Abstract
Soil effects on peak seismic acceleration and velocity are expressed by simple relations, in terms of five (5) basic site and excitation parameters: the fundamental vibration periods of the soil $T_S$ and the bedrock $T_b$, the predominant excitation period $T_e$, the peak seismic acceleration at outcropping bedrock $a_{b,\text{max}}$, and the number of significant cycles $n$. Furthermore, relations are proposed for the estimation of $T_S$ in terms of the soil thickness $H$, the average shear wave velocity of the soil $V_{S,o}$ and $a_{b,\text{max}}$. All relations were established in two steps: (a) the basic parameters were first identified through a simplified analytical simulation of the site excitation, and (b) the effect of each parameter was subsequently estimated from a statistical analysis of relevant data from more than 700 one-dimensional equivalent-linear seismic ground response analyses. The soil profiles used in the numerical analyses correspond to natural sites, while the seismic excitations originate from actual seismic motion recordings. Comparison with strong motion recordings, from seven (7) case studies, shows that the accuracy of the proposed relations is comparable to that of the equivalent-linear method. Hence they can be readily used as a quick alternative for routine applications, or in cases where numerical methods cannot be easily implemented, such as in seismic microzonation studies of wide areas that are commonly performed with the aid of G.I.S. systems (e.g. [21]).

Currently available empirical methods offer the advantage of immediate and fairly inexpensive application, but often fail to provide the accuracy required for engineering applications [3]. On the other hand, numerical methods are more accurate, but their application to routine projects is limited by the time and cost required in order to collect all necessary input data and to perform the analyses. To fill this gap, a set of relations is established here and in a companion paper [5], which draws upon the theory of 1-D seismic wave propagation in order to identify the basic factors affecting soil response and to evaluate their influence. In practical terms, such relations could prove useful in routine projects, or in cases where numerical methods cannot be easily implemented, such as in seismic microzonation studies of wide areas that are commonly performed with the aid of G.I.S. systems (e.g. [21]).

The new relations are based on data from equivalent-linear site response analyses, performed to simulate actual seismic excitations and natural soil conditions. In this way, the values of all parameters varied within a wide range, making a multivariable regression analysis of the data reliable. To guide the regression analysis, the basic parameters of the relations were first identified by means of an analytical solution for uniform soil and harmonic base excitation. To verify their validity for practical applications, an extensive evaluation is presented here pertaining to comparisons with strong motion data and related numerical analyses for seven (7) case studies.

1. INTRODUCTION

Soil alters the characteristics of seismic waves, in such a way that the amplitude as well as the frequency content of seismic motions on the free soil surface and on the free surface of the bedrock is different. Although it is well understood today that the topography of the ground and that of the bedrock basin are sometimes equally important for the definition of seismic ground motions, soil effects are the first to be considered in practical applications.

In broad terms, the methods used for this purpose may be divided into two categories:

a) Empirical, which correlate seismic motion characteristics from actual recordings with soil conditions at the recording site. Soil conditions are commonly characterized either in pure geological terms (e.g. [15], [6], [1], [24]) or with the aid of some representative soil parameter such as the average shear wave velocity (e.g. [19], [9], [7], [8]).

b) Numerical, which employ wave propagation theory either in the frequency or in the time domain and may simulate the details of any given soil profile and seismic excitation (e.g. [14], [17], [18], [22]).
This paper focuses upon soil effects on the peak ground acceleration, the peak ground velocity and the fundamental site period. The effect of soil on elastic response spectra is the subject of a companion paper [5].

2. LIST OF SYMBOLS

| Amplitude of outcropping bedrock to soil surface amplification ratio |
| Complex amplification ratio within soil |
| Complex amplification ratio within bedrock |
| Impedance ratio |
| Complex impedance ratio |
| Mass density of bedrock |
| Mass density of soil column |
| Critical damping ratio of bedrock |
| Critical damping ratio of soil column |
| Thickness of soil column |
| Elastic shear wave velocity in uniform soil |
| Elastic shear wave velocity in uniform soil |
| Complex shear wave velocity in soil |
| Average elastic shear wave velocity in soil column |
| Fundamental soil period |
| Complex fundamental soil period |
| Predominant excitation period |
| Shear wave velocity in (uniform) bedrock |
| Fundamental (uniform) bedrock period (=4HV_s) |
| Wave number for soil |
| Complex wave number for damped soil |
| Overall damping ratio of soil column |
| Wave number for bedrock |
| Peak horizontal acceleration at outcropping bedrock |
| Peak horizontal acceleration at soil surface |
| Outcropping bedrock to soil surface peak horizontal acceleration amplification ratio |
| Relative estimation error of A_a |
| Peak horizontal velocity at outcropping bedrock |
| Peak horizontal velocity at soil surface |
| Outcropping bedrock to soil surface peak horizontal velocity amplification ratio |
| Relative estimation error of A_y |
| Number of equivalent uniform cycles of excitation |
| Cyclic shear strain amplitude |

3. PARAMETER IDENTIFICATION

The basic parameters contributing to soil effects may be identified with the aid of one-dimensional wave propagation theory for a uniform, visco-elastic soil and bedrock profile under harmonic base excitation. Given the algebra outlined in the Appendix, the ratio of the amplitude of motion at the free ground surface to that at the outcropping bedrock is expressed as:

\[
|A_s| = \frac{\exp\left[\frac{\xi_b T_b}{T_s} \left(\frac{\pi T_s}{2 T_e}\right)\right]}{\cos\left(\frac{\pi T_s}{2 T_e}\right) + ia^* \sin\left(\frac{\pi T_s}{2 T_e}\right)}
\]

(3.1)

where \((\rho_S, \xi_S)\) and \((\rho_b, \xi_b)\) denote the pairs of mass density and damping for the soil and the bedrock, \(T_s\) and \(T_b\) are the fundamental vibration periods of columns of soil and bedrock of the same thickness \(H\), \(T_s\) is the excitation period, \(T_s = T_s(1-i\xi_S)\), while

\[
a = \frac{\rho_S T_b}{\rho_b T_s} \quad \text{and} \quad a^* = a \frac{1 + i\xi_S}{1 - i\xi_S}.
\]

