On a Characterization of Competitive Equilibrium Allocations in Differential Information Economies

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Abstract. We consider a finite-agent economy with differential information where the set of states of nature is finite. By introducing the notion of private non-dominated allocations, we characterize competitive (Radner) equilibrium allocations. The first and second welfare theorems for differential information economies are particular cases of the above equivalence result. Moreover, we show that continuum of agents economies with differential information and discrete ones can be considered equivalent with respect to competitive (Radner) equilibrium allocations. Also an extension of Vind's (1972) theorem to a differential information economy is provided.

Key words: Radner equilibrium, differential information economy, private non-dominated allocations.

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1 Introduction

An exchange economy with differential information (or a Radner-type economy) consists of a finite set of agents each of whom is characterized by a random utility function, a random initial endowment, a private information set and a prior. For such an economy Radner (1968) defined a notion of a Walrasian equilibrium, called here Radner equilibrium. This notion is analogous to the Walrasian equilibrium in the Arrow-Debreu-McKenzie deterministic model. The Walrasian equilibrium notion for an economy with differential information (Radner equilibrium) is of interest because it captures trades under asymmetric information. The purpose of this paper is two-fold:

Firstly, we provide a characterization of a Radner equilibrium in terms of non-dominated allocations. The notion of non-dominated allocation we introduce states that, it is not possible for the grand coalition to redistribute their initial endowments using their own private information and make each member of the grand coalition better off (in terms of their expected utility). Since agents do not necessarily exchange their own information, we call those allocations private non-dominated allocations. Thus, private non-dominated allocations have similar features with the private core (Yannelis [1991]).

Secondly, following the approach of García-Curtín and Herves-Beloso [1993] we show that a differential information economy with a continuum of agents can be interpreted as a discrete differential information economy in which only a finite number of different agent's characteristics can be distinguished. In particular, we show that a price-consumption pair is a Radner equilibrium for a discrete economy with differential information, if and only if, it is also a Radner equilibrium for the associated continuum differential information economy. Thus, continuum of agents differential information economies and discrete ones can be considered equivalent with respect to Radner equilibria.

It should be pointed out that the characterization of Radner equilibrium as private non-dominated allocations allows us to obtain as corollaries the first and second welfare theorems for differential information economies.

From a technical viewpoint our theorem on the characterization of Radner equilibrium not only gives as corollaries the welfare theorems but also provides new proofs. In particular, our argument relies on an extension of a result of Vind [1972] to a differential information economy setting.

The paper proceeds as follows: Section 2 contains the main concepts in a differential information economy with finitely many agents. Section 3 is focused on a continuum of agents differential information economy and we show that continuum of agents differential information economies and discrete ones can be considered equivalent with respect to Radner equilibria. Moreover, in the Section 3 an extension of Vind's [1972] result is given for a differential information economy. Finally, Section 4 contains a characterization of Radner equilibrium allocations as private non-dominated allocations.

2 Differential Information Economies with a Finite Number of Agents

Let us consider a Radner-type exchange economy $\mathcal{E}$ with differential information (see Radner (1968, 1982)). Let $(\Omega, \mathcal{F})$ be a measurable space, where Ω denotes the states of nature of the world and $\mathcal{F}$ denotes the set of all events. Hence, $(\Omega, \mathcal{F})$ describes, the exogenous uncertainty. The set of states of nature, Ω, is finite and there is a finite number of goods, ℓ, per state. N is the set of n traders or agents and $\mathbb{R}^\ell_+$ will denote the commodity space which is the positive orthant of $\mathbb{R}^\ell$.

The economy extends over two time periods $\tau = 0, 1$. Consumption takes place at $\tau = 1$. At $\tau = 0$ there is an uncertainty over the state of nature and the agents make contracts (agreements) that may be contingent on the realized state of nature at period $\tau = 1$ (that is, ex ante contract arrangement).

A differential information exchange economy $\mathcal{E}$ with a finite number of agents is defined by $\{((\Omega, \mathcal{F}), X_i, \mathcal{F}_i, U_i, e_i, q) : i = 1, \ldots, n\}$, where:

1. $X_i : \Omega \to 2^{\mathbb{R}^\ell_+}$ is the set valued function giving the *random consumption set* of agent i.

