The geometry of finite equilibrium data sets

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Abstract

We investigate the geometry of finite data sets defined by equilibrium prices, income distributions, and total resources. We show that the equilibrium condition imposes no restriction if total resources are held constant or constrained to remain collinear, a property robust to small perturbations of the total resources. When there are restrictions imposed by the equilibrium condition (because total resources are for example highly variable), a first step in understanding the geometry of equilibrium data sets is achieved with our proof of the arcconnectedness of that set.

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1. Introduction

It has been shown by Brown and Matzkin in [5] that finite collections of data consisting of price equilibria associated with individual endowments cannot be totally arbitrary. Their proof is based on an example defined by a two good two consumer economy. It follows from this property that the theory of general equilibrium is falsifiable. If the philosophical and scientific implications of falsifiability lay far beyond the scope of the current paper, one can nevertheless investigate how easy it is to formulate a "falsifiable statement" within the setup of general equilibrium theory. Assuming that one can define a "measure" on the set of "statements," how big is then the subset of "falsifiable statements?" This question is less rhetorical than it may seem in view of a recent result of Snyder [9]. This author shows that, if the number of consumers is larger than or equal to the number of goods, there are no restrictions on finite data sets consisting of total resources and equilibrium prices—equilibrium prices are associated here with some individual endowments compatible with the total resources. The property discovered by Snyder is very much in line with the essence of the Sonnenschein-Mantel-Debreu theorem [6]. In

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a companion result, however, Snyder shows that finite data sets cannot be totally arbitrary if income distribution is taken as (exogenously) given. The global picture of equilibrium data sets is therefore certainly complex. In addition, these two properties underline the importance of income distributions and total resources when it comes to explaining properties of finite equilibrium data sets.

The goal of this paper is to improve our understanding of the geometry of equilibrium data sets. We show that if total resources are held constant, or restricted to remain collinear, no further restrictions apply to equilibrium data sets. In other words, the sets of equilibrium data coincide with the full space. This property does not require the number of consumers to be larger than or equal to the number of goods. In addition, this property is robust to small perturbations of the total resources. Therefore, if the variations of total resources are small, or if the growth rate of total resources is almost the same for all goods, again no restrictions apply to equilibrium data sets. In such cases, there are no falsifying equilibrium data sets.

When a set is a strict subset of some other set or, more specifically, a topological subspace of some topological space, for example some Euclidean space, some information on the geometry of the subspace is provided by algebraic topology constructs like the various homology and cohomology groups of the subspace. The zero homology group being directly related to the number of connected components of the subspace, a very first step in the computation of these groups is the determination of whether the subspace is arcconnected. In this regard, we prove in this paper that equilibrium data sets that are strictly smaller than the full sets (which therefore supposes change rates of total resources sufficiently variable across the various goods), are arcconnected. (This property is trivially satisfied if the equilibrium data set coincides with the full data set.)

The paper is organized as follows. In Section 2, we recall the main assumptions and definitions, and set the notation. In Section 3, we state and prove the property that, if total resources are constant or collinear, there are no further restrictions on equilibrium datasets. In Section 4, we prove the arcconnectedness of the set of T-tuples of equilibrium datasets (where T is some arbitrarily chosen integer) when that set is strictly smaller than the full set (of data sets). Only the most standard concepts of set topology like arcconnectedness are required for reading this paper. They can be found in, for example, [7]. Though algebraic topology provides some motivation for this research, no knowledge of algebraic topology is needed.

2. Definitions, assumptions, and notation

A mathematical definition: Sets of T-tuples with distinct coordinates

We start with a mathematical definition. Let Y be some set and T be some finite integer. The Cartesian product Y^T consists by definition of the ordered T-tuples $(y^1, y^2, \dots, y^t, \dots, y^T)$ with $y^t \in Y$ for $1 \leq t \leq T$. We define the subset $Y^{(T)}$ of Y^T as consisting of the T-tuples whose components are all distinct: $y^t \neq y^{t'}$ for $1 \leq t, t' \leq T$.

