1.6
Humus soil	1.4
Clay	1.3
Loam	1.0
Sand	0.9
Midorikawa(1987)	
Holocene	3.0
Pleistocene	2.1
Quaternary volcanic rocks	1.6
Miocene	1.5
Pre-Tertiary	1.0

Attenuation Relationships for α_{max}, V_{max}, ...

π.χ. Sabetta & Pugliese (1987)

$$\log \alpha_{max} = 0{,}31\,M_S - \log\sqrt{R^2+5{,}8^2} + 0{,}17\,S - 1{,}56$$

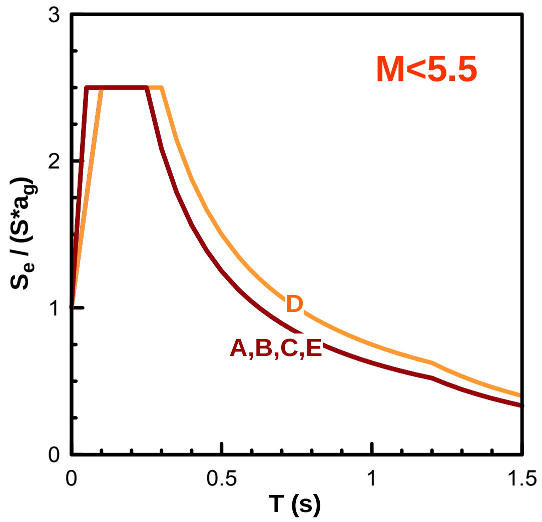
$$\log V_{max} = 0{,}46\,M_S - \log\sqrt{R^2+3{,}6^2} + 0{,}13\,S - 0{,}71$$

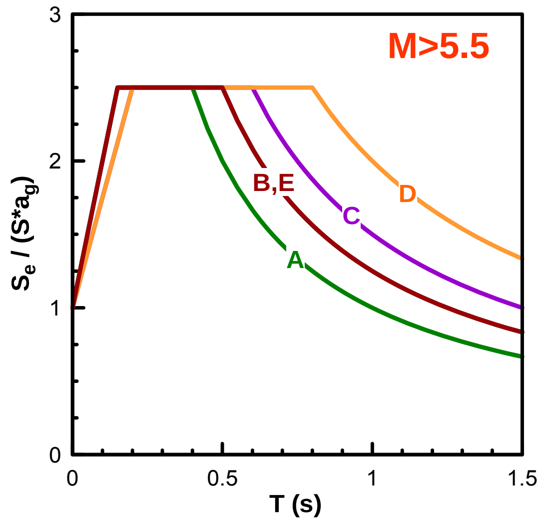
$$S = \begin{cases} 0 & \text{για "ΒΡΑΧΟ"} \\ 1 & \text{για "ΕΔΑΦΟΣ"} \end{cases}$$

$$\frac{\alpha_{max}^{ΕΔ.}}{\alpha_{max}^{ΒΡ.}} = 10^{0,17} = 1{,}50$$

$$\frac{V_{max}^{ΕΔ.}}{V_{max}^{ΒΡ.}} = 10^{0,13} = 1{,}35$$

Recent Seismic Codes: π.χ. EC - 8





	S		T_B		T_C	
	M>5,5	M<5,5	M>5,5	M<5,5	M>5,5	M<5,5
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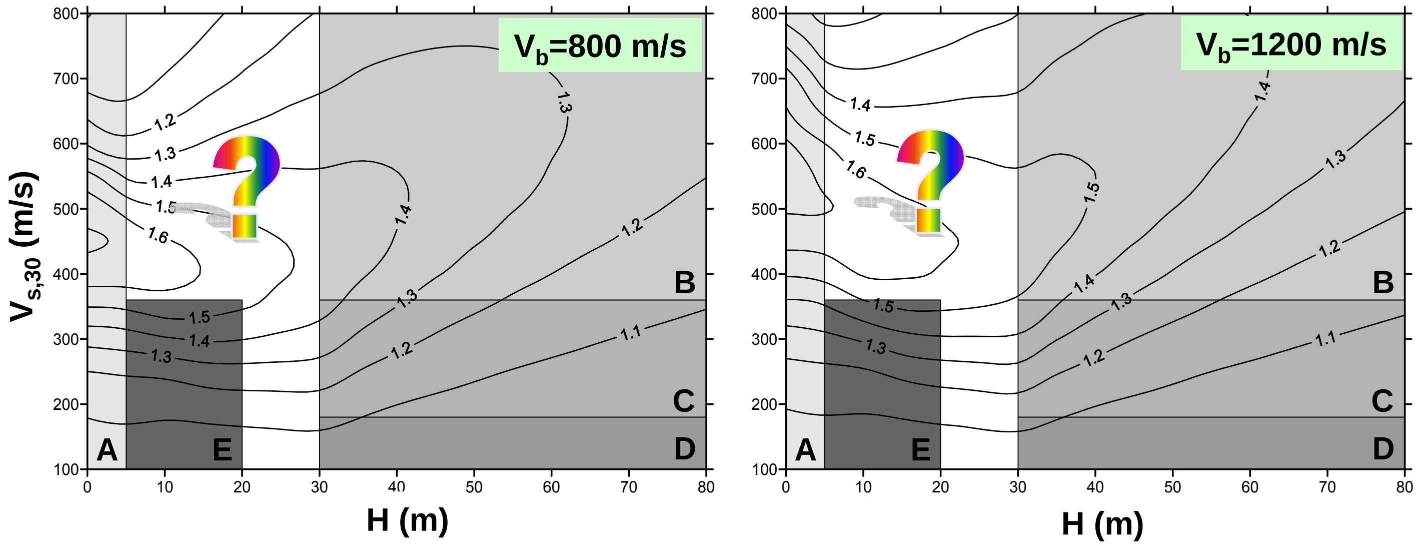
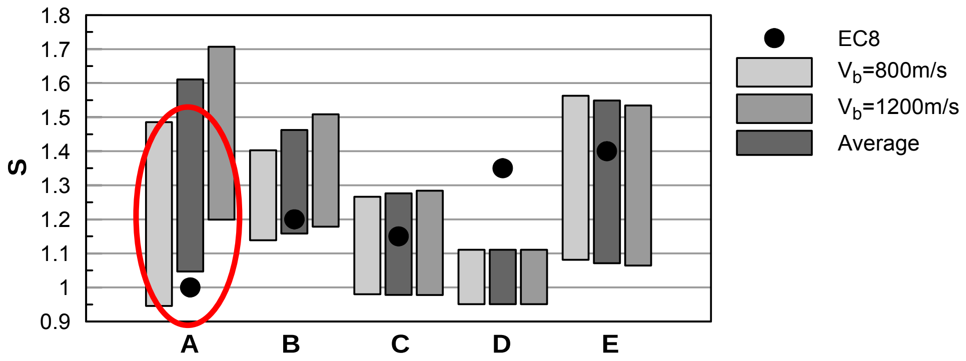
B	1.2	1.35	0.15	0.05	0.5	0.25
C	1.15	1.5	0.20	0.10	0.6	0.25
D	1.35	1.8	0.20	0.10	0.8	0.30
E	1.4	1.6	0.15	0.05	0.5	0.25

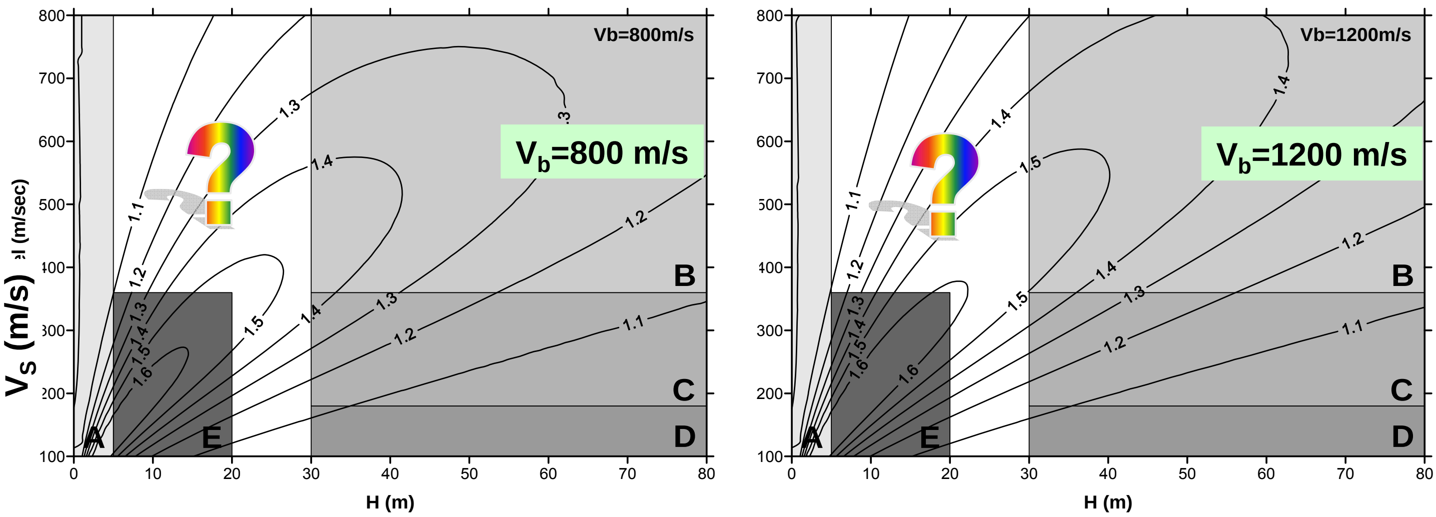
Ground type	**Description of stratigraphic profile**	**Parameters**		
		$V_{s,30}$ **(m/s)**	N_{SPT} **(blows/30cm)**	c_u **(kPa)**
A	Rock or other rock-like geological formation, including at most 5 m of weaker material at the surface	> 800	_	_
B	Deposits of very dense sand, gravel, or very stiff clay, at least several tens of m in thickness, characterised by a gradual increase of mechanical properties with depth	360 – 800	> 50	> 250
C	Deep deposits of dense or medium-dense sand, gravel or stiff clay with thickness from several tens to many hundreds of m	180 – 360	15 - 50	70 - 250
D	Deposits of loose-to-medium cohesionless soil (with or without some soft cohesive layers), or of predominantly soft-to-firm cohesive soil	< 180	< 15	< 70
E	A soil profile consisting of a surface alluvium layer with $V_{s,30}$ values of type C or D and thickness varying between about 5 m and 20 m, underlain by stiffer material with $V_{s,30}$ > 800 m/s			
S_1	Deposits consisting – or containing a layer at least 10 m thick – of soft clays/silts with high plasticity index (PI > 40) and high water content	< 100 (indicative)	_	10 - 20
S_2	Deposits of liquefiable soils, of sensitive clays, or any other soil profile not included in types A –E or S_1			

Soil Factors – Soil Categories EC 8– Effect of VS,30



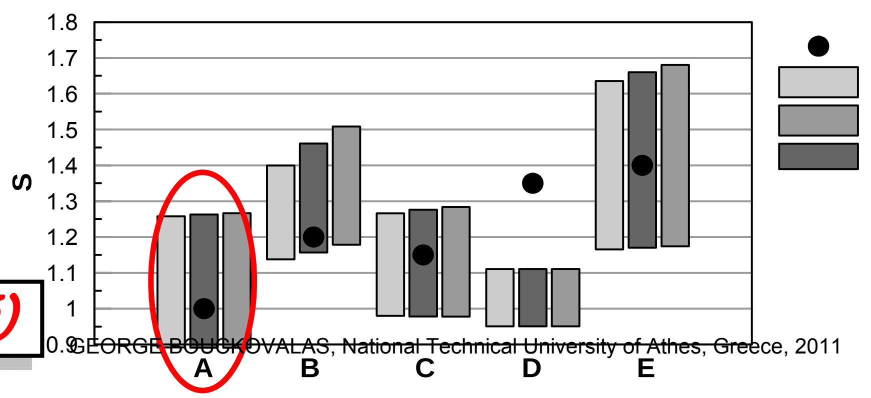


(M > 5.5)

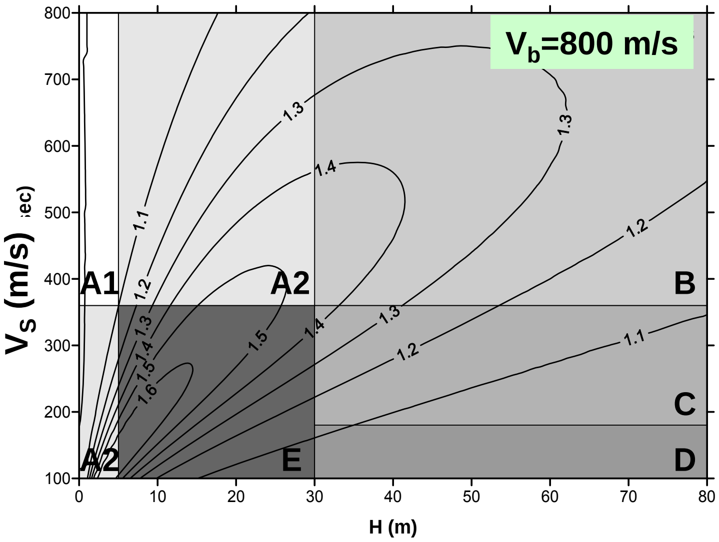


H (m)

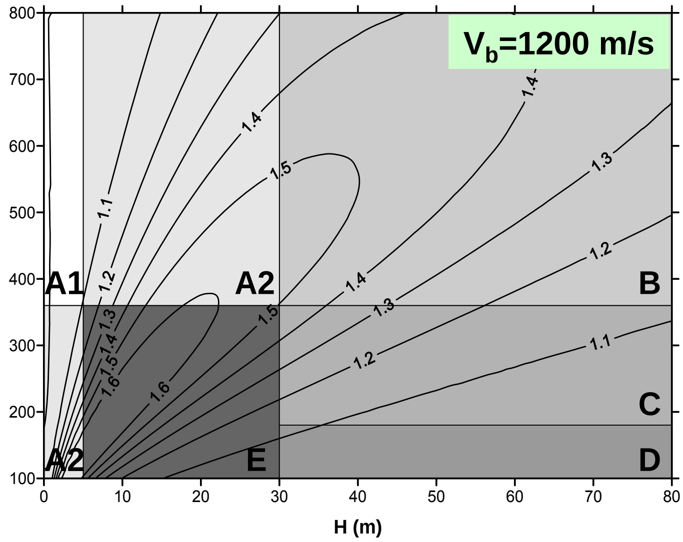
H (m)



(M > 5.5)



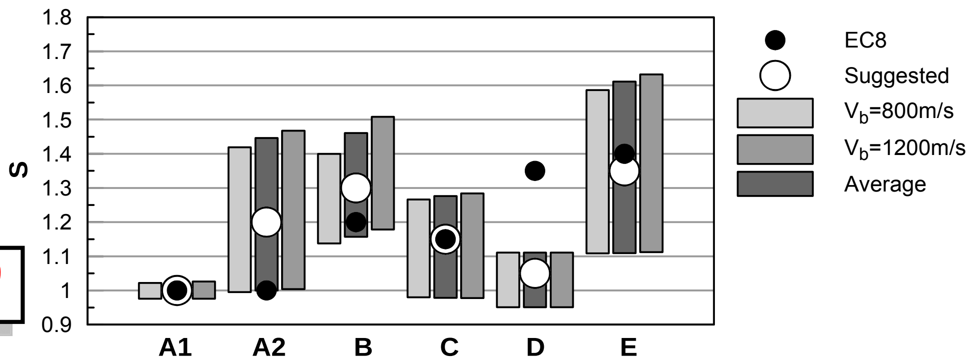
Vb=800 m/s
VS (m/s)
H (m)
A1
A2
B
C
D
E



Vb=1200 m/s
H (m)
A1
A2
B
C
D
E

H (m)

H (m)