2. $\mathcal{F}_i$ is a partition of Ω, denoting the private information of agent i;

3. $U_i : \Omega \times \mathbb{R}^\ell_+ \to \mathbb{R}$ is the *random utility function* of agent i;

4. $e_i : \Omega \to \mathbb{R}^\ell_+$ is the *random initial endowment* of agent i, assumed to be constant on elements of $\mathcal{F}_i$, with $e_i(\omega) \in \mathbb{R}^\ell_+$ for all $\omega \in \Omega$;

5. q is a probability function on Ω giving the *prior* of every agent. It is assumed that q is positive on all elements of Ω.

We will refer to a function with domain Ω, constant on elements of $\mathcal{F}_i$, as $\mathcal{F}_i$-measurable, although, strictly speaking, measurability is with respect to the σ-algebra generated by the partition. We can think of such a function as delivering information to trader i which does not permit discrimination between the states of nature belonging to any element of $\mathcal{F}_i$.

For any $x : \Omega \to \mathbb{R}^\ell_+$, the *ex ante expected utility* of agent i is given by

$$V_i(x) = \sum_{\omega \in \Omega} U_i(\omega, x(\omega)) q(\omega).$$

Let L_{X_i} denote the set of all $\mathcal{F}_i$-measurable selections from the random consumption set of agent i, that is:

$$L_{X_i} = \left\{ x_i : \Omega \to \mathbb{R}^\ell, \text{ such that } x_i \text{ is } \mathcal{F}_i\text{-measurable and } x_i(\omega) \in X_i(\omega) \text{ for all } \omega \right\}.$$

Let $L_X = \prod_{i=1}^{n} L_{X_i}$. Any element x in L_X is called an *allocation.* An allocation $x \in L_X$ is said to be *feasible* if $\sum_{i=1}^{n} x_i \leq \sum_{i=1}^{n} e_i$.

A coalition $S \subset N$ *private blocks* an allocation $x \in L_X$ if there exists $(y_i)_{i \in S} \in \prod_{i \in S} L_{x_i}$ such that $\sum_{i \in S} y_i \leq \sum_{i \in S} e_i$ and $V_i(y_i) > V_i(x_i)$ for every $i \in S$.

The *private core* of the differential information exchange economy $\mathcal{E}$ is the set of all feasible allocations which are not private blocked by any coalition.

Next we shall define a Walrasian equilibrium notion in the sense of Radner. In order to do so, we need the following. A *price system* is a $\mathcal{F}$-measurable, non-zero function $p : \Omega \to \mathbb{R}^\ell_+$. For a price system p, the budget set of agent i is given by

$$B_i(p) = \left\{ x_i \in L_{X_i}, \text{ such that } \sum_{\omega \in \Omega} p(\omega) x_i(\omega) \leq \sum_{\omega \in \Omega} p(\omega) e_i(\omega) \right\}.$$

Notice that the budget constraint is across the states of nature.

Definition 2.1 *A pair (p, x), where p is a price system and $x = (x_1, \ldots, x_n) \in L_X$ is an allocation, is a Radner equilibrium if* *(i) for all i the consumption function maximizes V_i on B_i,*

(ii) $\sum_{i=1}^{n} x_i \leq \sum_{i=1}^{n} e_i$ *(free disposal), and*

(iii) $\sum_{\omega \in \Omega} p(\omega) \sum_{i=1}^{n} x_i(\omega) = \sum_{\omega \in \Omega} p(\omega) \sum_{i=1}^{n} e_i(\omega).$

Radner equilibrium is an ex ante concept. Notice that we assume free disposal. It is well known that if we impose the condition of non-free disposal then a Radner equilibrium might not exist with positive prices (see, for example, Glycopantis, Muir and Yannelis (2002) and Einy and Shitovitz (2002)).

In this paper, we will assume that $X_i(\omega) = \mathbb{R}^{\ell}_{+}$ for every agent i and for every $\omega \in \Omega$; and we state the following assumptions on endowments and preferences:

(A.1) $e_i(\omega) \gg 0$ for all i and for all $\omega \in \Omega$.

