04/03/2024

Fundamentals of Radiometry & Photometry

Optical Engineering

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Radiometry and Photometry

What is Radiometry?

Radiometry is the field of metrology related to the physical measurement of the properties of electromagnetic radiation, including visible light (usually in the range between **0.01-1000 μm** – UV, Visible,IR).

https://www.bipm.org/metrology/photometry-radiometry/#:~:text=Radiometry%20is%20the%20field%20of,terms%20of%20brightness%20and%20colour.

What is Photometry?

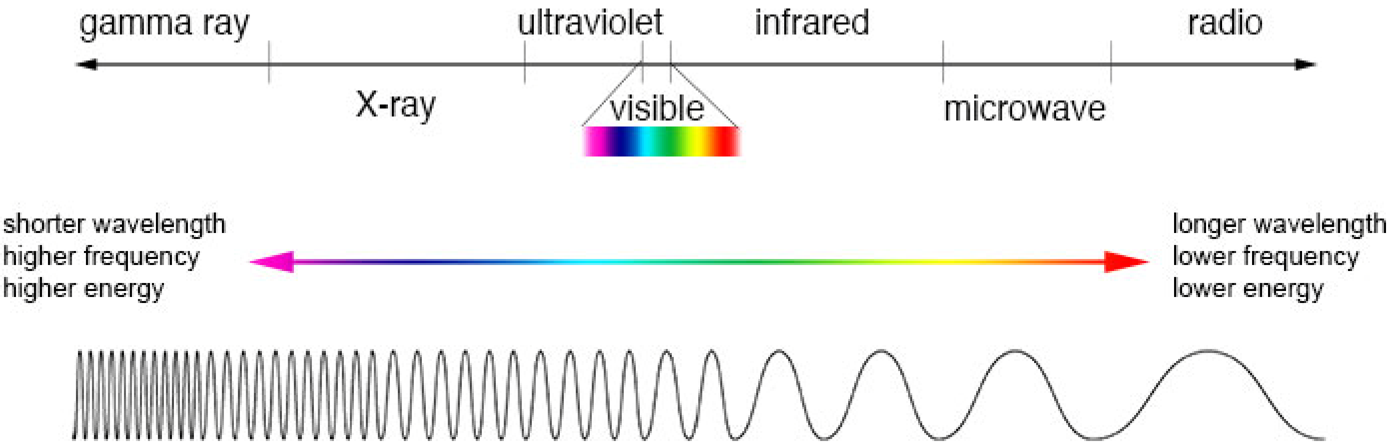
Photometry is the science of **measuring visible light (360-830 nm)** in units that are weighted according to the sensitivity of the human eye. It is a **quantitative** science based on a statistical model of the human visual response to light - that is, our perception of light - under carefully controlled conditions.

https://andor.oxinst.com/learning/view/article/radiometry-photometry

Radiometric and photometric measurements are of importance for a wide range of industries and applications, including the lighting, space, semiconductor, photovoltaic, optical communication, automotive and color industries, displays and imaging. Other emerging fields include appearance, terahertz applications, photonics, and quantum-based information.