Goods and prices

There is a finite number ℓ of goods. Let $p = (p_1, p_2, \ldots, p_{\ell-1}, p_\ell) \in \mathbb{R}^\ell_{++}$ be the price vector. We normalize the price vector p by picking the ℓ-th commodity as the numeraire, which is equivalent to setting $p_\ell = 1$. Let S denote the set of strictly positive normalized price vectors.

Individual preferences and demands

A consumer is characterized by a preference preordering represented by a utility function u_i defined on the strictly positive orthant $X = \mathbb{R}^\ell_{++}$ and an endowment vector $\omega_i \in X$. The utility function $u_i : X \to \mathbb{R}$ satisfies the standard assumptions of smooth equilibrium analysis, i.e., is smooth, monotone ($Du_i(x_i) \in X$ for any $x_i \in X$), smoothly strictly quasi-concave (the inequality $X^t D^2 u_i(x_i) X \geq 0$ and equality $X^t Du_i(x_i) = 0$ have a unique solution $X = 0$), and every indifference set $\{x_i \in X \mid u_i(x_i) = u_i\}$ is closed in $\mathbb{R}^\ell$ for any $u_i \in \mathbb{R}$.

Given any price vector $p \in S$ and endowment $\omega_i \in X$, consumer i's demand $f_i(p, p \cdot \omega_i)$ maximizes the utility $u_i(x_i)$ subject to the budget constraint $p \cdot x_i = p \cdot \omega_i$.

The SARP property and the existence of a utility function

A classical issue in consumer theory is the characterization of collections of prices and commodity bundles (p^t, x_i^t), with t varying from 1 to T, such that there exists a utility function $u_i : X \to \mathbb{R}$ that satisfies the assumptions of the previous section and such that x_i satisfies the relation $x_i = f_i(p, p \cdot x_i)$. Such a characterization exists and is known as the Strong Axiom of Revealed Preference, often called the SARP property. It takes the following form:

For any collection j_1, j_2, $\ldots$, j_n taken between 1 and T and such that the following inequalities

$$p^{j_1} \cdot x_i^{j_1} \leq p^{j_1} \cdot x_i^{j_2}, p^{j_2} \cdot x_i^{j_2} \leq p^{j_2} \cdot x_i^{j_3}, \ldots, p^{j_{n-1}} \cdot x_i^{j_{n-1}} \geq p^{j_{n-1}} \cdot x_i^{j_n} \quad (1)$$

are satisfied, the strict inequality

$$p^{j_n} \cdot x_i^{j_n} > p^{j_n} \cdot x_i^{j_1}$$

is also satisfied. For a proof, see [8].

Equilibrium and the equilibrium manifold

There is a finite number m of consumers. Let $\Omega = X^m$ denote the set of individual endowments. The price vector $p \in S$ is an equilibrium price vector for the collection of individual endowments $\omega = (\omega_1, \omega_2, \ldots, \omega_m)$ if there is equality of aggregate supply and demand for that price vector:

$$\sum_i f_i(p, p \cdot \omega_i) = \sum_i \omega_i. \quad (2)$$

One then calls the pair $(p, \omega) \in S \times \Omega$ an equilibrium.

The set E of equilibria (p, ω) is then a dimension $m\ell$ smooth submanifold of $S \times \Omega$ whose global structure (arcconnectedness, simple connectedness, contractibility, and diffeomorphism with a Euclidean space for example) has been investigated by one of the coauthors in [1], [2], and [3]. It follows from these global properties that not all dimension $m\ell$ smooth manifolds can be identified to equilibrium manifolds of exchange economies.

