Equivalent-uniform soil model for the seismic response analysis of sites improved with inclusions

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ABSTRACT: This paper studies how 1D equivalent-linear analyses may be accurately used for simulating the seismic response of sites improved with inclusions (e.g. gravel columns). For this purpose, the composite ground is modeled as an equivalent-uniform material, whose dynamic non-linear properties are a function of the respective properties of the natural soil and the inclusion material (e.g. gravel), as well as the replacement ratio of the composite ground. The equivalent-uniform model of the composite ground has a theoretical basis and is verified by comparing parametric results from pertinent 1D and 2D seismic ground response analyses, performed with the equivalent-linear method, for small and high intensity motions. Emphasis is put on replacement and solidification of cohesive soils using a grid of reinforcing columns (inclusions).

1 INTRODUCTION

In cases that the soil is either too soft or too loose, its use as a foundation layer is very often preceded by its improvement with inclusions that are materialized via replacement (e.g. vibro-flotation) or solidification (e.g. soil mixing) methods. The improved site is composite and has more or less different mechanical properties than the natural soil. The amount of differentiation depends on the inclusion material and the replacement ratio of the improvement geometry. For example, a soft clay site improved with gravel piles for the reduction of the anticipated settlements has a different seismic response than the natural soft clay site. Nevertheless, this fact is very often neglected in the seismic design of the superstructure, since taking it into account requires the performance of at least 2D (not to mention 3D) seismic ground response analyses. Moreover, opting for not performing any analyses is often based on the *ad hoc* assumption that the effect of improvement is beneficial for the superstructure, a fact that is not necessarily true

More accurate design could be achieved if the necessary ground response analysis of the improved site could be performed via simpler methodologies, like 1D equivalent-linear analysis (e.g. using SHAKE91, Idriss and Sun 1992). In an attempt to allow for such analyses, this paper proposes a methodology for modeling the composite (non-uniform) ground as an equivalent-uniform material that if it is subjected to the same base excitation leads to the same overall seismic motion at the ground surface. The methodology calibrates the shear modulus G degradation and damping ξ increase curves with the amplitude of the cyclic shear strain γ of the equivalent-uniform material, as a function of the respective curves of the natural soil and the inclusion material, as well as the replacement ratio a_r of the composite ground. The emphasis is put on grid-like improvement geometries that are usually modeled in 2D plane strain analyses like a series of embedded soldier pile walls.

The methodology has a theoretical basis and is verified by comparing parametric results from 2D and 1D equivalent-linear analyses that assume uniform soft soil and improvement material properties from the ground surface to the base. Despite the simplicity of the analyses, the proposed equivalent-uniform soil model is considered appropriate for use for non-uniform material properties, given the use of the appropriate per depth value of the improvement-to-soil maximum shear stiffness ratio $K_o = G_{io}/G_{so}$.

Moreover, it is shown to be accurate for sites excited by any earthquake intensity and predominant frequency and approximately accurate for non-uniform improvement geometries (e.g. shallow improvement in deep soft soil deposit or narrow improvement zone in extensive soft soil deposit). Sole exception to this rule is that the use of the proposed equivalent-uniform model is considered appropriate for the improvement of soft soils that do not exhibit excess pore pressure buildup and parallel drainage, a coupled mechanism of fluid flow and deformation that was not addressed in the performed analyses.

2 SEISMIC GROUND SURFACE RESPONSE OF AN IMPROVED SITE

The 2D and 1D analyses in this paper were performed with *QUAD4M* (Hudson et al 1994) and *Shake91* (Idriss and Sun 1992), respectively, two (2) commercial codes that perform an equivalent-linear analysis, the former in the time domain while the latter in the frequency domain. Before proceeding to the analysis of improved sites, it was considered necessary to establish that the two (2) codes produce identical results for the benchmark case of the 1D vertical S wave propagation through a uniform horizontal soil layer over rigid bedrock. This was achieved by disallowing vertical motion of the lateral boundaries of the 2D mesh, whose width was at least 5 times longer than the depth H of the uniform soil column. This type of lateral boundary conditions that are set far enough from the area of interest is the optimal solution for the numerical code at hand, which does not offer the user absorbing or free-field boundaries, as other codes do.