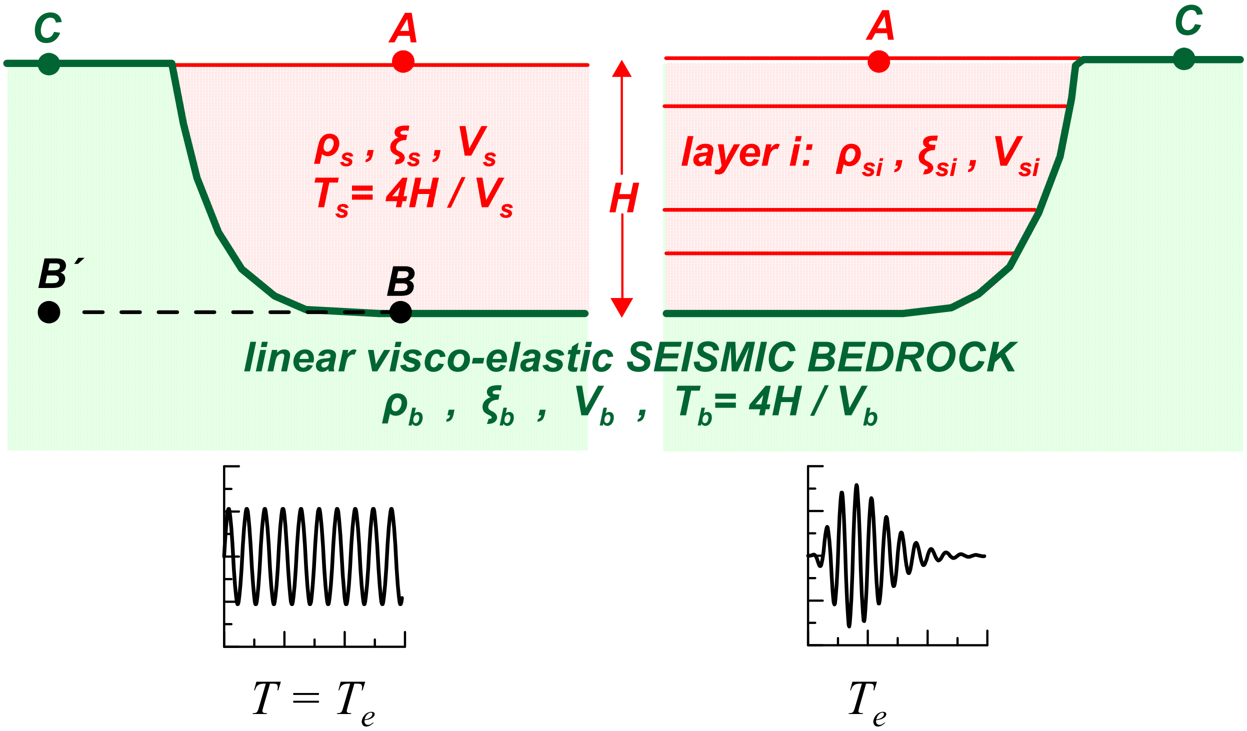
S
A1
A2
B
C
D
E
EC8
Suggested
Vb=800m/s
Vb=1200m/s
Average

(M > 5.5)

Semi-analytical relationships
(*Bouckovalas & Papadimitriou, 2003*)

IDEALIZED conditions — *linear visco-elastic soil*

REAL conditions — *nonlinear visco-elastic soil*



Semi-analytical relationships
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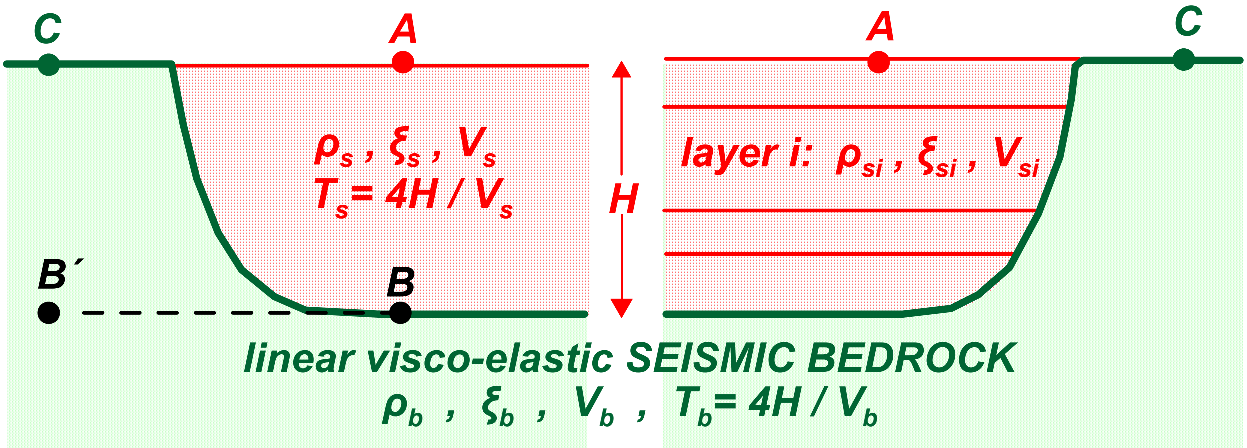
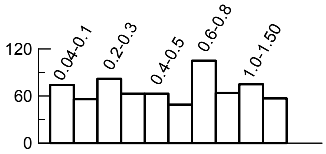


$$\text{Soil 'Amplification'} = \frac{\text{motion at A}}{\text{motion at C}}$$

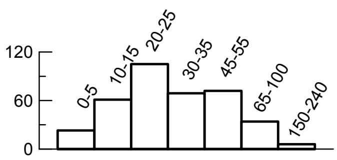
Database with more than 700 SHAKE-type analyses

Site Characteristics

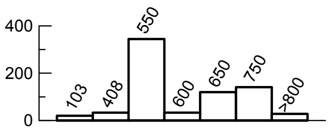
T_s = 0.04–3.33 s



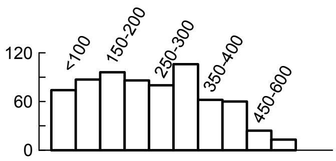
H = 3.5–240 m



V_b = 100–1000 m/s

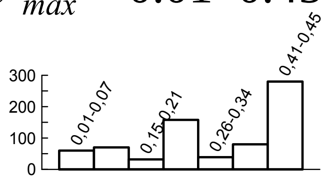


$\overline{V}_s$ = 50–700 m/s

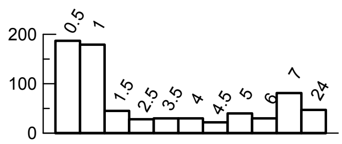


Excitation Characteristics

$a^b{}_{max}$ = 0.01–0.45g



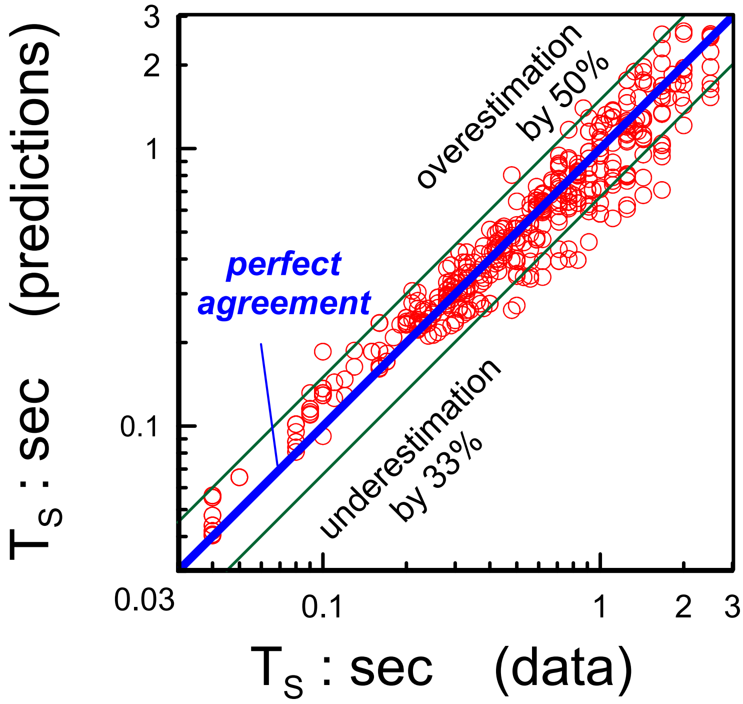
n = 0.5–24



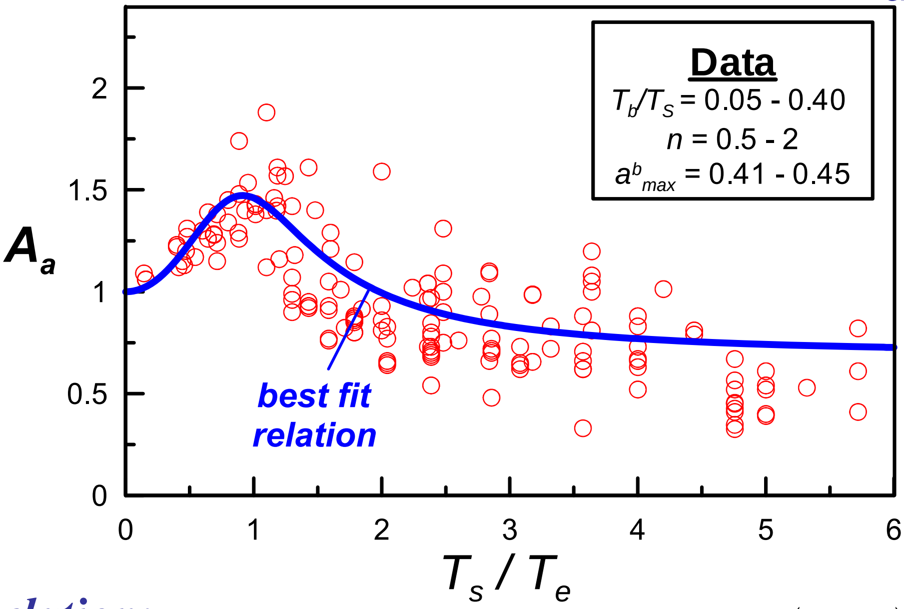
Step 1: Non-linear Soil Period, T_s

best fit relation:

$$T_s = T_{s,o}\sqrt{1+5330\frac{\left(a^b{}_{max}\right)^{1.04}}{\left(\overline{V}_{s,o}\right)^{1.3}}}$$



Step 2: 'Amplification' of peak ground acceleration, A_a



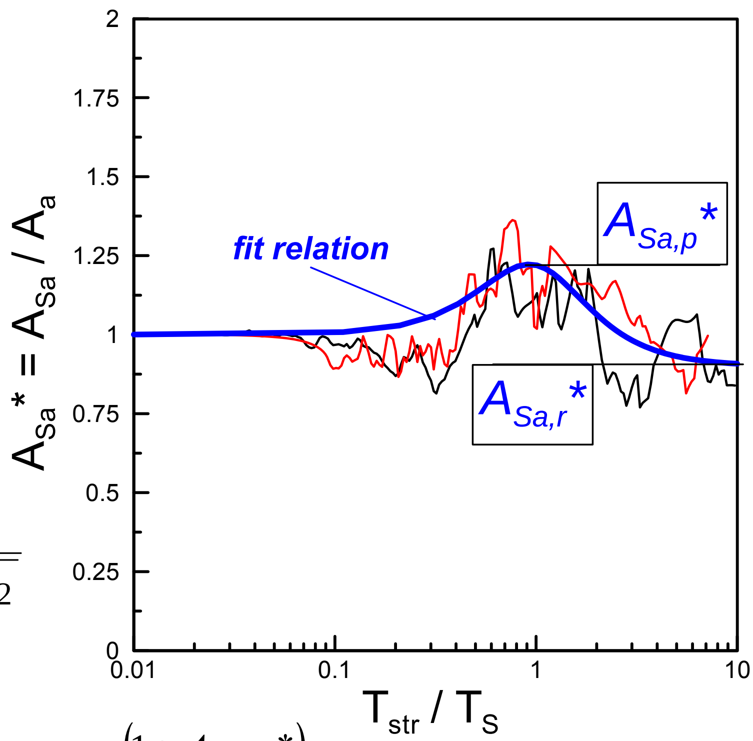
best fit relation:

$$A_a = \frac{1 + C_1 (T_s/T_e)^2}{\sqrt{\left[1-(T_s/T_e)^2\right]^2 + C_2^2 (T_s/T_e)^2}}$$

$$C_1 = 1.2\left(a^b{}_{max}\right)^{-0.17}\left[\frac{\sqrt{n}}{1+\sqrt{n}}\right]$$

$$C_2 = 1.05 + 0.57\frac{T_b}{T_s}$$

Step 3: 'Amplification' of normalized Elastic Response Spectra, A_{Sa}*

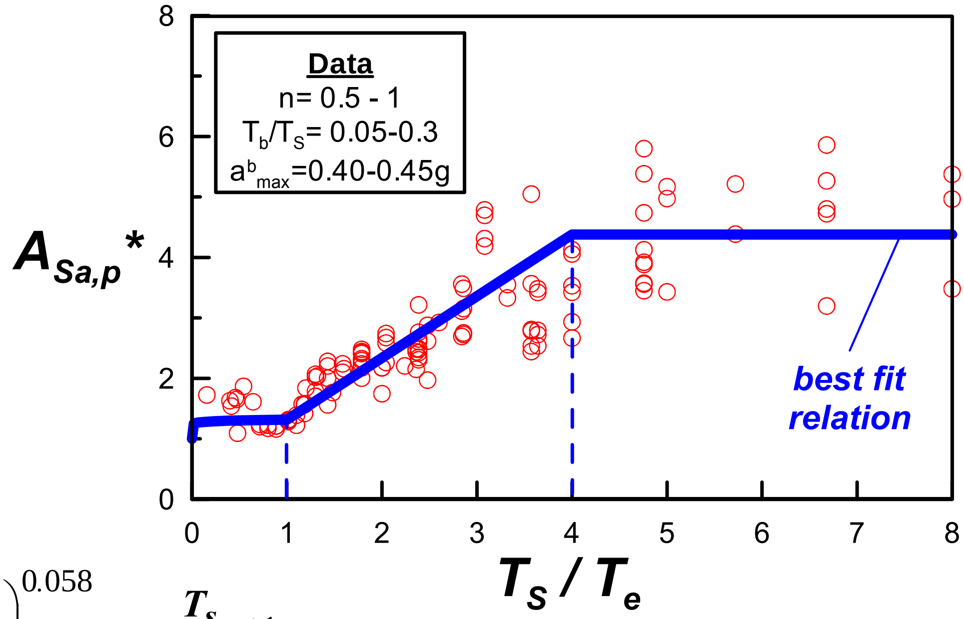


fit relation:

$$A_{Sa}{}^* = \frac{1 + B_1 (T_{str}/T_s)^2}{\sqrt{\left[1-(T_{str}/T_s)^2\right]^2 + B_2^2 (T_{str}/T_s)^2}}$$

$$B_1 = A_{Sa,r}{}^* \qquad B_2 = \frac{\left(1 + A_{Sa,r}{}^*\right)}{A_{Sa,p}{}^*}$$

Peak Spectral 'Amplification', $A_{Sa,p}$*

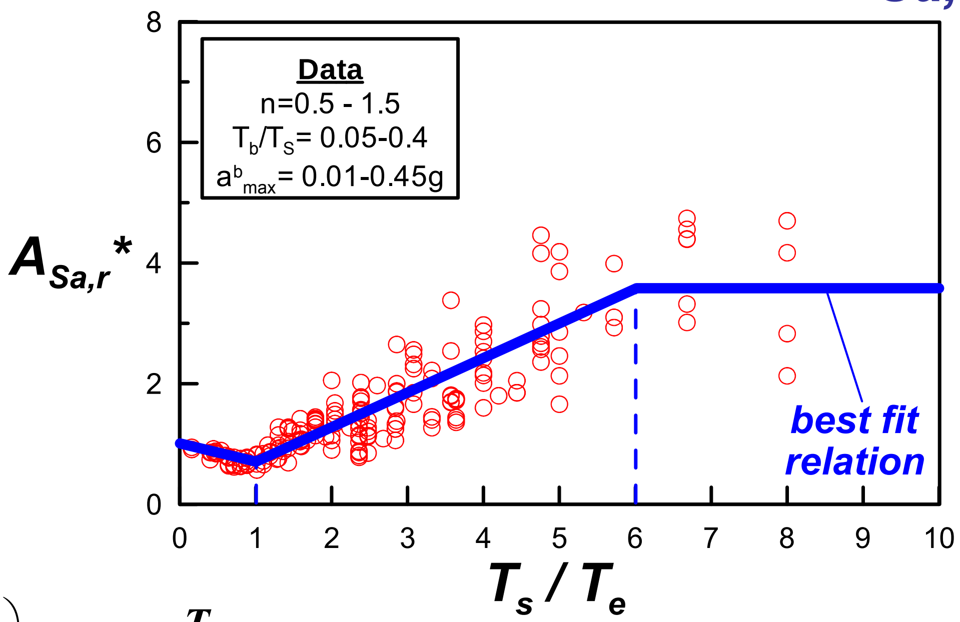


best fit relation:

$$A_{Sa,p}^* = \begin{cases} 1+0.318\left(\dfrac{T_s}{T_e}\right)^{0.058}, & \dfrac{T_s}{T_e} \le 1 \\ 1.318 + D\left(\dfrac{T_s}{T_e} - 1\right), & 1 \le \dfrac{T_s}{T_e} \le 4 \\ 1.318 + 3D, & 4 \le \dfrac{T_s}{T_e} \end{cases}$$

$$D = 0.279\left(\frac{T_b}{T_s}\right)^{-0.504} n^{-0.613}$$

Residual Spectral 'Amplification', $A_{Sa,r}$*



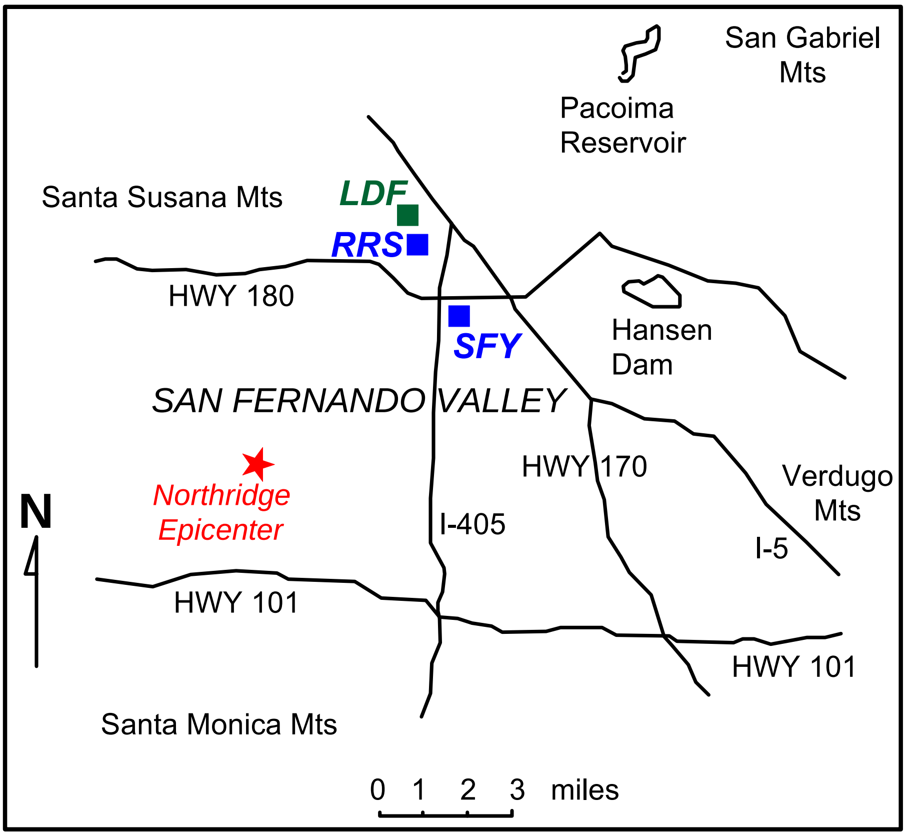
best fit relation:

$$A_{Sa,r}^* = \begin{cases} 1-0.302\left(\dfrac{T_s}{T_e}\right), & \dfrac{T_s}{T_e} \le 1 \\ 0.698 + F\left(\dfrac{T_s}{T_e} - 1\right), & 1 \le \dfrac{T_s}{T_e} \le 6 \\ 0.698 + 5F, & 6 \le \dfrac{T_s}{T_e} \end{cases}$$