(A.2) For all i and ω, $U_i(\omega, \cdot) : \mathbb{R}^{\ell}_{+} \to \mathbb{R}$ is continuous, monotone and concave.

(A.3) For all i, $U_i(\cdot, x) : \Omega \to \mathbb{R}$ is measurable.

Remark. Assumption (A.1) is often replaced by $\sum_{i=1}^{n} e_i(\omega) \gg 0$ for all $\omega \in \Omega$ together with irreducibility (i.e., the endowment of every coalition is desired).

Definition 2.2 *An allocation $x \in L_X$ (feasible or not) is private dominated (or private blocked by the grand coalition) in the economy $\mathcal{E}$ if there exits a feasible allocation $y \in L_X$, such that $V_i(y_i) > V_i(x_i)$ for every $i = 1, \ldots, n$.*

Observe that despite the fact that the whole coalition of agents get together they do not necessarily share their own information. On the contrary, the redistribution of the initial endowments are based only on their own private information. Hence, a feasible and non-dominated allocation reflects the private information of each agent and has the property that the coalition of all the agents can not redistribute their initial endowments, based on their own private information, and make every individual better off.

3 A Continuum Approach

3.1 Radner Equilibria in Continuum Economies

In this section, we interpret differential information economies with n agents as continuum economies where the ith agent is the representative of infinitely many identical agents (See García -Cutrín and Hervés-Beloso (1993) for the case of Arrow-Debreu economies). For it, let us associate to the differential information economy $\mathcal{E}$ (described in the previous section) a continuum economy $\mathcal{E}_c$ with differential information in which only a finite number of different agents can be distinguished.

Given $\mathcal{E} \equiv \{((\Omega, \mathcal{F}), X_i, \mathcal{F}_i, U_i, e_i, q) : i = 1, \ldots, n\}$, the associated atomless economy $\mathcal{E}_c$ is defined as follows. The set of agents is represented by the unit real interval $I = [0,1] = \bigcup_{i=1}^{n} I_i$, where $I_i = \left[\frac{i-1}{n}, \frac{i}{n}\right)$, if $i \neq n$, and $I_n = \left[\frac{n-1}{n}, 1\right]$. We consider the Lebesgue measure μ on the Borel subsets of I. Each agent $t \in I$ is characterized by her private information which is described by a partition $\mathcal{F}_t$ of Ω, where $\mathcal{F}_t = \mathcal{F}_i$ for every $t \in I_i$; her consumption set $X_t(\omega) = \mathbb{R}^{\ell}_{+}$ for every $\omega \in \Omega$; her random initial endowment $e(t, \cdot) = e_i$ for every $t \in I_i$ and her expected utility function $V_t = V_i$ for every $t \in I_i$. We will refer to agents belonging to the subinterval I_i as agents of type i.

An *allocation* is a function $f : I \times \Omega \to \mathbb{R}^{\ell}_{+}$, such that for every $\omega \in \Omega$ the function $f(\cdot, \omega)$ is μ-integrable on I.

An allocation f is *feasible* for the economy $\mathcal{E}_c$ if:

(i) for almost all $t \in I$ the function $f(t, \cdot)$ is $\mathcal{F}_t$-measurable, and

(ii) $\int_I f(t, \omega) d\mu(t) \leq \int_I e(t, \omega) d\mu(t)$ for all $\omega \in \Omega$.

A *coalition* in $\mathcal{E}_c$ is a Borel subset of I. A coalition $S \subset I$, with $\mu(S) > 0$, *private blocks* an allocation f if there exists an allocation g such that $g(t, \cdot)$ is $\mathcal{F}_t$-measurable for all $t \in S$, $\int_S g(t, \omega) d\mu(t) \leq \int_S e(t, \omega) d\mu(t)$ for every $\omega \in \Omega$ and $V_t(g(t, \cdot)) > V_t(f(t, \cdot))$ for every $t \in S$.

The *private core* of the economy $\mathcal{E}_c$ is the set of feasible allocations which are not private blocked by any coalition.