Equilibrium data sets

From a practical standpoint, one observes only finite collections of points belonging to the equilibrium manifold E. This raises the question of whether such finite collections of points satisfy properties that would not be satisfied by arbitrary collections of points. Brown and Matzkin have established in [5] that the set of such collections (of points that belong to the equilibrium manifold E) is indeed strictly smaller than the set of similar collections where prices do not have to equate aggregate supply and demand. The Brown and Matzkin property, however, tells us little about the "geometry" of the set of T-tuples of equilibrium pairs as a subset of the set $(S \times \Omega)^T$.

Equilibrium triples or the price–income distribution–total resource equilibria

Since we want to highlight the role of total resources and income distributions in the properties of equilibrium data sets, we first parameterize equilibrium data by prices, income distributions, and total resources.

We say that the vector of total resources $r \in \mathbb{R}^{\ell}_{++}$ is is compatible with the price vector $p \in S$ and the income distribution $(w_i) \in \mathbb{R}^m_{++}$ if the equality

$$\sum_i w_i = p \cdot r \tag{3}$$

is satisfied. Note that, once this equality is satisfied, there exists for every consumer i individual endowments $\omega_i \in X$ that satisfy the conditions $\sum_i \omega_i = r$ and $p \cdot \omega_i = w_i$. It suffices to pick $\omega_i = (w_i / p \cdot r) r$. Note, however, that the pair $\big(p, (\omega_i)\big)$ is not necessarily an equilibrium. This leads us to consider in the set $\mathcal{B}$ of triplets $b = \big(p, (w_i), r\big)$ that satisfy equality (3) the triplets that satisfy the equation

$$\sum_i f_i(p, w_i) = r. \tag{4}$$

We call such triples price-income distribution-total resource *equilibria* or, more simply, *equilibrium triplets*. They extend to the case of variable total resources $r \in X$ the price-income equilibria considered in [4].

We denote by $\mathcal{E}$ the subset of $\mathcal{B}$ consisting of equilibrium triples.

It is obvious that if the pair $\big(p, (\omega_i)\big)$ is an equilibrium pair in the sense that equation (2) is satisfied, then the triple $b = \big(p, (w_i), r\big)$ where $w_i = p \cdot \omega_i$ for every consumer i, and $r = \sum_i \omega_i$ is an equilibrium triple. Conversely, given the equilibrium triple $b = (p, (w_i), r)$, then any pair $\big(p, (\omega_i)\big)$ that is compatible with the triple is an equilibrium pair in the sense that equation (2) is satisfied. The concepts of equilibrium pairs and equilibrium triples are therefore equivalent because one can go from one to the other and conversely. The concept of equilibrium triple presents the advantage of making apparent from the very definition income distributions and total resources.

Equilibrium data sets

Let T denote a finite number of distinct observations. This leads us to consider the two subsets $\mathcal{E}^{(T)}$ and $\mathcal{B}^{(T)}$ consisting of T distinct triples and equilibrium triples respectively. The main result of Brown and Matzkin in [5] can be reformulated in our setup of equilibrium triples as saying that there exist finite numbers T such that the two sets are unequal:

$$\mathcal{E}^{(T)} \neq \mathcal{B}^{(T)}.$$

Our goal in this paper is to get some better understanding of the set of T different equilibrium triples $\mathcal{E}^{(T)}$ as a subset of the set $\mathcal{B}^{(T)}$.

3. The case of fixed and collinear total resources

A lemma about totally ordered commodity bundles

The commodity space $\mathbb{R}^\ell$ is partially ordered by the condition $x \leq y$ equivalent to $y - x \in X$. We say that a collection x^1, x^2, ..., x^T of T commodity bundles is *totally ordered* if the partial order of $\mathbb{R}^\ell$ induces a total order of the collection of the T elements x^1, x^2, ..., x^T.