For small values of the impedance ratio \((a)\) and the critical damping ratio \((\xi_S)\), Eq. (3.1) may be approximated by:

\[
|A_s| = \left|\frac{\exp\left[\frac{\xi_b T_b}{T_s} \left(\frac{\pi T_s}{2 T_e}\right)\right]}{\cos\left(\frac{\pi T_s}{2 T_e}\right)}\right| \left[1 + \frac{\xi_{c,S}}{\frac{2 \pi}{T_e}}\right]^{2}
\]

(3.2)

where

\[
\xi_{c,S} = \xi_S + \frac{2}{\pi} \frac{T_e}{T_s}
\]

(3.3)

These analytical relations refer to the steady state response of the soil profile. Accounting for the transient phase of the response, the amplification ratio \(A_s\) has to be related to the duration of the excitation, or to the number of cycles \(n\). Thus, \(A_s\) can be considered overall a function of five independent factors: \(T_s/T_e\), \(T_s/T_b\), \(\rho_s/\rho_b\), \(\xi_S/\xi_b\) and \(n\). Furthermore, for non-linear soils, the peak acceleration of the seismic excitation \(a_{\text{max}}\) should be added to the above factors, as it affects both the \(T_s\) and the \(\xi_S\) of the soil.

In this study, priority is given to the effect of four (4) of these factors: \(T_s/T_e\), \(T_s/T_b\), \(a_{\text{max}}\) and \(n\). The effect of the remaining factors is overlooked since \(\xi_s\) and \(\rho_s/\rho_b\) show little variability, while \(\xi_S\) is mostly a function of earthquake-induced shear strains and, in turn, of \(a_{\text{max}}\) and \(n\).

4. DATABASE AND STATISTICS

The proposed relations are based on a multivariable regression analysis of the input data and the results of more than 700 numerical analyses of seismic ground response. The site model used for these analyses consists of a number of horizontal soil layers, with non-linear visco-elastic response, resting upon a uniform, linear visco-elastic bedrock. Computations follow the equivalent-linear method, assuming vertical propagation of earthquake-induced shear
waves from the seismic bedrock to the ground surface and vice-versa [22, 14]. This method has been the standard analysis tool worldwide for a long time, and consequently its advantages and limitations are better understood than those of alternative true non-linear approaches (e.g. [17], [18]). Furthermore, its overall accuracy for low and moderate levels of ground shaking has been directly or indirectly demonstrated in a number of recent case studies, through comparison with data from seismic array recordings (e.g. [11], [20], [10], [12], [23]).

The more than 700 equivalent linear analyses were performed using 12 different (base-line corrected) seismic records as outcropping bedrock excitations. The characteristics of the recordings are presented in Table 1 and in Figures 1 and 2. No alteration was introduced to the records, other than the scaling to the desired value of $a_{\text{max}}$ per analysis, depending on the project at hand. Defining as predominant period $T_p$ of the excitation the period for which its spectral acceleration $S_a$ (for 5% critical damping ratio) takes its peak value, observe that $T_p$ in the excitations used varies from 0.1 to 0.8 sec, capturing a wide spectrum of potential seismic events.

For each excitation, $n$ was estimated as the number of cycles in the time-history that exceed a level of acceleration equal to $a_{\text{max}}(M-1)/10$, where $M$ is the earthquake magnitude. This empirical “rule of thumb” is an extension of the relation between equivalent uniform and maximum shear strains adopted in the numerical analyses of seismic ground response [14]. Observe that for the excitations used, $n = 0.5$ to 24, again capturing a wide spectrum of potential seismic events.

In the majority of the analyses, the seismic bedrock was...
defined within Neogene or older geological formations, with shear wave velocity $V_s = 550$ m/s and $\rho_g = 2.2$ Mg/m$^3$.

The analyses were performed for 105 soil sites where geotechnical investigations were available, including measurements (mainly crosshole) of shear wave velocity, as part of major infrastructure development projects in Greece (e.g. microzonation studies, design of motorways, gas and oil transmission pipelines). Each site was usually analyzed for 2 to 4 excitations and 1 to 2 levels of $a_{max}^b$, depending on the project at hand.

Soil non-linearity was introduced in terms of the shear modulus degradation and hysteretic damping ratio curves [25], for soils of different plasticity (e.g. Fig. 9). With few exceptions, the soil profiles considered in this study consisted of cohesionless non-plastic sand, silt or gravel, as well as of low plasticity clays and marls. Hence the theoretical predictions refer to soil layers of plasticity index between 0 and 50%. Such soils exhibit a higher potential for non-linearity relative to more plastic clays.

The overall variability of the site conditions is outlined in Table 2, by means of the range and frequency distribution of the major site characteristics. In addition, this table provides the range and frequency distribution of the main parameters that affect soil amplification, namely: $T/T_s$, $T/T_c$, $a_{max}^b$ and $n$.

Table 1: Characteristics of recordings.