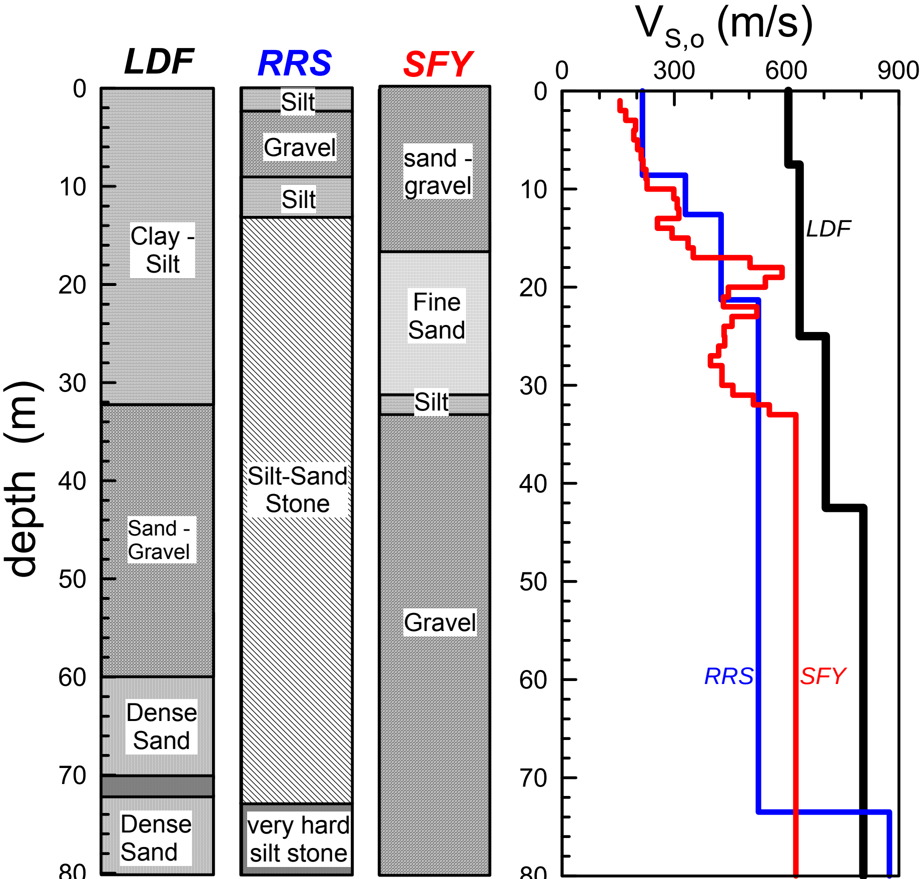
$$F = 0.189\left(\frac{T_b}{T_s}\right)^{-0.474} n^{-0.406}$$

VERIFICATION CASE STUDY

3 sites during the Northridge 1994 earthquake (M_w=6.7)



Soil conditions at recording sites

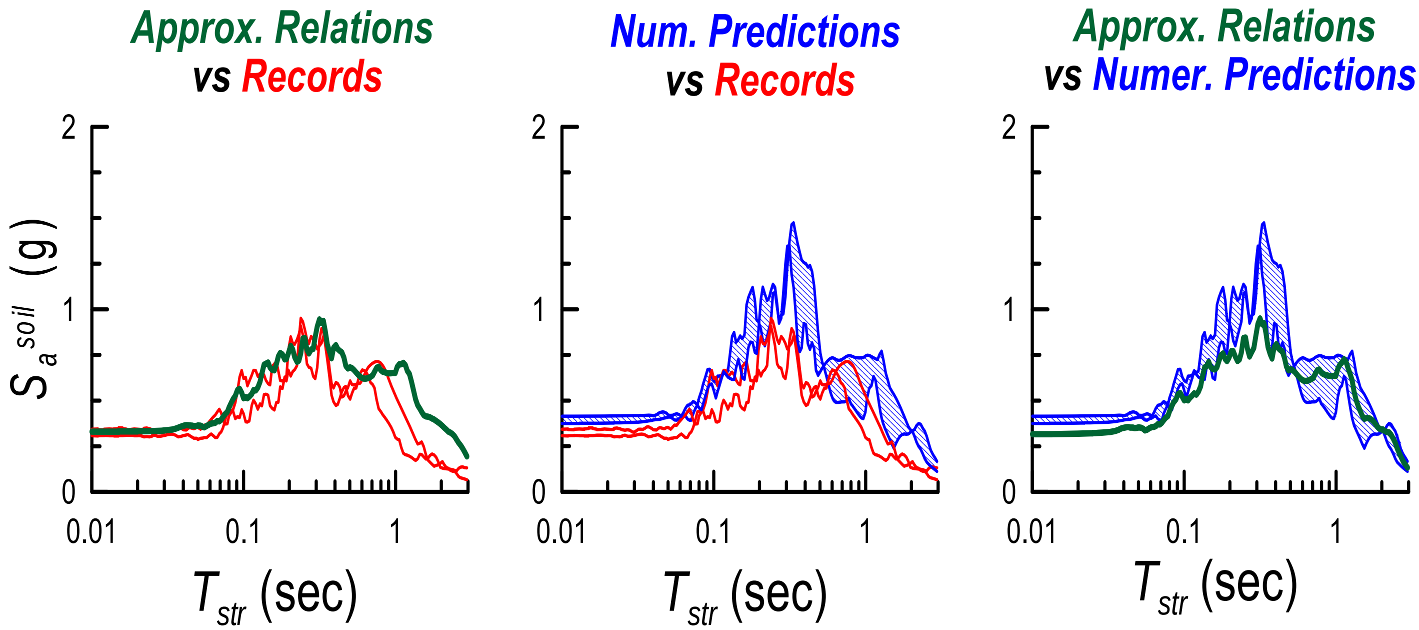


LDF:* ‘bedrock’ *site

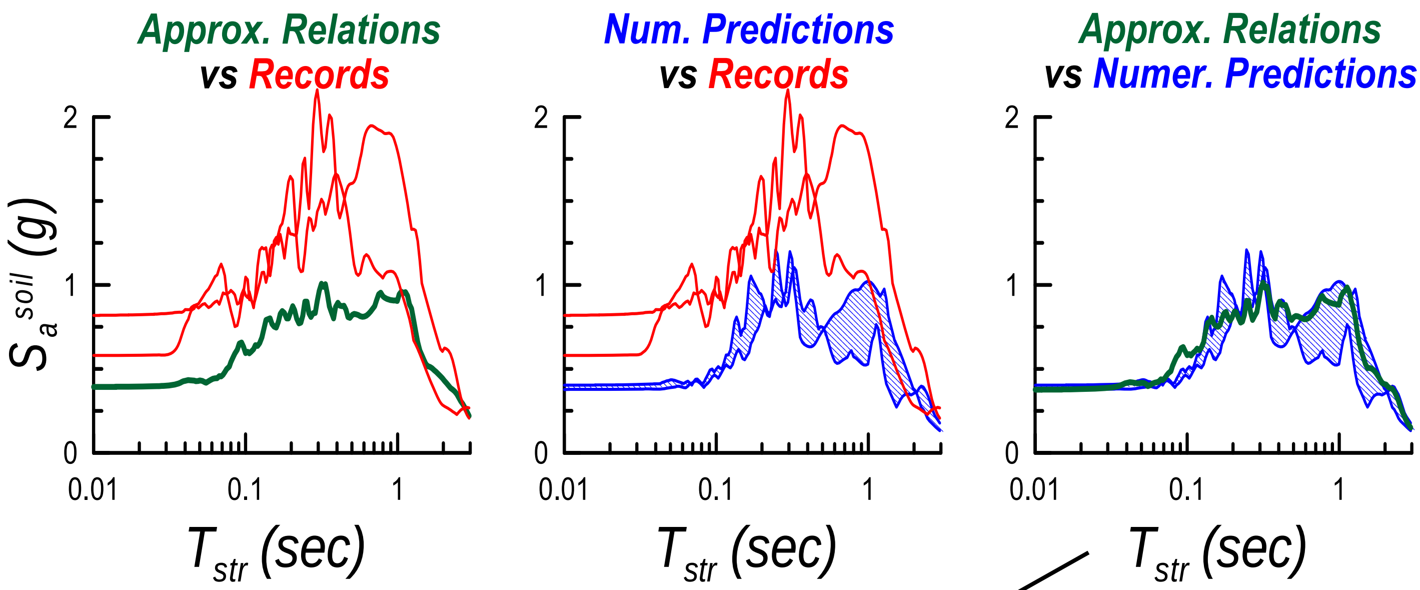
RRS
SFY } ***‘soil’*** *sites*

(Gibbs et al. 1999, Cultrera et al. 1999)

Site response at **SFY** (Arleta Fire Station)



Site response at **RRS** (Rinaldi Receiving Station)



Approx. Relations $\simeq$ Numerical Predictions

Homework 4.1:
Soil effects in Lefkada, Greece (2003) earthquake

The accompanying figures provide the basic data with regard to the recent (2003) strong motion recording in the island of Lefkada:

-Acceleration time histories and elastic response spectra (5% structural damping) from the two horizontal seismic motion recordings on the ground surface.

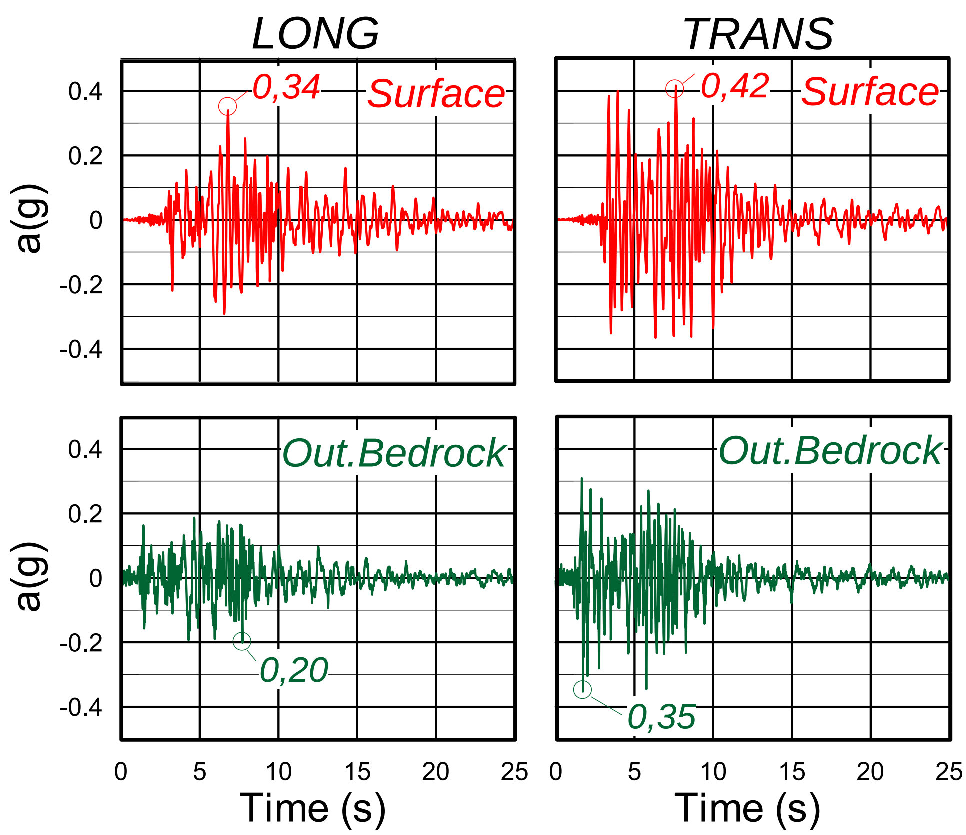
-Acceleration time histories and elastic response spectra (5% structural damping) for the two horizontal seismic motion recordings on the surface of the outcropping bedrock, as computed with a non-linear numerical analysis

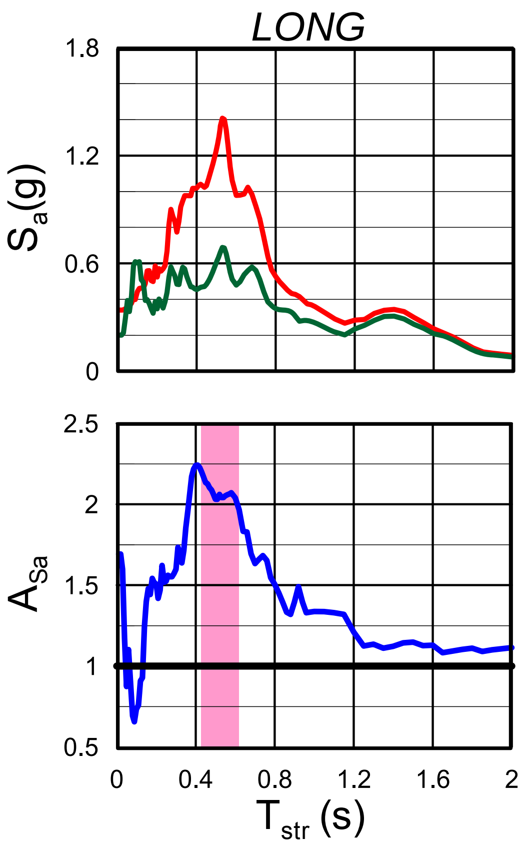
-Soil profile at the recording site.

Using the semi-analytical relationships, COMPUTE the ground surface spectral accelerations, at 5-6 representative structural periods, using the seismic recordings at the outcropping bedrock as input.

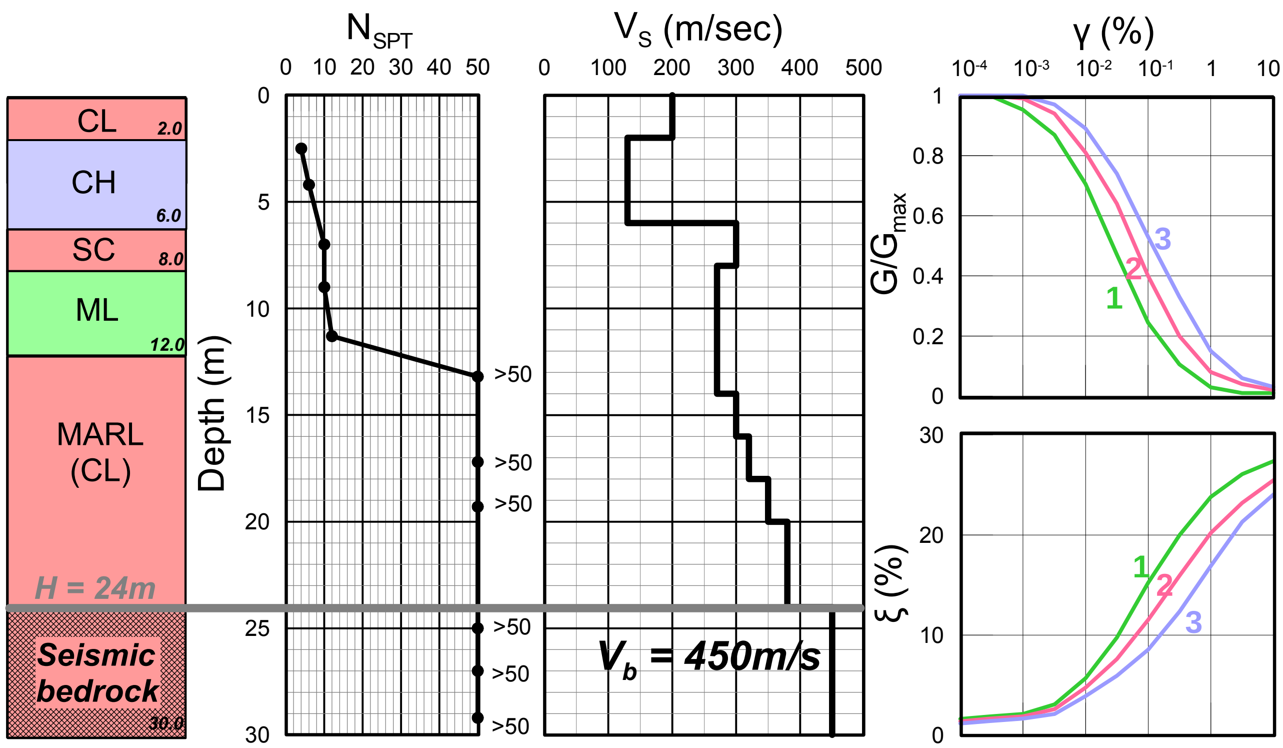
Compare with the actual recordings and comment on causes of any observed differences.