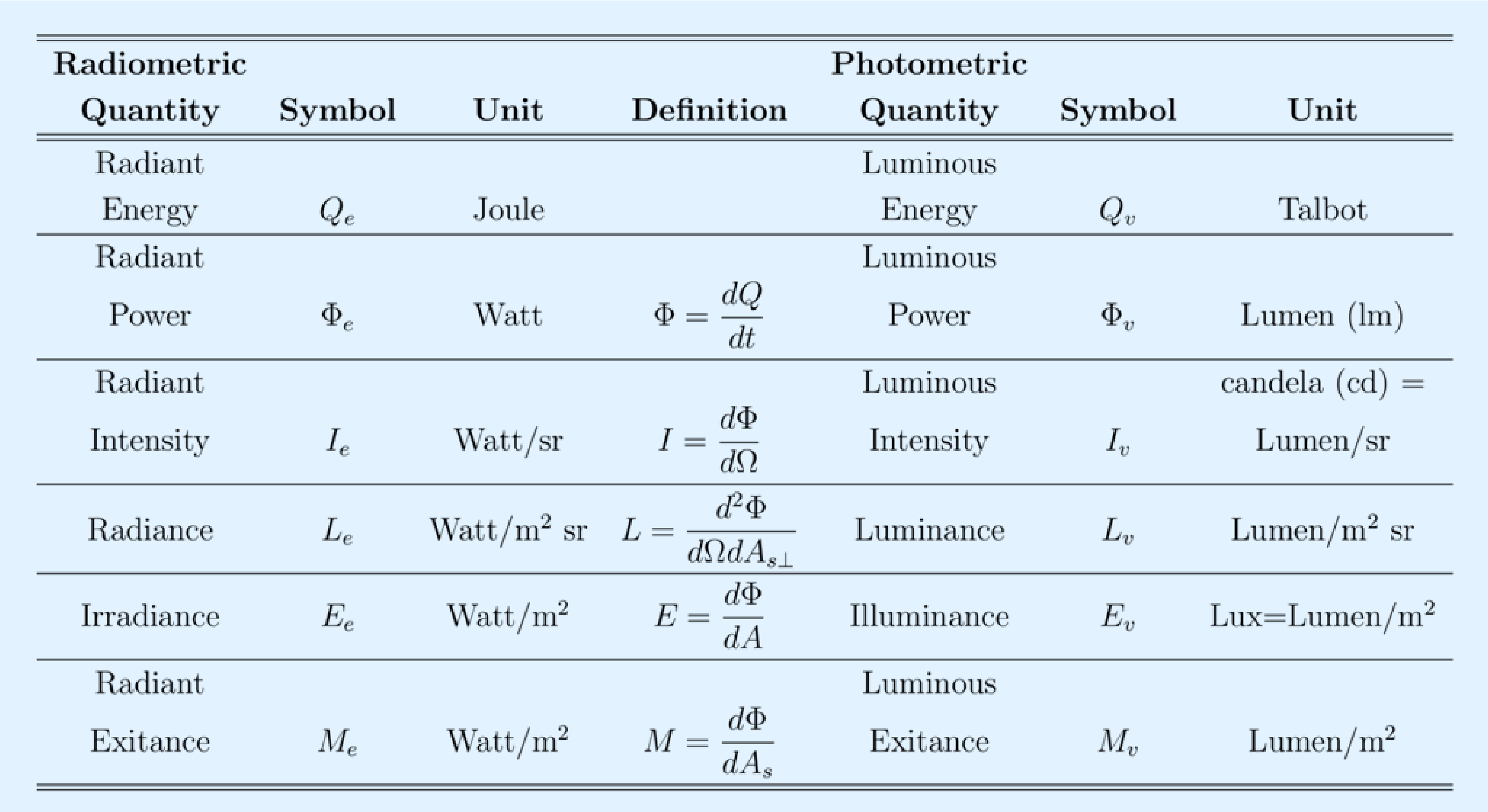
https://www.bipm.org/metrology/photometry-radiometry/#:~:text=Radiometry%20is%20the%20field%20of,terms%20of%20brightness%20and%20colour.

