ABSTRACTS

On Operators whose Compactness Properties are defined by Order by Safak Alpay

Abstract. Let E be a Banach lattice. A subset B of E is called order bounded if there exist a,b in E such that $a \le x \le b$ for each $x \in B$. Considering E in E'', the bidual of E, a subset B of E is called b-order bounded in E if it is order bounded in the Banach lattice E''. A bounded linear operator $T: E \to X$ is called o-weakly compact if T(B) is relatively weakly compact for each order bounded set E in E is called b-weakly compact if E is relatively weakly compact for each b-order bounded subset E in E is called an operator of strong type E if E if E is the band generated by E in E. Let these spaces be denoted by E is the band generated by E in E. Let these spaces be denoted by E is the space of weakly compact operators between E and E. In general, we have

$$W(E,X) \subseteq W_{st}(E,X) \subseteq W_b(E,X) \subseteq W_o(E,X)$$

between the spaces introduced above where containments may be strict. We study the pair of spaces where equalities hold above and show how the equalities of these spaces help to characterize various properties of E and X.

Vector lattices associated with ordered vector spaces

by Richard Becker

Abstract. Let V be a real, Archimedian ordered, vector space, whose positive cone V^+ satisfies $V = V^+ - V^+$. To V we can associate a Dedekind complete vector lattice W containing V (by abuse of notations), using cuts theory. When V is a vector lattice, W is the Dedekind complete vector lattice associated to V. In the case when V has an order unit the determination of W is already known.

Let $W_0 \subset W$ be the vector lattice generated by V. We study W_0 in the case where the cone C of all positive linear forms on V separates the elements of V. Moreover, we assume that $v \in V$ belongs to V^+ iff $\ell(v) \geq 0$ for every $\ell \in C$. The determination of W_0 involves the extreme rays of C. Let $\mathcal{E}_g(C)$ be the union of these rays. When C is the closed convex hull of $\mathcal{E}_g(C)$ we determine W_0 in terms of functions on $\mathcal{E}_g(C)$. In any case we determine the cone of all positive linear forms on W_0 in terms of conical measures on C, in the sense of G.Choquet. We formulate several problems within the framework of well-capped cones and ice-cream cones.

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R.Becker, Vector Lattices Associated with Ordered Vector Spaces, Mediterr. J. Math. 7, (2010), 313-322.

Price uncertainty and the existence of financial equilibrium

by Lionel de Boisdeffre

Abstract. We consider a pure exchange economy, with incomplete financial markets, where agents face an 'exogenous uncertainty', on the future state of nature, and an 'endogenous uncertainty', on the future price in each random state. Namely, every agent forms price anticipations on each future spot market, distributed along an idiosyncratic probability law. At a sequential equilibrium, all agents expect the 'true' price as a possible outcome and elect optimal strategies at the first period, which clear on all markets at every time period. We show that, provided the endogenous uncertainty is large enough, a sequential equilibrium exists under standard conditions, for all types of financial structures (i.e., with real, nominal and mixed assets). This result suggests that standard existence problems of sequential equilibrium models, following Hart (1975), stem from the single price expectation assumption.

Disjointness preserving operators on spaces of continuous functions

by Karim Boulabiar

Abstract. Let C(X) indicate the Riesz space of all real-valued continuous functions on a Tychonoff space. We characterize the unital Riesz subspaces L of C(X) such that any order bounded disjointness preserving operator on L is a weighted composition map.

Polynomials on Banach lattices

by Gerard Buskes

Abstract. We discuss joint work with Q. Bu on homogeneous polynomials on Banach lattices.

Invariant subspaces of weakly compact-friendly operators

by Mert Caglar

Abstract. Motivated by a problem of Abramovich, Aliprantis and Burkinshaw, we study invariant subspaces of weakly compact-friendly operators on Banach lattices with topologically full center. As an instance, we prove that if a non-zero weakly compact-friendly operator B on a Banach lattice with topologically full center is locally quasi-nilpotent, then the super right-commutant $|B\rangle$ of B has a non-trivial closed invariant ideal.