(*NOTE: Choose the LONG component of seismic motion for your computations*)





Soil profile at the recording site



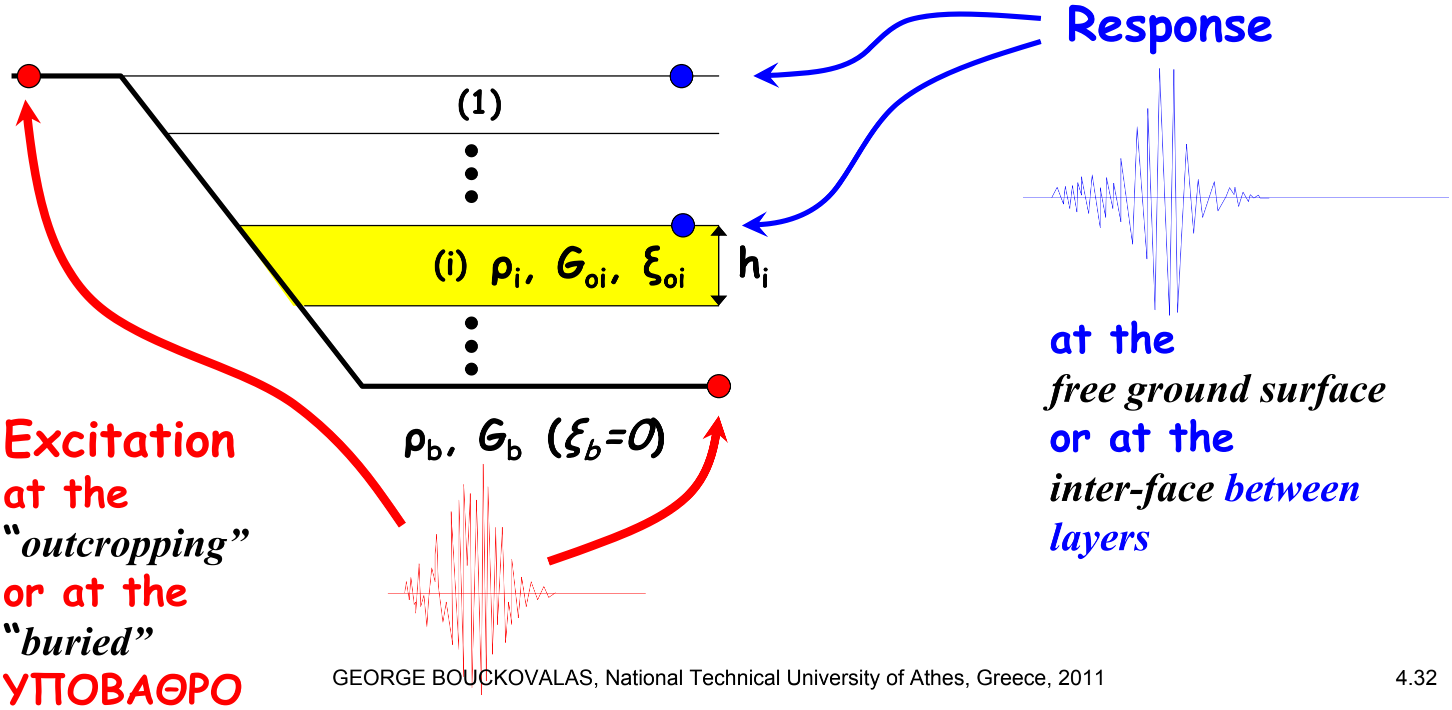
4.4 NUMERICAL METHODS

- NON-LINEAR
  TIME DOMAIN ANALYSIS

- EQUIVALENT LINEAR
  FREQUENCY DOMAIN ANALYSIS

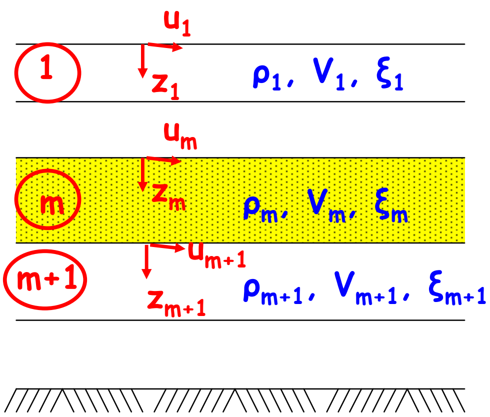
EQUIVALENT-LINEAR ANALYSIS

(also known as complex response method in the frequency domain)



This numerical method is essentially based on the analytical solution for . . .

NONUNIFORM (layered) visco-elastic soil on flexible bedrock.



motion at layer m

$$u_m = \left(A_m e^{ik_m^* z_m} + B_m e^{-ik_m^* z_m}\right)e^{i\omega t}$$

$$\tau_m = G_m^* \frac{\partial u_m}{\partial z_m} = iG_m^* k_m^* \left(A_m e^{ik_m^* z_m} - B_m e^{-ik_m^* z_m}\right)e^{i\omega t}$$

Boundary constraints at the free ground surface

$$\tau_{1,0} = 0 \Rightarrow A_1 = B_1$$

$$u_1 = 2A_1 \cos k_1^* z_1 e^{i\omega t}$$

$$\tau_1 = 2A_1\left(iG_1^* k_1^*\right)\sin k_1^* z_1 e^{i\omega t}$$

Boundary constraints between layers

$$u_{m+1,0} = u_{m,h_m} \Rightarrow A_{m+1} + B_{m+1} = A_m e^{ik_m^* h_m} + B_m e^{-ik_m^* h_m}$$

$$\tau_{m+1,0} = \tau_{m,h_m} \Rightarrow A_{m+1} - B_{m+1} = \frac{k_m^* G_m^*}{k_{m+1}^* G_{m+1}^*}\left(A_m e^{ik_m^* h_m} - B_m e^{-ik_m^* h_m}\right)$$

and finally

$$A_{m+1`} = \frac{1}{2} A_m \left(1 + a_m^*\right) e^{ik_m^* h_m} + \frac{1}{2} B_m \left(1 - a_m^*\right) e^{-ik_m^* h_m}$$

$$B_{m+1`} = \frac{1}{2} A_m \left(1 - a_m^*\right) e^{ik_m^* h_m} + \frac{1}{2} B_m \left(1 + a_m^*\right) e^{-ik_m^* h_m}$$

$$\acute{o}\pi o \upsilon \; a_m^* = \frac{\rho_m V_m^*}{\rho_{m+1} V_{m+1}^*}$$

Sequential application, from the free ground surface (i=1) to deeper layers (i=m)

i=1

$$A_2 = \frac{1}{2}A_1\left(1+a_1^*\right)e^{ik_1^*h_1} + \frac{1}{2}A_1\left(1-a_1^*\right)e^{-ik_1^*h_1}$$
$$= A_1\left(\cos k_1^*h_1 + ia_1^*\sin k_1^*h_1\right)$$
$$B_2 = \frac{1}{2}A_1\left(1-a_1^*\right)e^{ik_1^*h_1} + \frac{1}{2}A_1\left(1+a_1^*\right)e^{-ik_1^*h_1}$$
$$= A_1\left(\cos k_1^*h_1 - ia_1^*\sin k_1^*h_1\right)$$

or, briefly

$$A_2 = f_1\left(k_1^*h_1\right)A_1$$
$$B_2 = g_1\left(k_1^*h_1\right)A_1$$

i=2

$$A_3 = \frac{1}{2}A_2\left(1+a_2^*\right)e^{ik_2^*h_2} + \frac{1}{2}B_2\left(1-a_2^*\right)e^{-ik_2^*h_2}$$
$$= A_1\left[\frac{1}{2}a_1\left(1+a_2^*\right)e^{ik_2^*h_2} + \frac{1}{2}\beta_1\left(1-a_2^*\right)e^{-ik_2^*h_2}\right]$$
$$B_3 = A_1\left[\frac{1}{2}a_1\left(1-a_2^*\right)e^{ik_2^*h_2} + \frac{1}{2}\beta_1\left(1+a_2^*\right)e^{-ik_2^*h_2}\right]$$

or, briefly

$$A_3 = f_2\left(k_1^*h_1, k_2^*h_2\right)A_1$$
$$B_3 = g_2\left(k_1^*h_1, k_2^*h_2\right)A_1$$

i=m ⇨

$$A_m = A_1 . f_{m-1}(k_1^*h_1,\ k_2^*h_2, \dots,\ k_m^*h_m)$$
$$B_m = A_1 . g_{m-1}(k_1^*h_1,\ k_2^*h_2, \dots,\ k_m^*h_m)$$

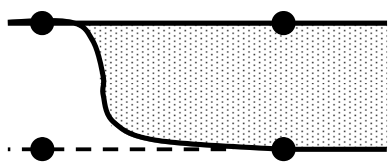
Considering that $A_1=B_1$, the soil amplification factor becomes:

$$\left|F_3(\omega)\right| = \left|\frac{u_A}{u_B}\right| = \left|\frac{A_1+B_1}{A_m+B_m}\right| = \left|\frac{2}{f_{m-1}+g_{m-1}}\right|$$

$$\left|F_4(\omega)\right| = \left|\frac{u_A}{u_\Gamma}\right| = \left|\frac{u_A}{uB}\right| \Big/ \left|\frac{u_\Gamma}{u_{B'}}\right| \Rightarrow$$

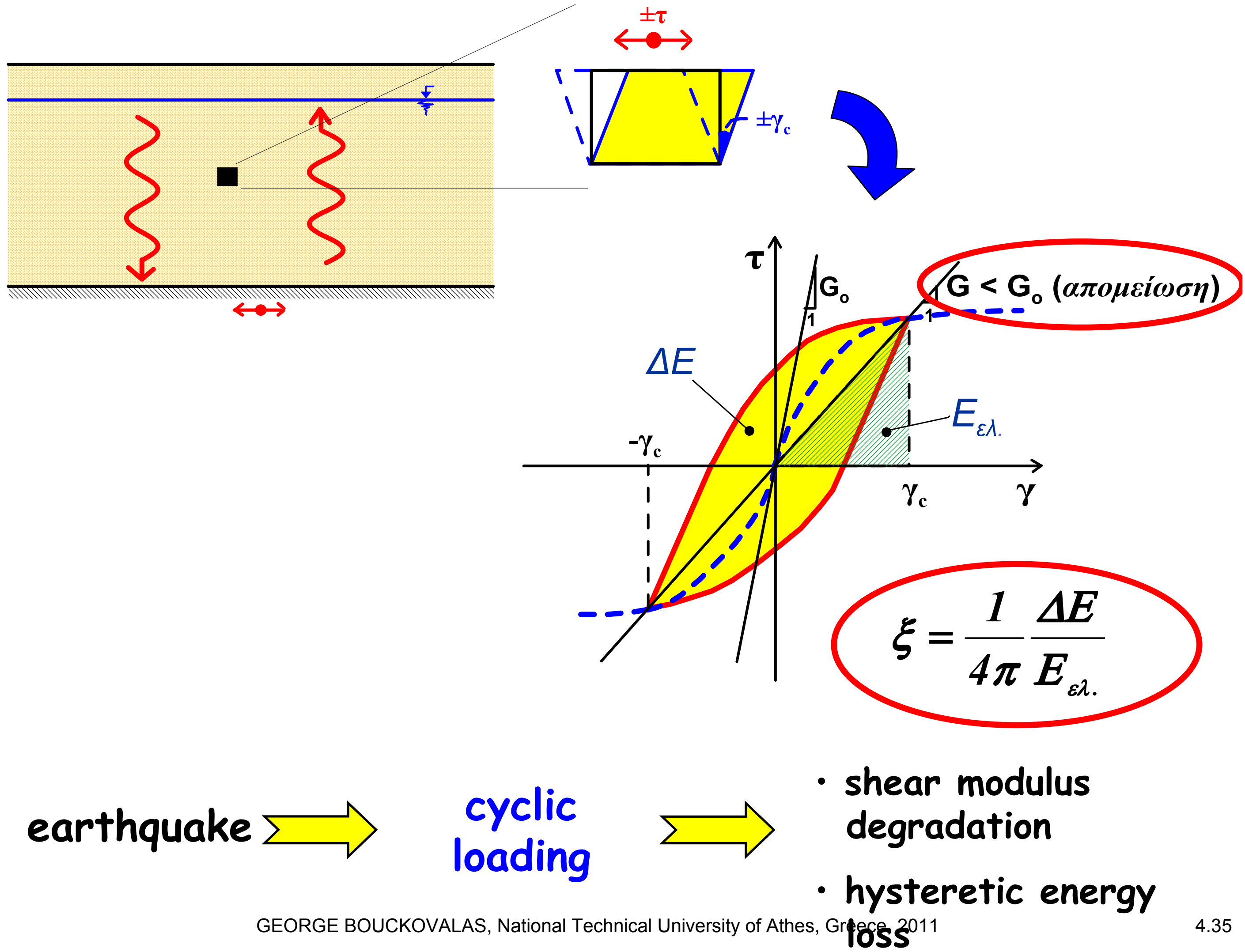
$$\left|F_4(\omega)\right| = \left|F_3(\omega)\right| \bullet \sqrt{cos^2\left(k_bH\right) + \left(\xi_b k_b H\right)^2}$$

$$\left(u_B \approx u_{B'}\right)$$

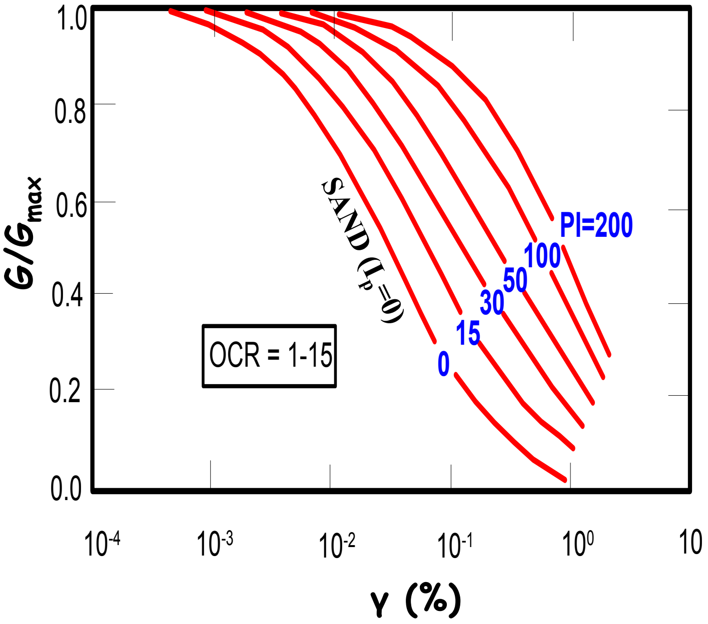


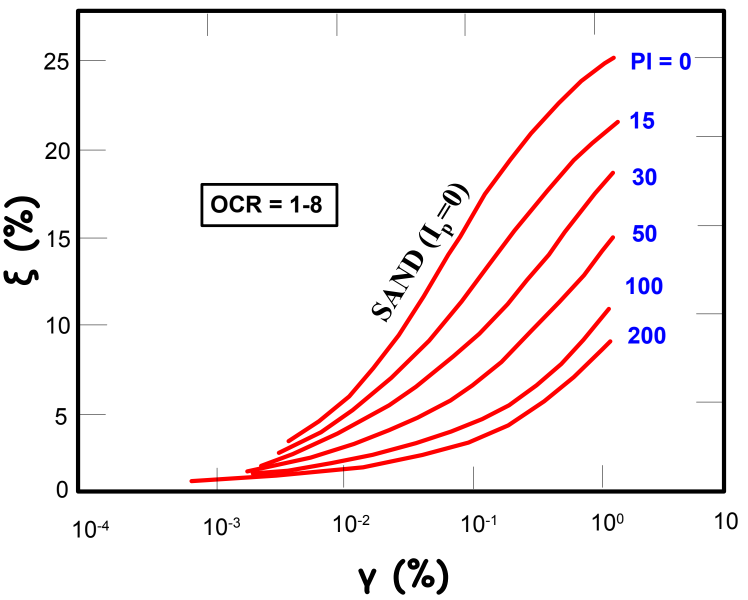
The basic input data required for this type of analysis are the following:

- Acceleration time history for the seismic excitation.
- The elastic shear modulus $G_{o,b}$ and the specific mass density ρ_b of the seismic bedrock
- The depth and the thickness of each soil layer i
- The elastic shear modulus $G_{o,i}$ and the specific mass density ρ_i of each soil layer i
- Experimental curves describing the variation of the shear modulus and hysteretic damping ratios with cyclic shear strain amplitude (G/G_o-γ and ξ-γ curves)



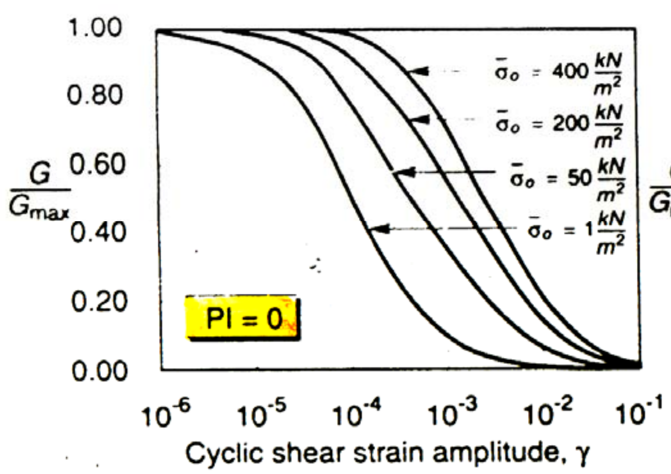
Experimental curves for the G/Go-γ and ξ-γ relations (Vucetic & Dobry, 1991)

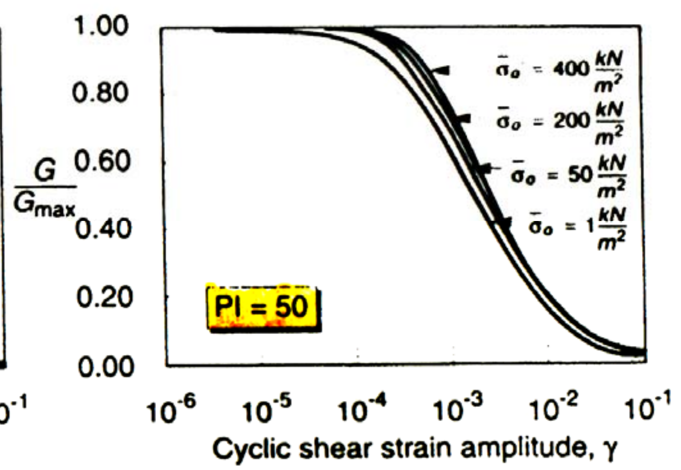




Effect of soil type (through I_P ή PI)

Effect of effective consolidation stress on the G/Go-γ and ξ-γ relations





(Ishibashi 1992)

$$\frac{G}{G_{max}} = K(\gamma, PI)\left(\sigma_m'\right)^{m(\gamma,PI)-m_o}$$

$$K(\gamma, PI) = 0.5\left\{1+\tanh\left[\ln\left(\frac{0.000102+n(PI)}{\gamma}\right)^{0.492}\right]\right\}$$

$$m(\gamma, PI)-m_o = 0.272\left\{1-\tanh\left[\ln\left(\frac{0.000556}{\gamma}\right)^{0.4}\right]\right\}\exp\left(-0.0145\,PI^{1.3}\right)$$

$$n(PI) = \begin{cases} 0.0 & \gamma\iota\alpha\ PI = 0 \\ 3.37\times10^{-6}\,PI^{1.404} & \gamma\iota\alpha\ 0 < PI \le 15 \\ 7.0\times10^{-7}\,PI^{1.976} & \gamma\iota\alpha\ 15 < PI \le 70 \\ 2.7\times10^{-5}\,PI^{1.115} & \gamma\iota\alpha\ PI > 70 \end{cases}$$

Solution Sequence

1st Step: Fourier analysis (transform) of the seismic excitation into harmonic components

2nd Step: $G_l = G_{o,l}$ και $\xi_l = \xi_o$

3rd Step: Computation of transfer functions $F_{i,j}$ for each soil layer and each harmonic excitation component

4th Step: Computation of ground response for each harmonic excitation component

5th Step: Inverse Fourier analysis (transform) of the harmonic ground response components for the computation of the seismic ground response

6th Step: Computation of maximum shear strain amplitude γ_{max} at the middepth of each soil layer

7th Step: Computation of the shear modulus G and damping ratio ξ values which correspond to 2/3 γ_{max}.

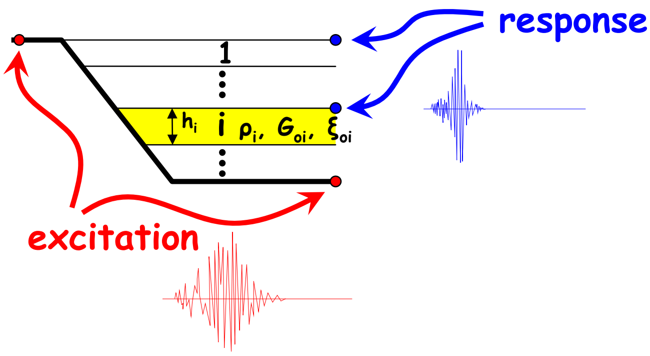
4.4 NUMERICAL METHODS

- NON-LINEAR
  TIME DOMAIN ANALYSIS

- EQUIVALENT LINEAR
  FREQUENCY DOMAIN ANALYSIS

NON-LINEAR ANALYSIS

(time domain analysis)



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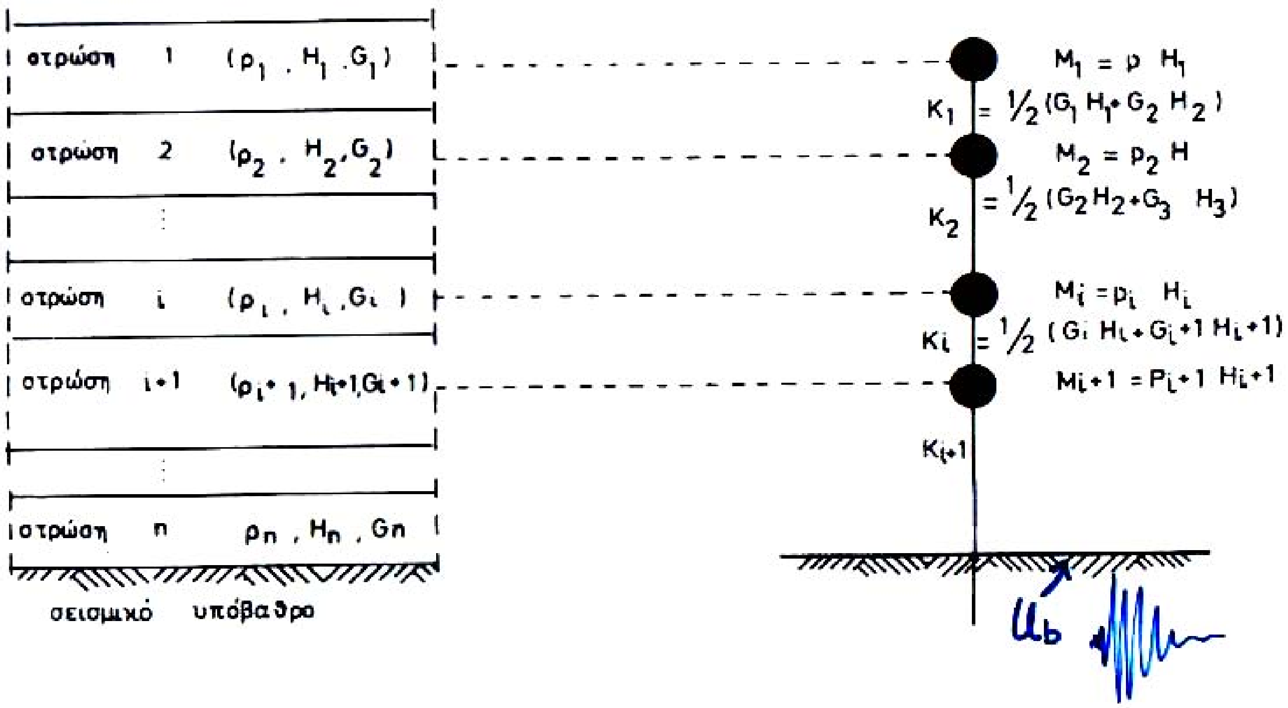
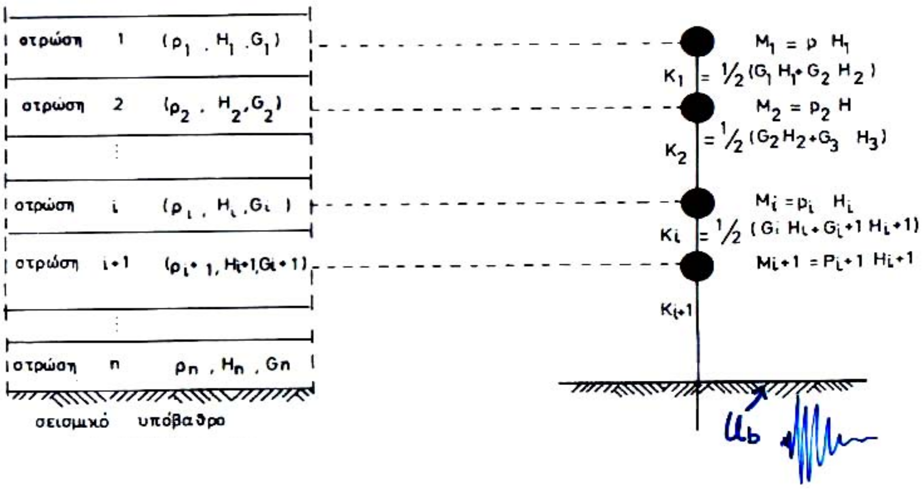
- Acceleration time history for the seismic excitation.
- The elastic shear modulus $G_{o,b}$ and the specific mass density ρ_b of the seismic bedrock
- The depth and the thickness of each soil layer i
- The elastic shear modulus $G_{o,i}$ and the specific mass density ρ_i of each soil layer i
- The shear stress-strain relationship (τ-γ) for monotonic and cyclic loading of each soil layer

The layered soil profile is discretized and simulated as a system of lamped masses and visco-elastic springs . . .



The layered soil profile is discretized and simulated as a system of lamped masses and visco-elastic springs . . .



$$M\ddot{X} + KX = -M\ddot{U}_b$$

with
$$M = \begin{bmatrix} M_1 & & & \\ & M_2 & & \\ & & \ddots & \\ & & & M_n \end{bmatrix} \qquad K = \begin{bmatrix} K_1 & -K_1 & 0 & 0 \\ -K_1 & K_1 + K_2 & -K_2 & 0 \\ 0 & -K_2 & K_2 + K_3 & -K_3 \\ 0 & 0 & -K_3 & \ddots \end{bmatrix}$$

Solution of the previous differential equation is achieved with time integration, for given initial conditions and a given time history of base excitation $U_b(t)$.

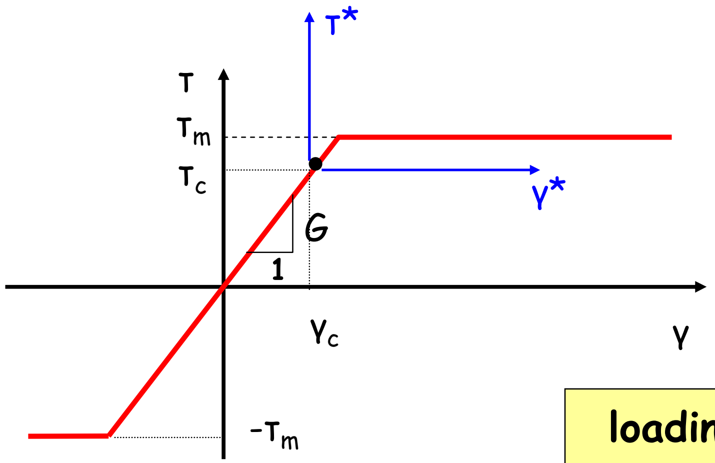
Given that $G=f(\tau$ ή $\gamma)$, it is evident that also $K=f(U)$. Hence, we deal with a non-linear differential equation, which has to be solved incrementally, in very small time steps (small enough to ensure convergence).

Regardless of the solution algorithm, it is mandatory to define the shear stress-strain relationship (τ-γ) which controls the dynamic response (loading-unloading-reloading) of each soil layer.

In fact, the accuracy of the numerical predictions is very sensitive to the adopted τ-γ relationsip, a fact that most users either are not aware of or …. choose to overlook.

"VERY SIMPLE" HYSTERETIC MODELS

(a) Elastic – perfectly plastic

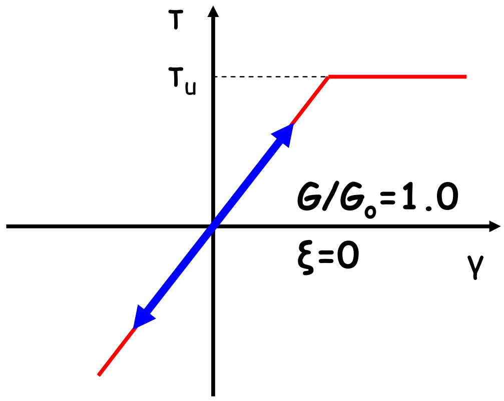


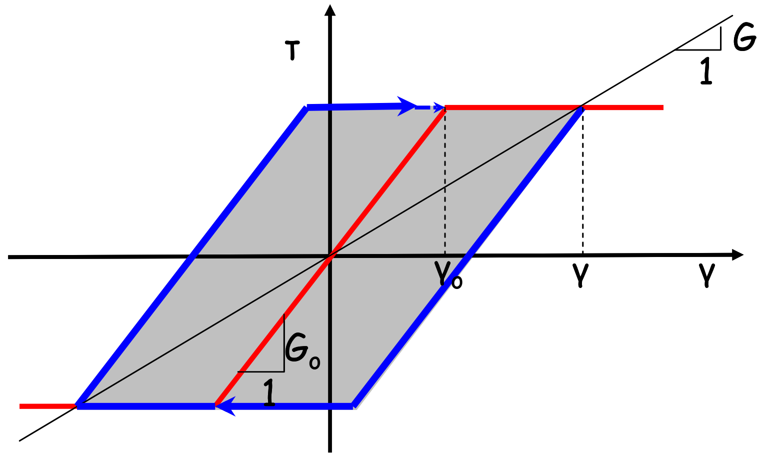
loading: $-\tau_m \leq \tau = \gamma\, G \leq \tau_m$

Unloading from (τ_c,γ_c): $-\tau^*_m \leq \tau^* = \gamma^*\, G \leq \tau^*_m$

where $\tau^*=\tau-\tau_c$, $\gamma^*=\gamma-\gamma_c$ και $\tau_m^*=\tau_m+|\tau_c|$

Loop for $\tau \le \tau_m$





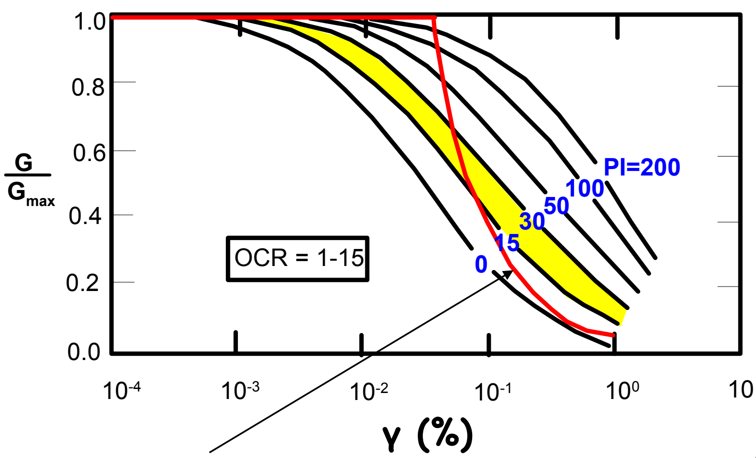
Loop for $\tau \ge \tau_m$

$$\left.\begin{aligned} G &= \frac{\tau_m}{\gamma} \\ G_o &= \frac{\tau_m}{\gamma_o} \end{aligned}\right\} \frac{G}{G_o} = \frac{\gamma_o}{\gamma},$$

$$\xi = \frac{1}{4\pi}\frac{\Delta E}{E_{\varepsilon\lambda}} = \frac{1}{4\pi}\frac{4\tau_m(\gamma-\gamma_o)}{\frac{1}{2}\tau_m\gamma} = \frac{2}{\pi}\left(1-\frac{\gamma_o}{\gamma}\right)$$