Radiometry and Photometry



https://imagine.gsfc.nasa.gov/Images/science/EM_spectrum_compare_level1_lg.jpg

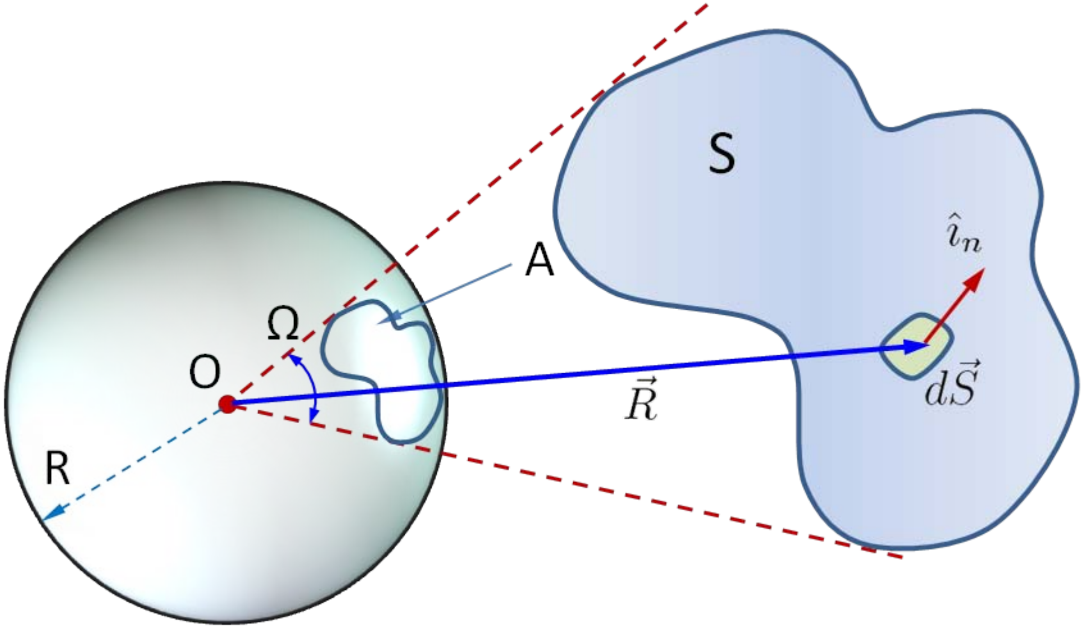
Radiometric and Photometric Quantities



Radiometric Quantity	Symbol	Unit	Definition	Photometric Quantity	Symbol	Unit
Radiant Energy	Q_e	Joule		Luminous Energy	Q_v	Talbot
Radiant Power	Φ_e	Watt	$\Phi = \dfrac{dQ}{dt}$	Luminous Power	Φ_v	Lumen (lm)
Radiant Intensity	I_e	Watt/sr	$I = \dfrac{d\Phi}{d\Omega}$	Luminous Intensity	I_v	candela (cd) = Lumen/sr
Radiance	L_e	Watt/m^2 sr	$L = \dfrac{d^2\Phi}{d\Omega dA_{s\perp}}$	Luminance	L_v	Lumen/m^2 sr
Irradiance	E_e	Watt/m^2	$E = \dfrac{d\Phi}{dA}$	Illuminance	E_v	Lux=Lumen/m^2
Radiant Exitance	M_e	Watt/m^2	$M = \dfrac{d\Phi}{dA_s}$	Luminous Exitance	M_v	Lumen/m^2

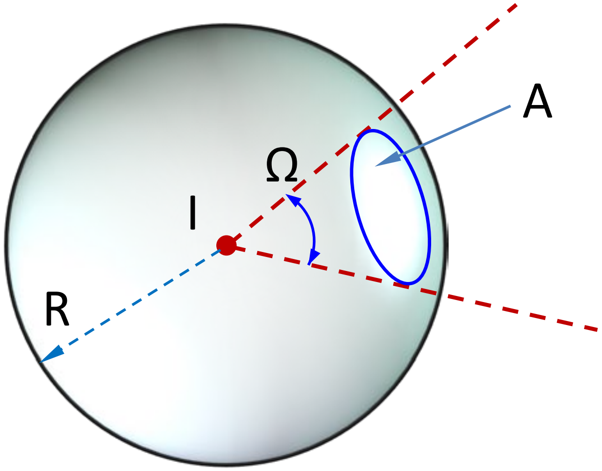
English Terms	**Greek Terms**
Radiant Energy	Ακτινοβολούμενη Ενέργεια
Luminous Energy	Φωτεινή Ενέργεια
Radiant Power	Ακτινοβολούμενη Ισχύς
Luminous Power	Φωτεινή Ισχύς
Radiant Intensity	Ακτινοβολούμενη ‘Ενταση
Luminous Intensity	Φωτεινή Ένταση
Radiance	Ακτινοβολία
Luminance	Φωτεινότητα (Λαμπρότητα)
Irradiance	Ένταση Ακτινοβολίας
Illuminance	Ένταση Φωτισμού
Radiant Exitance	Αφετική Ικανότητα Ακτινοβολίας
Luminous Exitance	Αφετική Ικανότητα Φωτεινής Ακτινοβολίας

Solid Angle Definition



$$\boxed{\Omega = \int_S \frac{d\vec{S} \cdot \hat{\imath}_R}{|\vec{R}|^2} = \int_S \frac{\hat{\imath}_n \cdot \hat{\imath}_R dS}{|\vec{R}|^2} = \int_A \frac{dA}{R^2}}$$

