

NUMERICAL EVALUATION OF LATERAL SPREADING DISPLACEMENTS IN LAYERED SOILS

Alexandros VALSAMIS¹, George BOUCKOVALAS², Vasiliki DIMITRIADI³

ABSTRACT

The seismic design of earthworks and foundations against liquefaction-induced lateral spreading requires knowledge of the associated maximum ground displacement and its variation with depth. This article focuses upon the numerical prediction of these basic design parameters for the case of a gently inclined ground surface and a uniform liquefiable soil profile or a layered profile with non-liquefiable soil inclusions. The comparisons show a reasonable overall consistency between empirical and numerical predictions of the anticipated maximum ground displacement, provided that the effect of layering is properly taken into account in estimating the total thickness of liquefiable soil. On the other hand, the pattern of ground displacement variation with depth seems to be seriously affected by the presence of non-liquefiable soil layers, on top and within the liquefiable soil profile. Guidelines and an approximate methodology are provided in order to account for these effects in the case of simple 2-and 4-layered soil profiles.

Keywords: Liquefaction, Lateral Spreading, Ground Displacements, Prediction methods.

INTRODUCTION

"Lateral spreading" refers to the development of large horizontal ground displacements due to earthquake induced liquefaction, in the case of even small free ground surface inclination or small topographic irregularities (e.g. river and lake banks). Recent earthquakes have shown that this phenomenon is of significant practical importance for civil engineering structures (quay walls, bridge piers, etc) as it imposes considerable lateral loads and may lead to wide spread failures. There are methods today which can be used for the safe design of such structures against lateral spreading. However, their efficiency depends greatly on our ability to estimate the anticipated lateral ground displacements and their distribution with depth. In this context, the scope of this paper is to explore the capacity of presently available empirical and numerical methods to predict these design parameters. Emphasis is given to the common practical case of layered soil profiles, with an interchange of liquefiable and non-liquefiable soil layers.

Based on a literature survey, it was possible to collect eight (8) independent empirical relations for the prediction of ground surface displacements, three (3) having a "seismological" origin (Rauch & Martin 2000, Youd et al. 2002 and Zhang & Zhao 2005) and the remaining five (5) having a "engineering" origin (Hamada 1986, Shamoto et al. 1998, Hamada 1999, Zhang et al. 2004, Aydan et al. 2005). Furthermore, two different assumptions were located regarding the displacement distribution with depth: the linear (e.g. Sento et al., 1999) and the sinusoidal (e.g. Ishihara & Cubrinovski 1998, Tokimatsu 1999, Towhata 2005). Note that the terms "seismological" and "engineering" used earlier

¹ Civil Engineer, Graduate student, National Technical University of Athens, Greece, Email: <u>valsamis@central.ntua.gr</u>

² Professor, School of Civil Engineering, National Technical University of Athens, Greece, Email: <u>gbouck@central.ntua.gr</u>

³ Civil Engineer, National Technical University of Athens, Greece

are definitely informal, and are merely used here in order to distinguish the methods which rely on a Magnitude (M) and epicentral distance (R) description of the intensity of ground shaking from those which use the peak seismic motion acceleration and velocity (PGA and PGV) instead.

In addition to the empirical relations, a large number of parametric numerical analyses were performed, for a wide variety of seismic excitations, liquefiable soil properties and layering. The methodology which is used for the numerical analyses has been recently developed at NTUA, with the aim to predict large ground and foundation displacement induced by liquefaction, as it is presented in more detail by Andrianopoulos et al. (2007) in another paper of this conference. The accuracy of selected empirical methods, as well as the numerical predictions for the specific case of lateral spreading is evaluated herein, through comparison with results from seventeen (17) well documented centrifuge tests, also collected from the literature.

EMPIRICAL RELATIONS FOR LATERAL SPREADING

Relations for Ground Surface Displacement

Mainly due to length limitations, only three of the available eight empirical methods for the prediction of maximum ground surface displacements will be discussed in the following, namely those proposed by Youd et al. (2002), Hamada (1999) and Shamoto et al. (1998).