<table>
<thead>
<tr>
<th>#</th>
<th>Name (date)</th>
<th>$a_{max}$ (g)</th>
<th>$V_{max}$ (cm/s)</th>
<th>M</th>
<th>$R$ (km)</th>
<th>Comp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Coyote Lake (8-6-79)</td>
<td>0.321</td>
<td>25.1</td>
<td>5.8</td>
<td>9.6</td>
<td>N320°</td>
</tr>
<tr>
<td>B</td>
<td>Japan-236</td>
<td>0.168</td>
<td>6.1</td>
<td>8.4</td>
<td>185</td>
<td>NS</td>
</tr>
<tr>
<td>C</td>
<td>Alkyonides (24-2-81)</td>
<td>0.301</td>
<td>24.4</td>
<td>6.7</td>
<td>32</td>
<td>Trans</td>
</tr>
<tr>
<td>D</td>
<td>San Fernando (9-2-71)</td>
<td>0.346</td>
<td>14.5</td>
<td>6.5</td>
<td>23.1</td>
<td>N21°</td>
</tr>
<tr>
<td>E</td>
<td>Coyote Lake (8-6-79)</td>
<td>0.417</td>
<td>43.7</td>
<td>5.8</td>
<td>9.6</td>
<td>N230°</td>
</tr>
<tr>
<td>F</td>
<td>Kalamata (12-9-86)</td>
<td>0.273</td>
<td>23.6</td>
<td>6.2</td>
<td>4</td>
<td>EW</td>
</tr>
<tr>
<td>G</td>
<td>San Fernando (9-2-71)</td>
<td>0.278</td>
<td>12.5</td>
<td>6.5</td>
<td>23.1</td>
<td>N291°</td>
</tr>
<tr>
<td>H</td>
<td>Cephalonia (17-1-83)</td>
<td>0.142</td>
<td>8.4</td>
<td>7.0</td>
<td>34</td>
<td>NS</td>
</tr>
<tr>
<td>I</td>
<td>Pyrgos (26-3-93)</td>
<td>0.454</td>
<td>19.3</td>
<td>5.5</td>
<td>3</td>
<td>Trans</td>
</tr>
<tr>
<td>J</td>
<td>Parkfield (27-6-66)</td>
<td>0.264</td>
<td>14.2</td>
<td>6.1</td>
<td>43.9</td>
<td>N295°</td>
</tr>
<tr>
<td>K</td>
<td>Cephalonia (23-3-83)</td>
<td>0.239</td>
<td>9.8</td>
<td>6.2</td>
<td>13</td>
<td>EW</td>
</tr>
<tr>
<td>L</td>
<td>Aigion (15-6-95)</td>
<td>0.543</td>
<td>48.1</td>
<td>6.2</td>
<td>18</td>
<td>N150°</td>
</tr>
</tbody>
</table>

Observe that site characteristics as well as soil amplification parameters in Table 2 cover a wide range of values, typical for the great majority of potential cases. The general form of the proposed relations was defined in advance of the statistical analysis of the relevant data, from a joint evaluation of appropriate analytical solutions (e.g. Eq. 3.2) and the trends exhibited by the numerical predictions themselves. Then, a multivariable regression analysis of the entire database, according to the Newton-Raphson iterative procedure, calibrated the 3 to 5 constants of each of the relations. Appropriate weighting was introduced in the statistical analysis to counterbalance the non-uniformity of the database, especially in connection to $a_{max}^b$, an independent variable with significant influence on all aspects of the response, since it is related to soil non-linearity. In what follows, the results of the equivalent-linear analyses are denoted as data, although they are also simulations and not actual recorded data.

5. PEAK GROUND ACCELERATION

Fig. 3 shows the variation of the relative amplification ratio for the peak ground acceleration, denoted hereafter as $A_a$, with the normalized soil period $T/T_s$. In this figure, the data are presented in pairs of groups, by maintaining 2 of the remaining free variables fixed within a narrow range, and significantly changing the third variable. Specifically, all data in Fig. 3a correspond to fixed $T/T_s$ and $n$ values, but significantly different values of $a_{max}^b = 0.01$ to 0.14g (moderate shaking) and $a_{max}^b = 0.4$ to 0.45g (strong shaking). In Fig. 3b, differences in the data correspond to significantly different values of $n = 0.5$ to 1 (impulse-like motions) and $n = 4$ to 6 (long duration motions). Finally, in Fig. 3c, the differences correspond to significantly different values of $T_s/T_5 = 0.05$ to 0.4 (high contrast profiles) and $T_s/T_5 = 0.5$ to 0.9 (low contrast profiles).

Observe that the effect of normalized site period $T_s/T_5$ is similar in all figures. Namely, $A_a$ tends to 1.0 as $T_s/T_5$ tends to zero, it becomes maximum close to $T_s/T_5$=1.0 and it decreases gradually as $T_s/T_5$ exceeds 1.0. This trend is strongly reminiscent of the response of single degree of freedom (SDOF or mass-spring-dashpot) systems subjected to harmonic base excitation. Hence, drawing upon the theory of SDOF vibrations under support excitation (e.g. [2], [13]), the data in Fig. 3 have been fitted by:

$$A_a = \frac{1 + C_{1,a} (T_s / T_c)^2}{\sqrt{1 - (T_s / T_c)^2} + C_{2,a} (T_s / T_c)^2} \quad (5.1)$$

According to Eq. (5.1), $A_a$ takes the following characteristic values:

$$A_a = \begin{cases} 1.0 & \text{for } T_s / T_c = 0 \\ \left(1 + C_{1,a} / C_{2,a}\right) & \text{for } T_s / T_c = 1 \\ C_{1,a} & \text{for } T_s / T_c \to \infty \end{cases} \quad (5.2)$$

The fact that $A_a$ tends to a fixed, nonzero value at large
normalized site periods $T_s/T_e$ is the only basic difference from the response of a SDOF system, which eventually diminishes to zero. This differentiation was conservatively introduced in order to take into account the contribution to the response of the higher modes of vibration.