The basic prerequisite for using 1D seismic ground analysis for an improved composite ground is that the actual seismic ground surface response shows negligible spatial variability. If that is the case, then it has the potential to be approximated by an "average" ground surface response, that of an equivalent-uniform material. In order to answer this question, the improvement geometry of Fig.1 was simulated by 2D analyses, with a base excitation that imposes vertically propagating SV waves.

The soft soil and the inclusions are assumed linear visco-elastic materials with $G_{s,o}$ = 80MPa and $G_{i,o}$ = 800MPa having a total depth H = 10m that are imposed to a seismic excitation with peak acceleration at outcropping bedrock equal to 0.15g.

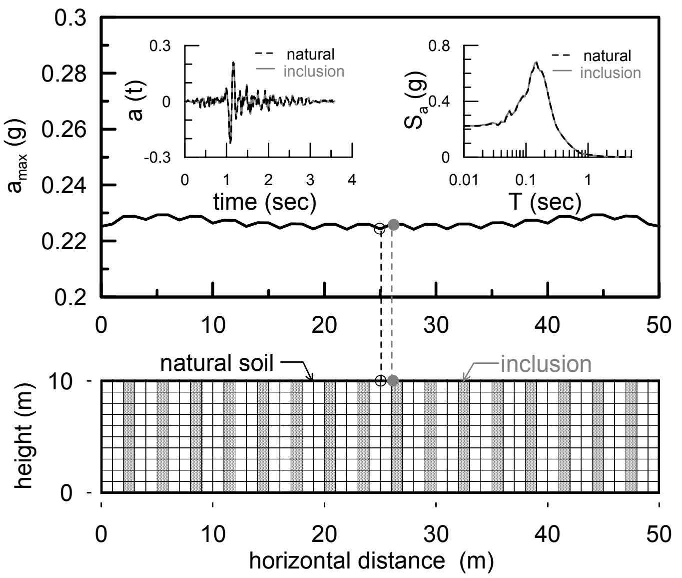

Figure 1. Finite element mesh and results for the variation of seismic acceleration at ground surface from 2D analysis

As shown in this figure, the peak horizontal acceleration a_{max} at various points of the ground surface show small variability (0.22 – 0.23g). This is also depicted in the time-histories and the elastic response spectra of 2 neighboring nodes at the center of the improvement geometry, one on natural soil and the other on the inclusion that show practically identical results. Hence, the ground surface response can be considered as practically uniform and as such, it can potentially be estimated without the use of a 2D analysis, but via a 1D analysis of the seismic response of an equivalent-uniform soil column of the same depth H.

The question that arises is what are the dynamic properties of this equivalent-uniform material? This is the subject of the following paragraphs.

3 EQUIVALENT-UNIFORM MATERIAL FOR LINEAR ANALYSES

The improvement configuration of any 2D analysis may be identified in terms of two (2) parameters: a) the improvement-to-soil maximum shear stiffness ratio $K_o = G_{i,o}/G_{s,o}$ (>> 1), and b) the improvement area ratio $a_r = d' / s$ (< 1), where d′ is the width of the embedded soldier pile walls of the analysis and s their center-to-center interdistance. For the example of Fig.1, a_r = 1m/3m ≅ 33% and K_o = 800/80 = 10. Note that the actual values of d′ and s are not important for the seismic ground response, but only the improvement area ratio, i.e. $a_r = d'/s$. This is shown in Fig.2 that compares the elastic response spectra at ground surface from three (3) analyses with the same K_o (=10) value and the same a_r (= 15%) value that are excited by the same input motion.

As a first approximation, the maximum shear modulus $G_{eq,o}$ of the equivalent-uniform material can be estimated by assuming that the vertically propagating SV waves impose the same shear strain γ to both the improvement inclusion and the neighboring natural soil (e.g. Baez & Martin 1993).