Youd and Perkins (1987), based on several case histories from large earthquakes in the Western United States and Alaska, proposed one of the first simple, "seismological" relations. Later on, Bartlett and Youd (1992, 1995) enriched the above database, gathering a total of 467 displacement measurements from Japan and U.S. earthquakes. Based on these data, Youd et al. (2002) consequently proposed relations (1) and (2) for the computation of liquefaction-induced maximum lateral displacements in the case of a gently sloping ground and a (free-face) step topography respectively:

$$\log D_{H} = -16.213 + 1.532M - 1.406 \log R^{*} - 0.012R + 0.338 \log S + + 0.540 \log T_{15} + 3.413 \log(100 - F_{15}) - 0.795 \log(D_{5015} + 0.1mm)$$
(1)

$$\log D_{H} = -16.713 + 1.532M - 1.406 \log R^{*} - 0.012R + 0.592 \log W + + 0.540 \log T_{15} + 3.413 \log(100 - F_{15}) - 0.795 \log(D_{5015} + 0.1mm)$$
(2)

| where, | $D_{H}(m)$ | is the lateral ground displacement |
|--------|------------------------|--|
| | Μ | is the moment magnitude of the earthquake |
| | R (km) | is the nearest horizontal or map distance from the site to the seismic |
| | | energy source |
| | $R^{*}=R_{o}+R(km)$ | where $R_0 = 10^{(0.89M-5.64)}$ |
| | $T_{15}(m)$ | is the cumulative thickness of saturated granular layers with corrected |
| | | blow counts, $(N_1)_{60}$, less than 15 |
| | F_{15} (%) | is the average fines content for granular materials included within T_{15} |
| | D ₅₀₁₅ (mm) | is the average mean grain size for granular materials within T_{15} |
| | S (%) | is the ground slope |
| | W (%) | is the free-face ratio defined as the height (H) of the free face divided by |
| | | the distance (L) from the base of the free face to the point of interest |

The earliest "engineering" relation which was located in the literature was that proposed by Hamada (1986). More recently Hamada (1999) upgraded his relation by indirectly including the effects of soil density and earthquake intensity:

$$D_{H} = \frac{0.0125(H)^{0.5} \mathcal{G}}{\overline{N}^{0.88}} \sum a_{i}^{0.48} T_{i}$$
(3)

| where, | H (m) | is the thickness of the liquefied soil layer, and |
|--------|-----------------|--|
| | θ (%) | is the ground inclination |
| | N | is the average corrected SPT blow count for the liquefied layer. |
| | $\alpha_i(gal)$ | is the mean horizontal acceleration in the i part of the acceleration time history |
| | $T_i(sec)$ | is the time length of the i part of the acceleration time history |

Following an entirely different approach, Shamoto et al. (1998), correlate the maximum horizontal displacement with the residual shear strain γ_r after post-liquefaction drained consolidation or with $(\gamma_r)_{max}$ which is the residual shear strain potential corresponding to any given value of the maximum double shear strain amplitude γ_{max} :

$$D_{H} = \int_{0}^{H} \int \gamma_{r} dz \approx 0.16 \int_{0}^{H} \int (\gamma_{r})_{\max} dz$$
(4)

where z is the current depth (m), while $(\gamma_r)_{max}$ is estimated on the basis of empirical charts in terms of the corrected SPT blow count \overline{N} and the shear stress ratio τ/σ' imposed by the of the earthquake.

Relations for Ground Displacement Variation with Depth

Two different relations for the prediction of the ground displacement variation with depth are encountered in the literature. The first implies a linear decrease with depth, from a maximum value at the ground surface to zero at the maximum depth of liquefied soil (e.g. Sento et al., 1999). The second relation is similar, only that the decrease is sinusoidal and can be expressed as (Ishihara & Cubrinovski 1998, Tokimatsu 1999, Towhata 2005):

$$D(z) = D_H \sin \frac{\pi (H-z)}{2H}$$
(5)

where, z(m) is the current depth and

H (m) is the thickness of the liquefied soil layer

The above relations are strictly applicable to a single liquefied soil layers. In presence of a nonliquefiable soil layer on top of the liquefiable one, it is reasonably assumed that the non-liquefied surface layer follows the maximum horizontal displacement of the liquefied layer immediately underneath. The possible extension of these relations to multi-layered formations, where there is interchange between liquefiable and non-liquefiable layers, is not clear.

