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Non-Dominated allocations characterize Walasian equilibria in Finite Economies*

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Abstract. In this paper, we consider a pure exchange economy with a finite number of agents and commodities. By considering a notion of dominated allocations, we show that the Walrasian equilibrium set is characterized by the non-dominated allocations in a precise class of economies nearby the initial economy. The first and second welfare theorems are particular cases of this equivalence result.

Key words: Walrasian equilibrium, efficiency, dominated allocations.

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1 Introduction

In this paper, we consider a pure exchange economy $\mathcal{E}$, with a finite number of agents and commodities, where the initial endowments are $\omega = (\omega_1, \dots, \omega_n)$.

Given a feasible allocation x in the economy $\mathcal{E}$, we define a class of pure exchange economies which differs from $\mathcal{E}$ in the initial endowments that, in these new economies, are given by a convex combination of x_i and ω_i for every agent i.

We say that x is dominated in an economy if it is blocked by the coalition of all the agents.

We identify the finite economy with a continuum economy with a finite number of types of agents. We use Vind's (1972) remark on core allocations to show that non-dominated allocations in this class of economies characterize Walrasian equilibria in the initial economy $\mathcal{E}$.

We remark that the first and second welfare theorems are particular cases of this equivalence result.

2 The Equivalence Result

Let us consider a pure exchange economy $\mathcal{E}$ with a finite number of consumers and ℓ commodities.

Each agent $i \in \{1, \dots, n\}$ is characterized by her consumption set $X_i = \mathbb{R}^\ell_+$, her initial endowment $\omega_i \in \mathbb{R}^\ell_+$, and her preference relation $\succeq_i$.

We state the following assumptions on endowments and preference relations:

(H.1) The total endowment is strictly positive, that is, $\sum_{i=1}^{n} \omega_i \gg 0$.

(H.2) For every consumer i, the preference relation $\succeq_i$ is continuous, monotone and convex.

Note that the continuity of preferences implies that each preference relation $\succeq_i$ is represented by a continuous and monotone utility function $U_i : \mathbb{R}^\ell_+ \to \mathbb{R}$ (see Eilenberg (1941), Debreu (1954) or Debreu (1983) pp. 105-110).

Then, the economy is defined by $\mathcal{E} \equiv (X = \mathbb{R}^\ell, (U_i, \omega_i), i = 1, \ldots, n)$

An allocation $x = (x_1, \ldots, x_n)$ is feasible if $\sum_{i=1}^{n} x_i \leq \sum_{i=1}^{n} \omega_i$.

A competitive (or Walrasian) equilibrium for the economy $\mathcal{E}$ is a pair (p, x), where p is a price system and x is a feasible allocation such that for every agent i, the consumption bundle x_i maximizes U_i on the budget set $\{x \in \mathbb{R}^\ell_+ | p \cdot x \leq p \cdot \omega_i\}$.

A coalition S blocks an allocation x if there exists $y = (y_i, i \in S)$, such that $\sum_{i \in S} y_i \leq \sum_{i \in S} \omega_i$ and $y_i \succ_i x_i$ for every $i \in S$.

Definition 2.1 *An allocation (feasible or not in the economy $\mathcal{E}$) is dominated in the economy $\mathcal{E}$ if the coalition of all agents can improve upon it. That is, an allocation x is dominated in the economy $\mathcal{E}$ if the whole coalition blocks x.*

Note that the non-dominated and feasible allocations in the economy $\mathcal{E}$ are, precisely, the Pareto optimal or efficient allocations.

Given an allocation x and a vector $a = (a_1, \ldots, a_n)$, with $0 \leq a_i \leq 1$, let $\mathcal{E}(a, x)$ be a pure exchange economy which coincides with $\mathcal{E}$ except for the initial endowments that are given by $\omega(a, x) = a\omega + (\mathbf{1} - a)x$.

That is, $\mathcal{E}(a, x) \equiv \left(X = \mathbb{R}^\ell, (U_i,\ \omega_i(a_i, x_i) = a_i\omega_i + (1 - a_i)x_i), i = 1, \ldots, n\right)$

Theorem 2.1 *x is a walrasian allocation for the economy $\mathcal{E}$ if and only if x is a non-dominated allocation in every economy $\mathcal{E}(a, x)$.*

Proof. Let (p, x) be a walrasian equilibrium for the economy $\mathcal{E}$. Suppose that there exists $a = (a_1, \ldots, a_n)$, such that x is dominated in the economy $\mathcal{E}(a, x)$. Then, there exists $y = (y_1, \ldots, y_n)$ such that $\sum_{i=1}^{n} y_i = \sum_{i=1}^{n} \omega_i(a_i, x_i)$ and $U_i(y_i) > U_i(x_i)$ for every agent $i \in \{1, \ldots, n\}$. These inequalities imply that $p \cdot y_i > p \cdot \omega_i$, which is a contradiction with the feasibility of y in the economy $\mathcal{E}(a, x)$.

Let x be a non-dominated allocation for every economy $\mathcal{E}(a, x)$. Let f a step function on the real interval $I = [0, 1]$, defined by $f(t) = x_i$ if $t \in I_i = \left[\frac{i-1}{n}, \frac{i}{n}\right)$, if $i \neq n$, and $f(t) = x_n$ if $t \in I_n = \left[\frac{n-1}{n}, 1\right]$. Analogously, let $\omega(t) = \omega_i$ if $t \in I_i$, if $i \neq n$, and $\omega(t) = x_n$ if $t \in I_n$. Let μ the Lebesgue measure on I.

Assume that x is not a walrasian allocation for the economy $\mathcal{E}$. Then, the step allocation f given by x is not a Walrasian allocation for the associated continuum economy with n different types of agents (see García and Hervés (1993) for details). Applying Core-Walras equivalence (see Aumann (1964, 1966)), we have that f does not belong to the core of the associated continuum economy. Even more, there exists a coalition $S \subset I = [0,1]$, with $\mu(S) > 1 - \frac{1}{n}$, and there exists g, such that $\int_S g(t)d\mu(t) \leq \int_S \omega(t)d\mu(t)$ and $U_i(g(t)) > U_i(f(t))$ for almost all $t \in I_i$ for every $i = 1, \dots n$ (see Vind (1972)). Let $S_i = S \bigcap I_i$ and $a_i = \mu(S_i)$.

Now, in the finite economy, let us consider the allocation $(g_1, \dots, g_n)$, where $g_i = \dfrac{1}{\mu(S_i)} \displaystyle\int_{S_i} g(t)d\mu(t)$.

By convexity of preferences, $U_i\left(a_i g_i + (1 - a_i)x_i\right) > U_i(x_i)$, for every agent $i \in \{1, \dots, n\}$. Note that, since $\mu(S) > 1 - \frac{1}{n}$, we have that $a_i > 0$ for every i.

Therefore, x is a dominated allocation in the economy $\mathcal{E}(a.x)$, which is a contradiction.

Q.E.D.

This result provides a precise characterization of the walrasian equilibria in finite economies as non-dominated allocations.

Note that the first welfare theorem is an immediate consequence of the result above.

Moreover, note also that taking $x = \omega$, we obtain, exactly, the second welfare theorem.

Therefore, both welfare theorems are, obviously, particular cases or immediate interpretations of the result we have obtained.

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