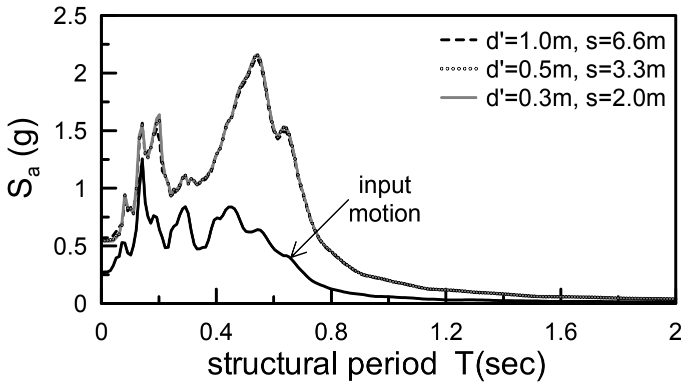

Figure 2. Elastic response spectra (5% damping) at ground surface for 3 different improvement configurations, but the same area replacement ratio a_r.

In such a case, this common value of γ is assigned to the equivalent-uniform material as well. The estimation of $G_{eq,o}$ is enabled by estimating the total shear force $F_{eq,o}$ imposed by the SV wave on a unit cell composed by an inclusion and its neighboring soil (see Fig.3a):

$$F_{eq,o} = F_{s,o} + F_{i,o} \Rightarrow \tau_{eq,o} = \tau_{s,o}(1-a_r) + \tau_{i,o}a_r \quad (1)$$

Given the aforementioned equality of shear strains γ in the inclusion, the soil and the equivalent-uniform material, Eq.(1) leads to:

$$G_{eq,o} = G_{s,o}[(1-a_r) + K_o a_r] \quad (2)$$

As a second approximation, the $G_{eq,o}$ of the equivalent-uniform material can be estimated indirectly by assuming two (2) materials in sequence, under the same normal stress σ. This 1-D loading leads to deformations d_1 and d_2 in the two materials, two values that are related to their Young's moduli E_1 and E_2 and their initial lengths L_1 and L_2, respectively. In this case, an equivalent-uniform material would have a total length $L=L_1+L_2$ and a total deformation $d=d_1+d_2$, that would be interrelated via the Young's modulus E_{eq} of the equivalent-uniform material. Based on elasticity theory, the values of E_1, E_2 and E_{eq} are interrelated as:

$$E_1 = \sigma\frac{L_1}{d_1} \quad ; \quad E_2 = \sigma\frac{L_2}{d_2} \quad ; \quad E_{eq} = \sigma\frac{L_1 + L_2}{d_1 + d_2} \quad (3)$$

Introducing E_1 and E_2 into E_{eq} of Eq.(3), and after some algebra, leads to the following relation for E_{eq}:

$$E_{eq} = E_2\frac{(E_1/E_2)}{a_r + (1-a_r)(E_1/E_2)} \quad (4)$$

where $a_r = L_1/(L_1+L_2)$. By assigning the 1-D physical analogy, to the physical problem of earthquake-induced shearing of the inclusion (material 1) and the natural soil (material 2), the common value of σ is replaced by a common value of τ and the values of E_1, E_2 and E_{eq} by $G_{i,o}$, $G_{s,o}$ and $G_{eq,o}$. Hence, the $G_{eq,o}$ is given by:

$$G_{eq,o} = G_{s,o}\frac{K_o}{a_r + (1-a_r)K_o} = G_{s,o}M_o \quad (5)$$

where $M_o \geq 1$, is a dimensionless multiplier of $G_{s,o}$. In such a case, the common value of τ leads to different values of γ for the inclusion and the natural soil, where $\gamma_{i,o} = \gamma_{s,o}/K_o$, as shown schematically in Fig.3b.

Eqs (2) and (5) provide two analytical approaches for the value of the equivalent-uniform maximum shear modulus $G_{eq,o}$. In order to ascertain which of the 2 approaches is more appropriate for use in "equivalent" 1D analyses, Fig.4a presents the results from linear 1D analyses that were conducted with $G_{eq,o}$ equal to 320 and 114.3MPa.