Note that given a price system $p : \Omega \to \mathbb{R}^{\ell}_{+}$, the budget set of agent $t \in I$ is $B_t(p) = B_i(p)$ for every $t \in I_i$.

Definition 3.1 *A competitive equilibrium (or Radner equilibrium) for the continuum economy $\mathcal{E}_c$ is a pair (p, f), where p is a price system and f is a feasible allocation such that:*

(i) for almost all $t \in I$ the function $f(t, \cdot)$ maximizes V_t on $B_t(p)$, and

(ii) $\sum_{\omega\in\Omega} p(\omega) \int_I f(t,\omega)d\mu(t) = \sum_{\omega\in\Omega} p(\omega) \int_I e(t,\omega)d\mu(t).$

Remark. It follows from the continuity of $U_i(\omega, x)$ in x and measurability in ω that $U_i(\cdot,\cdot)$ is jointly measurable. Hence, under (A.2) and (A.3), the associated continuum economy $\mathcal{E}_c$ satisfies all the assumptions of the equivalence theorem of Radner equilibria and private core (see, for example, Einy, Moreno and Shitovitz (2001)).

Let us consider the differential information economy $\mathcal{E}$ with n agents and the associated continuum economy $\mathcal{E}_c$ with n different types of agents. An allocation $x = (x_1, \ldots, x_n)$ in the economy $\mathcal{E}$ can be interpreted as an allocation f in $\mathcal{E}_c$, where f is the step function defined by $f(t, \cdot) = x_i$ for every agent $t \in I_i$. Reciprocally, an allocation f in $\mathcal{E}_c$ can be interpreted as an allocation $x = (x_1, \ldots, x_n)$ in $\mathcal{E}$, where $x_i = \dfrac{1}{\mu(I_i)} \int_{I_i} f(t,\cdot)d\mu(t)$.

We will show that the continuum and the discrete approach can be considered equivalent with respect to Radner equilibrium. In order to prove this result we will need the following lemma.

Lemma 3.1 *Let $S \subset I_i$, with $\mu(S) > 0$, and $z : \Omega \to \mathbb{R}^\ell_+$. Let $f : S\times\Omega \to \mathbb{R}^\ell_+$ be a function such that $V_i(f(t,\cdot)) > V_i(z)$ for every $t \in S$. Then, under assumption (A.2), $V_i(h) > V_i(z)$, where $h(\omega) = \dfrac{1}{\mu(S)} \int_S f(t,\omega)d\mu(t)$.*

Proof: Let $\varepsilon > 0$. By Lusin's theorem, there exists a compact set $K \subset S$, with $\mu(K) > \mu(S) - \varepsilon$, such that $f(\cdot,\omega)$ is continuous on K for every $\omega \in \Omega$. Hence, there exists $y : \Omega \to \mathbb{R}^\ell_+$, such that $V_i(f(t,\cdot)) \geq V_i(y) > V_i(z)$ for all $t \in K$.

On the other hand $\dfrac{1}{\mu(K)} \int_K f(t,\omega)d\mu(t) \in \overline{co}(f(K,\omega))$, for every $\omega \in \Omega$; where $\overline{co}(f(K,\omega))$ denotes the closed convex hull of $f(K,\omega)$ (see Diestel and Uhl (1977), p. 48, for details). By (A.2), $\overline{co}(f(K,\cdot)) \subset \{a : \Omega \to \mathbb{R}^\ell_+$ such that $V_i(a) \geq V_i(y)\}$. Thus $V_i(h) \geq V_i(y) > V_i(z)$. Analogously, if $\mu(S \setminus K) > 0$, then $V_i(\widehat{h}) \geq V_i(z)$, where $\widehat{h}(\omega) = \dfrac{1}{\mu(S\setminus K)} \int_{S\setminus K} f(t,\omega)d\mu(t)$.

Now, for every $\omega \in \Omega$, we can write $h(\omega)$ as follows

$$\frac{\mu(K)}{\mu(S)}\left(\frac{1}{\mu(K)}\int_K f(t,\omega)d\mu(t)\right)+\frac{\mu(S)-\mu(K)}{\mu(S)}\left(\frac{1}{\mu(S\setminus K)}\int_{S\setminus K} f(t,\omega)d\mu(t)\right)$$

Therefore, by (A.2), we conclude that $V_i(h) > V_i(z)$.