NUMERICAL SIMULATION OF LATERAL SPREADING

As part of this study, lateral spreading displacements due to gently inclined ground were reproduced numerically, in a large number of paramentric analyses for various combinations of ground surface inclination, soil layering, relative density of liquefiable sand, as well as peak seismic excitation accelation and frequency. The numerical analyses were performed with the aid of the non-linear Finite Difference method combined with an effective stress constitutive model which can simulate the static and the dynamic response of cohesionless soils, including liquefaction. In brief, this is a bounding surface model, with a vanished elastic region, which was built based upon the Critical State Theory framework (Papadimitriou et al., 2001, Andrianopoulos et al. 2006a, 2006b and 2007). One of its main characteristics is that the monotonic and cyclic response of soils is described using a single set parameters which is soil-specific, but does not depend on the initial stress and density conditions. A more explicit presentation of the constitutive model and its numerical implementation to liquefaction related problems is provided by Andrianopoulos et al. (2007) in this conference.

In order to evaluate the capacity of the aforementioned numerical methodology to predict the relatively large displacements induced by lateral spreading, it was systematically used to reproduce the results of several relevant centrifuge tests, summarized in Table 1. Note that, the total (prototype) soil thickness assumed in all tests was 10m. Nevertheless, for Taboada & Dobry (1998) and Sharp et al (2003) this thickness corresponds entirely to liquefiable sand, while for Abdoun (1998) the soil profile is layered, consisting of 2m of non-liquefiable partially cemented Nevada sand above and beneath a 6m thick liquefiable layer of Nevada sand. The discretized soil profile which was used for the numerical simulation of the centrifuge tests is shown in Figure 1. It consists of 280 equal square elements, 1.0x1.0m in dimension, with the lateral boundaries tied to one-another in order to ensure that they will have the same horizontal displacements, as is the case with shaking into a laminar box container. The seismic excitation was imposed as an acceleration time history at the base of the above soil profile.

| Test name | Publication | Pore Pressure fluid | D _r (%) | Slope angle | a _{max} (in base) | N _{cycle} | f (Hz) | Thick. of liq. layer | Lateral ground disp.(cm) |
|-----------|------------------------|---------------------------|-----------------------|----------------|-------------------------------|--------------------|---------------|-------------------------|--------------------------------|
| M2-1 | Taboada & Dobry (1998) | water | 40-45 | 2 | 0.18 | 21.5 | 2 | 10.0 | 44.0 |
| M2-2 | Taboada & Dobry (1998) | water | 40-45 | 1.94 | 0.23 | 22 | 2 | 10.0 | 47.0 |
| M2-3 | Taboada & Dobry (1998) | water | 40-45 | 2.18 | 0.46 | 22.5 | 2 | 10.0 | 97.0 |
| M2-4 | Taboada & Dobry (1998) | water | 40-45 | 2.07 | 0.19 | 22 | 1 | 10.0 | 61.0 |
| M2-5 | Taboada & Dobry (1998) | water | 40-45 | 2 | 0.25 | 22 | 1 | 10.0 | 68.0 |
| M2a-3 | Taboada & Dobry (1998) | water | 40-45 | 0.6 | 0.28 | 21.5 | 2 | 10.0 | 12.2 |
| M2a-4 | Taboada & Dobry (1998) | water | 40-45 | 0.6 | 0.26 | 22 | 2 | 10.0 | 14.8 |
| M2b-5 | Taboada & Dobry (1998) | water | 40-45 | 0.8 | 0.40 | 22.5 | 2 | 10.0 | 30.0 |
| M2c-6 | Taboada & Dobry (1998) | water | 40-45 | 3.95 | 0.17 | 21.5 | 2 | 10.0 | 72.5 |
| LAM1 | Abdoun (1998) | water | 40 | 2 | 0.3 | 40.5 | 2 | 6.0 | 80.0 |
| LAM2 | Abdoun (1998) | water | 40 | 2 | 0.3 | 40.5 | 2 | 6.0 | 80.0 |
| L45V-2-10 | Sharp et al (2003) | viscous | 45 | 2 | 0.23 | 20 | 2 | 10.0 | 66.0 |
| L45V-4-10 | Sharp et al (2003) | viscous | 45 | 2 | 0.41 | 20 | 2 | 10.0 | 87.0 |
| L65V-2-10 | Sharp et al (2003) | viscous | 65 | 2 | 0.20 | 20 | 2 | 10.0 | 28.0 |
| L65V-4-10 | Sharp et al (2003) | viscous | 65 | 2 | 0.38 | 20 | 2 | 10.0 | 63.0 |
| L75V-2-10 | Sharp et al (2003) | viscous | 75 | 2 | 0.21 | 20 | 2 | 10.0 | 23.0 |
| L75V-4-10 | Sharp et al (2003) | viscous | 75 | 2 | 0.38 | 20 | 2 | 10.0 | 47.0 |