Point Source



$$I = \frac{d\Phi}{d\Omega}$$

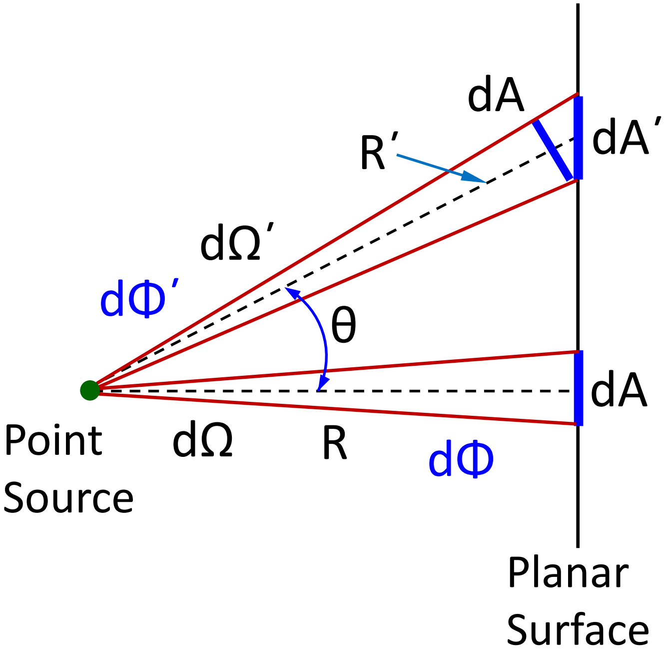
$$d\Omega = \frac{dA}{R^2}$$

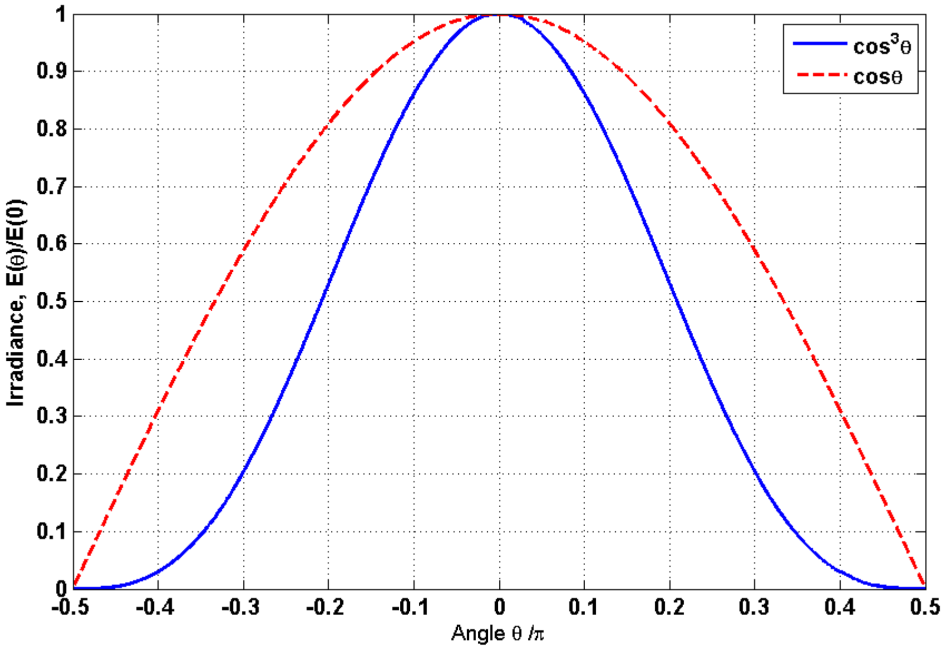
The intensity *I* of a point source is constant!

Point Source (continue)

