Equivalence between 2D and 3D numerical simulations of the seismic response of improved sites

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ABSTRACT: This paper studies how 2D numerical analyses may be accurately used for simulating the truly 3D problem of the seismic response of improved sites. Specifically, parametric results are compared from pertinent 3D and 2D seismic ground response analyses and a methodology is proposed for replacing the time-consuming (and usually unavailable) 3D analyses with "equivalent" 2D analyses that closely simulate the results of the 3D analyses. Emphasis is put on replacement and solidification methods of soft (cohesive) soils and on three (3) improvement geometries: a) an embedded soldier pile wall, b) a grid of columns/piles and c) a grid of closed square cells

1 INTRODUCTION

The use of 3D numerical analyses in geotechnical earthquake engineering is very scarce. For practitioners such analyses are considered a luxury, since they are very consuming in terms of time and computational effort. In addition, the commercially available 3D codes for performing numerical analysis of geotechnical earthquake engineering problems are very few and usually these codes have a smaller potential than commercial 2D codes. For example, 2D codes offer the use of advanced constitutive models or element types that are not found in the libraries of 3D codes. Hence, the numerical research in geotechnical earthquake engineering has been historically based on the use of (1-D and) 2D analyses.

In an attempt to bridge this gap, this paper studies how 2D numerical analyses may be accurately used for simulating the truly 3D problem of the seismic response of improved sites. The emphasis is put on three (3) distinct geometries of soil improvement, namely: a) an embedded soldier pile wall, b) a grid of columns/piles and c) a grid of closed square cells. These geometries are usually materialized via replacement (e.g. vibro-replacement) or solidification methods (e.g. Deep Soil Mixing, DSM). In particular, grids of closed square cells are usually constructed using a solidification method, while the other two (2) geometries are materialized by either improvement method.

This paper compares parametric results from 3D and 2D seismic ground response analyses of improved sites and proposes a methodology for replacing the costly 3D analyses with "equivalent" 2D analyses that closely simulate the results of the 3D analyses. In more detail, this methodology proposes a transformation of the 3D actual improvement geometry to an "equivalent" 2D geometry that if it is subjected to the same base excitation leads to the same seismic motion at the ground surface.

The performed analyses are visco-elastic and assume uniform soft soil and improvement properties from the ground surface to the base. Despite the simplicity of the analyses, the proposed 3D to 2D transformation is considered appropriate for use for non-uniform and non-linear material properties, since it is not affected by non geometric parameters like the improvement-to-soil shear stiffness ratio $K = G_i/G_s$, the predominant period of the base excitation T_e and the Rayleigh damping of the visco-elastic analyses. Sole exception to this rule is that the methodology is considered appropriate for the improvement of soft soils that do not exhibit excess pore pressure buildup and parallel drainage, a coupled mechanism of fluid flow and deformation that was not addressed in the performed analyses.

2 CALIBRATION OF NUMERICAL CODES

The 3D and 2D analyses in this paper were performed with *FLAC3D* (Itasca Inc 1997) and *FLAC* (Itasca Inc 1993), respectively, two (2) commercial codes that use the finite difference method in performing a time domain analysis. Before proceeding to the analysis of improved sites, it was considered necessary to establish that the two (2) codes produce identical results for the benchmark case of the 1D

vertical wave propagation through a uniform horizontal soil layer over rigid bedrock. For this purpose, a mesh of 1x10x1 cubic elements (in the x, z and y directions respectively) was constructed in *FLAC3D* and free field boundaries were assigned at all lateral faces. Note that in this paper the x and y directions are horizontal and the z is vertical. In comparison, a 2D analysis was performed in *FLAC* and a purely 1D analysis was performed with *Shake91* (Idriss and Sun 1992). In all analyses, the base excitation was applied as an acceleration time history of the base in the x (horizontal) direction. The comparison of results for the seismic response at ground surface (in terms of acceleration time history and elastic response spectrum) showed perfect agreement for all three (3) analyses.