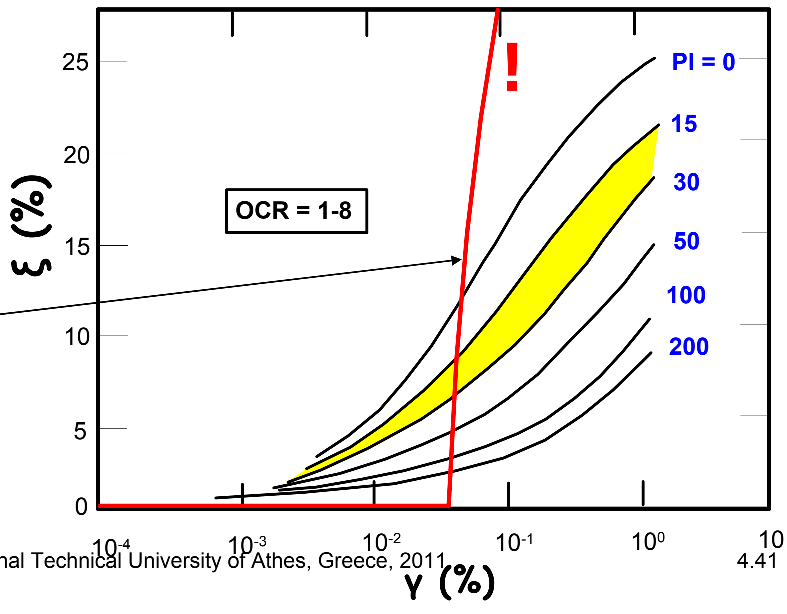
$$\xi = \frac{2}{\pi}\left(1-\frac{G}{G_o}\right) = 0 \sim 0.65$$

Comparison with experimental data



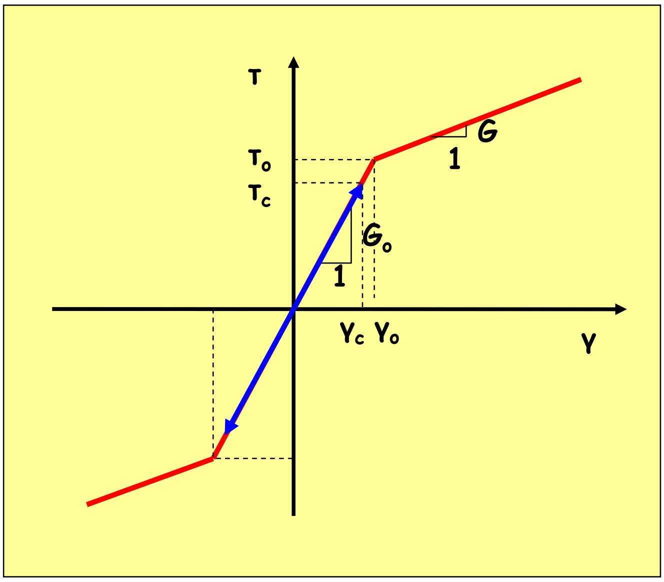
RAPID degradation of shear modulus G . . .

and, even worst, RAPID & EXTREME increase of the critical damping ratio ξ



What are the consequences of these differences for the prediction of the seismic ground response?

(β) Bi-linear elastoplastic

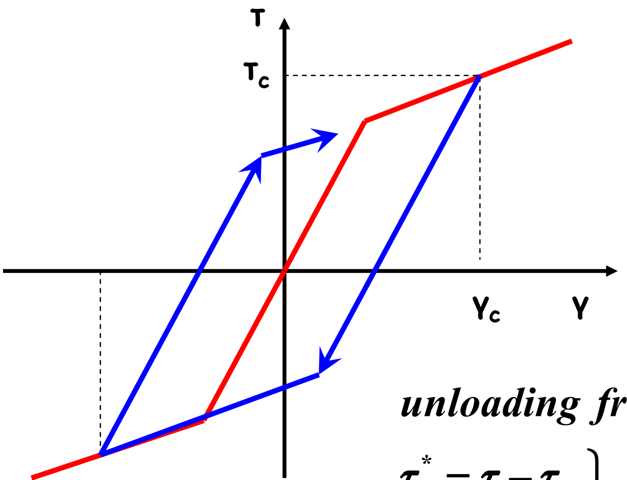


(a) $|\tau| \le |\tau_o|$

$$\gamma_c = \frac{\tau}{G_o} \qquad \textit{loading}$$

$$\gamma - \gamma_c = \frac{\tau - \tau_c}{G_o} \qquad \textit{unloading – reloading}$$

$G/G_o=1$
$\xi=0$



(b) $|\tau| \ge |\tau_o|$

$$\textit{loading}: \quad \gamma - \gamma_o = \frac{\tau - \tau_o}{G_1} \Rightarrow \gamma = \gamma_o + \frac{\tau - \tau_o}{G_1}$$

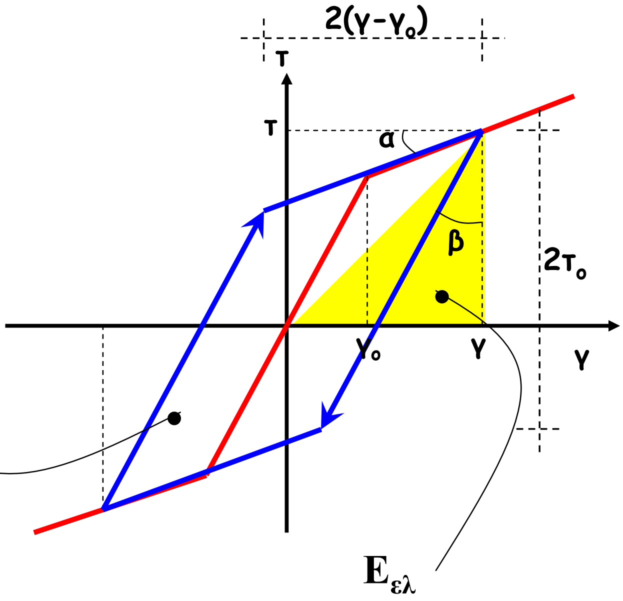
unloading from γ_c, τ_c :

$$\left.\begin{aligned} \tau^* &= \tau - \tau_c \\ \gamma^* &= \gamma - \gamma_c \\ \tau_o^* &= 2\tau_o \end{aligned}\right\} \quad G = \frac{\tau}{\gamma} \Rightarrow \left.\begin{aligned} \tau &= G\gamma \\ \tau_o &= G_o\gamma_o \end{aligned}\right\} \quad \gamma = \gamma_o + \frac{G\gamma - G_o\gamma_o}{G_1}$$

$$\gamma = \gamma_o + \frac{G/G_o\,\gamma - \gamma_o}{G_1/G_o} \Rightarrow G/G_o\,\gamma - \gamma_o + G_1/G_o\,\gamma_o = G_1/G_o\,\gamma$$

$$G/G_o = \frac{\gamma_o}{\gamma}\left(1 - G_1/G_o\right) + G_1/G_o$$

$$\xi = \frac{1}{4\pi}\frac{\Delta E}{E_{\varepsilon\lambda}}$$



$$\Delta E = \frac{2(\gamma-\gamma_o)}{\cos\alpha}\frac{2\tau_o}{\cos\beta}\cos(\alpha+\beta) = \frac{2(\gamma-\gamma_o)2\tau_o}{\cos\alpha\cos\beta}(\cos\alpha\cos\beta - \sin\alpha\sin\beta)$$

$$= 4(\gamma-\gamma_o)\tau_o(1-\tan\alpha\tan\beta)$$

$$\tan\alpha = G_1$$

$$\tan\beta = \tan(90-\beta') = \cot\beta' = \frac{1}{G_o}$$

$$\Delta E = 4(\gamma-\gamma_o)\tau_o\left(1-\frac{G_1}{G_o}\right)$$

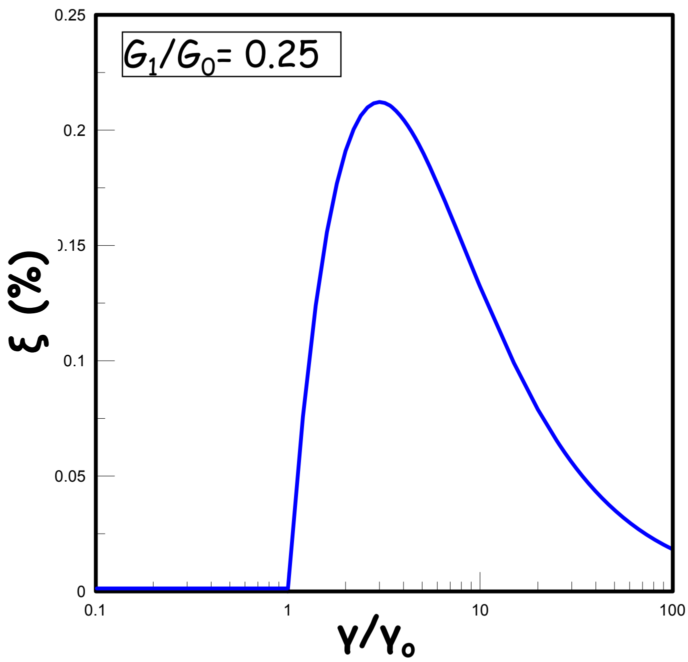
$$\Delta E = 4(\gamma-\gamma_o)\tau_o\left(1-\frac{G_1}{G_o}\right)$$

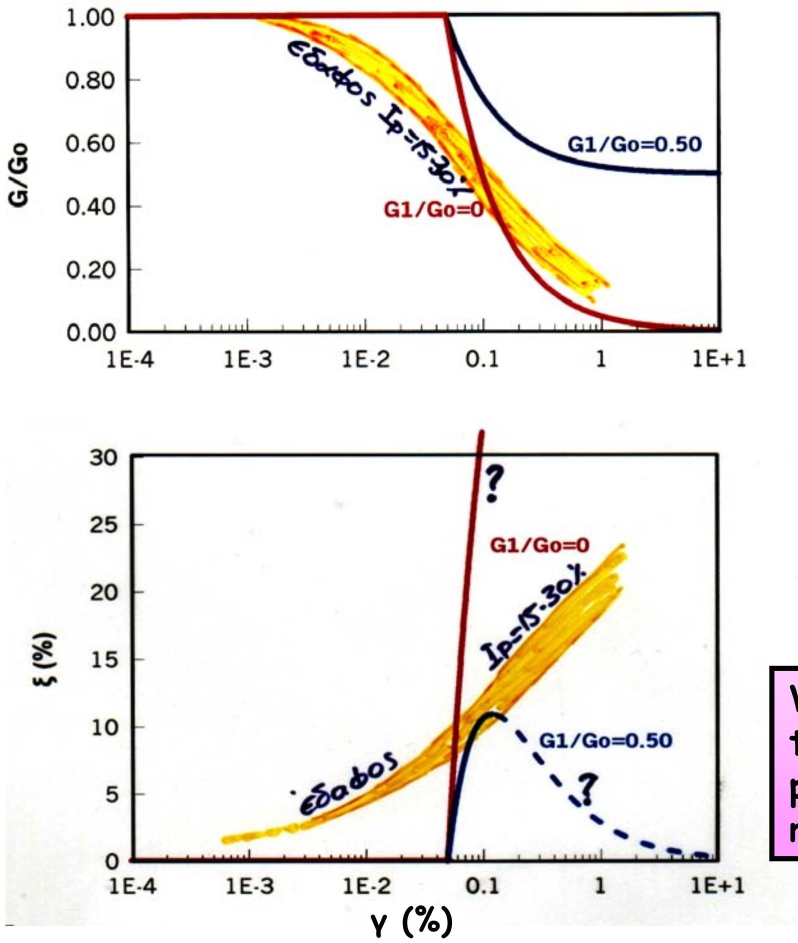
$$E = \frac{1}{2}\tau\gamma$$

$$\xi = \frac{1}{4\pi}\frac{4(\gamma-\gamma_o)\tau_o\left(1-\frac{G_1}{G_o}\right)}{\frac{1}{2}\tau\gamma} =$$

$$= \frac{2}{\pi}\left(1-\frac{\gamma_o}{\gamma}\right)\frac{G_o\gamma_o}{G\gamma}\left(1-\frac{G_1}{G_o}\right) \Rightarrow$$

$$\xi = \frac{2}{\pi}\frac{\left(1-\frac{\gamma_o}{\gamma}\right)\frac{\gamma_o}{\gamma}\left(1-\frac{G_1}{G_o}\right)}{\frac{G_1}{G_o}+\frac{\gamma_o}{\gamma}\left(1-\frac{G_1}{G_o}\right)}$$



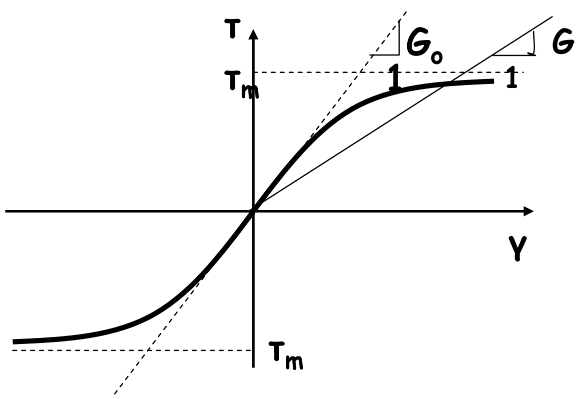


Comparison with experimental data

What are the consequences of the observed deviations for the prediction of seismic ground response?

"SIMPLE" HYSTERETIC MODELS

(α) The "hyperbolic" model



monotonic
loading - unloading

$$\gamma = \frac{\tau}{G_o}\frac{1}{1-\frac{|\tau|}{|\tau_m|}}$$

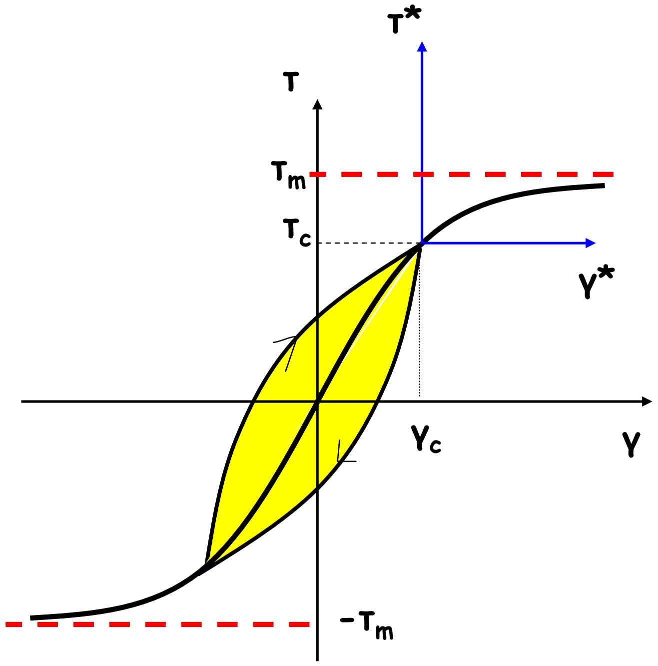
ή $$\tau = \frac{G_o\gamma}{1+\frac{G_o}{|\tau_m|}|\gamma|} \Rightarrow$$

$$G/G_o = \left[1+\left(G_o/|\tau_m|\right)|\gamma|\right]^{-1}$$

The above relation for the G/G_o ratio is more or less valid regardless of the unloading-reloading scheme which will be chosen to simulate cyclic loading (see the following paragraphs)

Cycllic unloading – reloading according to MASING (1926)

(it is popular as it always leads to closed and symmetric loops)