Q.E.D.

We are now ready to state our first result.

Theorem 3.1 Assume that, for every $i = 1, \ldots, n$, (A.2) holds for the random utility function U_i. Then, (x, p) is a Radner equilibrium for the discrete economy $\mathcal{E}$ if and only if (f, p) is a Radner equilibrium for the associated continuum economy $\mathcal{E}_c$.

Proof: Let $((x_1, \ldots, x_n), p)$ a Radner equilibrium for $\mathcal{E}$. Then, for every state $\omega \in \Omega$, $\int_I f(t,\omega)d\mu(t) = \sum_{i=1}^{n} \mu(I_i)x_i(\omega) \le \sum_{i=1}^{n} \mu(I_i)e_i(\omega) = \int_I e(t,\omega)d\mu(t)$ and, for all $t \in I_i$, the consumption function $f(t,\cdot)$ maximizes V_t on $B_t(p) = B_i(p)$. Therefore, (f, p) is a Radner equilibrium for the continuum economy $\mathcal{E}_c$.

Conversely, let (f, p) be a Radner equilibrium for $\mathcal{E}_c$. Then, $x = (x_1, \ldots, x_n)$, with $x_i = \frac{1}{\mu(I_i)}\int_{I_i} f(t,\cdot)d\mu(t)$, is a feasible allocation in the economy $\mathcal{E}$. Since $\sum_{\omega\in\Omega} p(\omega)x_i(\omega) = \sum_{\omega\in\Omega} \frac{1}{\mu(I_i)}\int_{I_i} p(\omega)f(t,\omega)d\mu(t) \le \sum_{\omega\in\Omega} p(\omega)e_i(\omega)$, we have that $x_i \in B_i(p)$ for every agent i. By Lemma 3.1, if $V_i(z) > V_i(x_i)$ then $V_i(z) > V_i(f(t,\cdot))$ for every $t \in S \subset I_i$, $\mu(S) > 0$; and thus $\sum_{\omega\in\Omega} p(\omega)z(\omega) > \sum_{\omega\in\Omega} p(\omega)e_i(\omega)$.

Q.E.D.

3.2 An Extension of Vind's Theorem

We now extend Vind's (1972) result to cover the private core. To this end, we need the following lemma.

Lemma 3.2 *Assume that assumption (A.1) holds and preferences are continuous and strictly monotone. Then, if the allocation f does not belong to the private core of the associated continuum economy $\mathcal{E}_c$, it is blocked by a coalition A via the allocation g with* $\int_A (e(t,\omega) - g(t,\omega))\, d\mu(t) \gg 0$.

Proof: Let f be an allocation which does not belong to the private core of $\mathcal{E}_c$. Then, there exist a coalition A and an allocation $h : A \times \Omega \to \mathbb{R}^\ell_+$, such that

$\int_A h(t,\omega)d\mu(t) \le \int_A e(t,\omega)d\mu(t)$ for every $\omega \in \Omega$ and $V_t(h(t,\cdot)) > V_t(f(t,\cdot))$ for every agent $t \in A$.

Let us show that we can take h such that $\int_A h(t,\omega)d\mu(t) \gg 0$ for every $\omega \in \Omega$. For each commodity j and each state $\omega \in \Omega$, let us define

$$H(j,\omega) = \{t \in A \text{ such that } h_j(t,\omega) = 0\}.$$

Assume that there exists (j_0, ω_0) such that $\mu(H(j_0,\omega_0)) = \mu(A)$. This implies that there is a type of agents i_0 such that $h_{j_0}(t,\omega_0) = 0$ for every $t \in A_{i_0} = A \bigcap I_{i_0}$. Since $h(t,\cdot)$ is $\mathcal{F}_t$-measurable for every $t \in A$, $h_{j_o}(t,\omega) = 0$, for every $t \in A_{i_0}$ and for every $\omega \in E_{i_0}(\omega_0)$, where $E_{i_0}(\omega_0)$ is the subset of the partition $\mathcal{F}_{i_0}$ to which ω_0 belongs.