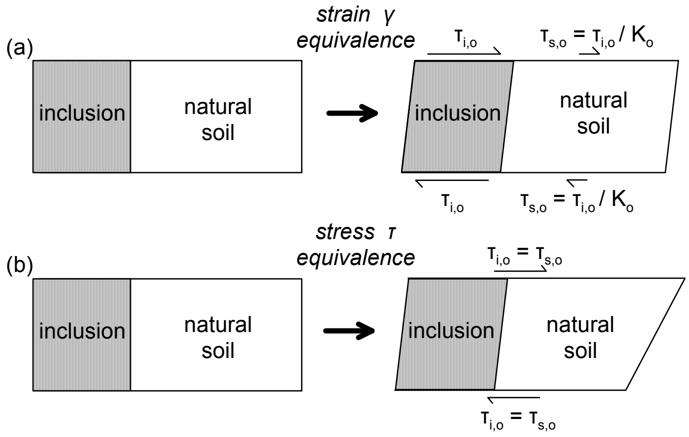

Figure 3. Schematic illustration of deformed soil-inclusion cell based on the assumptions of strain and stress equivalence

These two (2) values of $G_{eq,o}$ result from using Eqs (2) and (5), respectively, for $G_{s,o}$ = 80MPa, K_o = 10 and a_r = 33%. Furthermore, Fig.4a includes the 1D results for the response of the natural soil deposit ($G_{s,o}$ = 80MPa), as well as the results from the 2D analysis of the composite improved ground (the same from Fig.1). It is deduced that a 1D analysis with an equivalent-uniform material whose G value is estimated via Eq.(5) practically duplicates the a_{max} variation at the ground surface of the improved ground, while the use of Eq.(2) leads to erroneous results. The same holds for the amplification of the elastic response spectra S_a, whose typical comparison for case of Fig.1 is given in Fig.4b. Obviously, fine-tuning of the value of $G_{eq,o}$ could lead to an even better match of the 2D results for both the a_{max} variation and the response spectral amplification.

Fig.4 presents an example of how one could back-estimate $G_{eq,o}$ for a specific set of $G_{s,o}$, K_o and a_r. Repeating the same exercise for various values of K_o (= 3.33, 15) and a_r = 2.5 – 94% leads to the summary plot of Fig.5 (a total of 67 cases). For comparison with these numerical estimates of $G_{eq,o}$, the lines procuring from Eqs (2) and (5) are also presented, showing that the accuracy of Eq.(5) is universal, independently of the value of K_o and a_r. Based on this result, it is deduced that the maximum stiffness $G_{eq,o}$ of the improved site is much less affected by the area replacement ratio a_r, than what Eq.(2) of Baez and Martin (1993) predicts.

It should be underlined that the use of Eq.(5) for the estimation of $G_{eq,o}$ is not restricted to specific ranges of K_o and a_r, since it has been based on a simplistic theoretical model of the loading of a composite cell of natural soil and a stiffer inclusion and verified from numerical analyses. Yet, it still requires verification from insitu (or centrifuge) measurements from actual cases.

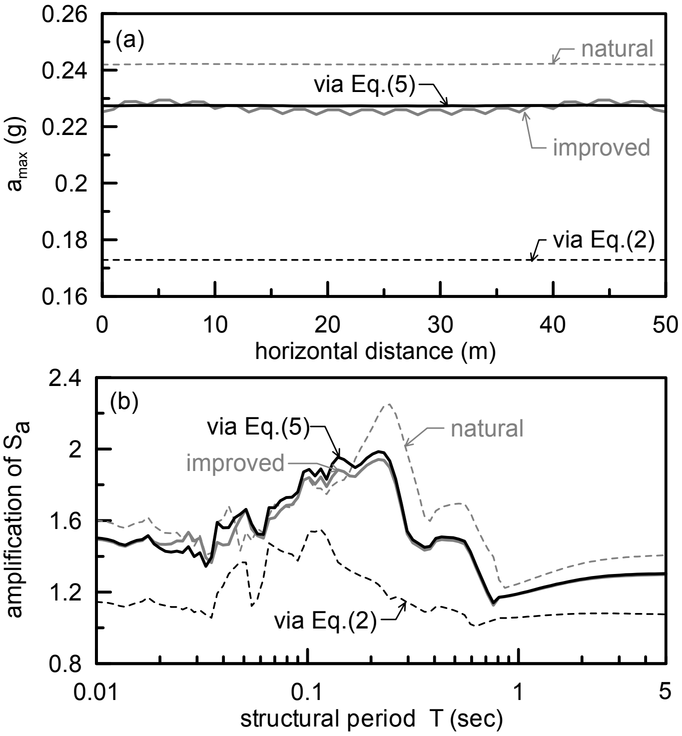

Figure 4. Comparison of results from 1D analyses with equivalent-uniform materials to results from a 2D analysis of composite improved ground, in terms of: a) a_{max} variation with horizontal distance, b) S_a amplification for typical surface location.