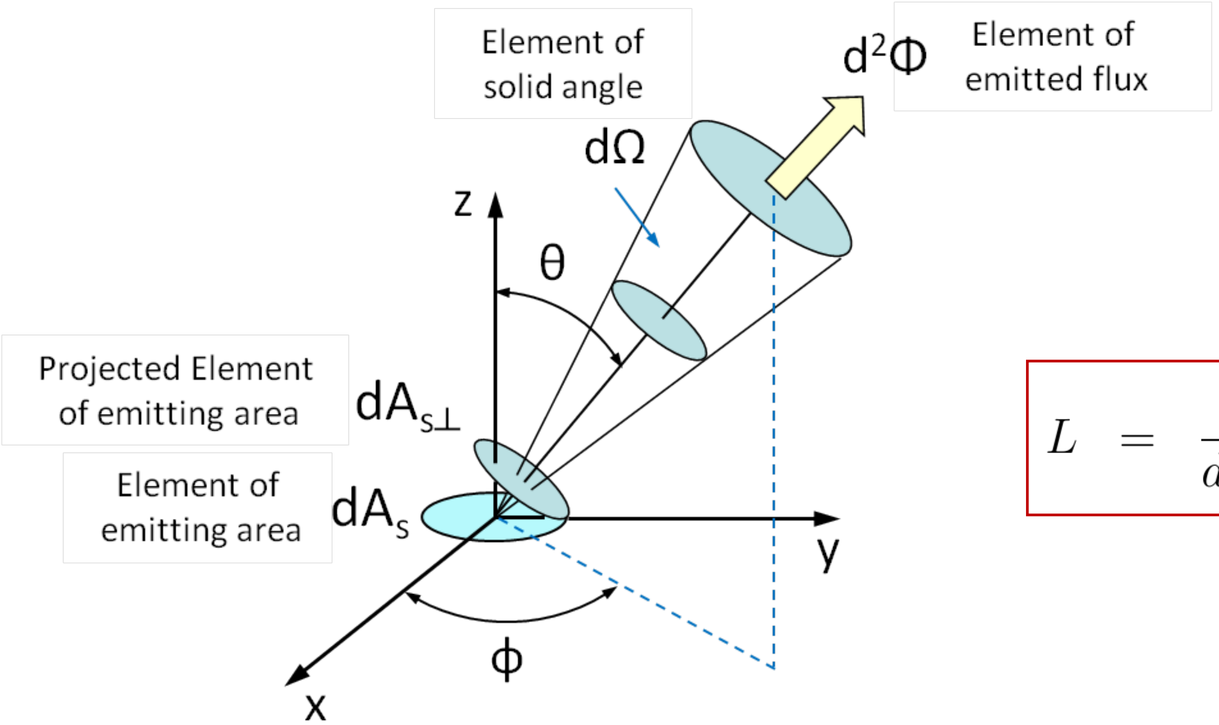
Cosine-to-the-third Irradiance Falloff

$$
\begin{aligned}
E(\theta=0,\phi) &= \frac{d\Phi}{dA} = \frac{I\,d\Omega}{dA} = \frac{I}{R^2} = E_0 \\
dA' &= \frac{dA}{\cos\theta} \\
R' &= \frac{R}{\cos\theta}\,, \qquad d\Omega' = \frac{dA}{R'^2} \\
E(\theta,\phi) &= \frac{d\Phi'}{dA'} = \frac{I\,d\Omega'}{dA/\cos\theta} = \frac{I\cos\theta}{R'^2} = \frac{I}{R^2}\cos^3\theta
\end{aligned}
$$





Radiance and Luminance

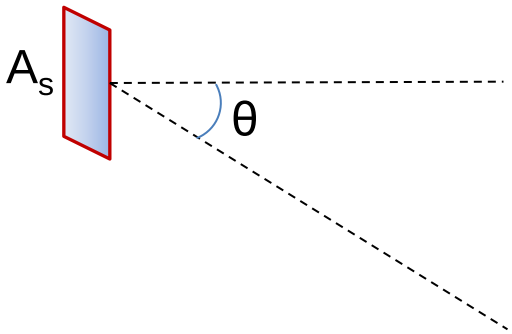


$$\boxed{L \;=\; \frac{d^2\Phi}{dA_{s\perp}\,d\Omega} = \frac{d^2\Phi}{dA_s\cos\theta\, d\Omega}}$$