 Table 1. Summary of published centrifuge tests



Figure 1. Finite Difference mesh used for the numerical analyses



Figure 2. Typical comparison of numerical predictions and centrifuge test results



Figure 3. Numerical prediction of lateral ground displacements vs centrifuge test measurements

The numerical predictions are compared to centrifuge test measurements in Figures 2 and 3. In more detail, Figure 2 depicts a typical comparison between predicted and recorded time histories of horizontal displacement (x_d), excess pore pressure (Δu) and horizontal acceleration (x_{acc}) for centrifuge test M2-1 (Taboada & Dobry, 1998). Moreover, Figure 3 compares the predicted and recorded maximum ground displacements from all centrifuge tests listed in Table 1. These comparisons show a reasonably good, qualitative but also quantitative, consistency. The tendency of the numerical predictions to exceed recordings, observed in Figure 3, could be the result of the artificial restraint

imposed to large ground displacements by the latex membrane used to prevent leakage of the pore fluid through the walls of laminar box containers.

Using the numerical model described above we performed a total of forty (40) parametric analyses, for ground surface inclination $\theta = 0.5$ to 4 degrees, relative density of sand Dr = 35% to 90%, maximum horizontal base acceleration $a_{max} = 0.04$ to 0.4g and predominant frequency of shaking f = 1 to 10Hz. The basic engineering input parameters for these analyses are summarized in the Appendix. Note that only half of the analyses refer to uniform sand, while the other half refer to layered soil profiles with two to four alternating liquefiable and non-liquefiable layers, as shown in Figure 4. In the following, the numerical predictions of lateral ground displacements are compared to empirical predictions obtained with the empirical relations which were briefly presented before. The aim of this comparison is to check whether these two widely different approaches provide consistent results, not only for uniform but for layered soil profiles as well.



Figure 4. Types of soil profiles used in the numerical analyses (see also the Appendix)

PREDICTION OF GROUND DISPLACEMENTS

Ground Surface Displacement

The comparison between empirical and numerical predictions of lateral ground surface displacement is shown in Figures 5a, 5b and 5c for uniform, as well as for layered soil profiles. Note that, unlike the "engineering" methods of Shamoto et al. and Hamada, which could be applied directly using the input parameters summarized in the Appendix, the seismological method of Youd et al. required indirect definition of an equivalent seismic magnitude (M) and epicentral distance (R). The first of these parameters was more or less arbitrarily chosen as M = 6.8, corresponding to a common, yet strong earthquake, which is compatible with the duration of the applied shaking. The epicentral distance R was subsequently estimated from M and a_{max} , using the relevant attenuation relations of Sabetta and Pugliese (1987) for soft soil conditions.

Furthermore, the following assumptions were found most appropriate for extending the above empirical methods to layered soil profiles:

- For the methods of Youd et al. and Hamada, the required soil thickness was equal to the total thickness of the liquefied soil layers (i.e. excluding the thickness of any intermediate non-liquefiable soil layer).
- On the other hand, for the method of Shamoto et al., the required soil thickness was equal to the maximum depth of liquefaction, measured from the free ground surface.