Furthermore, before performing numerical analyses of improved sites with *FLAC3D* it is important to establish its accuracy for another benchmark case that involves purely 2D improvement geometry. For this purpose, the case of a vertical diaphragm wall of thickness d=1m, height H=10m and G_i=540MPa was selected that is embedded in 10m soft soil with G_s=18MPa that lies over rigid bedrock. The vertical wall is aligned along the yz plane. This problem was first analyzed with *FLAC* using a 80x10 mesh of square 1m wide elements discretizing the xz plane and having free field lateral boundaries. In this analysis, the vertical diaphragm wall is simulated by a centered column of 10 elements with a different value of shear modulus (K=G_i/G_s=30). Using *FLAC3D* for the same physical problem and for comparison purposes entails the use of cubic 1m wide elements that construct a mesh that is surrounded by free field lateral boundaries. This mesh consists of 80x10xY elements, with Y the number of elements necessary for duplicating the results of the 2D analysis. The correct value of Y was estimated by performing parametric 3D analyses for Y=1, 6, 10, 20, 80.

In all 3D and 2D analyses, the seismic excitation was a Chang's signal with T_e=0.1sec that was applied as an acceleration time history at the nodes

of the base of the mesh in the x direction. Of interest for comparison purposes is the amplification of the peak acceleration in the x direction at the ground surface a_{max} due to existence of the diaphragm wall along the yz plane, i.e. the amplification of a_{max} as compared to the free field (soft soil) response. Hence, Fig 1a presents contours of the amplification of a_{max} from a 3D analysis where Y=80m. The details of the contours (e.g. the numerical values) are not important here. What is important is that these contours clearly show that the ground response varies only along the x direction and is not a function of the y distance, a fact that underlines the 2D character of the problem.

This is better depicted in Fig 1b that compares the same results along the x axis, where the 3D results compare perfectly with the 2D results. The question that arises is whether it is actually necessary to perform such a time consuming 3D analysis with 64000 (= 80x10x80) elements in order to achieve the accuracy of a 2D analysis. The answer is given again in Fig 1b, where the results from two (2) more 3D analyses are presented where Y=20 and Y=1, respectively. Observe that for Y=20 the results are again identical to those for Y=80, and that even for Y=1 the differentiations are marginal. It is concluded that for 2D improvement geometries in the xz plane, a 3D analysis with 1 element in the y direction and free field lateral boundaries suffices.

3   2D ANALYSIS OF SOLDIER PILE WALL

An embedded soldier pile wall in the y direction has a top view as shown in Fig 2. It consists of a series of improvement piles/columns of diameter d that are equally spaced at a center-to-center distance D along the y direction. Obviously, when D=d the soldier pile wall becomes a diaphragm wall that has a purely 2D geometry, as discussed in the previous section. Nevertheless, in general, a soldier pile wall along the y direction with applied base acceleration time history along the x direction is a 3D problem.
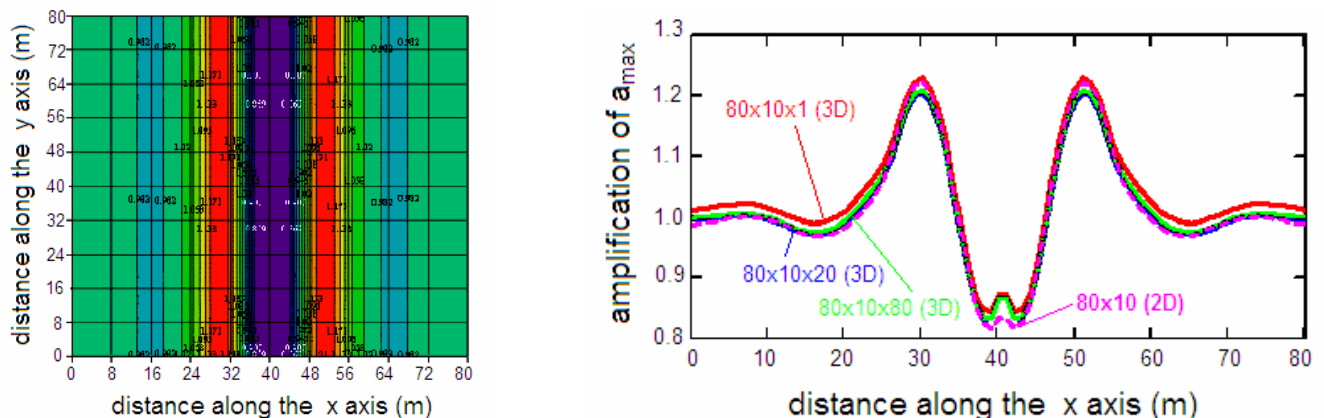