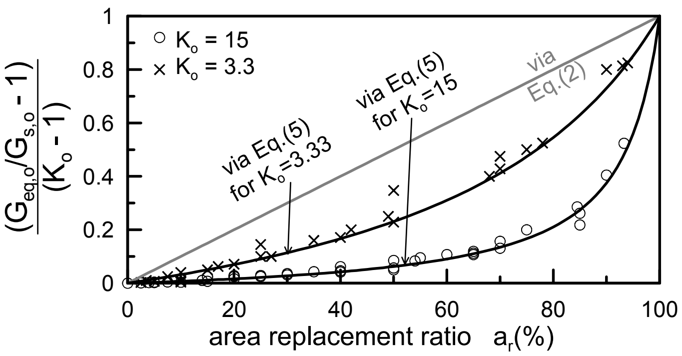

Figure 5. Comparison between numerical estimates of $G_{eq,o}$ and the two (2) analytical approaches

4 EQUIVALENT-UNIFORM MATERIAL FOR NON-LINEAR ANALYSES

The shear stress τ equivalence between the inclusion and the neighboring natural soil holds true for both linear and non-linear analyses. Therefore, Eq. (5) may be re-written as:

$$G_{eq} = G_s \frac{K}{a_r + (1 - a_r)K} = G_s M \tag{6}$$

where G_{eq}, G_s, K (= G_i/G_s) and M are the non-linear (and strain-level dependent) counterparts of $G_{eq,o}$, $G_{s,o}$, K_o (= $G_{i,o}/G_{s,o}$) and M_o of Eq.(5). Due to the τ equivalence, the strains γ_i and γ_s that control the values of G_i and G_s, respectively, are not equal. In particular, since $\tau_i = \tau_s$ it holds that $\gamma_s = K\gamma_i$. By defining γ_{eq} as the shear strain of the equivalent-uniform soil, Eq. (6) may be written as:

$$G_{eq}(\gamma_{eq}) = G_s(\gamma_s) \frac{K(\gamma_{eq})}{a_r + (1 - a_r)\; K(\gamma_{eq})} \tag{7}$$

where:

$$\gamma_{eq} = \gamma_s \left[1 - \left(1 - \frac{1}{K(\gamma_{eq})} \right) a_r \right] \tag{8}$$

Eqs (7) and (8) show that the estimation of the G_{eq}-γ_{eq} curve to be used in non-linear 1D analysis cannot be performed directly, but requires iterations on the basis of the G_s-γ_s and the G_i-γ_i curves of the natural soil and the inclusion materials. Given Eq.(5), the foregoing calculations may be performed on the basis of the normalized degradation curves, i.e. the $G_{eq}/G_{eq,o}$-γ_{eq} curve may be estimated on the basis of the $G_s/G_{s,o}$-γ_s and the $G_i/G_{i,o}$-γ_i curves of the natural soil and the inclusion materials.

In more detail, the iterative procedure is performed in steps, i.e. for any given value of γ_s, the following are successively estimated:

- the $G_s/G_{s,o}$ value from the $G_s/G_{s,o}$-γ_s curve,
- the $G_i/G_{i,o}$ value from the $G_i/G_{i,o}$-γ_i curve, on the basis of $\gamma_i = \gamma_s/K_{ini}$ (the first estimate of which can be $K_{ini} = K_o$)
- the $K_{fin} = G_i/G_s = K_o(G_i/G_{i,o})/(G_s/G_{s,o})$ value, which is then used for re-estimating the γ_s value.