It may be observed that the agreement between the two different sets of predictions is reasonable for all layering cases, in the sense that there is definitely scatter between the data points, but no average bias resulting in a systematic over- or under-prediction of the lateral ground displacements. Special mention is deserved for the comparison with the empirical predictions according to Hamada (1999) which is clearly associated with the least data scatter.



Figure 5. Comparison between numerical and empirical predictions of lateral ground surface displacements: (a) uniform profile, (b) 2-layered profile and (c) 4-layered profile.

Displacement Variation with Depth

The comparisons regarding the displacement variation with depth are shown in Figures 6a and 6b for the uniform and the 2-layer soil profile, and in Figure 7 for the 4-layer soil profiles. Note that the "basic" run referred in 7a we correspond to run no. 24 of the Appendix. Also note that, in all figures:

- The horizontal ground displacement at a given depth D(z) has been normalized against the maximum horizontal displacement D_H , while depth z has been normalized against the respective maximum depth of liquefied soil H_{max} . The definitions of z and H_{max} are given schematically in each figure, as they depend upon layering.
- The comparison includes the linear and the sinusoidal variations proposed in the literature, any relevant measurements from centrifuge tests, as well as the range of numerical predictions.



Figure 6. Displacement variation with depth for (a) uniform and (b) 2-layered soil profiles



Figure 7. Displacement variation with depth for 4-layered soil profiles

For the uniform soil profile, all different sets of data agree for a sinusoidal variation with depth. However, in the presence of a non-liquefiable soil cap, the numerical predictions systematically deviate from the sinusoidal variation towards the linear one, while the experimental data (one centrifuge test) remain close to the sinusoidal variation. The picture becomes considerably more complex for layered soil profiles where ground displacements remain practically constant within the non-liquefiable soil layers while they vary more or less linearly within the liquefiable layers. Thus, to predict the variation with depth in the latter case, it is necessary to predict first the proportion of the maximum ground displacement which is attributed to the lower and the upper liquefiable soil layers. From a statistical analysis of the numerical predictions, it appears that the maximum ground displacement D_H may be written in terms of the maximum displacement of the lower and the upper liquefiable layers, $D_{H, L}$ and $D_{H, U}$ respectively as:

$$D_{HL} = m \cdot D_H \tag{6}$$

$$D_{H,U} = (1 - m) \cdot D_{H,U} \tag{7}$$

where

$$n = \frac{1}{1 + 0.60(H_{II} / H_{I})}$$
(8)

and H_U and H_L denote the thickness of the upper and the lower liquefiable layer respectively. The accuracy of the simple relation of the estimation of m is verified through the comparison of Figure 8.

ľ



Figure 8. Comparison of approximate (Eqs. 6- 8) and numerical prediction of the displacement ratio of 4-layer profiles

CONCLUSIONS

In the previous sections, comparisons were shown between numerical and empirical predictions, as well as centrifuge tests measurements regarding the maximum ground displacement induced by lateral spreading and its variation with depth. Although not complete, these comparisons show that:

- (a) Numerical analysis methods have now reached a level of sophistication where they can be trusted for (very) large lateral displacement computations related to liquefaction-induced lateral spreading. However, due to the required expertise, such computations are still aimed at complex soil geometries and special projects involving interaction between the spreading ground and the foundation.
- (b) For simpler applications, reasonable estimates of the maximum ground displacements can be obtained with available empirical methods, of either "seismological" or "engineering" origin. Nevertheless, in the case of liquefiable non liquefiable soil layer interchange, special attention should be given to the proper selection of the equivalent liquefiable soil thickness which is required by each relation. Relevant guidelines for three commonly used such methods are provided in the text.
- (c) Lateral ground displacements always become maximum at the ground surface and diminish at the maximum depth of liquefied soil. The variation between these two extreme values is closely sinusoidal for uniform profiles of liquefiable soil, but tend to become more linear in the presence of a non liquefiable soil cap.

(d) For layered soil profiles, with a liquefiable – non liquefiable layer interchange, ground displacements remain practically constant within the non-liquefiable soil layers while they vary more or less linearly within the liquefiable layers. A simple methodology to describe this variation in the case of 4-layered profiles is provided in the text.