Figure 1.   Amplification of peak ground acceleration a_{max} from 3D seismic response analyses of a diaphragm wall (d=1m), in terms of: a) contours from an analysis with a 80x10x80 mesh, b) variation along the x axis from the reference 2D and all 3D analyses.
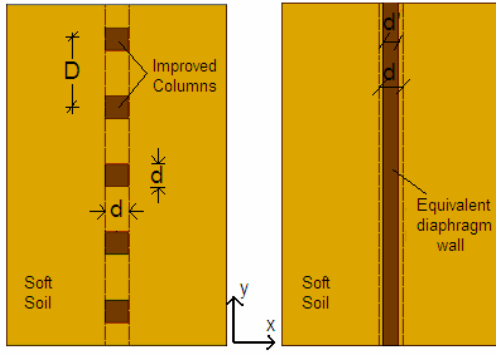
Figure 2. Top views of 3D and equivalent 2D geometries of an embedded soldier pile wall

This is shown in Fig 3 that presents the contours of the amplification of a_{max} from a 3D seismic response analysis of an embedded soldier pile wall with d=1m, D=4m and K=30 in 10m of soft soil.

The analysis was performed with a mesh of 80x10x80 cubic (1m wide) elements and the soft soil properties and base excitation characteristics described in the calibration section of this paper. As in Fig 1, the details of the contours in Fig 3 are not important. What is important is to observe that the amplification of a_{max} along the axis of the wall is not uniform, a fact that underlines the 3D character of the problem. Yet, at small distances perpendicular to the wall axis (e.g. 4 – 5m), symmetry of the seismic ground response is established, i.e. the ground response is the same irrespective of the value of the y distance. It is this symmetry that allows for a potential 2D analysis of an "equivalent" diaphragm wall of thickness d´, as shown in Fig 3b.

The question that arises is whether one could know *a priori* the accurate value of d´ for any given set of d and D. The answer to this question is given in this paper using the format of Fig 3, i.e. by comparing any given 3D analysis (for specific values of d and D) to pertinent 2D analyses with various d´ values. In this way, any given set of d and D is related to a unique d´ value. By repeating such nu-

merical experiments, the authors constructed a database of (d, D, d´) triplets. Yet, a general rule for estimating d´ without the need of a 3D analysis had yet to be devised. Such a rule should be based on the form of deformation that the seismic ground excitation applies to an improved ground in its 3D actual geometry and its 2D "equivalent". In this effort, one could borrow knowledge from beam theory and by doing so three (3) different approaches were examined here, and compared to the database of the numerical experiments. In more detail:

3.1 *Area (A) equivalence*

This approach assumes equivalence between the cross sectional areas A of the improved ground in the 3D and the "equivalent" 2D geometries, i.e. $A_{3D}=A_{2D}$. Based on Fig 2, the cross sectional areas of improved ground in a distance D along y are:

$$A_{3D} = d^2 \quad ; \quad A_{2D} = d'D \tag{1}$$

Based on Eq.(1), the d´ is given by:

$$d' = d^2/D \tag{2}$$

This A equivalence has been traditionally used for calculations of consolidation rates (e.g. Barron 1948) and settlements (e.g. Priebe 1976) of improved ground. In terms of the latter, it implies an equivalence of the axial (vertical) stiffness between the 3D and 2D geometries.

3.2 *Moment of inertia (I) equivalence*

This approach assumes equivalence between the moments of inertia I of the cross sections of the improved ground in the 3D and the "equivalent" 2D geometries, i.e. $I_{3D}=I_{2D}$. Based on Fig 2, the moments of inertia of the cross sections of improved ground in a distance D along y are:

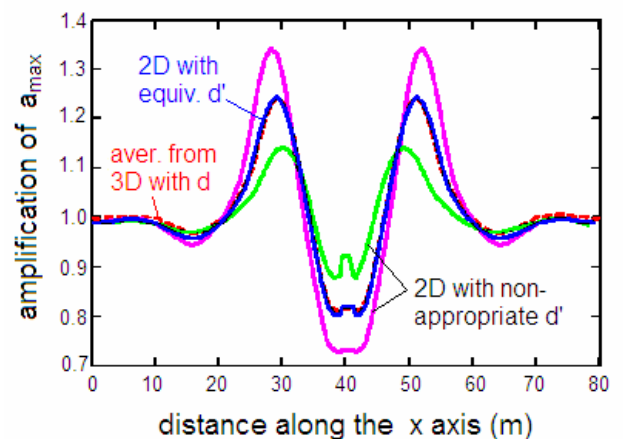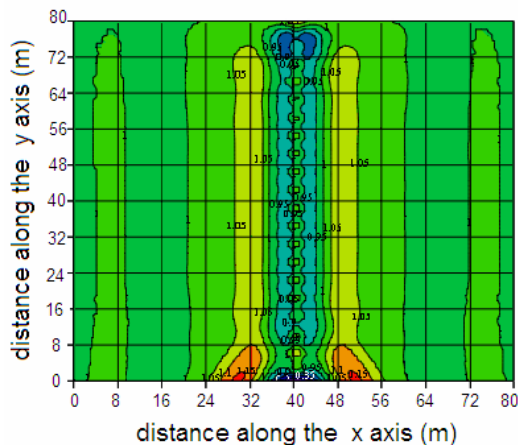$$I_{3D} = d^4/12 \quad ; \quad I_{2D} = d'^3D/12 \tag{3}$$

Figure 3. Amplification of peak ground acceleration a_{max} from 3D and 2D seismic response analyses of an embedded soldier pile wall (d=1m, D=4m), in terms of: a) contours from the 3D analysis (80x10x80 mesh), b) variation along the x axis from the reference 3D analysis (average response) and the various 2D analyses.

Based on Eq.(3), the d′ is given by:

$$d' = \left(d^4/D\right)^{1/3} \qquad (4)$$

This I equivalence is being used in practice for 2D static plane strain analyses of excavations with retaining (soldier pile) walls and implies an equivalence of the bending stiffness between the 3D and 2D geometries.

3.3 Section modulus (W) equivalence

This approach assumes equivalence between the section moduli W of the cross sections of the improved ground in the 3D and the "equivalent" 2D geometries, i.e. $W_{3D}=W_{2D}$. Based on Fig 2, the section moduli of the cross sections of improved ground in a distance D along y are:

$$W_{3D} = d^3/6 \quad ; \quad W_{2D} = d'^2 D/6 \qquad (5)$$

Based on Eq.(5), the d′ is given by:

$$d' = \left(d^3/D\right)^{1/2} \qquad (6)$$

As deduced by Eqs (2), (4) and (6), the W equivalence yields values of d′ that are intermediate between the values from the I equivalence and the A equivalence. As such, the W equivalence may be viewed empirically as an "overall" stiffness equivalence between the 3D and 2D geometries, that have neither an axial nor a bending stiffness equivalence.
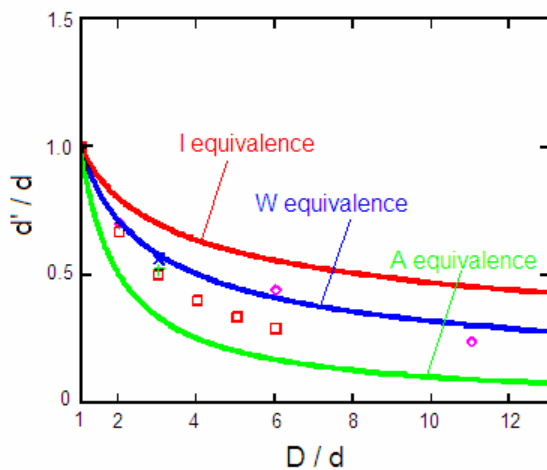

Figure 4. Comparison of analytical predictions of "equivalent" diaphragm wall thickness d′ and their estimates from numerical experiments for an embedded soldier pile wall

3.4 Back estimation of d′

Fig 4 compares the results of the numerical experiments (symbols) to the pertinent predictions from the 3 analytical approaches. It is concluded, that the W equivalence provides the best fit to the numerical estimates of d′, while the I and A equivalences serve as a upper and lower limits, respectively. This can be attributed to the fact that during shaking, the re-

sponse of a level ground layer (and its improvement inclusions) reminds that of a shear beam, whose vibration is more confined than that of a bending beam (I equivalence), while it is irrelevant to an axial (vertical) vibration implied by an A equivalence.

Note that the use of Eq. (6) for estimating d′ is appropriate, irrespective of the improvement geometry (d and D), the improvement method (K) and the predominant period of the seismic excitation (T_e). This is due to the fact that the numerical experiments summarized in Fig 4 correspond to 3D analyses with the following characteristics: d = 1, 2m – D/d = 2, 3, 4, 5, 6, 11 – K = 15, 30 – T_e = 0.1, 0.2sec.