This iterative procedure for any given value of γ_s is continued until convergence ($K_{fin} = K_{ini}$). Then, given the converged value of K (= $K_{ini} = K_{fin}$) the values of G_{eq} and γ_{eq} are estimated from Eqs (6) and (8) on the basis of G_s and γ_s. Repeating this iterative procedure for the whole range of γ_s values, constructs the whole $G_{eq}/G_{eq,o}$-γ_{eq} curve.

The calculations were performed for the well-established degradation curves of Vucetic & Dobry (1991), i.e. the $G_i/G_{i,o}$-γ_i curve being that for I_p = 0% and the $G_s/G_{s,o}$-γ_s curve alternatively being that for I_p = 15% and 30%. Performing such calculations for various values of K_o and a_r showed that the $G_{eq}/G_{eq,o}$-γ_{eq} curve is very little dependent on the $G_i/G_{i,o}$-γ_i curve and practically comes about by a translation of the $G_s/G_{s,o}$-γ_s curve to smaller values of γ. Fig. 6 shows an example of such a translation for K_o = 15 and a_r = 33% for the natural soil curve being that for I_p = 30%.

Detailed analysis of the foregoing calculations showed that this translation can be assumed to be parallel, i.e. $G_{eq}/G_{eq,o}(\gamma_{eq}) = G_s/G_{s,o}(\gamma_s)$, with γ_{eq} being related to γ_s. Since the γ_{eq} in question refers to the G degradation curve, it is hereafter denoted as $\gamma_{eq,G}$ and is interrelated to γ_s according to the following empirical equation:

$$\gamma_{eq,G} = \gamma_s \left[1 - \left(1 - \frac{1}{K_o^{0.85}} \right) a_r \right] \quad (9)$$

Note that Eq.(9) is practically an empirical form of Eq.(8) that came about by best-fitting the iteratively estimated $G_{eq}/G_{eq,o}$-γ_{eq} curve for intermediate strain levels, i.e. around strains where $G_{eq}/G_{eq,o}(\gamma_{eq}) = G_s/G_{s,o}(\gamma_s) = 0.5$ ($\gamma = 0.01 - 0.1\%$) that are of primary concern for this problem in practice.

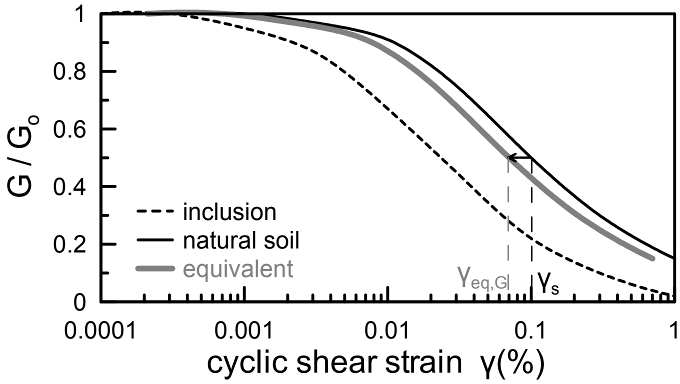

Figure 6. Shear modulus degradation curves for the natural soil, the inclusion and the equivalent-uniform material

The damping ratio increase curve of the equivalent-uniform material, ξ_{eq}-γ_{eq}, is again based on the respective curves for the natural soil, ξ_s-γ_s and the inclusion material, ξ_i-γ_i. Its estimation process is based on the general form of Eq.(6), where the shear modulus values are introduced in terms of their complex forms, i.e. $G_{eq}^* = G_{eq}(1+2i\xi_{eq})$, $G_i^* = G_i(1+2i\xi_i)$ and $G_s^* = G_s(1+2i\xi_s)$. Appropriate algebraic manipulations lead to the following relation between the various ξ values:

$$\xi_{eq} = \xi_s (1 - a_r) M + \xi_i a_r \frac{M}{K} \quad (10)$$

where K ($=G_i/G_s$) and M are the non-linear counterparts of K_o and M_o of Eq.(5).