From “Introduction of Radiometry and Photometry”, McCluney , Artech House 1994

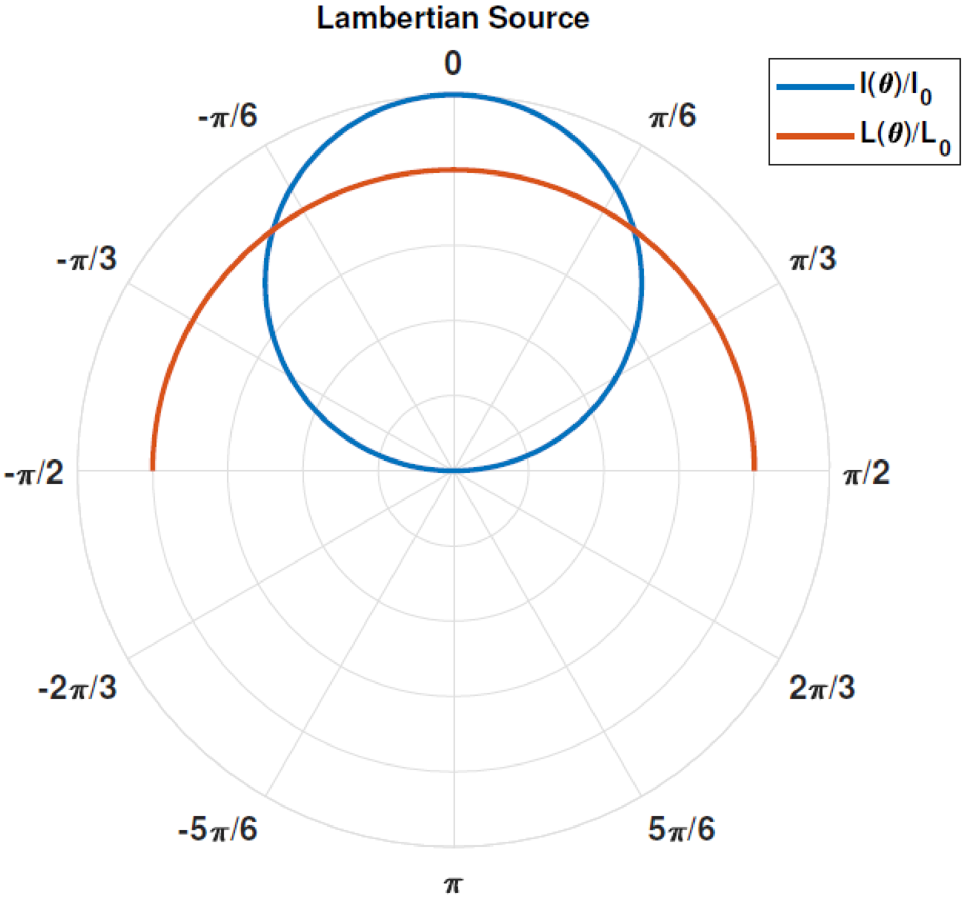
Lambertian Source

Extended Source



Lambert's Law

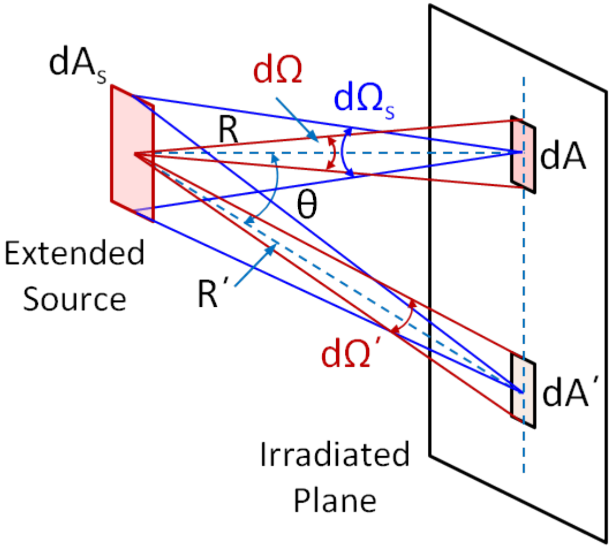
$$\boxed{\begin{aligned} L(\theta) &= L_0 \\ I(\theta) &= I_0 \cos\theta \end{aligned}}$$



Intensity and Radiance

Extended Source (Lambertian)

Cosine-to-the-fourth Irradiance Falloff



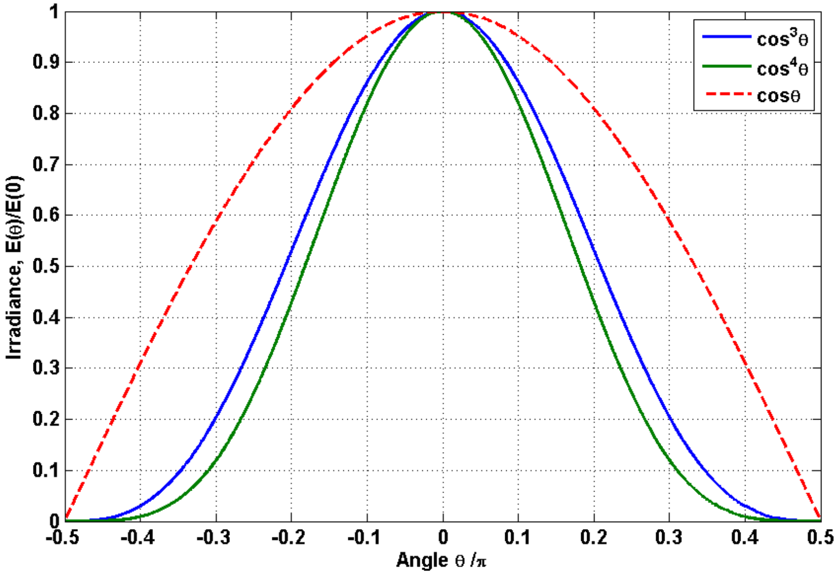
$$dE(\theta=0) = \frac{d^2\Phi}{dA} = \frac{L d\Omega dA_s}{dA} = L\,\frac{dA}{R^2}\frac{dA_s}{dA} = L\,\frac{dA_s}{R^2}$$