![Graph](image)

Figure 3: Effect of site and excitation parameters on $A_\alpha$, Σήμα 3: Επίδραση παραμέτρων εδάφους και διέγερσης στο $A_\alpha$

In general, the coefficients $C_{1,\alpha}$ and $C_{2,\alpha}$ should be expressed as functions of the three (3) remaining independent variables, i.e. $a_{\text{max}}$, $T_s/T_e$ and $n$. However, the data in Fig. 3 show that $C_{1,\alpha}$ i.e. the asymptotic value of $A_\alpha$ at large normalized soil periods, is not affected by $T_s/T_e$ but increases consistently as $n$ and $a_{\text{max}}$ become higher and lower, respectively. Moreover, the value of $A_\alpha$ at resonance increases with increasing $n$ and decreasing $a_{\text{max}}$. Hence, $C_{1,\alpha}$ can be expressed as a function of $a_{\text{max}}$ and $n$ only:

$$C_{1,\alpha} = d_{1,\alpha} \left( \frac{a_{\text{max}}}{g} \right)^{d_{2,\alpha}} g(n)$$  \hspace{1cm} (5.3)

with

$$g(n) = \frac{n^{d_{3,\alpha}}}{1 + n^{d_{4,\alpha}}}$$  \hspace{1cm} (5.4)

and $d_{1,\alpha}, d_{2,\alpha}<0$, $d_{3,\alpha}>0$. Note that the general form of $g$ provides an asymptotic increase of $C_{1,\alpha}$ and $A_\alpha$ towards the steady state values, at a large number of cycles $n$. This effect resembles the transient response of SDOF systems in resonance conditions (e.g. [13]), and is also consistent with the response displayed by the data in Fig. 3b.

Finally, the data in Fig. 3c indicate that the peak value of $A_\alpha$ tends rather to decrease as the normalized period of the bedrock $T_s/T_e$ becomes higher. This is reasonable since $T_s/T_e$ represents essentially the contrast in dynamic stiffness between the soil and the underlying bedrock, and it is consequently a measure of the radiation damping.

Table 2: Range and Frequency Distribution of Parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H$ (m)</td>
<td>3.5 – 240</td>
<td><img src="image" alt="Graph" /></td>
</tr>
<tr>
<td>$V_{S,0}$ (m/s)</td>
<td>50 – 700</td>
<td><img src="image" alt="Graph" /></td>
</tr>
<tr>
<td>$V_b$ (m/s)</td>
<td>100 – 1000</td>
<td><img src="image" alt="Graph" /></td>
</tr>
<tr>
<td>$T_s$ (sec)</td>
<td>0.04 – 3.33</td>
<td><img src="image" alt="Graph" /></td>
</tr>
<tr>
<td>$T_b$ (sec)</td>
<td>0.02 – 1.75</td>
<td><img src="image" alt="Graph" /></td>
</tr>
<tr>
<td>$T_b/T_S$</td>
<td>0.05 – 0.95</td>
<td><img src="image" alt="Graph" /></td>
</tr>
<tr>
<td>$T_S/T_b$</td>
<td>0.06 – 13.3</td>
<td><img src="image" alt="Graph" /></td>
</tr>
<tr>
<td>$a_{\text{max}}$ (g)</td>
<td>0.01 – 0.45</td>
<td><img src="image" alt="Graph" /></td>
</tr>
<tr>
<td>$n$ (cycles)</td>
<td>0.5 – 24</td>
<td><img src="image" alt="Graph" /></td>
</tr>
</tbody>
</table>

To simulate this effect, $C_{1,\alpha}$ was correlated with $T_s/T_e$ through a linear relation, of the same form as Eq. (3.3), which describes the equivalent critical damping ratio:
\[ C_{2,a} = d_{4,a} + d_{5,a} \frac{T_b}{T_s} \]  
(5.5)

with \(d_{4,a}, d_{5,a} > 0\).

The constants in Eqs (5.3), (5.4) and (5.5) were determined from a statistical analysis of all available data. This procedure led to a best fit relation for \(d_{1,a} = 1.20\), \(d_{2,a} = -0.17\), \(d_{3,a} = 0.50\), \(d_{4,a} = 1.05\) and \(d_{5,a} = 0.57\). Practically, this best-fit relation ensures that \(A_a\) is over-predicted in 50% of the cases and under-predicted in the remaining 50% (median value). Note that by changing to \(d_{1,a} = 1.75\) and retaining the values of the other 4 constants, Eq. (5.1) produces conservative upper bound estimates, i.e. ensures overprediction of \(A_a\) in 84% of the cases in the database, for added conservatism.

Fig. 4 presents a one-to-one comparison of \(A_a\) values, based on the proposed relations (predictions) and the equivalent-linear analyses for all the cases in the database (data). This means that after statistically calibrating Eq. (5.1) for best fit and upper bound predictions, an a posteriori prediction of \(A_a\) for all the cases in the database was performed, as an index of the overall predictive ability of the relation. In addition, Fig. 5 presents another comparison of the best-fit predictions and the data, in terms of the relative error \(R_{A_a}\), defined as the difference between approximate predictions of \(A_a\) and data normalized with respect to the latter. Observe that the relative error of the best-fit relation has practically no bias with respect to the included parameters and that the standard deviation of the error in predicting \(A_a\) is +24%. Anyway, some conservative overprediction of \(A_a\) is expected for very flexible soil profiles and relatively intense shaking. On the other hand, Fig.4b shows that that the proposed upper bound relation provides a consistent overprediction of the whole range of data, and can be used instead of the best-fit relation if significant conservatism must be incorporated in the design.

6. PEAK GROUND VELOCITY

Numerical predictions for the relative amplification of peak ground velocity \(A_v\), are plotted in Fig. 6 against the normalized soil period \(T_s/T_e\). Specifically, Figs. 6a, 6b and 6c show examples of the effects of the bedrock to soil period ratio \(T_b/T_s\), the peak bedrock acceleration \(a_{b,\text{max}}\) and the number of equivalent cycles \(n\), respectively. The effect of the various factors on \(A_v\) is similar as in the case of \(A_a\), except for two main differences.

The first is that \(A_v\) is not consistently affected by the duration of the seismic motion, Fig. 6c. The second difference is that the maximum amplification of the velocity occurs at normalized soil periods \(T_s/T_e \approx 1\). This is because the predominant period of the velocity time history of seismic motions is usually higher than that of the corresponding acceleration time history (e.g. peak spectral velocity usually occurs at larger structural periods than the peak spectral acceleration in tri-logarithmic plots of elastic response spectra).
Figure 6: Effect of site and excitation parameters on $A_v$.