Following a similar iterative procedure as that for the G degradation curve for various values of a_r and K_o it was deduced that the ξ_{eq}-γ_{eq} curve is very little dependent on the ξ_i-γ_i curve and practically results from a translation of the ξ_s-γ_s curve to smaller values of γ. Fig. 7 shows an example of such a translation for $K_o = 15$ and $a_r = 33\%$ for the natural soil curve being that for $I_p = 30\%$.

Detailed analysis of the foregoing calculations showed that this translation can be assumed to be parallel, i.e. $\xi_{eq}(\gamma_{eq}) = \xi_s(\gamma_s)$, with γ_{eq} being related to γ_s via the following empirical form of Eq.(11):

$$\gamma_{eq,\xi} = \gamma_s \left[1 - \left(1 - \frac{1}{K_o^{0.975}} \right) a_r \right] \quad (11)$$

As for Eq.(9), the empirical estimate of the $\gamma_{eq,\xi}/\gamma_s$ ratio of Eq.(11) came about by best-fitting the iteratively estimated ξ_{eq}-γ_{eq} curve for intermediate strain levels, i.e. around strains where $\xi_{eq}(\gamma_{eq}) = \xi_s(\gamma_s) = 8\%$ ($\gamma = 0.01 - 0.1\%$), that are of primary concern for this problem in practice.

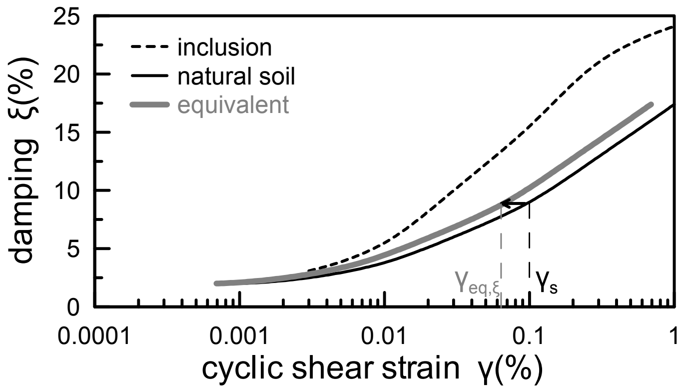

Figure 7. Damping increase curves for the natural soil, the inclusion and the equivalent-uniform material

5 VERIFICATION OF THE EQUIVALENT-UNIFORM SOIL MODEL

This section presents verification runs for the equivalent-uniform model of improved ground, whose calibration process is described in the foregoing sections. The emphasis here is on non-linear response, since Fig.4 has already presented an example of the accuracy of the proposed methodology for linear analyses.

In particular, the case of an improved site with depth H = 20m, $K_o = 30$ and $a_r = 30\%$ is assumed. The non-linear properties of the natural soil and the inclusions are introduced via the G/G_o-γ and ξ-γ curves of Vucetic & Dobry (1991) for $I_p = 30\%$ and 0%, respectively. This site subjected to an intense seismic excitation with a peak horizontal acceleration of 0.27g that induces non-linear behavior of the soil. Figs 8a & 8b compare the elastic response spectra and the spectral amplification, respectively, at the ground surface from 3 analyses performed with the foregoing seismic excitation. These 2D analyses refer to: a) the natural soil site, b) the improved (composite) site and c) the equivalent-uniform improved site. It is observed that the analysis for the equivalent-uniform site yields practically the same elastic response spectrum at the ground surface as that for the improved (composite) site. This is an example of the accuracy of the proposed methodology for non-linear analyses.

In addition, this figure provides insight to the effect of the improvement on the ground surface response, which is of primary interest for civil engineering works.