Given $\omega \in E_i(\omega_0)$, either

(i) $\int_{A\setminus A_{i_0}} h_{j_0}(t,\omega)d\mu(t) = \int_A e_{j_0}(t,\omega)d\mu(t)$ holds, or

(ii) $\int_A e_{j_0}(t,\omega)d\mu(t) - \int_{A\setminus A_{i_0}} h_{j_0}(t,\omega)d\mu(t) = \varepsilon(\omega) > 0$ holds.

Let $\Upsilon = \{\omega \in E_{i_0}(\omega_0)$ such that *(i)* holds $\}$. If $\omega \in \Upsilon$, there exists a type of agents $i(\omega)$ such that $\int_{A \bigcap I_{i(\omega)}} h_{j_0}(t,\omega)d\mu(t) > 0$. Then, by continuity and monotonicity of preferences and applying Lusin's theorem, we conclude that for each $\omega \in \Upsilon$ there exists a compact set $B(\omega) \subset A_{i(\omega)} = A \bigcap I_{i(\omega)}$, with $\mu(B(\omega)) > \mu(A_{i(\omega)}) - \varepsilon$, and there exists $\delta(\omega) > 0$, such that $V_t(\widehat{h}(t,\cdot)) > V_t(h(t,\cdot))$ for every $t \in B = \bigcup_{\omega\in\Upsilon} B(\omega)$, where

$$\widehat{h}_j(t,\omega) = \begin{cases} h_{j_0}(t,\omega) - \delta(\omega) & \text{if } j = j_0,\ \omega \in \Upsilon \text{ and } t \in B(\omega) \\ \\ h_j(t,\omega) & \text{otherwise.} \end{cases}$$

Let $\beta = \min\limits_{\omega\in E_{i_0}(\omega_0)} \left\{ (\varepsilon(\omega))_{\omega\notin\Upsilon}, (\delta(\omega)\mu\left(B(\omega)\right))_{\omega\in\Upsilon} \right\}$. Note that β is a strictly positive real number. Let us define the allocation $\widehat{g} : A \times \Omega \to \mathbb{R}^{\ell}_{+}$ as follows:

$$\widehat{g}_j(t,\omega) = \begin{cases} \widehat{h}_j(t,\omega) & \text{if } \omega \in \Upsilon \text{ and } t \in B \\ \\ \dfrac{\beta}{\mu(A_{i_0})} & \text{if } j = j_0,\ \omega \in E_{i_0}(\omega_0) \text{ and } t \in A_{i_0} \\ \\ h_j(t,\omega) & \text{otherwise.} \end{cases}$$

By construction, A blocks f via $\widehat{g}$ and $\int_A \widehat{g}_{j_0}(t,\omega)d\mu(t) > 0$ for every $\omega \in E_{i_0}(\omega_0)$.

Applying the argument above as many times as necessary, we can take $\overline{g}$ such that A blocks f via $\overline{g}$ and $\int_A \overline{g}(t,\omega)d\mu(t) \gg 0$, for every ω. If the inequality $\int_A \overline{g}(t,\cdot)d\mu(t) \ll \int_A e(t,\cdot)d\mu(t)$ does not hold, there exist a commodity j_1 and a state ω_1 such that $\int_A \overline{g}_{j_1}(t,\omega_1)d\mu(t) = \int_A e_{j_1}(t,\omega_1)d\mu(t)$. Then, for some type i of agents $\overline{g}_{j_1}(t,\omega) > 0$ for every $t \in B_i \subset A_i$, with $\mu(B_i) > 0$. By continuity of preferences, we can take $\varepsilon : B_i \to \mathbb{R}_+$, with $\int_{B_i} \varepsilon(t)d\mu(t) > 0$, such that the coalition A blocks f via the allocation g given by

$$g_j(t,\omega) = \begin{cases} \overline{g}_{j_1}(t,\omega_1) - \varepsilon(t) & \text{if } t \in B_i,\ j = j_1 \text{ and } \omega \in E_i(\omega_1) \\ \\ \overline{g}_j(t,\omega) & \text{otherwise.} \end{cases}$$

Now, take the set $\{(j,\omega) \in (\{1,\dots,\ell\} \times \Omega) \setminus (\{j_1\} \times E_i(\omega_1))\}$ and apply the previous argument. In this way, we can construct an allocation g such that A blocks f via g and $\int_A g(t,\omega)d\mu(t) \ll \int_A e(t,\omega)d\mu(t)$ for every state $\omega \in \Omega$.