According to the data in Fig. 6, the peak amplification occurs at approximately $T_s \approx 1.50 T_e$. Hence, Eq. (5.1) is re-written as:

$$A_v = \frac{1}{\sqrt{ [1 - (T_s / 1.5 T_e)^2] + C_{2,v}^2 (T_v / 1.5 T_e)^2}}$$  \hspace{1cm} (6.1)

where:

$$C_{1,v} = d_{1,v} \left( \frac{a_{\text{max}}}{g} \right)$$  \hspace{1cm} (6.2)

$$C_{2,v} = d_{3,v} + d_{4,v} \frac{T_B}{T_s}$$  \hspace{1cm} (6.3)

with $d_{2,v} < 0$ and $d_{3,v} d_{4,v} > 0$.

The constants in Eqs (6.2) and (6.3) were again determined from a multi-variable regression analysis of all available data. According to this, the best fit relation provides a median value of $A_v$, but by changing to $d_{1,v}=1.25$ and retaining the values of the other 3 constants, Eq. (6.1) produces conservative upper bound estimates, i.e. ensures overprediction of $A_v$ in 84% of the cases in the database, for added conservatism.

The predictions of $A_v$ are evaluated in Figs. 7 and 8, in the same format as that used to evaluate $A_f$. In this case, the best-fit predictions agree well with the data for $A_v > 1$, with...
the unbiased relative error $R_\gamma$, having a standard deviation of +19.9%. Anyway, some conservative overprediction of $A_\gamma$ may be expected for very flexible soil profiles, over stiff bedrock and relatively intense seismic excitations, similarly to $A_\gamma$. Finally, note in Fig. 8b that the proposed upper bound fit does provide a consistent overprediction of the whole range of $A_\gamma$ data.

7. FUNDAMENTAL SOIL PERIOD

In order to apply the previous relations in practice, one has to provide the peak acceleration at the outcropping bedrock $a_{\text{max}}^b$, the predominant period of the excitation $T_s$, as well as the fundamental vibration periods for the bedrock $T_b$ and for the soil $T_s$. Among these parameters, $a_{\text{max}}^b$ and $T_b$ are usually provided by a seismic hazard study while $T_s$ is related by definition to the soil thickness $H$ and the elastic shear wave velocity of the bedrock $V_s$ (i.e. $T_s=4H/V_s$). In contrast, estimation of $T_s$ is not equally straightforward, even if the variation of elastic shear wave velocity with depth is known. This is mostly due to the fact that soil response during shaking is non-linear and consequently the fundamental period $T_s$ is related to the applied shear stresses and strains in addition to the elastic soil properties. Fig. 9 shows typical experimental curves for the variation of the shear modulus $G$, normalized against the elastic shear modulus $G_o$, with applied cyclic shear strain amplitude $\gamma$, for soils with different plasticity index $I_p$ [25]. Analytically, these curves are approximately expressed as:

$$\frac{G}{G_o} = \frac{1}{1 + \kappa \gamma^\lambda}$$  \hspace{1cm} (7.1)

For instance, Eq. (7.1) with $\kappa = 6$, $\lambda = 0.91$ and $\gamma$ in (%) is compared to the experimental curve for $I_p = 30\%$ in Fig. 9. In terms of shear wave velocities, Eq. (7.1) becomes:

$$\left(\frac{V_s}{V_{s,o}}\right)^2 = \frac{1}{1 + \kappa \gamma^\lambda}$$  \hspace{1cm} (7.2)

where $V_s$ denotes the shear wave velocity for cyclic shear strain amplitude $\gamma$, while $V_{s,o}$ denotes its value for $\gamma < 10^{-5}$. Based on Eq. (7.2), a general relation for the fundamental soil period $T_s$ is:

$$\left(\frac{T_s}{T_{s,o}}\right)^2 = 1 + \kappa \gamma^\lambda$$  \hspace{1cm} (7.3)

where $T_{s,o}$ denotes the elastic soil period (for $\gamma < 10^{-5}$).

As a first approximation, the $\gamma$ may be related to $a_{\text{max}}^b$, as an index of the shaking intensity, and the average elastic shear wave velocity $V_{s,o}$ (=4$H/T_{s,o}$), as an index of the dynamic soil stiffness. Hence, Eq. (7.3) is written as:

$$\left(\frac{T_s}{T_{s,o}}\right)^2 = 1 + d_{1,T} \left(V_{s,o}\right)^{d_{2,T}} (\frac{a_{\text{max}} b}{g})^{d_{3,T}}$$  \hspace{1cm} (7.4)

with $d_{1,T}, d_{3,T} > 0$ and $d_{2,T} < 0$.

The values of these constants were estimated from a multi-variable regression analysis, as: $d_{1,T} = 5330$, $d_{2,T} = -1.30$ and $d_{3,T} = 1.04$.

Fig. 10 presents a one-to-one comparison between $T_s$ predictions and results from the analyses (data). Fig. 11 provides the relative error $R_\gamma$ as a function of the two basic input parameters $a_{\text{max}} b$ and $V_{s,o}$. It is argued that Eq. (7.4) follows closely the trends of the data, in qualitative and quantitative terms (standard deviation +24.3%).

Figure 9: Variation of $G$ with shear strain $\gamma$.
\textit{Σχήμα 9: Μεταβολή του G με την διατμητική παραμόρφωση $\gamma$.}

Figure 10: Comparison between predictions and data for $T_s$.
\textit{Σχήμα 10: Σύγκριση προβλέψεων - δεδομένων για το $T_s$.}
8. DISCUSSION

The proposed relations estimate soil effects on peak ground acceleration and velocity in terms of four (4) basic parameters: period ratios $T_1/T_s$ and $T_2/T_s$, $\alpha_{max}$ and $n$. These four parameters are treated as independent, given that their covariance coefficient is in all cases smaller than 0.15 (note that a covariance coefficient equal to 1.0 corresponds to total dependence between two variables). According to the statistical analysis, the effect of $T_1/T_s$ is the most systematic and pronounced, at least for low values of $T_2/T_s$ (< 0.40). The effects of the remaining parameters are relatively less significant.