This is joint work with Tunç Mısırlıoğlu of Istanbul Kültür University.

σ -Weak orthomorphisms

by Elmiloud Chil

Abstract. In this tolk we introduce a new class of weak orthomorphisms, so

called, σ -weak orthomorphisms. We prove that for a uniformly complete vector lattice, σ -weak orthomorphisms and σ -extended orthomorphisms coincide.

Dedekind Completion of C(X)

by Nicolae Dăneț

Abstract. The vector lattice C(X) of all real-valued continuous functions on a topological space X is not Dedekind complete. In this talk I present some methods to construct the Dedekind completion of C(X), for X a Hausdorff completely regular topological space.

First I show that Anguelov's construction [1] with Hausdorff continuous (in the sense of B. Sendov) interval-valued functions on X can be obtained via an order isomorphism from an earlier result of Horn [5, 3]. Secondly I give an alternative construction which used classes of real-valued quasi-continuous functions [2]. Thirdly I discuss the case when X is a compact metric space. In this situation the Dedekind completion of C(X) coincides with the metric completion of S. N. Samborskii [6].

All these constructions are possible using Hausdorff continuous intervalvalued functions since these type of functions are nothing else that Dedekind cuts in C(X) [4].

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Common Extensions of Riesz Homomorphisms

by Rodica-Mihaela Dăneț

Abstract. In this paper we will give sufficient conditions for the existence of a Riesz homomorphism which simultaneously extends a family of Riesz homomorphisms in the vector lattices setting.

Positive operators on strongly normal ordered normed spaces

by Eduard Emel'yanov

Abstract. We investigate the notion of the strong normality of positive cones in ordered normed spaces and give several applications to positive operators on normed spaces ordered by strongly normal cones.

Personalized equilibria and the grand coalition

by Maria Gabriella Graziano

Abstract. We provide cooperative characterizations of equilibria in economies with an infinite dimensional commodity space using the veto power of the grand coalition. The setting is quite general: the commodity space is an ordered locally convex space which need not be a vector lattice; competitive equilibria are decentralized using non-linear value functions arising from personalized pricing systems (compare [1]). Following the veto mechanism introduced by [2], we show that personalized equilibria are exactly allocations that cannot be blocked by the grand coalition: 1) in a family of economies in which the initial endowments of agents are perturbed in a certain direction; 2) in the original economy, when to each agent a share of participation in the coalition is assigned. We discuss extensions and applications of the results.

A joint article with Achille Basile.

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Riesz completions by functional embedding

by Onno van Gaans

Abstract. The functional embedding of an Archimedean partially ordered vector space X with strong order unit is obtained by endowing the space with the norm induced by the unit and defining S to be the closure in weak-star topology of the set of extreme points of the positive functionals that are one at the order unit. Then X is naturally embedded in the space C(S) of continuous functions on the compact set S with compatibility of the vector space and order structures. This embedding is called the functional embedding. It turns out that X is order dense in C(S). Hence the Riesz completion of X is a Riesz subspace of C(S). Moreover, disjointness of elements in X corresponds to the usual disjointness of their images in C(S). There are various spaces X for which C(S) can fairly easily be determined, for instance spaces with polyhedral cones, symmetric matrices with the order induced by the cone of positive definite

matrices, and spaces with Lorentz cones. We will illustrate how the embedding can be used to study disjointness and anti-lattice properties.

On commutative and semi-commutative positive operators

by Niushan Gao

Abstract.In this talk, we will discuss some properties of positive operators on Banach lattices that commute with a positive ideal irreducible operator and a positive compact operator. A generalization of the Perron-Frobenius theorem will be given. We will also present a solution to an open question of Drnovšek et al that if A, B > 0 with one of them compact, and $AB \ge BA$ or $AB \le BA$, then C := AB - BA is quasi-nilpotent.