$$E(\theta=0) = E_0 = \int_{A_s} L\,\frac{dA_s}{R^2} = L\int_{A_s}\frac{dA_s}{R^2} \simeq L\Omega$$

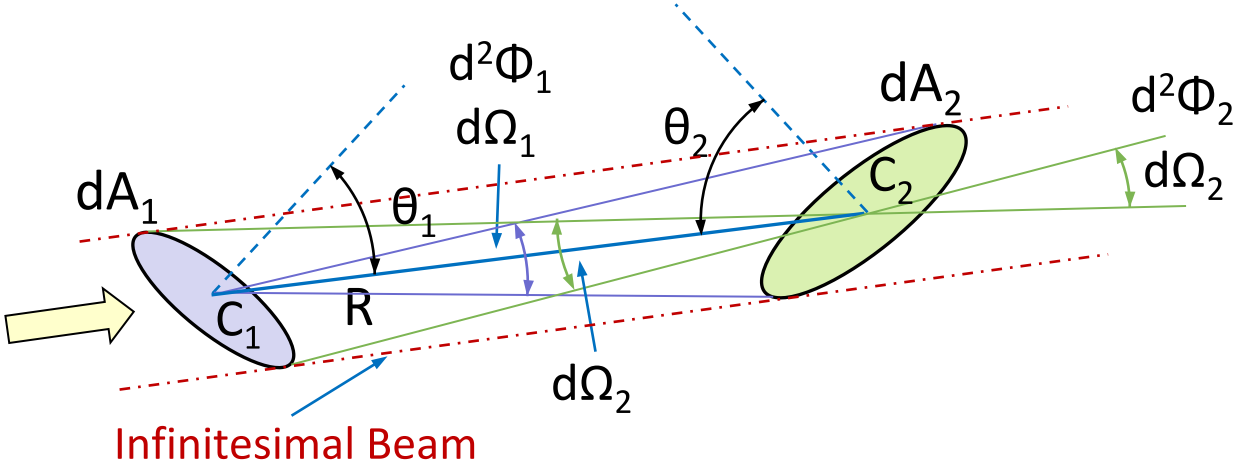
$$dE(\theta) = \frac{d^2\Phi'}{dA'} = \frac{L d\Omega' dA_s\cos\theta}{dA'} = L\,\frac{dA'\cos\theta}{R'^2}\frac{dA_s\cos\theta}{dA'} = L\,\frac{dA_s}{R^2}\cos^4\theta,$$

$$E(\theta) = \int_{A_s} L\,\frac{dA_s}{R^2}\cos^4\theta = L\int_{A_s}\frac{dA_s}{R^2}\cos^4\theta \simeq L\Omega\cos^4\theta = E_0\cos^4\theta.$$

$$\boxed{E(\theta) = E(0)\cos^4\theta = L\Omega\cos^4\theta}$$



Radiance/Luminance Conservation



$$d^2\Phi_1 = L_1 dA_1 \cos\theta_1 d\Omega_1 = L_1 dA_1 \cos\theta_1 \left(\frac{dA_2 \cos\theta_2}{R^2}\right) = L_1 \frac{dA_1 dA_2 \cos\theta_1 \cos\theta_2}{R^2}$$