Q.E.D.

Proposition 3.1 *Assume that (A.1)-(A-3) hold. If a feasible allocation f is not in the private core of the associated continuum economy $\mathcal{E}_c$ then for any α, with $0 < \alpha < 1$ there exists a blocking coalition S with $\mu(S) = \alpha$.*

Proof: Let f be a feasible allocation which is blocked by a coalition $A \subset I$ via g. Then, $g(t,\cdot)$ is $\mathcal{F}_t$-measurable for all $t \in A$, $\int_A g(t,\omega)d\mu(t) \leq \int_A e(t,\omega)d\mu(t)$ for every $\omega \in \Omega$ and $V_t(g(t,\cdot)) > V_t(f(t,\cdot))$ for every $t \in A$. In view of Lemma 3.2, we can take $g : A \times \Omega \to \mathbb{R}^\ell_+$ such that, $\int_A (g(t,\omega) - e(t,\omega))d\mu(t) = -z(\omega)$, with $z(\omega) = (z_1(\omega),\dots,z_\ell(\omega)) \gg 0$ for each $\omega \in \Omega$.

By assumption (A.2), for every ε, with $0 < \varepsilon < 1$, $V_t(\varepsilon g(t,\cdot) + (1-\varepsilon)f(t,\cdot)) > V_t(f(t,\cdot))$ for every $t \in A$.

Applying the Liapunov convexity theorem, there exists $B \subset I \setminus A$, such that $\mu(B) = (1-\varepsilon)\mu(I \setminus A)$ and

$$\int_B (g(t,\cdot) - e(t,\cdot))d\mu(t) = (1-\varepsilon)\int_{I\setminus A} (g(t,\cdot) - e(t,\cdot))d\mu(t).$$

Let $\delta = (\delta_1,\dots,\delta_\ell) \in \mathbb{R}^\ell_+$ given by $\delta_j = \min\{z_j(\omega), \omega \in \Omega\}$. The coalition $S = A \bigcup B$ blocks f via the allocation h given by

$$h(t,\cdot) = \begin{cases} \varepsilon g(t,\cdot) + (1-\varepsilon)f(t,\cdot) & \text{if } t \in A \\ f(t,\cdot) + \dfrac{\varepsilon}{\mu(B)}\,\delta & \text{if } t \in B. \end{cases}$$

Since $\mu(S) = \mu(A) + (1-\varepsilon)\mu(I \setminus A)$, we have constructed an arbitrarily large coalition blocking f.

Q.E.D.

4 Equivalence Result

Given an allocation $x \in L_X$ and a vector $a = (a_1, \ldots, a_n)$, with $0 \leq a_i \leq 1$, let $\mathcal{E}(a, x)$ be a differential information exchange economy which coincides with $\mathcal{E}$ except for the initial endowment of each agent that is given by

$$e_i(a_i, x_i) = a_i e_i + (1 - a_i) x_i,$$

where $e_i, x_i \in L_{X_i}$. That is,

$$\mathcal{E}(a, x) \equiv \{((\Omega, \mathcal{F}), X_i, \mathcal{F}_i, U_i, e_i(a_i, x_i) = a_i e_i + (1 - a_i) x_i, q) : i = 1, \ldots, n\}.$$

Theorem 4.1 *let $\mathcal{E}$ be a differential information exchange economy satisfying assumptions (A.1)-(A.3). Then $x \in L_X$ is a Radner equilibrium allocation if and only if x is a private non-dominated allocation for every economy $\mathcal{E}(a, x)$.*