Fixing properties of operators on Banach spaces of continuous functions

by Ioannis Gasparis

Abstract. We shall discuss some extensions of results due to Bourgain and Rosenthal, concerning operators on spaces of continuous functions, and their connection to the complementation problem for spaces of continuous functions.

Compactness of power of strictly singular operators on Banach lattices by Francisco L. Hernandez

Abstract. The compactness of the iterates of strictly singular operators on Banach lattices E is analyzed. Conditions on the behavior of the disjoint sequences in E are given in order that every strictly singular operator T have square T^2 compact or to be Dunford-Pettis. A Banach lattice is disjointly homogeneous if every pair of disjoint normalized sequence have equivalent subsequences. We also provide examples of rearrangement invariant spaces of lattice indices equal to p ($1 \le p < \infty$) with strictly singular non power-compact operators.

Joint work with J. Flores, E. Semenov and P. Tradacete.

Decomposing positive representations in L^p -spaces for Polish transformation groups.

by Marcel de Jeu

Abstract. One of the conceptual highlights in the theory of unitary group representations is the possibility to decompose any such representation in a separable Hilbert space into irreducibles. There are many natural examples of positive isometric representations of groups in Banach lattices: is there a decomposition theorem into irreducibles for these representation as well, where now the ordering is also taken into account? In this talk, we will indicate how such a theorem can, in fact, be obtained for positive isometric representations on L^p -spaces as associated with Polish transformation groups. The decomposition is then in terms of Banach lattice bundles, rather than in terms of direct integrals

as in the unitary case. This is joint work with Jan Rozendaal.

Characterization of bands in pre-Riesz spaces by means of functional representation

by Anke Kalauch

Abstract. In an Archimedean directed partially ordered vector space X, bands are defined by means of disjointness and studied with the aid of the Riesz completion of X. If X has an order unit, its Riesz completion is given by a functional representation, i.e. as a subspace of $C(\Omega)$, where Ω is a compact Hausdorff space. It will be shown that bands in X can be characterized in terms of subsets of Ω . The result is applied to spaces with polyhedral cones. Furthermore two methods to extend a band in X to the functional representation are established, and it is shown how the carriers of the band and its extensions are related.

New smoothness conditions on Riesz space with applications to nonadditive measure theory

by Jun Kawabe

Abstract. In 1974, Sugeno introduced the notion of fuzzy measures and integrals to evaluate nonadditive or nonlinear quality in systems engineering. In the same year, Dobrakov independently introduced the notion of submeasures to refine measure theory further. Both fuzzy measures and submeasures are special kinds of nonadditive measures, and their studies have stimulated engineers' and mathematicians' interest in nonadditive measure theory.

The classical theorems, such as the Egoroff theorem, the Alexandroff theorem, and some convergence theorems of Lebesgue integrals, are fundamental and important to develop measure theory. Therefore, many researchers continue to try to obtain their successful analogues in nonadditive measure theory.

When we try to develop nonadditive measure theory in Riesz space, along with the nonadditivity of measures, we confront some tougher problems due to the ε -argument, which is useful in measure theory, not working well in a general Riesz space. In this talk, instead of the ε -argument, we introduce and impose some new smoothness conditions on a Riesz space (asymptotic Egoroff property, monotone function continuity property, and so on) to obtain successful analogues of some important results in real-valued nonadditive measure theory.

This work is supported by Grant-in-Aid for Scientific Research No. 2354019200, Japan Society for the Promotion of Science (JSPS).

Strong Monotonicity of Spectral Radius of Positive Operators

by Arkady Kitover

Abstract. A part of the famous Perron - Frobenius theorem states that if A and B are matrices with nonnegative elements, $A \leq B$, A is irreducible, and

 $\rho(A) = \rho(B)$ where $\rho(A)$ means the spectral radius of A then A = B. It was proved by Perron in 1907 for matrices with strictly positive elements and by Frobenius in 1912 for arbitrary irreducible matrices.

In the talk we show that the statement above can be extended on a very large class of positive band irreducible operators on Banach lattices.