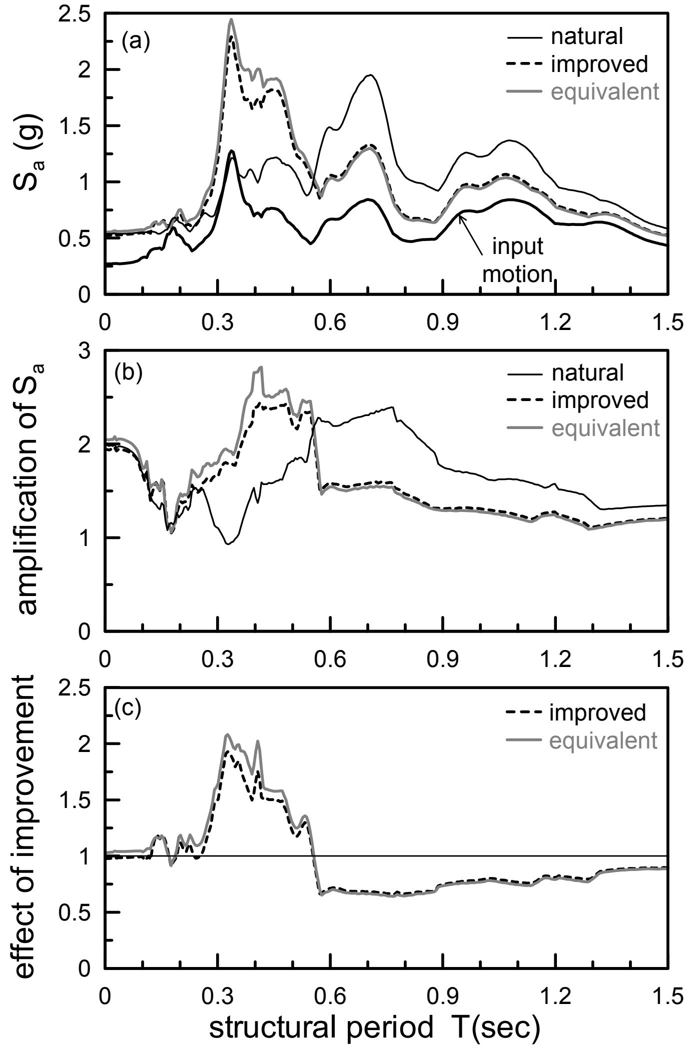

Figure 8. Comparison of seismic response at ground surface for a natural soil site and its improvement analyzed via a composite and an equivalent-uniform model: a) elastic response spectra S_a, b) amplification of S_a, c) effect of improvement on S_a.

In particular, Fig 8 shows that the ground response of the natural soil may be very different from that of the improved ground. In particular, Fig 8b shows that although the peak horizontal acceleration is more or less the same, the spectral values are different. This is better depicted in Fig 8c that shows the effect of the improvement on the spectral ordinates. It is observed that the improvement de-amplifies the motion at large structural periods (larger than 0.6s), but amplifies it for intermediate periods (between 0.25 – 0.5s). This is something expected for most cases in practice, where the fundamental period of the natural soil site (here $T_s \cong 0.7$s) is much larger than the predominant period of the excitation (here $T_e \cong 0.33$s). The reason is that the improvement introduces stiff inclusions in the soft soil that reduce the fundamental period of the site (here to 0.45s approximately) and bring it closer to the predominant period of the excitation.

Based on the above it is deduced that an amplification of spectral ordinates due to the improvement is expected at periods around the (reduced) fundamental period of the improved site. In addition, Fig 8 shows that this selective spectral amplification may reach a factor of 2. Although the phenomenon of selective spectral amplification is qualitatively expected in all cases where the predominant period T_e of the excitation is smaller than the fundamental period of the natural soil site T_s, the factor of 2 presented in Fig 8 must be considered an extremely high value that came about due to resonance phenomena, since the predominant period T_e of the excitation is quite similar to the reduced fundamental period of the improved site.

6 CONCLUSIONS

This paper shows that 2D seismic response analyses of improved sites may be accurately replaced by 1D analyses for an equivalent-uniform material, whose dynamic properties are a function of the respective properties of the natural soil, the inclusion material and the area replacement ratio. The proposed calibration process of an equivalent-uniform material and its use for 1D equivalent-linear analysis (e.g. using *Shake91*, Idriss and Sun 1992) has been shown to effectively duplicate the results of respective 2D analyses for the composite improved site, and this for both linear and non-linear ground response conditions. In addition, it is shown that the improvement itself is not necessarily beneficial for the seismic response at ground surface, since it may lead to selective spectral amplification at periods around the fundamental period of the improved site. These results have been produced by numerical analyses and still require verification from insitu (or centrifuge) measurements from actual cases.

7 ACKNOWLEDGEMENTS

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