The effect of soil non-linearity on $T_s$ is estimated as a function of two (2) obviously independent variables: the average small-strain shear wave velocity $V_{S,a}$ of the soil, and $\alpha_{max}$. In this case, the statistical analysis indicates that the importance of these variables is broadly equivalent.

In addition to the evaluation of the proposed relations against the numerical predictions used in the statistical analysis, their accuracy was verified in a series of case studies (not included in the database): a) two (2) sites in the San Fernando Valley during the Northridge earthquake (January 17th 1994), and b) five (5) seismic events recorded by the SMART-1 accelerometer array in Taiwan. Full details of the site characteristics (e.g. geology, $V_s$ profile with depth) and the recordings (i.e. earthquake magnitude, elastic response spectra) for these case studies are provided in [5]. In this paper, the information regarding the site characteristics and the recordings for the seven (7) case studies is outlined in Table 3, in the processed form of the parameters entering the proposed relations. Observe that the sites and the seismic events considered cover a wide range of potential cases, rendering this evaluation representative for a wide range of applications in practice.

![Figure 11: Relative error of proposed relation for $T_s$.](image)

**Figure 11:** Relative error of proposed relation for $T_s$.

Σχήμα 11: Σχετικό λάθος της προτεινόμενης σχέσης για το $T_s$.

![Figure 12: Evaluation of best fit (f) and upper bound (o) relations against seismic recordings and numerical predictions](image)

**Figure 12:** Evaluation of best fit (f) and upper bound (o) relations against seismic recordings and numerical predictions: (a), (b) & (c) for $A_a$; (d), (e) & (f) for $A_V$.

Σχήμα 12: Αποτίμηση βέλτιστων σχέσεων (f) και σχέσεων άνω ορίου (o) εναντίον σεισμικών καταγραφών και αριθμητικών προβλέψεων: (a), (b) & (c) για το $A_a$ - (d), (e) & (f) για το $A_V$. 

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*Note: The figures and tables are placeholders as the actual images are not provided in the text.*
For each one of the seven (7) case studies, soil amplification was estimated by three methods: a) direct calculation from the actual recorded time-histories in the surface of nearby ‘soil’ and the ‘bedrock’ sites, b) approximate calculations via the best fit proposed relations, and c) numerical calculations with the equivalent-linear method [14]. The three sets of data are compared to each other in Fig. 12 (solid circles). Observe first that the approximate relations estimate the recorded $A_s$ and $A_b$ with a safety factor equal to 2 and no systematic bias (Figs 12a and 12d). Note also that the same level of accuracy is obtained from the simulations with the equivalent-linear method (Figs 12b & 12e). This is clearly an indication of the widely acknowledged difficulties encountered when field data are interpreted on the basis of theoretical models. Finally, note in Figs 12c & 12f that the approximate best fit predictions fall consistently within ±45% of their numerical counterparts, without any systematic bias. On the other hand, the comparisons for approximate upper bound predictions in the same figure (hollow circles) show that these consistently overpredict numerical and recorded data, ensuring a reasonable level of conservatism.

9. CONCLUSIONS

A set of approximate relations has been established to evaluate soil effects on $a_{max}$ and $V_{max}$, and also to estimate the basic site parameter, i.e. the non-linear soil period $T_s$. The basic parameters of the relations are identified through a simplified analytical simulation of the problem. Their effect is quantified via a statistical analysis of data obtained from over 700 equivalent-linear analyses of seismic ground response and not from seismic recordings. In summary, it was found that:

a) Soil effects on $a_{max}$ are a function of the period ratios $T_b/T_s$ and $T_b/T_{so}$, the peak bedrock acceleration $a_{max}$ and the equivalent number of cycles $n$. Soil effects on $V_{max}$ depend on the same parameters, except for $n$.

b) The non-linear soil period $T_s$ is related with the elastic soil period $T_{so}$, the average elastic shear wave velocity $V_{so}$ over the entire soil depth, and $a_{max}$.

c) Predictions obtained from the proposed best-fit relations are usually within ±19.9 to 24.3% from the respective estimates from equivalent-linear analyses. On the other hand, the upper bound relations provide consistently conservative upper bound estimates of the numerical results.

d) Evaluation of the proposed relations in well-documented cases of actual recordings and numerical analyses that are not included in the database verified the above safety margins.

Nevertheless, the accuracy of the proposed relations is conceptually related to, and limited by, the accuracy of the equivalent-linear method ([14], [22]) used to obtain the numerical predictions in the database. Furthermore, their application should be limited to cases where the site and excitation characteristics fall within the limits summarized in Table 2. Further details concerning the relations and their verification can be found in [4]. Overall, the relations should be considered as approximate, aimed at the preliminary evaluation of soil effects. In addition, they can be used as a user-friendly alternative to the equivalent-linear method, when the latter is too cumbersome to implement, as in GIS-aided microzonation studies [21].

10. ACKNOWLEDGEMENTS

This research was funded by the Earthquake Protection and Planning Organization of Greece (O.A.Σ.Π.). Professor G. Gazetas has contributed valuable comments on the paper presentation, while fellow Civil Engineers Thomas Panourgias, Michael Klovavas and Niki Kringos assisted with the statistical analysis and the verification of the relations. These contributions are gratefully acknowledged.