Risk Measures on Ordered Banach Spaces

by Christos E. Kountzakis

Abstract. Partially ordered Banach spaces are the appropriate framework in order to study the family of coherent risk measures which are at least discussed and partially put in use in finance and actuarial models during last ten years. We present this framework of study both with some results that characterize these spaces.

Boolean valued analysis approach to injective Banach lattices

by Anatoly Kusraev

Abstract. Every injective Banach lattice embeds into an appropriate Boolean valued model, becoming an AL-space. This result together with the fundamental principles of Boolean valued set theory provides a Boolean valued transfer principle from AL-spaces to injective Banach lattices: Each theorem about AL-spaces within Zermelo-Fraenkel set theory has an analog for injective Banach lattices interpreted as the Boolean valued AL-spaces.

Finite Elements in Lattice Ordered Algebras

by Helena Malinowski

Abstract. Finite elements are already well-known and studied in the context of vector lattices. We study them in lattice ordered algebras, preferably in f-algebras, and in product algebras. The additional structure of an associative multiplication leads to new questions and some new properties concerning the collections of finite, totally finite and self-majorizing elements.

In many cases the order ideal of finite elements turns out to be even an algebraic ideal.

If the multiplicative unit exists, the latter plays an important role in the investigation of finite elements. We apply some of our results concerning unitary f-algebras (i.e. f-algebras with a multiplicative unit) to the case of orthomorphisms.

Semiprime non-unitary f-algebras \mathcal{A} are embeddable into the set of all orthomorphisms on \mathcal{A} . If in this case \mathcal{A} is a Banach lattice, then we obtain that all elements of \mathcal{A} are self-majorizing. We reveal some facts concerning finite elements, their multiplication and their square roots in a class of non-unitary f-algebras satisfying the so-called weak factorization property.

For the product of special f-algebras we show that an element is finite in

the algebra if and only if its power is finite in the product algebra. Analogous results are obtained for totally finite and self-majorizing elements.

A joint article with Martin R. Weber

A Jordan-Hölder theorem for Riesz spaces

by Miek Messerschmidt

Abstract. he Jordan-Hölder theorem for modules states that for a module, M, if there exists a finite maximal chain of submodules of M,

$$0 = M_0 \subsetneq M_1 \subsetneq \ldots \subsetneq M_n = M,$$

then any chain of submodules can be extended to a maximal chain $\{N_i\}_{i=0}^n$ necessarily of length n, which is equivalent to the chain $\{M_j\}_{j=0}^n$, in the sense that there exists a permutation σ of $\{1,\ldots,n\}$ such that $M_j/M_{j-1} \simeq N_{\sigma(j)}/N_{\sigma(j)-1}$.

For Riesz spaces, i.e., for vector lattices, the notion of a submodule does not take the ordering into account, and one is led to consider, e.g., invariant ideals and invariant bands. We will explain to which extent there again is a Jordan-Hölder theorem for composition series involving such order-theoretic invariant objects, including a discussion of (order)irreducibility of the action on the quotients corresponding to a maximal chain, and the essential uniqueness of the set of quotients.

Vector measures and Mackey topologies

by Marian Nowak

Abstract. Let Σ be a σ -algebra of subsets of a non empty set Ω . Let $B(\Sigma)$ be the Banach lattice of all bounded Σ -measurable real-valued functions defined on Ω , equipped with the natural Mackey topology $\tau(B(\Sigma), ca(\Sigma))$. We study $(\tau(B(\Sigma), ca(\Sigma)), \xi)$ -continuous linear operators from $B(\Sigma)$ to a quasicomplete locally convex space (E, ξ) . A generalized Nikodym convergence theorem and a Vitali-Hahn-Saks type theorem for operators on $B(\Sigma)$ are obtained. It is shown that the space $(B(\Sigma), \tau(B(\Sigma), ca(\Sigma)))$ has the strict Dunford-Pettis property. Moreover, a Yosida-Hewitt type decomposition for weakly compact operators on $B(\Sigma)$ is given.