4 2D ANALYSIS OF GRID OF PILES

Fig 5 presents the contours of the amplification of amax from a 3D seismic response analysis of a grid of 19x19 improvement piles with d=1m, D=4m and K=30 embedded in 10m of soft soil over rigid bedrock. The analysis was performed with a mesh of 80x10x80 cubic (1m wide) elements and the soft soil properties and base excitation characteristics described in the calibration section of this paper.

As above, the numerical details of Fig 5 are unimportant here. Of importance is to observe that the amplification of a_{max} at the ground surface is not uniform, a fact that underlines the 3D character of the seismic response of a grid of columns.
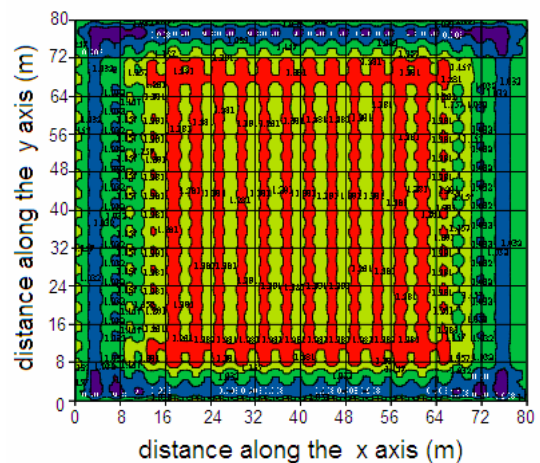

Figure 5. Contours of amplification of peak acceleration a_{max} from a 3D seismic response analysis (80x10x80 mesh) of a 19x19 grid of improvement piles (d=1, D=4m)

Yet, far from the horizontal mesh boundaries (e.g. for x and y ranging from 20 to 60m), symmetry of the seismic response is established, i.e. the response is the same irrespective of the value of y.

This symmetry provides the potential for performing "equivalent" 2D analyses, using the methodology devised for a single soldier pile wall in the previous section. In other words, a grid of improvement piles (of diameter d at a spacing D in 3D) may be viewed as a series of "equivalent" diaphragm

walls of thickness d´ at a centerline-to-centerline distance D that may be analyzed in 2D.

Following the same methodology as in the case of the soldier pile wall, the value of d´ for the 2D analyses of a grid of columns was estimated by comparing the ground surface response of reference 3D analyses to trial-and-error 2D analyses. Hence, Fig 6 compares the results from the foregoing numerical experiments to the three (3) analytical estimates of d´ summarized by Eqs (2), (4) and (6). Observe that this comparison shows that the use of Eq. (6), i.e. the W equivalence, gives again the best fit to the 3D response.

Note that the use of Eq (6) for estimating d´ for the 2D analyses of a grid of piles is valid, irrespective of the geometry (d and D) and the method (K) of improvement. This is due to the fact that the numerical experiments summarized in Fig 6 correspond to 3D analyses with the following characteristics: $d = 1, 2m - D/d = 3, 4, 6 - K = 15, 30$.
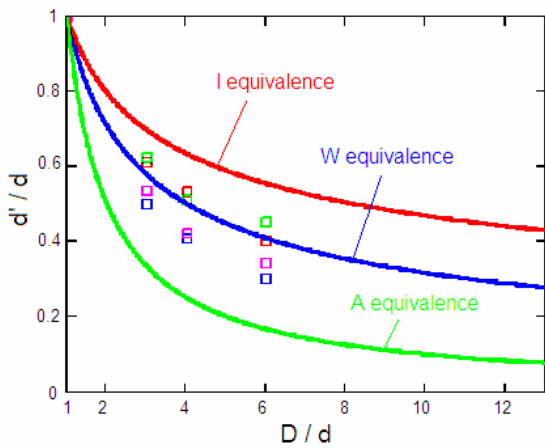

Figure 6. Comparison of analytical predictions for the "equivalent" diaphragm wall thickness d´ to their estimates from numerical experiments for a grid of improvement piles
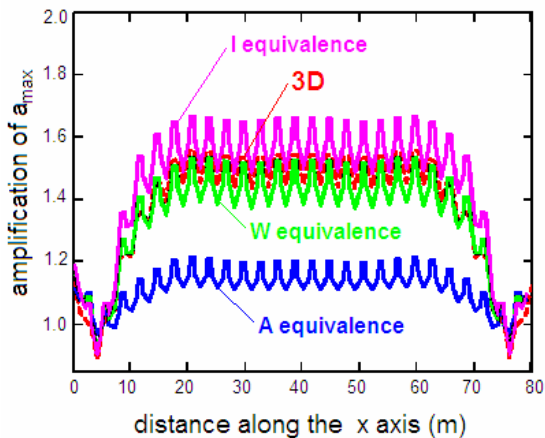

Figure 7. Exemplary estimate of error in the amplification of a_{max} from 2D seismic response analyses of a 27x27 grid of improvement piles.