11. APPENDIX

Analysis of soil effects for harmonic excitation and uniform visco-elastic soil and bedrock conditions

Based on one-dimensional wave propagation theory, the amplification of the seismic excitation from the outcropping bedrock to the free soil surface, in the simplified case of a uniform soil and bedrock site, is expressed as (e.g. [16]):

$$A_{s,0} = \frac{1}{\cos k_s^* H + ia^* \sin k_s^* H} \tag{I.1}$$

where

$$k_s^* H = \frac{\omega H}{V_s}, \quad V_s^* = V_s'(1 + i\xi_s^*), \quad a^* = \alpha \frac{1 + i\xi_s^*}{1 + \xi_p^*}.$$
and
\[ a = \frac{\rho_s}{\rho_b} \frac{T_b}{T_s} \]

Taking into account that for small values of \( \xi_s \) (i.e., \( \xi_s < 0.10 \)); \( k_s H \approx k_s H (1 - i \xi_s) \) and that by definition \( \cos(ix) = \cos(x) \) and \( \sin(ix) = i \sin(x) \), the complex trigonometric terms in Eq. (I.1) are written as:
\[
\cos(k_s H) \approx \cos(k_s H (1 - i \xi_s)) = (\cos(k_s H) \cosh(\xi_s k_s H) + i \sin(k_s H) \sinh(\xi_s k_s H))
\]
and:
\[
\sin(k_s H) \approx \sin(k_s H (1 - i \xi_s)) = (\sin(k_s H) \cosh(\xi_s k_s H) - i \cos(k_s H) \sinh(\xi_s k_s H))
\]

Furthermore, for small values of \( \xi_s \),
\[
a^* = a \frac{1 + i \xi_s}{1 + i \xi_b} \approx a[1 + i(\xi_s - \xi_b)]
\]

Hence, Eq. (I.1) is expanded as shown below (Eq. I.5):
\[
\frac{1}{A_{s,o}} = \cos(k_s H) \cosh(\xi_s k_s H) + a \cos(k_s H) \sinh(\xi_s k_s H) - a(\xi_s - \xi_b) \sin(k_s H) \cosh(\xi_s k_s H)
\]
\[
+ i \left[ \sin(k_s H) \sinh(\xi_s k_s H) + a \sin(k_s H) \cosh(\xi_s k_s H) + a(\xi_s - \xi_b) \cos(k_s H) \sinh(\xi_s k_s H) \right]
\]
and consequently (Eq. I.6):
\[
\left| \frac{1}{A_{s,o}} \right|^2 = \cos^2(k_s H) + \sinh^2(\xi_s k_s H) + a^2 \left[ \sin^2(k_s H) + \sin^2(\xi_s k_s H) \right] + a(\xi_s - \xi_b) \cos(k_s H) \sinh(\xi_s k_s H) -
\]
\[
2a^2 \left[ \cosh(\xi_s k_s H) \sinh(\xi_s k_s H) - \right. \left. (\xi_s - \xi_b) \cos(k_s H) \sinh(\xi_s k_s H) \right]
\]

For the special case where the bedrock and the soil have the same properties (i.e., \( a = 1 \), \( \xi_s = \xi_b \)), Eq. (I.6) simplifies to:
\[
\left| \frac{1}{A_{s,o}} \right|^2 = \cos^2(k_s H) + \sinh^2(\xi_s k_s H) + \exp(\xi_s k_s H) \quad (I.7)
\]

Furthermore, for low values of \( \xi_s \) and \( \xi_b \), as well as high contrast between the shear wave velocities of the bedrock and the soil (i.e., low a values), Eq. (I.6) may be written as:
\[
\left| \frac{1}{A_{s,o}} \right|^2 = \frac{1}{\cos^2(k_s H) + \xi_s^2 \sinh^2(k_s H)}
\]

or, introducing the excitation period \( T_e = 2\pi/\omega \) and the predominant soil period \( T_s = 4H/V_s \):
\[
\left| \frac{1}{A_{s,o}} \right|^2 = \frac{1}{\cos^2\left(\frac{\pi T_s}{2 T_e} - \left(\frac{\xi_s}{\pi T_s} \frac{\pi T_s}{2 T_e}\right)^2 \right)}
\]
where:
\[
\xi_{e,s} = \xi_s + 2\frac{a T_e}{\pi T_s}
\]

Eqs. (I.7) and (I.9) provide the amplification ratios for seismic waves propagating vertically within the seismic bedrock and within the soil. Hence, the relative amplification ratio, from the free surface of the seismic bedrock to that of the soil is given in complex function form as:
\[
\left| A_{s,o} \right| = \frac{\exp\left(\frac{\xi_b T_b}{T_s} \frac{\pi T_s}{2 T_e}\right) + i a^* \sin\left(\frac{\pi T_s}{2 T_e}\right)}{\cos\left(\frac{\pi T_s}{2 T_e}\right) + \left(\frac{\xi_{e,s} \pi T_s}{2 T_s}\right)^2}
\]
or, approximately:
\[
\left| A_{s,o} \right| = \frac{\exp\left(\frac{\xi_b T_b}{T_s} \frac{\pi T_s}{2 T_e}\right) \frac{\pi T_s}{2 T_e}}{\cos\left(\frac{\pi T_s}{2 T_e}\right) + \left(\frac{\xi_{e,s} \pi T_s}{2 T_s}\right)^2}
\]

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Ι. Μέγιστη Σεισμική Επίταχυνση και Ταχύτητα

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Οι κατάστασεις της έδαφους και της θάλασσας μετασχηματίζονται στον κόσμο των οικονομικών συστημάτων, των τεχνικών παραμετρών και των επιστημονικών κοστών. Αυτό οδηγεί στην ανάγκη ένταξης των οικονομικών και τεχνικών παραμέτρων στην ανάλυση της εδαφικής ενίσχυσης.