Lemma 1. *Let x_i^1, x_i^2, ..., x_i^T be a fully ordered sequence of* distinct *commodity bundles. Then, for any arbitrary sequence of price vectors p^1, p^2, ..., p^T, the T pairs (p^t, x_i^t) satisfy the SARP property.*

Proof. We can assume without loss of generality that we have

$$x_i^1 \leq x_i^2 \leq \ldots \leq x_i^T.$$

Note that the inequality $x_i^t \leq x_i^{t'}$ combined with the inequality $x_i^t \neq x_i^{t'}$ implies, for any price vector p^t, the strict inequality

$$p^t \cdot x_i^t < p^t \cdot x_i^{t'}.$$

Assume now that the following inequalities

$$p^{j_1} \cdot x_i^{j_1} \leq p^{j_1} \cdot x_i^{j_2}, p^{j_2} \cdot x_i^{j_2} \leq p^{j_2} \cdot x_i^{j_3}, \ldots, p^{j_{n-1}} \cdot x_i^{j_{n-1}} \geq p^{j_{n-1}} \cdot x_i^{j_n} \quad (5)$$

are satisfied. It then suffices that we show that the strict inequality

$$p^{j_n} \cdot x_i^{j_n} > p^{j_n} \cdot x_i^{j_1}$$

is satisfied in order to establish the SARP property.

It follows from the complete ordering property that the inequalities (5) are compatible with only one full ordering of the commodity bundles, namely:

$$x_i^{j_1} \leq x_i^{j_2} \leq \ldots \leq x_i^{j_n},$$

which implies the ordering

$$x_i^{j_1} \leq x_i^{j_n}.$$

It follows from this inequality combined with the inequality $x_i^{j_1} \neq x_i^{j_n}$ that the strict inequality

$$p^{j_n} \cdot x_i^{j_1} < p^{j_n} \cdot x_i^{j_n}$$

is then satisfied. □

Corollary 2. *Let $x_i^1, x_i^2, \ldots, x_i^n$ be a sequence of non-negative commodity bundles that are collinear with the origin. Then, for any arbitrary sequence of price vectors $p^1, p^2, \ldots, p^n$, the n pairs (p^j, x_i^j) satisfy the SARP property.*

Proof. It follows from the assumptions that the sequence is fully ordered. It then suffices to apply Lemma 1. □

The case of fixed total resources

Let $\mathcal{E}^{(T)}(r)$ (resp. $\mathcal{B}^{(T)}(r)$ denote the intersection of $\mathcal{E}^{(T)}$ (resp. $\mathcal{B}^{(T)}$) with the set $\big(S \times (\mathbb{R}_{++})^m \times \{r\}\big)^T$. The set $\mathcal{E}^{(T)}(r)$ therefore consists of T two by two distinct price-income distribution-total resource equilibrium triples for a vector of total resources $r \in \mathbb{R}^{\ell}_{++}$ that is constant.

Theorem 3. *We have*

$$\mathcal{E}^{(T)}(r) = \mathcal{B}^{(T)}(r)$$

Proof. Let $\big(p^t, (w_i^t), r^t\big)$ be a collection of T two by two distinct price-income distribution-total resource triples (which are not necessarily equilibrium triples). The idea of the proof is show that there exist commodity bundles x_i^t and utility functions u_i for i varying from 1 to m and t from 1 to T that satisfy the equalities $x_i^t = f_i(p^t, w_i^t)$ (where f_i is the demand function associated with the utility function u_i) and $\sum_i x_i^t = r^t$.

We first assume that the following additional property is satisfied:

$$\frac{w_i^t}{p^t \cdot r} \neq \frac{w_i^{t'}}{p^{t'} \cdot r} \tag{6}$$

for all t, t', and i.