Both Figs 4 and 6 show a significant difference between the d´ values from Eqs (2), (4) and (6). Yet, of importance for engineering purposes is how much

an erroneous estimate of d´ may affect the predicted seismic response. As an example, Fig 7 compares the "average" response of a grid of 27x27 improvement piles with $d=1m$, $D/d=3$ and $K=30$ as deduced by a 3D analysis (dashed line), to the respective 2D analyses using the three (3) analytical approaches (solid lines). It becomes obvious that the W equivalence (Eq. 6) gives the best fit to the 3D analysis, while the other two analyses have an error that could surpass 25%.

5 2D ANALYSIS OF GRID OF CLOSED CELLS

An embedded grid of closed square cells has a top view as shown in Fig 8a and is usually materialized using a solidification method (like the Deep Soil Mixing (DSM) denoted in Fig. 8a). It consists of two (2) series of DSM diaphragm walls (of thickness d) that are perpendicular to one another and which are equally spaced at a centerline-to-centerline distance D along the x and y directions. Obviously, when D=d the grid becomes a DSM block where all the soil has been solidified, a purely 2D geometry.
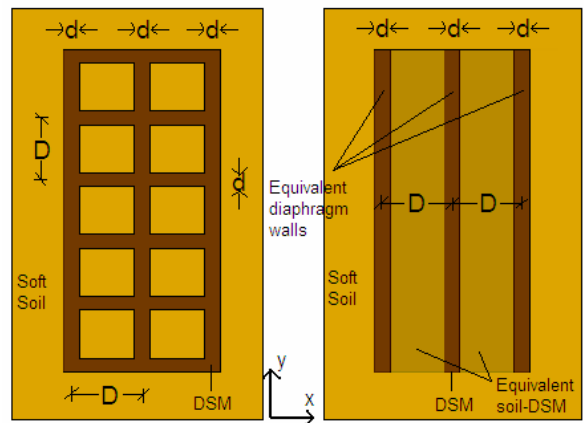

Figure 8. Top views of actual 3D and equivalent 2D geometries of a grid of closed improved cells

Nevertheless, in general, the seismic response of a soft soil improved with a DSM grid is a 3D problem. This is shown in Fig 9a that presents the contours of the amplification of a_{max} from a 3D seismic response analysis of a DSM grid with $d=1m$, $D=4m$ and $K=78$ in 10m of soft soil. The analysis was performed with a mesh of 100x10x80 cubic (1m wide) elements and the soft soil properties and base excitation characteristics described in the calibration section of this paper. Observe that the amplification of a_{max} on top of the DSM grid (from $x = 43$ to $x = 56m$) is not uniform, a fact that underlines the 3D character of the problem. Yet, immediately outside the DSM grid, symmetry of the seismic response is established, i.e. the response is the same irrespective of the value of y.

It is this symmetry that allows for a potential "equivalent" 2D analysis of the longitudinal series of

DSM walls (of thickness d at a distance D), as shown in Fig 8b. But, accurate 2D simulations require that the properties of the material between these DSM walls (denoted as "equivalent" soil-DSM in Fig 8b) are adjusted to account for the existence of the transverse DSM walls. This re-adjustment is performed in terms of the shear modulus G_{eq}, as:

$$G_{eq}/G_S = [1-(d/D)] + K(d/D) \qquad (7)$$

Eq (7) originates from appropriate analytical manipulations of an assumed GI equivalence of the material between the longitudinal DSM walls in the 3D and 2D configurations.