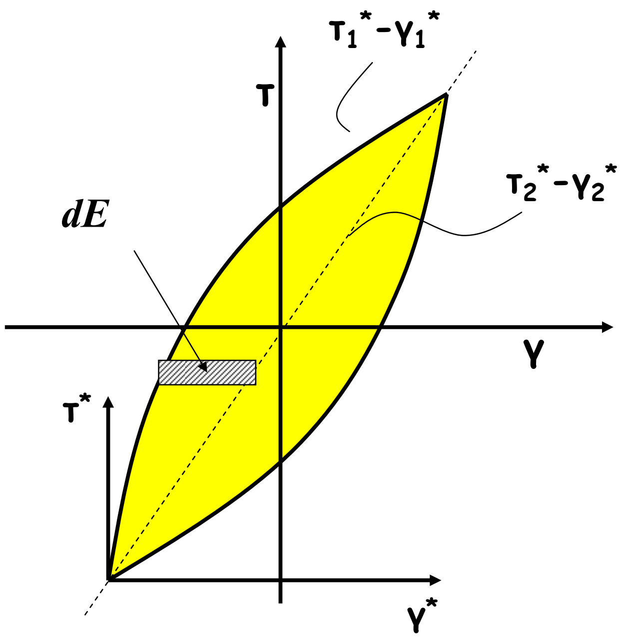


$\gamma^* = \gamma - \gamma_c$ $\quad \tau^* = \tau - \tau_c$

$G_o^* = G_o$ $\quad \tau_m^* = 2\tau_m$

$$\text{άρα} \quad \gamma - \gamma_c = \frac{\tau - \tau_c}{G_o} \frac{1}{1 - \frac{|\tau - \tau_c|}{2\tau_m}}$$

$$\text{ή} \quad \tau - \tau_c = \frac{G_o(\gamma - \gamma_c)}{1 + \left(\frac{G_o}{2\tau_m}\right)|\gamma - \gamma_c|}$$



$$\gamma_1^* = \frac{\tau^*}{G} = \frac{\tau^*}{G_o\left(1 - \frac{2\tau_c}{2\tau_m}\right)} = \frac{\tau^*}{G_o\left(1 - \frac{\tau_c}{\tau_m}\right)}$$

$$\gamma_1^* = \frac{\tau^*}{G_o} \frac{1}{1 - \frac{\tau^*}{2\tau_m}}$$

$$\frac{dE}{d\tau^*} = \gamma_2^* - \gamma_1^* = \frac{\tau^*}{G_o}\left[\frac{1}{1 - \frac{\tau^*}{2\tau_m}} - \frac{1}{1 - \frac{\tau_c}{\tau_m}}\right] =$$

$$= \frac{\tau^*}{G_o}\left[\frac{2\tau_m}{2\tau_m - \tau^*} - \frac{2\tau_m}{2\tau_m - 2\tau^*}\right] =$$

$$= \frac{\tau^*}{G_o}\left[\frac{2\tau_m^2 - 2\tau_m\tau_c - 2\tau_m^2 + \tau^*\tau_m}{(2\tau_m - \tau^*)(\tau_m - \tau_c)}\right]$$

$$\frac{dE}{d\tau^*} = \frac{\tau^* \tau_m}{G_o\left(\tau_m - \tau_c\right)} \frac{\left(\tau^* - 2\tau_c\right)}{2\tau_m - \tau^*} \Rightarrow$$

$$\Delta E = \frac{2\tau_m}{G_o\left(\tau_m - \tau_c\right)} \int_0^{2\tau_c} \frac{\tau^*\left(\tau^* - 2\tau_c\right)}{2\tau_m - \tau^*} d\tau^*$$

$$E = \frac{1}{2}\tau_c \gamma_c = \frac{1}{2}\tau_c^2 \big/ G = \frac{1}{2}\frac{\tau_c^2}{G_o\left(1 - \frac{\tau_c}{\tau_m}\right)}$$

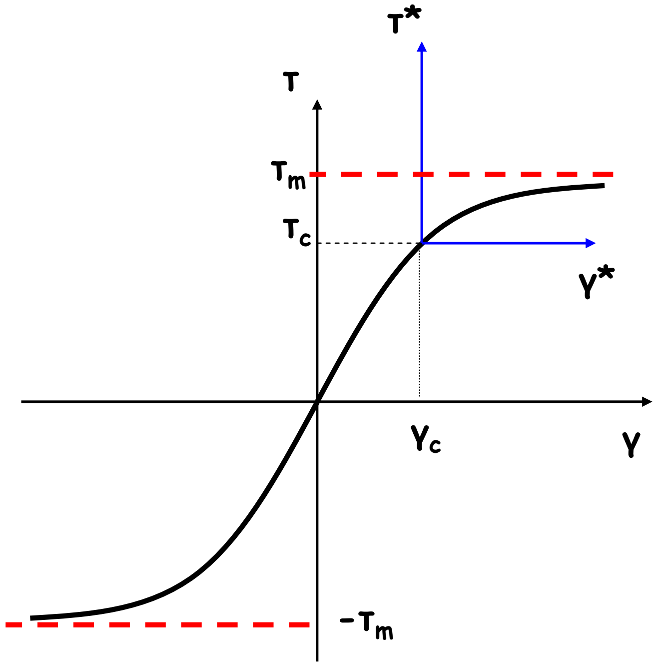
$$\xi = \frac{1}{4\pi}\frac{\Delta E}{E} = \frac{2}{\pi}\left(1 + 2\alpha\right) + \frac{4}{\pi}\left(\alpha^2 + \alpha\right)\ln\frac{G}{G_o} =$$

$$\xi = \frac{2}{\pi}\left[1 - 2\alpha \ln\frac{G}{G_o}\right]$$

$$with \quad a = \frac{\tau_m}{G_o}\frac{1}{\gamma} \quad \kappa\alpha\iota \quad \frac{G}{G_o} = \frac{1}{1 + 1/a}$$

Cyclic unloading – reloading according to PYKE (1979)

(it may be simpler, but does not provide closed and symmetric hysteresis loops)



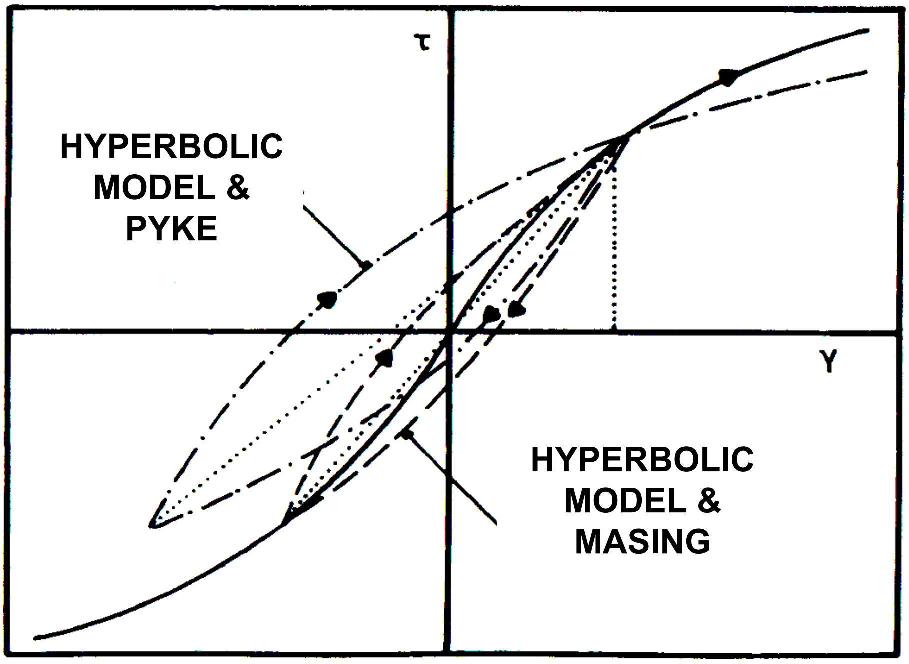
$$\gamma^* = \gamma - \gamma_c$$

$$\tau^* = \tau - \tau_c$$

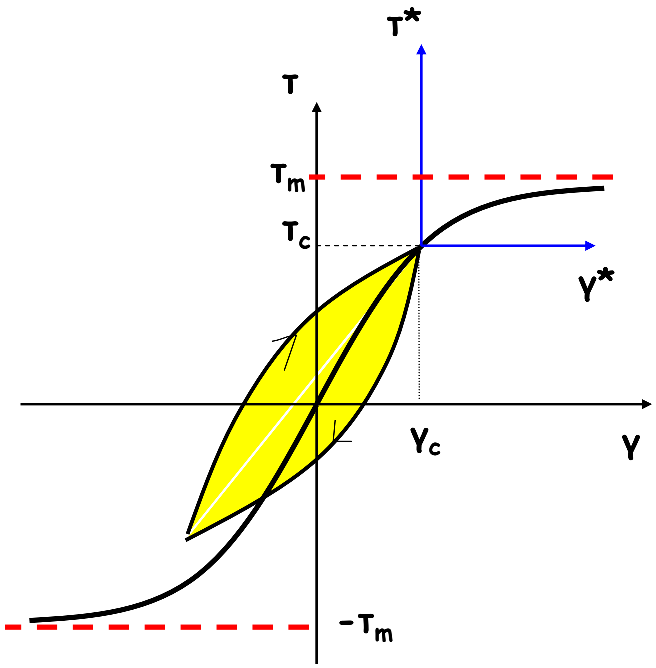
$$G_o^* = G_o$$

$$\tau_m^* = \tau_m + \left|\tau_c\right| = \tau_m\left(1 + \frac{\left|\tau_c\right|}{\tau_m}\right)$$

Representative hysteresis loops for the hyperbolic model with unloading-relaoding according to Masing and according to Pyke



Cyclic unloading – reloading according to PYKE (1979)



$$\gamma^* = \gamma - \gamma_c$$

$$\tau^* = \tau - \tau_c$$

$$G_o^* = G_o$$

$$\tau_m^* = \tau_m + |\tau_c| = \tau_m \left(1 + \frac{|\tau_c|}{\tau_m}\right)$$

$$\acute{\alpha}\rho\alpha \quad \gamma - \gamma_c = \frac{\tau - \tau_c}{G_o} \frac{1}{1 - \dfrac{|\tau - \tau_c|}{\tau_m \left(1 + \dfrac{|\tau_c|}{\tau_m}\right)}}$$

$$\acute{\eta} \qquad \tau - \tau_c = \frac{G_o (\gamma - \gamma_c)}{1 + \left(\dfrac{G_o}{\tau_m + |\tau_c|}\right) |\gamma - \gamma_c|}$$

similarly . . .

$$\frac{G}{G_o}=\frac{a^2(a+2)}{a^2+4a+2}$$

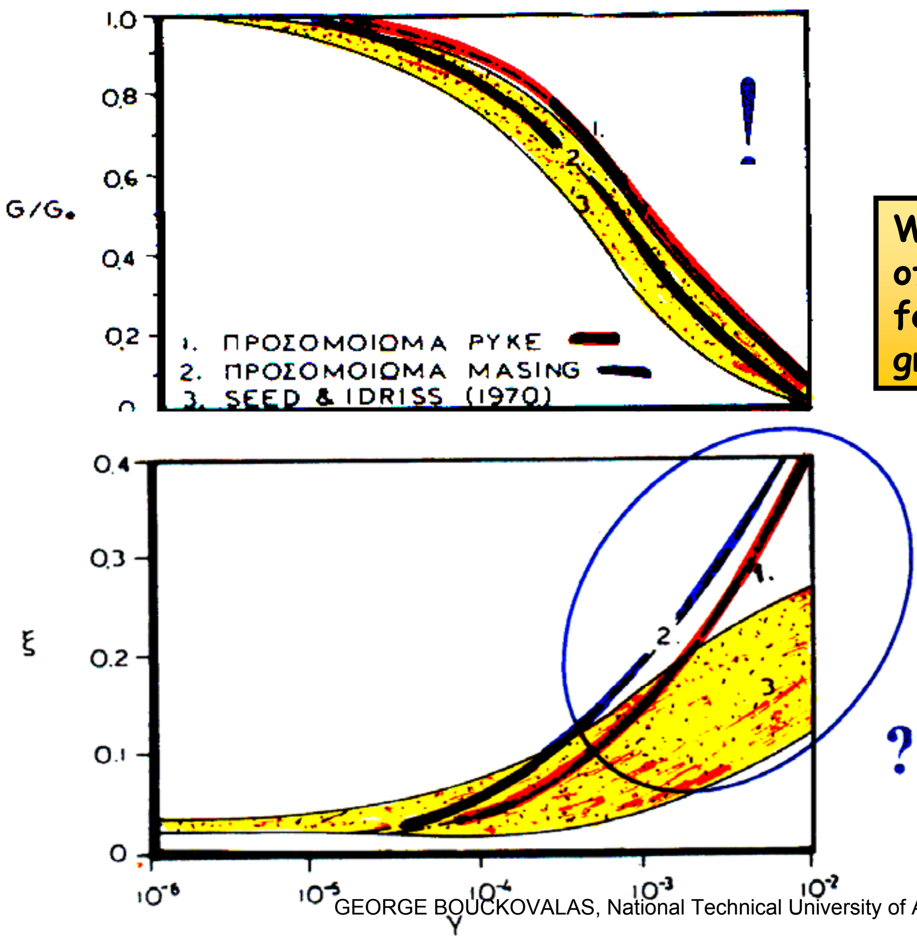
$$\xi=\frac{\alpha+1}{2\pi}\left[-\frac{4C_1}{C_1a+2}+2C_1+2C_2-C_1^2a\ln\left(1+\frac{2}{C_1a}\right)-C_2^2a\ln\left(1+\frac{2}{C_2a}\right)\right]$$

$$a=\frac{\tau_m}{G_o}\frac{1}{\gamma}$$

$$C_1=1+\frac{1}{a+1}$$

$$C_2=1-\frac{1}{a+1}+\frac{2a+4}{a^2+4a+2}$$

Comparison with experimental data …………



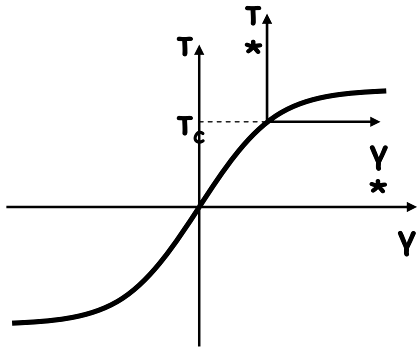
What are the consequences of the observed deviations for the prediction of seismic ground response?

(b) Ramberg-Osgood (1943)

Monotonic loading

$$\gamma = \frac{\tau}{G_{\max}}\left[1+\left(\frac{1}{a_y}-1\right)\left(\frac{|\tau|}{\tau_1}\right)^{w-1}\right]$$

$$for \quad \left.\begin{aligned}\alpha_y &= 0.64\\ w &= 2\end{aligned}\right\} \quad \gamma = \frac{\tau}{G_{\max}}\left(1+0.5625\frac{|\tau|}{\tau_1}\right)$$



Unloading-reloading

$$\gamma^* = \gamma - \gamma_c$$
$$\tau^* = \tau - \tau_c$$
$$\tau_1^* = 2\tau_1$$

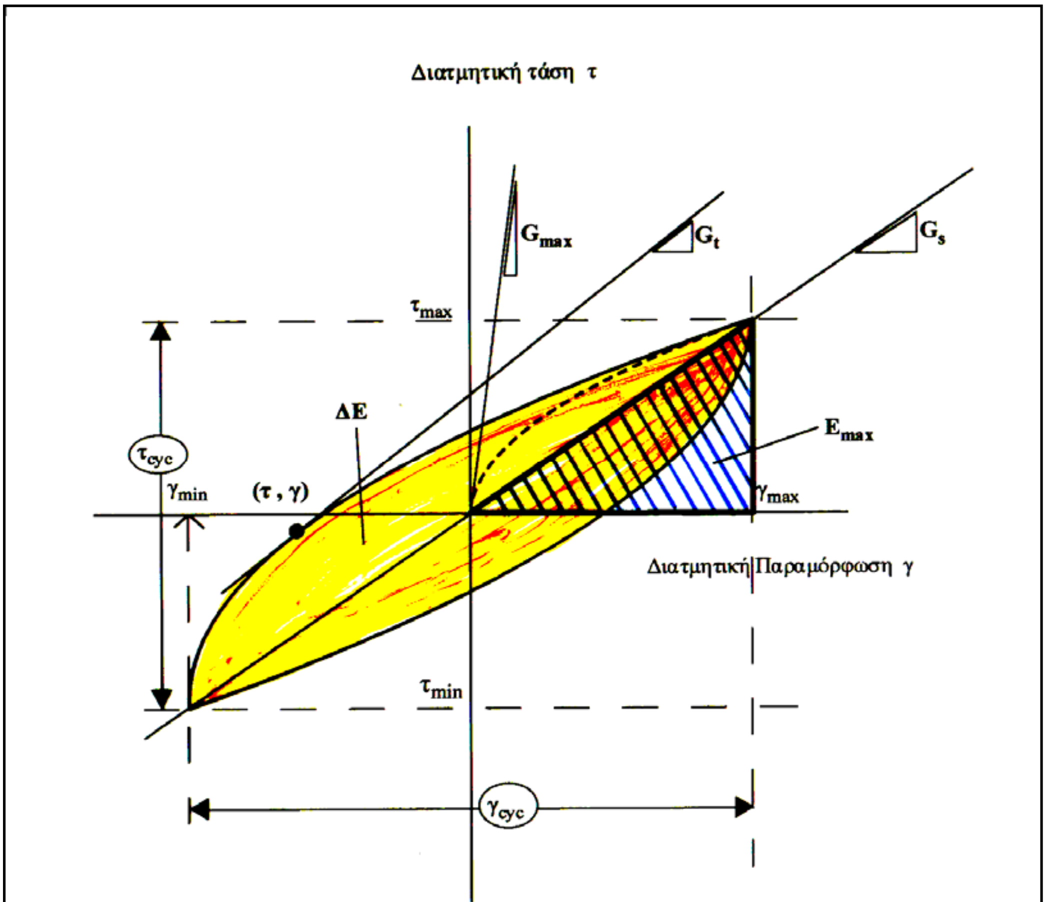
$$\frac{G}{G_{\max}} = \frac{1}{1+\left(\frac{1}{a_y}-1\right)\left|\frac{\tau_c}{\tau_1}\right|^{w-1}}$$

$$\xi = \frac{2}{\pi}\left(\frac{w-1}{w+1}\right)\left(1-\frac{G}{G_{\max}}\right)$$

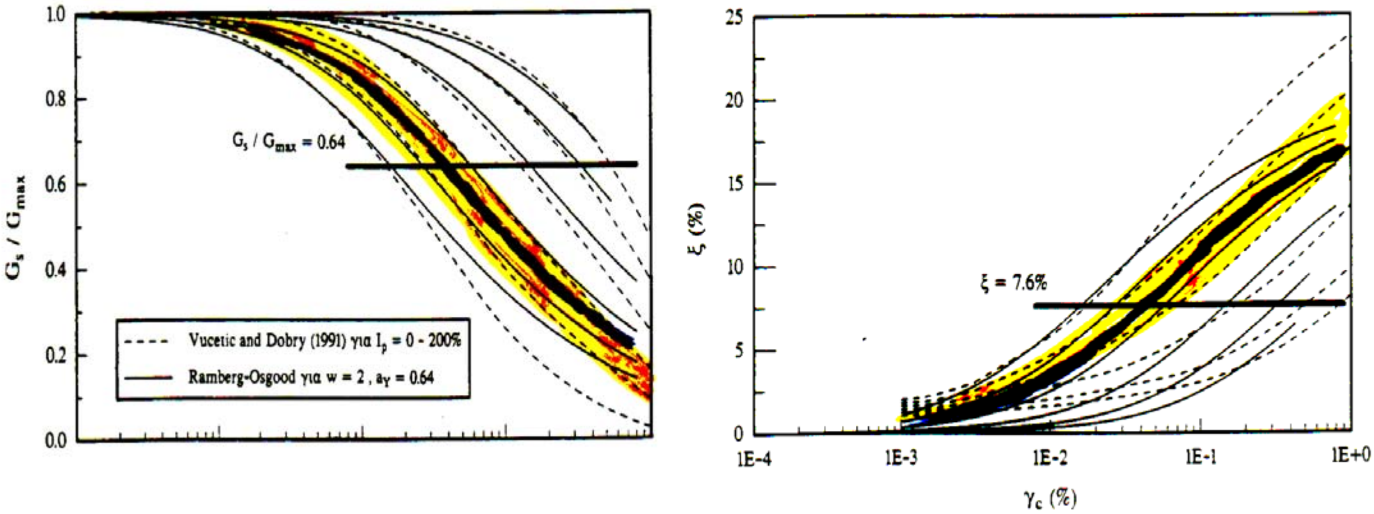
$$\left.\begin{aligned}a_y &= 0.64\\ w &= 2\end{aligned}\right\} \frac{G}{G_{max}} = \frac{1}{1+0.56\left|\frac{\tau_c}{\tau_1}\right|}$$

$$\xi = \frac{2}{3\pi}\left(1-G/G_{max}\right)$$

Representative hysteresis loop for Ramberg-Osgood model



Comparison with experimental data ....