Define

$$x_i^t = \frac{w_i^t}{p^t \cdot r} r.$$

By construction, all the vectors x_i^t for a given i are collinear with the positive vector $r \in \mathbb{R}^{\ell}_{++}$. It follows from inequalities (6) that the x_i^t's are all distinct for any given i. It then suffices to apply Lemma 1 to conclude that, for every i, the T pairs (p^t, x_i^t) (for t varying from 1 to T) satisfy the SARP property. This implies for every i between 1 and m the existence of a utility function u_i such that the equality $x_i^t = f_i(p^t, w_i^t)$ is satisfied. In addition, it follows from the formula defining x_i^t that we have

$$\sum_i x_i^t = \frac{\sum_i w_i^t}{p^t \cdot r} r = r,$$

which implies that the equality

$$\sum_i f_i(p^t, w_i^t) = r = r^t$$

is satisfied for t varying from 1 to T. This proves that such price-income distribution-total resource triples are indeed equilibrium triples for suitably defined utility functions.

The next step is to deal with situations where, for some consumer i, there exist t and t' such that the equality

$$\frac{w_i^t}{p^t \cdot r} = \frac{w_i^{t'}}{p^{t'} \cdot r}$$

is satisfied with $p^t \neq p^{t'}$.

Because of this equality, the commodity bundles $x_i^t = \dfrac{w_i^t}{p^t \cdot r}$ and $x_i^{t'} = \dfrac{w_i^{t'}}{p^{t'} \cdot r}$ are not good candidates for our construction because they are equal while the candidate "supporting" price vectors p^t and $p^{t'}$ are different. The idea is therefore to perturb x_i^t and $x_i^{t'}$ in such a way that the perturbed sequence $x_i'^1, x_i'^2, \dots, x_i'^T$ remains totally ordered, and the equalities $\sum_i x_i'^t = r^t$ are satisfied for all t's.

In order to do that, consider the line Δ_0 that passes through the origin and that is collinear with the vector $r \in X$. Let Δ be a line parallel to the line Δ_0 and sufficiently close to Δ_0 for the following property to be satisfied. The intersection points $x_i'^t$ and $x_i'^{t'}$ of Δ with the budget hyperplanes $p^t \cdot x_i^t = w_i^t$ and $p^{t'} \cdot x_i^{t'} = w_i^{t'}$ are distinct and, if we define $x_i'^{t''} = x_i^{t''}$ for $t'' \neq t, t'$, the sequence $x_i'^1, \dots, x_i'^T$ is completely ordered. This follows from the fact that the sequence $x_i^1, x_i^2, \dots,$ x_i^T is already fully ordered, with the elements x_i^t and $x_i^{t'}$ being identical. The new sequence is obtained by just perturbing those two identical elements: the elements $x_i'^t$ and $x_i't'$ can therefore be compared to all the other elements of the sequence provided the perturbation is small enough. In addition, thanks to the choice of the direction of the line Δ, these two elements are themselves ordered.

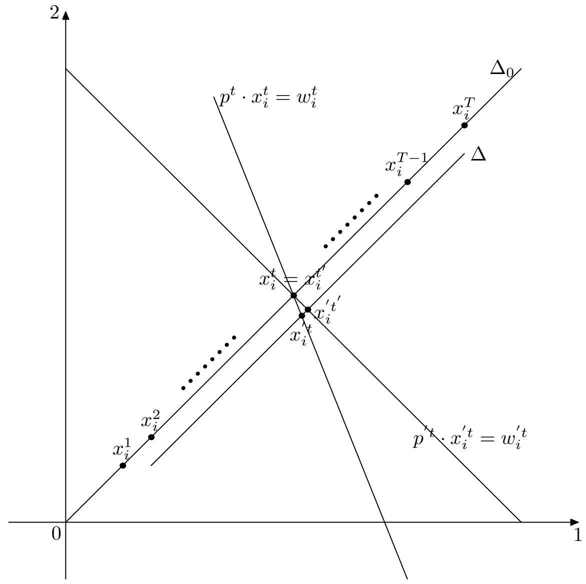

Figure 1: Perturbation of the collection $\{x_i^t\}$ for $1 \leq t \leq T$

The next step is to find another consumer j and to perturb the corresponding sequence x_j^1, x_j^2, …, x_j^T, so that the total resources remain constant. Therefore, we pick some arbitrary consumer j and define the new sequence $x_j'^1$, $x_j'^2$, …, $x_j'^T$ by

$$x_j'^t = x_j^t + (x_i^t - x_i'^t), x_j'^{t'} = x_j^{t'} + (x_i^{t'} - x_i'^{t'}), x_j'^{t''} = x_j^{t''}$$

with $t'' \neq t, t'$.