As an example, Fig 9b compares the ground surface responses from 2 analyses, the "average" from a 3D analysis and that from its "equivalent" 2D analysis, both of which pertain to a DSM grid with three (3) longitudinal walls (with d=1m, D/d=6 and K=78) that is embedded in 10m of soft soil. The 3D analysis was performed with a mesh of 100x10x80 cubic (1m wide) elements and the soft soil properties and base excitation characteristics described in the calibration section of this paper. The 2D analysis used a mesh of 100x10 square (1m wide) elements and an "equivalent" soil-DSM material calibrated according to Eq. (7). An excellent agreement is observed between the "average" 3D and the "equivalent" 2D analyses, especially in the area outside the DSM grid. Within the DSM grid, the comparison is merely satisfactory, but the agreement is much enhanced in more realistic DSM grids, where the D/d ratio rarely exceeds 4 – 5.
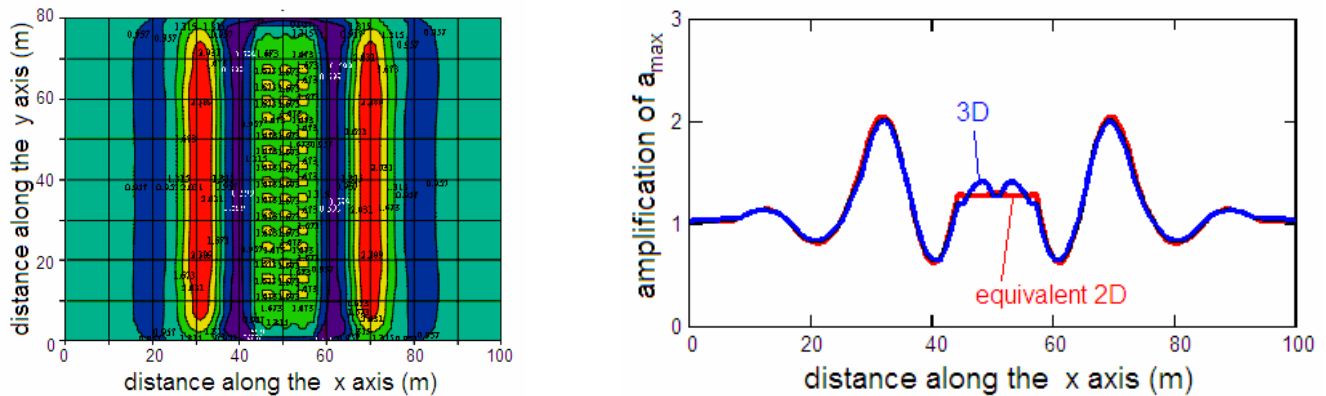

Figure 9. Amplification of peak ground acceleration a_{max} from 3D and 2D seismic response analyses of a grid of closed DSM cells (d=1m, D=4m), in terms of: a) its contours from the 3D analysis (100x10x80 mesh), b) its variation along the x axis from the foregoing 3D and the respective 2D analysis calibrated on the basis of Eq.(7).

6 CONCLUSIONS

This paper shows that 3D seismic response analyses of improved sites may be replaced with "equivalent" 2D analyses. This is achieved by a transformation of the 3D improvement geometry to an "equivalent" 2D, on the basis of equivalence of the section moduli W of the improvement inclusions. The proposed transformation is valid for improvement geometries in the form of a soldier pile wall and a grid of piles, irrespective of improvement method or excitation characteristics. For closed (DSM) cells, "equivalent" 2D analyses are also possible by adjusting the stiffness properties of the soil inside the cells to account for the transverse diaphragm walls. These results hold for improvement of soft soils that do not exhibit excess pore pressure buildup and parallel drainage, a coupled mechanism of fluid flow and deformation that was not addressed in the performed analyses. In closing, note that these results have been produced by numerical analyses and still require verification from insitu (or centrifuge) measurements from actual cases.

7 ACKNOWLEDGEMENTS

8 REFERENCES

Barron, R.A. 1948, Consolidation of fine-grained soils by drain wells. *Transactions of the American Society of Civil Engineers* 113: 718-742

Idriss, I.M. & Sun, J.I. 1992, *Shake91: A computer program for conducting equivalent linear seismic response analysis of horizontally layered soil deposits*. User's Guide, Center for Geotechnical Modeling, Civil Engineering Department, UC Davis

Itasca Consulting Group Inc 1993, *FLAC: Fast Lagrangian Analysis of Continua*. User's Manual.

Itasca Consulting Group Inc 1997, *FLAC3D: Fast Lagrangian Analysis of Continua in 3D*. User's Manual

Priebe, H.J. 1976, Abschatung des Setzungsverhaltens eines durch Stopfverdichtung verbesserten Baugrundes. *Die Bautechnik*, 5