Proof: Let (p, x) be a Radner equilibrium for the economy $\mathcal{E}$. Suppose that there exists $a = (a_1, \ldots, a_n)$, such that x is dominated in the economy $\mathcal{E}(a, x)$. Then, there exists $y = (y_1, \ldots, y_n)$ such that $\sum_{i=1}^{n} y_i \leq \sum_{i=1}^{n} e_i(a_i, x_i)$ and $V_i(y_i) > V_i(x_i)$ for every agent $i \in \{1, \ldots, n\}$. These inequalities imply that for every agent i, $\sum_{\omega \in \Omega} p(\omega) \cdot y_i(\omega) > \sum_{\omega \in \Omega} p(\omega) \cdot e_i(\omega)$, which is a contradiction with the feasibility of y in the economy $\mathcal{E}(a, x)$.

Let x be a private non-dominated allocation for every economy $\mathcal{E}(a, x)$. Let f a step function on the real interval $I = [0, 1]$, defined by $f(t) = x_i$ if $t \in I_i = \left[\frac{i-1}{n}, \frac{i}{n}\right)$, if $i \neq n$, and $f(t) = x_n$ if $t \in I_n = \left[\frac{n-1}{n}, 1\right]$.

Assume that x is not a Radner equilibrium allocation for the economy $\mathcal{E}$. Then, by Theorem 3.1, the step allocation f given by x is not a Radner equilibrium allocation for the associated continuum economy $\mathcal{E}_c$ with n different types of agents. Applying the equivalence between the private core and the set of Radner equilibrium allocations (see Einy, Moreno and Shitovitz (2001)), we have that f does not belong to the private core of the associated continuum economy. Even more, by Proposition 3.1, there exists a coalition $S \subset I = [0, 1]$, with

$\mu(S) > 1 - \frac{1}{n}$, and there exists $g : S \times \Omega \to \mathbb{R}_+^\ell$, such that $g(t, \cdot)$ is $\mathcal{F}_t$-measurable for every $t \in S$; $\int_S g(t, \cdot) d\mu(t) \leq \int_S e(t, \cdot) d\mu(t)$ and $V_t(g(t, \cdot)) > V_t(f(t, \cdot))$ for all $t \in S$. Let $S_i = S \bigcap I_i$ and $a_i = \mu(S_i)$.

Now, in the finite economy $\mathcal{E}$, let us consider the allocation $(g_1, \ldots, g_n)$, where $g_i = \dfrac{1}{\mu(S_i)} \displaystyle\int_{S_i} g(t, \cdot) d\mu(t)$. Since $g(t, \cdot)$ is $\mathcal{F}_t$-measurable for every $t \in S$, the function g_i is $\mathcal{F}_i$-measurable for every $i = 1, \ldots, n$.

By convexity of preferences, $V_i\left(a_i g_i + (1 - a_i) x_i\right) > V_i(x_i)$, for every agent $i \in \{1, \ldots, n\}$. Note that, since $\mu(S) > 1 - \frac{1}{n}$, we have that $a_i > 0$ for every i.

Therefore, x is a dominated allocation in the economy $\mathcal{E}(a, x)$, which is a contradiction.

Q.E.D.

Remark 4.1: The above result provides a precise characterization of the set of Radner equilibria in finite economies as private non-dominated allocations. Note that the first welfare theorem is an immediate consequence of the result above. Moreover, note that by taking $x_i = e_i$, for all i, we obtain, exactly, the second welfare theorem. Therefore, both welfare theorems are, obviously, particular cases of the above theorem.

Remark 4.2: As in was remarked in Section 2, we have allowed for free disposal since a Radner equilibrium may not exist with positive prices. However, by allowing for negative prices one can dispace with the free disposal assumption. Notice, that the private core equivalence theorem of Einy-Moreno-Shitovitz [2001] doesn't depend on whether or not the free disposal assumption holds. Indeed, as it is the case in the Aumann [1964, 1966] deterministic model with a continuum of agents, the assumptions which guarantee the core–Walras equivalence may not assure nonemptiness of the sets. The results obtained in this paper still hold without the free disposal assumption, but if we want to guarantee the existence of Radner equilibrium one must allow for negative prices (see Radner [1968]).

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