Using the same line of reasoning as above, we observe that the perturbation that defines the consumption bundles of consumer i can be made small enough for the new sequence to be fully ordered and the already distinct elements to remain distinct through the perturbation. In addition, the total resources are then equal, by construction, to the vector r . Overall, this construction reduces by at least one unit the number of non distinct commodity bundles. It then suffices to iterate this construction for every consumer i and pairs (t, t') such that $x_i^t = x_i^{t'}$. Eventually, one gets for each consumer a collection of ordered sequences of T elements that sum up to the vector of total resources $r \in X$. One then concludes with the application of Lemma 1. $\square$

Extension to the case of collinear total resources

Theorem 3 admits the following extension when total resources, instead of being constant, are collinear, i.e., the total resource vector r^t is collinear with some fixed vector r for $t = 1, 2, \ldots, T$.

Let us denote by $\mathcal{E}^{(T)}[r]$ the intersection of $\mathcal{E}^{(T)}$ with the set $\big(S \times (\mathbb{R}_{++})^m \times$

$\{r\mathbb{R}\}\big)^T$. The set $\mathcal{E}^{(T)}[r]$ then consists of the price-income-total resource equilibria for total resources $r^t \in \mathbb{R}_{++}r \subset X$. Similarly, we define the set $\mathcal{B}[r]$ as the set of price-income-total resources, with total resources collinear with $r \in X$.

Theorem 4. *We have*

$$\mathcal{E}^T[r] = \mathcal{B}^T[r]$$

Proof. It suffices to reproduce the same line of proof as in Theorem 3, a theorem that is actually a special case of the current theorem.

□

Remark 1. One checks readily that the proofs of Theorems 3 and 4 extend to the case where the total resources are not collinear, but only sufficiently close to being collinear, which includes as a special case variable total resources provided the variations are sufficiently small. From a practical standpoint, equilibrium data sets show no restrictions in these cases.

4. Connectedness of the set $\mathcal{E}^{(T)}$

In general, however, the set $\mathcal{E}^{(T)}$ of T-tuples of equilibrium triples is a strict subset of the set $\mathcal{B}^{(T)}$. The following theorem states a remarkable global topological property.

Theorem 5. *The set $\mathcal{E}^{(T)}$ is arcconnected.*

Recall that a subset C of a topological space is arcconnected if any two elements x and y of C can be linked by some continuous path belonging to the set C. This is the same thing as saying that there exists a continuous map $h : [0, 1] \to C$ such that $h(0) = x$ and $h(1) = y$.

Proof. The idea of the proof of the arcconnectedness of $\mathcal{E}^{(T)}$ consists therefore in the construction of a continuous path linking two arbitrarily given T-tuples of equilibrium triples b and b' defined by the collections $\big(p^t, (w_i^t), r^t\big)$ and $\big({p'}^t, ({w'}_i^t), {r'}^t\big)$ for t varying from 1 to T.

We first define the following set of T triples:

$$\big({p'}^t, ({w'}_i^t), \sum_i f_i({p'}^t, {w'}_i^t)\,\big)$$

for t varying from 1 to T. These are equilibrium triples associated with the preferences (or utility functions) corresponding to the demand functions f_i, with i varying from 1 to m.

Note that these T triples are necessarily two by two distinct so that the T-tuple they define, a T-tuple denoted by b'', does belong to the set $\mathcal{E}^{(T)}$. In order to connect b and b', it therefore suffices to connect b to b'' and b'' to b'.