ACKNOWLEDGEMENTS

We would like to thank the General Secretariat for Research and Technology (GSRT) of Greece for funding this research through the Research Project X-Soils ($\Delta\Pi$ -23).

REFERENCES

- Abdoun T. H. (1997) "Modelling of seismically induced lateral spreading of multi-layered soil and its effect on pile foundations", PHD Thesis, Rensselaer Polytechnic Institute, Troy, New York.
- Andrianopoulos, K.I. (2006), "Numerical modeling of static and dynamic behavior of elastoplastic soils", Doctorate Thesis, Department of Geotechnical Engineering, School of Civil Engineering, National Technical University of Athens (in Greek).
- Andrianopoulos, K.I., Papadimitriou, A.G., Bouckovalas, G.D. (2006a), "Numerical analysis of geostructures in a liquefaction regime", Proceedings, 1st European Conference on Earthquake Engineering and Seismology, Geneva, September, paper No. 1245
- Andrianopoulos, K.I., Papadimitriou, A.G. and Bouckovalas, G.D. (2006b), "Implementation of a bounding surface model for seismic response of sands", Proceedings of the 4th International FLAC Symposium on Numerical Modeling in Geomechanics, Madrid, Spain
- Andrianopoulos, K.I., Papadimitriou, A.G. and Bouckovalas, G.D. (2007), "Use of a new bounding surface model for the analysis of earthquake-induced liquefaction phenomena", paper no 1443, Proceedings of 4th International Conference on Earthquake Geotechnical Engineering.
- Aydan O., Atak O.V., Ulusay R., Hamada H, and Bardet P.J. (2005), "Ground deformations and lateral spreading around the shore of Sapanca lake induced by the 1999 Kocaeli earthquake", Proceedings of Geotechnical Earthquake Engineering Satellite Conference Osaka, Japan, 10 September.
- Bartlett F. S. and Youd T. L. (1992),"Empirical analysis of horizontal ground displacement generated by liquefaction-induced lateral spread", Tech. Rep. No. NCEER-92-0021, National Center for Earthquake Engineering Research, Buffalo, N.Y., pp. 114.
- Bartlett F. S. and Youd T. L. (1995),"Empirical prediction of liquefaction-induced lateral spread", Journal of Geotechnical Engineering, Vol. 121, No. 4, April, pp 317-329.
- Hamada M., Yasuda S., Isoyama R. and Emoto K. (1986), "Study on liquefaction induced permanent ground displacements", Association for the development of earthquake prediction, Tokyo, pp. 1-87.
- Hamada M. (1999), "Similitude law for liquefied-ground flow", Proceedings of the 7th U.S.-Japan Workshop on Earthquake Resistant design of lifeline facilities and countermeasures against soil liquefaction, pp. 191-205.
- Ishihara K. and Yoshimine M. (1992), "Evaluation of settlements in sand deposits following earthquakes", Soils and Foundations, Vol. 32, No. 1, pp. 173-188.
- Ishihara K. & Cubrinovski M. (1998), "Soil-pile interaction in liquefied deposits undergoing lateral spreading", XI Danube-European Conference, Croatia, May 1998
- Papadimitriou A., Bouckovalas G. and Dafalias Y. (2001), "A plasticity model for sand under small and large cyclic strains", Journal of Geotechnical and Geoenviromental Engineering, Vol.127, No. 11
- Rauch F. A. and Martin II R. J. (2000), "EPOLLS model for predicting average displacements on lateral spreads," Journal of Geotechnical and Geoenvironmental Engineering, Vol. 126, No. 4, April, pp 360-371.
- Sabetta, F. and A. Pugliese (1987), "Attenuation of peak horizontal acceleration and velocity from Italian strong-motion records", Bull. Seism. Soc. Am., 77, pp. 1491-1513.