On the "many more agents than commodities" interpretation of core-Walras equivalence

by Konrad Podczeck

Abstract. The first result presented extends a result by Tourky and Yannelis (JETh, 2001) that "many more agents than commodities" is a necessary condition for core-Walras equivalence from Hilbert spaces to general Banach spaces. Actually, in these results, the order of the commodity space is not taken as given, but as an object of construction. The second result presented shows that for C(K)-spaces with the natural lattice order, "many more agents than

commodities" need not be necessary for core-Walras equivalence. This shows, in particular, that the order of the commodity space matters for core-Walras equivalence.

Reflexive cones

by Ioannis Polyrakis

Abstract. A cone P of a Banach space X is reflexive if $U_+ = P \cap U$, where U is the unit ball of X is weakly compact. If the set $P \cap U$ is compact the cone P is strongly reflexive. We show that P is reflexive if and only if the positive cone of ℓ_1 is not embeddable in P. We also prove that the space X is reflexive if and only P and P^0 are reflexive where P^0 is the dual cone of P. If X is a Banach lattice with a positive Schauder basis $\{e_i\}$, we show that X_+ contains a strongly reflexive cone P so that $\overline{P-P}=X$. We also give different properties of reflexive cones and examples of reflexive cones which generate a dense subspace in X.

Joint work with E. Casini, E. Miglierina and F. Xanthos.

Topological Vector Spaces And Game Theory

by Demir Sindel

Abstract. Since the very beginning of Game Theory the topological vector spaces played an important role. The pioniering work of J. v. Neuman used L.E.J. Brouwer's fixed point theorem to prove the minimax theorem. The present work gives an outline of the game theory in the context of topological vector spaces. First the matrix games have been given having the stategies being elements of some functional spaces and elements of some topological vector spaces. The differential games are games given with the differential equations of state of the player's interests and the value of the game is extremum of a performance index. As the aim of the players the Pareto strategy, the Nash and the Stackelberg strategy has been studied. The games where the state of the game is given by integral and integro differential equations have been studied in the context of Banach spaces and Uryson spaces. The games where the state of the game is given by partial differential equations. The theorems of optimal control of distributed parameter control can be generalised to game theory of distributed parameter systems. Some conclusion for the future work has been outlined.

Strongly normal cones and the midpoint locally uniform rotundity

by Konstantin Storozhuk

Abstract. Let X be a normed ordered space and let $K = X_+$ be its positive cone An (order) interval (or conic segment) Let $\langle a,b\rangle = \{x\mid a\leq x\leq b\}$ be an oder interval.

Krein [1] introduced the notion of a normal cone. In our terminology, a cone

is normal if the function $x, y \mapsto dist(\langle 0, x \rangle, \langle 0, y \rangle)$, defined on the set $K \times K$, is continuous at (0,0). In [2,3], there appeared the strong normality condition: it means the continuity of this function on the entire $K \times K$. It was also indicated there that it was not known whether every normal cone is strongly normal.

We give examples of normal non-strongly normal cones. We characterize strong normality in terms of midpoint locally uniformly rotunduty (MLUR) of the hyperplane section of cone.

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Continious bundles of Banach lattices.

by Soslan Tabuev

Abstract. Continious bundle of Banach lattices is itself a Banach lattice if continious structure in it is lattice. Some structural properties of continuous bundles of Banach lattices will be presented.

Domination properties and operator ideals

by Pedro Tradacete

Abstract. In the framework of Banach lattices a central topic is the study of relations between order and Banach space structure.

Given an operator ideal \mathcal{I} we are interested in the following problems:

- **Domination problem:** Let $0 \le S \le T : E \to F$. Under which conditions on the Banach lattices E and F does $T \in \mathcal{I}$ imply $S \in \mathcal{I}$?
- Power problem: Let $0 \le S \le T : E \to E$. Does there exist $n \in \mathbb{N}$ such that if $T \in \mathcal{I}$ then $R^n \in \mathcal{I}$?