Path from b to b''

Here, the preferences (or utility functions) do not vary. One starts by constructing a continuous path linking $\big(p^t, (w_i^t)\big)_{1 \le t \le T}$ to $\big(p'^t, (w_i'^t)\big)_{1 \le t \le T}$, a path such that all coordinates remain different for the points in the path. (That such a construction is always possible is straightforward; for example, one can start with the line segment linking the two points. If, at some points, some coordinates become equal, it is easy to see that a small perturbation of the path will restore inequality of the coordinates.) Let $\big(p^t(\theta), (w_i^t(\theta))\big)_{1 \le t \le T}$, with $\theta \in [0, 1]$, denote the generic point of this path. One then defines

$$r^t(\theta) = \sum_i f_i\big(p^t(\theta), w_i^t(\theta)\big).$$

By definition, the triple

$$b(\theta) = \big(p^t(\theta), (w_i^t(\theta)), r^t(\theta)\big)$$

is an equilibrium triple. It follows from the continuity of the individual demand functions that this is a continuous path (for θ varying from 0 to 1) linking b to b''.

Path from b'' to b'

Here, the T-tuple $\big(p'^t, (w_i'^t)\big)_{1 \le t \le T}$ is kept fixed. Consider the points

$$x_i^t = f_i(p'^t, w_i'^t) \quad \text{and} \quad x_i'^t = f_i'(p'^t, w_i'^t)$$

in X. Let K be some convex compact subset of X that contains these $2T$ points in its interior. It follows from the assumptions regarding the preferences associated with the demand functions f_i and f_i' that these preferences can be represented by utility functions u_i and u_i' whose restrictions to the interior of the compact set K are strictly concave. Then, consider for $\theta \in [0, 1]$ the function

$$u_i(\theta) = (1 - \theta)u_i + \theta u_i'.$$

The restriction of the function $u_i(\theta)$ to the interior of the compact set K is strictly concave in addition to satisfying the assumptions that we impose on utility functions. The preference preordering defined on the compact K by the utility function $u_i(\theta) \mid K$ can be extended into a preference preordering defined on the strictly positive orthant X. Let $f_i(\theta)$ be the corresponding demand function. One checks readily that the restriction of the demand function $f_i(\theta)$ to the points $(p'^t, w_i'^t)$ varies continuously with $\theta \in [0, 1]$. Define

$$r^t(\theta) = \sum_i f_i(\theta)\big(p'^t, w_i'^t\big).$$

Then, the T-tuple

$$b(\theta) = \big(p'^t, (w_i'^t), r^t(\theta)\big)_{1 \le t \le T}$$

is a T-tuple made of distinct equilibrium triples, and varies continuously from b'' to b when θ varies from 0 to 1.

By piecing together the two continuous paths just constructed, one defines a continuous path linking b to b'.

$\square$

5. Concluding comments

Arcconnectedness is often the first global property investigated for a topological space. A positive result regarding arcconnectedness is then followed by investigations regarding simple connectedness, contractibility, and the determination of the homeomorphism or diffeomorphism type of the topological space under study. The study of the global properties of a topological space culminates with the computation of the various homology and cohomology groups of that space, something much more demanding than just checking connectedness (or arcconnectedness). It is likely, therefore, that, at variance with the approach followed in the current paper, any further step along this direction will have to exploit the semi-algebraic structure of the set $\mathcal{E}^{(T)}$, a remarkable property that follows readily from [5] as a consequence of the equivalence between Afriat's inequalities and the SARP property.

From a purely philosophical standpoint, the falsifiability issue requires only that the space of equilibrium data triples be strictly smaller that the set of all data triples. That these two sets are in fact identical when total resources are the same or almost the same or, more generally, collinear or almost collinear, shows us that the theory of general equilibrium, though falsifiable in principle, may turn out to be almost impossible to falsify using real world data, a point so far neglected by philosophers.

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