- Sento N., Goto K., Namba S., Kobayashi K., Oh-oka H. & Tokimatsu K. (1999), "Case study for pile foundation damaged by soil liquefaction at inland site of artificial island", Second international conference on earthquake geotechnical engineering, 21-25th June 1999, Lisbon, Portugal
- Shamoto Y., Zhang J. and Tokimatsu K. (1998), "New charts for predicting large residual postliquefaction ground deformation," Soil Dynamics and Earthquake Engineering, Vol. 17, February 18, pp 427-438.
- Sharp K. M., Dobry R. and Abdoun T. (2003) "Liquefaction centrifuge modelling of sands of different permeability", Journal of Geotechnical and Geoenvironmental Engineering, Vol. 129, No. 12, December 1, pp 1083-1091.
- Taboada V.M. and Dobry R. (1998) "Centrifuge modeling of earthquake-induced lateral spreading in sand", Journal of Geotechnical and Geoenvironmental Engineering, ASCE, Vol 124, No 12, pp. 1195-1206.
- Towhata I. (2005) "Development of geotechnical earthquake engineering in Japan", Heritage lecture, 16th International Conference on Soil Mechanics and Geotechnical Engineering (16ICSMGE), Osaka, Japan, pp. 251-291.
- Tokimatsu K. (1999), "Performance of pile foundations in laterally spreading soils", Proc. 2nd Intl. Conf. Earthquake Geotechnical Engineering (P. Seco e Pinto, ed.), Lisbon, Portugal, June 21-25, Vol 3, pp. 957-964
- Vucetic, M., and Dobry, R. (1991) Effect of soils plasticity on cyclic response, Journal of Goetechnical Engineering, ASCE, 117 (1), pp. 898-907.
- Youd L. T., Hansen M. C. and Bartlett F. S. (2002), "Revised multilinear regression equations for prediction of lateral spread displacement", Journal of Geotechnical and Geoenvironmental Engineering, Vol. 128, No. 12, December 1, pp 1007-1017.
- Youd L. T. and Perkins M. D. (1987), "Mapping of liquefaction severity index", Journal of Geotechnical Engineering, Vol. 113, No.11, November, pp 1374-1392.
- Zhang G., Robertson K. P. and Brachman I. W. R. (2004), "Estimating liquefaction-induced lateral displacements using the standard penetration test or cone penetration test," Journal of Geotechnical and Geoenvironmental Engineering, Vol 130, No. 8, August 1, pp 861-871.
- Zhang J. and Zhao X.J. (2005), "Empirical models for estimating liquefaction-induced lateral spread displacement," Soil Dynamics and Earthquake Engineering, Vol. 25, pp 439-450.