Fair agreement with the experimental data is possible, both for G/Go and ξ-γ, following a proper selection of the model parameters. This is not possible with any of the models presented earlier.

Final Comments on Numerical Methods

The nonlinear analysis with time integration is free from any basic approximations and, , in theory at least, it may provide higher accuracy. However, in practice, it is also subjected to some important limitations which need to be accounted for during application.

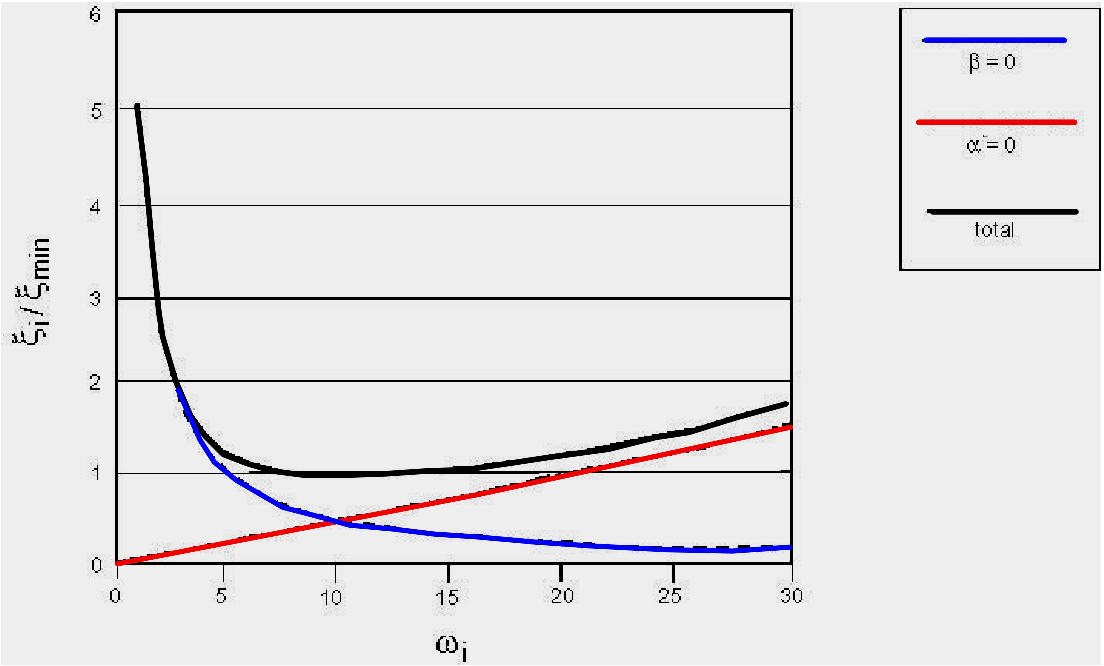
First, we must make sure that **the code that we are using is equipped with the proper constitutive model** for the simulation of cyclic soil response. In addition, extra caution is required for assigning the correct critical damping ratio ξ_o (or D_o) at very small shear strain amplitudes, since this parameter is frequency dependent (Rayleigh damping) and may obtain erroneously high values if we are not careful (e.g. see figure of next page).

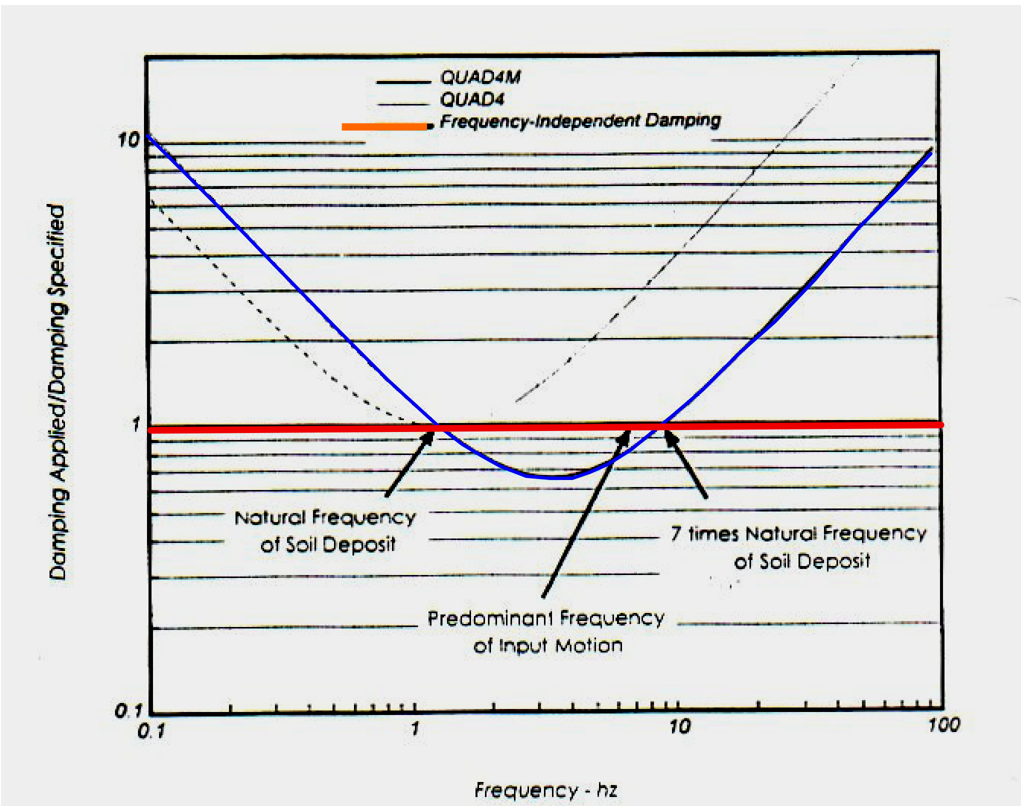
In any case, we have to admit that this methodology is the most reliable for applications where intense soil non-linearity is anticipated (e.g. very strong seismic excitations and/or very soft soils).

The basic advantage of the equivalent-linear analysis in the frequency domain is simplicity. On the other hand, this method **violates one basic law of Mechanics**: it applies supperposition of the harmonic components of ground response despite that soil response during seismic loading is non-linear. As a result, the contribution of high-frequency components is underestimated while the contribution of the low frequency components is over estimated. Nevertheless, these effects are not significant, and may be readily overlooked, as long as the shear strain amplitude in the ground does not exceed about 0.03-0.10%.

Example of Rayleighl damping ratio variation with excitation frequency

$$C = \alpha M + \beta K$$

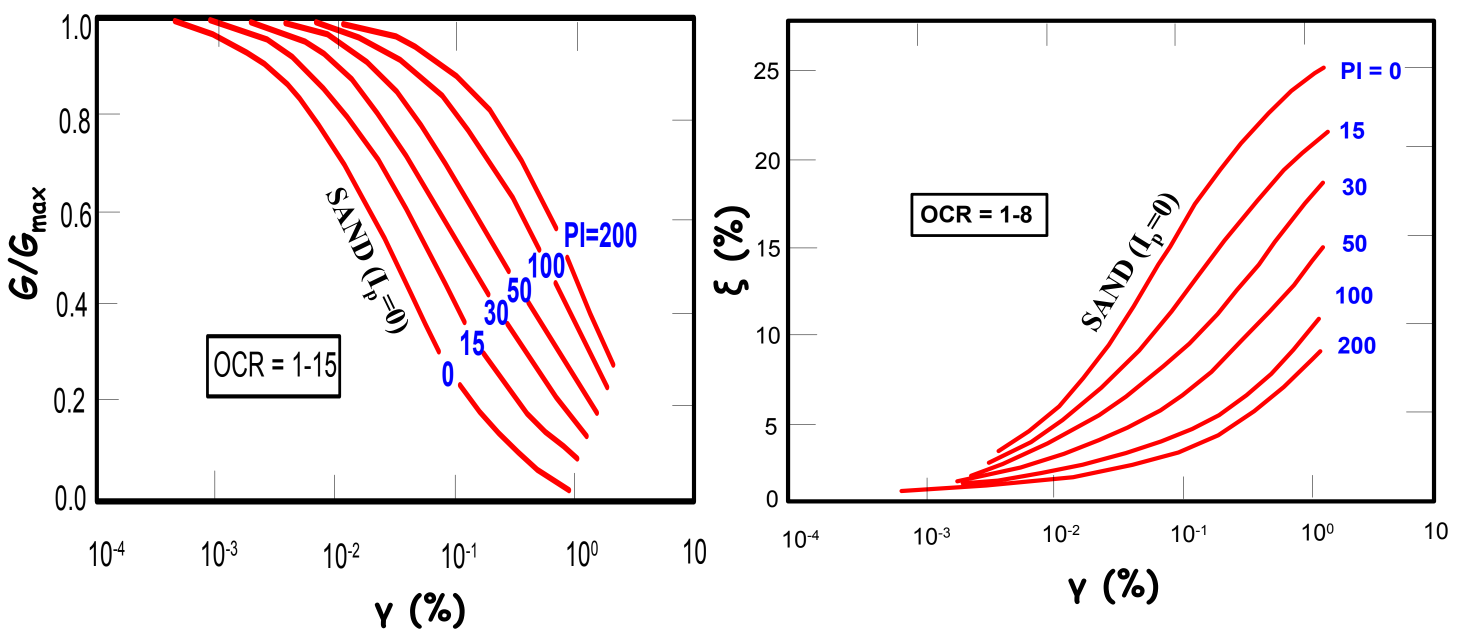




Homework 4.2

- Fit the elastic – perfectly plastic model to the experimental curves for G/G_{max} of Vuccetic & Dobry for G/G_{max}=0.50, for the different values of plasticity index PI. In the sequel, compare the theoretical and the experimental G/G_{max} – γ and ξ-γ relations.
- What is the maximum ξ value predicted with the various theoretical models presented here? How does this value compare to the maximum experimental values? How important are the observed differences for the prediction of seismic ground response?

Experimental curves for the G/Go-γ and ξ-γ relations *(Vucetic & Dobry, 1991)*



Effect of soil type (through I_P ή PI)

Homework 4.3:
Soil effects in Lefkada, Greece (2003) earthquake

The accompanying figures provide the basic data with regard to the recent (2003) strong motion recording in the island of Lefkada:

-Acceleration time histories and elastic response spectra (5% structural damping) from the two horizontal seismic motion recordings on the ground surface.

-Acceleration time histories and elastic response spectra (5% structural damping) for the two horizontal seismic motion recordings on the surface of the outcropping bedrock, as computed with a non-linear numerical analysis

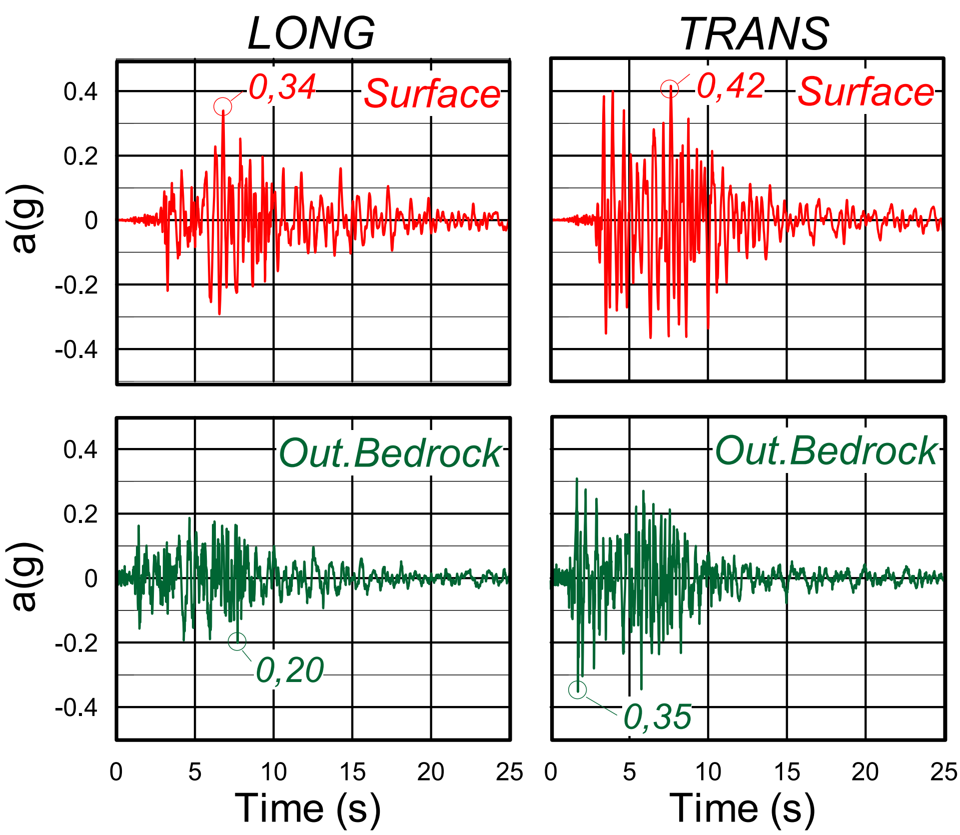
-Soil profile at the recording site.

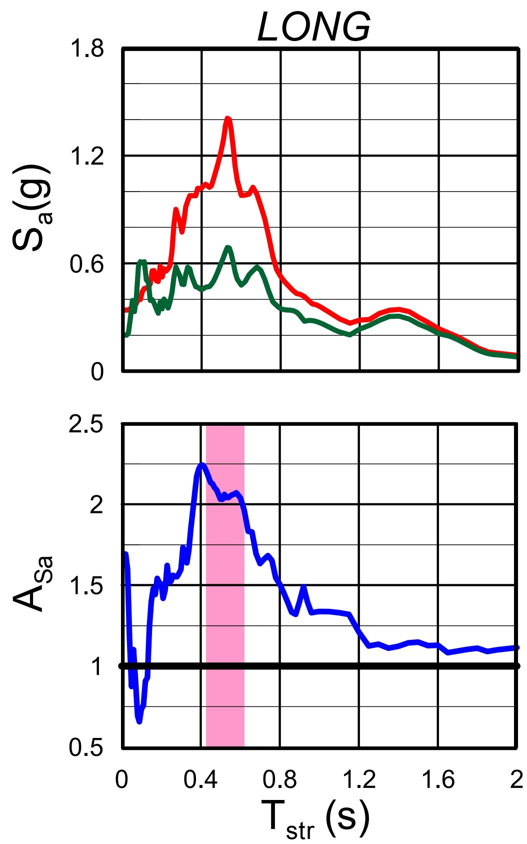
(a) Using the equivalent linear method of analysis, COMPUTE the peak seismic acceleration and the elastic response spectra at the free ground surface, using as input the seismic recordings at the outcropping bedrock.

Compare with the actual recordings and comment on causes of any observed differences.

(b) Repeat your computations assuming that soil response is elastoplastic (see Hwk 4.2) and compare with the predictions of (a) above. Comment on the observed differences.

(*NOTE: Choose the LONG component of seismic motion for your computations*)





Εδαφική Τομή στην Θέση καταγραφής