For instance, a classical result due to P. Dodds and D. Fremlin asserts that for the ideal of compact operators \mathcal{K} , the domination problem has a positive answer when E^* and F are order continuous and for the power problem, n=3 works (both being optimal). We will survey some old and new results for several operator ideals.

Self-majorizing Elements in Archimedean Vector Lattices

by Martin R. Weber

Abstract. A finite element in an Archimedean vector lattice is called self-majorizing if its modulus is a majorant, i.e. $|x| \wedge n|\varphi| \leq c_x |\varphi|$ holds for some $c_x > 0$ and all $n \in \mathbb{N}$. Such elements exist in many vector lattices and naturally occur in different contexts. They are also known as semi-order units as the modulus of a self-majorizing element is an order unit in the band generated by the element. In the talk the properties of these elements are studied systematically. They are characterized both by means of properties of the generated ideals as well as by topological properties of the space of maximal ideals. The relations between the ideals of finite, totally finite and self-majorizing elements of a vector lattice are studied. In several examples it is demonstrated how to find the self-majorizing elements. In a Banach lattice an element φ is self-majorizing, if and only if the ideal and the band both generated by φ coincide.

Joint work with K. Teichert

Projective Banach Lattices

by Anthony Wickstead

Abstract. A Banach lattice P is projective if whenever X is a Banach lattice, J a closed ideal in X and $Q: X \to X/J$ the quotient map then for every linear lattice homomorphism $T: P \to X/J$ and $\epsilon > 0$ there is a linear lattice homomorphism $\hat{T}: P \to X$ such that (i) $T = Q \circ \hat{T}$ and (ii) $\|\hat{T}\| \le (1+\epsilon)\|T\|$. We describe the relationship between projective and free Banach lattices and make a start on investigating which Banach lattices are projective.

On the algebraic sum of ideals and sublattices

by Witold Wnuk

Abstract. We will discuss the problem concerning closedness of X + Y for closed ideals and sublattices X, Y in a topological Riesz space. First of all, following ideas due to L. Drewnowski and A. Wilansky, we show that if X is an infinite dimensional and infinite codimensional closed ideal in a complete metrizable locally solid Riesz space and X does not contain any order copy of $\mathbb{R}^{\mathbb{N}}$ then there exists a closed, separable, discrete sublattice Y such that the topology induced on Y is order continuous, $X \cap Y = \{0\}$, and X + Y is not closed.

Compact groups of positive operators

by Marten Wortel

Abstract. In this talk we will examine groups of positive operators in certain Banach lattices which are compact in the strong operator topology. We will then apply this to the theory of strongly continuous positive representations of compact groups in these Banach lattices.

Grothendieck Ordered Banach Spaces

by Foivos Xanthos

Abstract. In this talk we generalize a classical result of (Seever, 1968) on Grothendieck spaces. In particular we prove that if X is an ordered Banach space with the countable interpolation property, X has an order unit and X_+ is closed and normal, then X is a Grothendieck space. Furthermore we give a characterization of Grothendieck spaces X according to the geometry of cones of X^* .

A joint article with Ioannis Polyrakis.

A projected dynamical systems approach to bargaining

by Athanasios N. Yannacopoulos

Abstract. We propose a projected gradient dynamical system as a model for a bargaining scheme for an asset for which the two interested agents have personal valuations which do not initially coincide. The personal valuations are formed using subjective beliefs concerning the future states of the world and the reservation prices are calculated using expected utility theory. The agents are not rigid concerning their subjective probabilities and are willing to update them under the pressure to reach finally an agreement concerning the asset. The proposed projected dynamical system, on the space of probability measures, provides a model for the evolution of the agents beliefs during the bargaining period and is constructed so that agreement is reached under the minimum possible deviation of both agents from their initial beliefs. The convergence results are shown using techniques from convex dynamics and Lyapunov function theory.

This is work in collaboration with D. Pinheiro, A. A. Pinto and S. Xanthopoulos.