Περίληψη
Η επίδραση του έδαφους στη μέγιστη σεισμική επιτάχυνση και ταχύτητα εκφράζεται μέσω απλών σχέσεων υπολογισμού, οι συνήθως πίνεται (5) βασικοί παράμετροι του έδαφους και της διάχυσης: τον ιόδεσηρον του έδαφους Σ, τον υποψάθρον Τ, τη διάχυση ζ. Επιπλέον, προτείνεται και μία σχέση για τον υπολογισμό της Σ ως συνάρτηση του δόνημα της Σ στην ποιότητα της εδαφικής πτερίδας S, της μέγιστης ταχύτητας σεισμικών κυμάτων στο έδαφος V, και της Σ. Όλες αυτές οι σχέσεις διατηρούν ακολούθαις δύο βασικές: (α) ένα παράμετρο αναγνωρίζει μέσω αναλυτικής προσερμοκοιτίζων της εδαφικής απόκρισης υπο-αριθμητικής διάχυσης (β) η επίδραση της κάθε παραμέτρου εκτιμάται μέσω στατιστικής ανάλυσης σχετικών δεδομένων από έναν από τα τεχνικά παραμέτρα. Οι σχέσεις, που προτείνονται εδώ, είναι πολύπαραμετρικές, και αντιπροσωπεύουν δύο βασικές κατηγορίες: Εμπειρικές και Αριθμητικές. Οι εμπειρικές σχέσεις, ενώ διατηρούν το χαρακτήρα της απλής και αμέσης επίδρασης σε τοπογραφικές παραμέτρους, επικεντρώνονται σε συγκεκριμένες ζώνες ή εμπειρίες: (π.χ. μεταφορά ηλεκτρικού ρυθμού σε τοπωνυμία). Οι αριθμητικές σχέσεις, όμως, υπολογίζουν με τη βοήθεια τους μονοπαραμετρικών και πολυπαραμετρικών παραμέτρων, σταχθέντα σε σχέσεις της άμεσης και απλής επίδρασης. Επιπλέον, προτείνεται και μία σχέση για τον υπολογισμό της έντασης της έδαφικής ενίσχυσης με τον αριθμό ισοδύναμων κύματος της έδαφικης ενίσχυσης, που μπορεί να χρησιμοποιηθεί ως μία εύχρηστη εναλλακτική της επίσης για τους παραπάνω παραμέτρους και την εκτέλεση της ανάλυσης.
συστήματα αγωγών, οδικά δίκτυα κ.λπ.).

Η προτυπωτική στη διατύπωση των νέων σχέσεων έγινε
tαι σε δύο κύριοι σημειώσεις. Κατά πρώτον, η επιλογή των
gεωτεχνικών - σεισμολογικών παραμέτρων έγινε με βάση
analytikés lósis kai kubatikis diáforés, για ομοιόμορφα
εξοδουλευτικά εδάφη και αρμονικές σεισμικές διέγερσες.
Κατά δεύτερον, η διατύπωση των σχέσεων έγινε μετά από
statistikì epexeirhiasia autoptelomástωn pléon των 700
aritmihtików análýoun σεισμικής απόκρισης του εδάφους,
για πραγματικές σεισμικές διέγερσες και φυσικά εδάφη,
ti ópotois échoun εκτελέσει οι συγγραφείς στο πλαίσιο τεχ
λογικών και ερευνητικών προγραμμάτων. Όλες οι
aritmihtikés análýoun éginan συμφωνα με την isodínyma
grammatiké méthodos, h oπoia einai h συνήθηση χρησιμοποι
úmeni káta tìn télleutía 30-etaía, δηλαδή με τα λογισμικά
SHAKE [22] kai kuriás το SHAKE91 [14]. Αντίστοιχη στη
statistikì epexeirhiasia seismologików dedomenon dein einai eìpì
to to paróntos eukútt. dodeýmenon óti μικρό μόνον ποσοστό
ton diathésmenos σεισμικών καταγραφών échi gíne se té
seis me gnóstos seismikò charaktéristikà.

To parón aríthmo anaféresetai stois schéseis edafikís eni
sychias gia tì mígysti seismikì epiástíthe aπò aπó tìn télleutía
της isodínyma stílhès, kai sto upolológh tìs idiwssteùs stoìs
aritmihtikès stílhès Tα, eìpì to συνωμό aríthmos [5] anaféresetai se análýoun
schéseis gia ta elástika fásmata apókrisis. Syngkerímena, gia tì
suyxisthèse twn aπò aπó την elèuthè seismikís anástasis (εισòdëia aπò aπó
την piò stín stílhè seismikís anástasis του edáfous oδικών
kai το upolológh aπò aπó tìn elèuthè seismikís anástasis (εισòdëia aπò aπó
την περίοδο της diégerhs του edáfous oδικών
η ισοδύναμη-καταγραφής. Τον τελευταίο 
tow idíou oδικών

Η ερευνητική μας προσπάθεια χρηματοδοτήθηκε από
to Oragnismo Antisismikou Sèk الدمato kai Prostasiais (Ω.Α.Σ.Π). Ο καθηγητής κ. Γ. Γκούκας συνέβαλε με χρήσι
ma sχολία επί της παρουσίασης, οι δυ συνάδελφοι πολίτικο
περιοδος της εδαφικής απόκρισης (Σχήμαta 4, 5, 6, 7, 8, 10 kai 11). Από τη σύγκριση αυτή
προκύπτει óti h apókri tis prosegéstikon apo tis
aritmihtikes ektrimèseis dein paroussázei megalè diáfora
(typikè apókri tis lóðous +20 -24%), ma to ti σημα
ntiko einai óti h oπoia apókri tis einai praktikòs tòu
xous, diégh th dein syxhídei me kápsiia apo tis
anéxoartítes pa

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ΕΥΧΑΡΙΣΤΙΕΣ

Η ερευνητική μας προσπάθεια χρηματοδοτήθηκε από τον Οργανισμό Αντισεισμικού Σχεδιασμού και Προστασίας (Ο.Α.Σ.Π). Ο καθηγητής κ. Γ. Γκούκας συνέβαλε με χρήσιμα σχόλια επί της παρουσίασης, οι δυσάδελφοι πολιτικοί μηχανικοί Θωμάς Παυρουρίας και Μιχάλης Κλούβας συνέβαλαν στη στατιστική επεξεργασία των δεδομένων. Επιπλέον, η επισκέψεις σπουδαιότητα πολιτικού μηχανικού και Niki Krinos (TU Delft) βοήθησε στη σύγκριση με τις πραγματικές καταγραφές. Τους ευχαριστούμε όλους θερμά.