| No | Slope a _{max} | | 1 | | Thickness | Thick. of | Lateral | | |
|-----|------------------------|-----------|--------------------|-----------|----------------|-----------|-----------------------|-----------|-----------|
| of | angle | (in base) | N _{cycle} | † (⊔⇒) | D _r | | of | liquefied | ground |
| run | (°) | (g) | | (ПZ) | (70) | layers | layers ⁽²⁾ | layer(s) | disp (cm) |
| 1 | 2 | 0,12 | 20 | 2 | 45,0 | 1 | 10 | 9 | 87.32 |
| 2 | 2 | 0,08 | 20 | 2 | 45,0 | 1 | 10 | 9 | 70,34 |
| 3 | 2 | 0,2 | 20 | 2 | 45,0 | 1 | 10 | 10 | 106,70 |
| 4 | 2 | 0,3 | 20 | 2 | 45,0 | 1 | 10 | 10 | 137,50 |
| 5 | 2 | 0,4 | 20 | 2 | 45,0 | 1 | 10 | 10 | 145,00 |
| 6 | 2 | 0,12 | 20 | 10 | 45,0 | 1 | 10 | 10 | 8,03 |
| 7 | 2 | 0,12 | 20 | 4 | 45,0 | 1 | 10 | 9 | 48,99 |
| 8 | 2 | 0,12 | 20 | 1,5 | 45,0 | 1 | 10 | 10 | 111,80 |
| 9 | 2 | 0,12 | 20 | 1 | 45,0 | 1 | 10 | 10 | 126,1 |
| 10 | 0.5 | 0,12 | 20 | 2 | 45,0 | 1 | 10 | 10 | 37,20 |
| 11 | 1 | 0,12 | 20 | 2 | 45,0 | 1 | 10 | 10 | 57,35 |
| 12 | 4 | 0,12 | 20 | 2 | 45,0 | 1 | 10 | 9 | 139,60 |
| 13 | 2 | 0,12 | 20 | 2 | 45,0 | 4 | 2/1/2/5 | 1up/5down | 56,66 |
| 14 | 2 | 0,12 | 20 | 2 | 35,0 | 1 | 10 | 10 | 145,20 |
| 15 | 2 | 0,12 | 20 | 2 | 55,0 | 1 | 10 | 9 | 60,90 |
| 16 | 2 | 0,12 | 20 | 2 | 65,0 | 1 | 10 | 9 | 53,50 |
| 17 | 2 | 0,12 | 20 | 2 | 75,0 | 1 | 10 | 9 | 41,00 |
| 18 | 2 | 0,12 | 20 | 2 | 45,0 | 2 | 1/9 | 8 | 86,30 |
| 19 | 2 | 0,12 | 20 | 2 | 45,0 | 2 | 2/8 | 7 | 82,30 |
| 20 | 2 | 0,12 | 20 | 2 | 45,0 | 2 | 4/6 | 5 | 70,40 |
| 21 | 2 | 0,04 | 20 | 2 | 45,0 | 1 | 10 | 9 | 46,10 |
| 22 | 2 | 0,12 | 20 | 2 | 90,0 | 1 | 10 | 8 | 35,40 |
| 23 | 2 | 0,04 | 20 | 2 | 90,0 | 1 | 10 | 7 | 13,80 |
| 24 | 2 | 0,12 | 20 | 2 | 45,0 | 4 | 2/3/2/3 | 3up/2down | 59,60 |
| 25 | 2 | 0,12 | 20 | 2 | 75,0 | 4 | 2/3/2/3 | 3up/2down | 32,30 |
| 26 | 1 | 0,12 | 20 | 2 | 45,0 | 4 | 2/3/2/3 | 1up/3down | 34,80 |
| 27 | 2 | 0,3 | 20 | 2 | 45,0 | 4 | 2/3/2/3 | 3up/3down | 93,80 |
| 28 | 2 | 0,12 | 20 | 1 | 45,0 | 4 | 2/3/2/3 | 3up/3down | 86,40 |
| 29 | 2 | 0,3 | 20 | 2 | 45,0 | 2 | 2/8 | 8 | 119,00 |
| 30 | 2 | 0,12 | 20 | 1 | 45,0 | 2 | 2/8 | 8 | 120,8 |
| 31 | 1 | 0,12 | 20 | 2 | 45,0 | 2 | 2/8 | 8 | 54,80 |
| 32 | 2 | 0,12 | 20 | 2 | 75,0 | 2 | 2/8 | 7 | 52,10 |
| 33 | 2 | 0,12 | 20 | 2 | 45,0 | 2 | 2/6 | 6 | 50,80 |
| 34 | 2 | 0,12 | 20 | 2 | 45,0 | 1 | 8 | 8 | 55,80 |
| 35 | 2 | 0,12 | 20 | 2 | 45,0 | 4 | 2/4/2/2 | 4up/1down | 47,40 |
| 36 | 2 | 0,12 | 20 | 2 | 45,0 | 4 | 2/2/2/4 | 2up/4down | 56,10 |
| 37 | 2 | 0,12 | 20 | 2 | 45,0 | 4 | 2/5/2/3 | 4up/3down | 84,40 |
| 38 | 2 | 0,12 | 20 | 2 | 45,0 | 4 | 2/6/2/2 | 6up/1down | 73,50 |
| 39 | 2 | 0,12 | 20 | 2 | 45,0 | 4 | 2/4/2/1 | 4up/1down | 42,10 |
| 40 | 2 | 0,12 | 20 | 2 | 45,0 | 4 | 2/5/2/1 | 5up/1down | 38,00 |

APPENDIX: List of parametric numerical analyses

(1) With reference to Figure 4
 (2) In multilayered profiles the thickness of the various layers is given in sequence of increasing depth