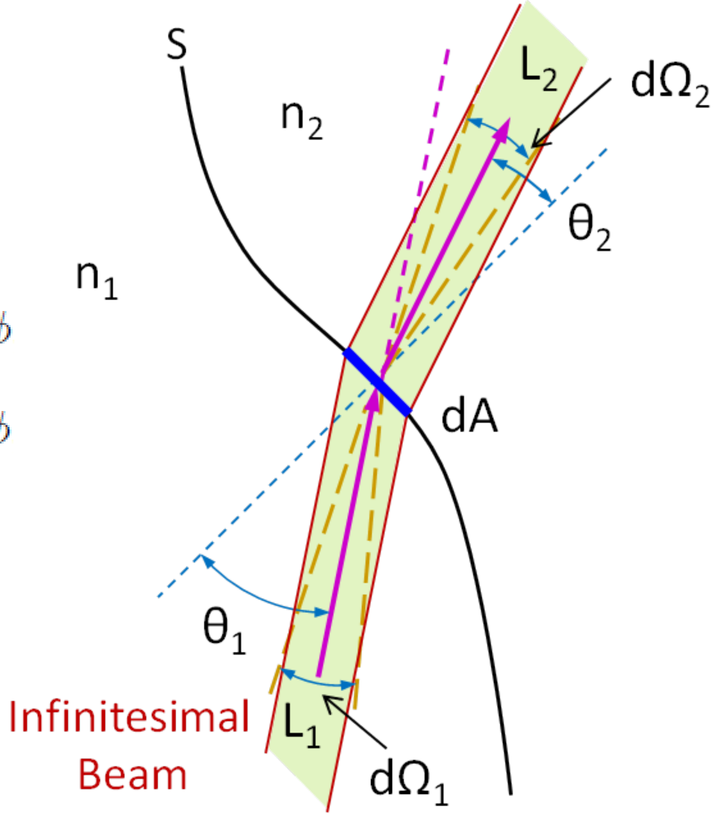
$$d^2\Phi_2 = L_2 dA_2 \cos\theta_2 d\Omega_2 = L_2 dA_2 \cos\theta_2 \left(\frac{dA_1 \cos\theta_1}{R^2}\right) = L_2 \frac{dA_2 dA_1 \cos\theta_2 \cos\theta_1}{R^2}.$$

$$\boxed{d^2\Phi_1 = d^2\Phi_2 \quad \Longrightarrow \quad L_1 = L_2}$$

Radiance/Luminance Conservation

$$d^2\Phi_1 = L_1 dA\cos\theta_1 d\Omega_1 = L_1 dA\cos\theta_1\sin\theta_1 d\theta_1 d\phi$$

$$d^2\Phi_2 = L_2 dA\cos\theta_2 d\Omega_2 = L_2 dA\cos\theta_2\sin\theta_2 d\theta_2 d\phi$$

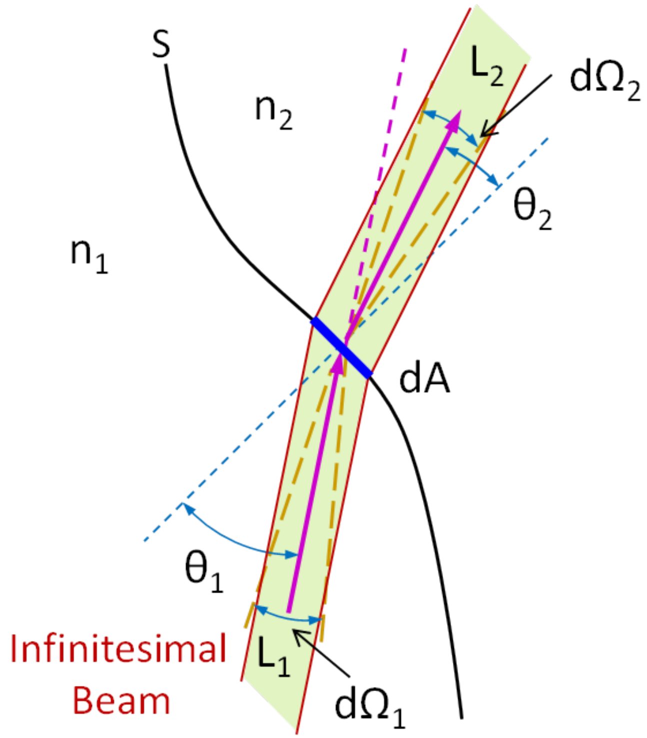


$$\begin{aligned} d^2\Phi_1 &= L_1\cos\theta_1 dA\sin\theta_1 d\theta_1 d\phi = d^2\Phi_2 = L_2\cos\theta_2 dA\sin\theta_2 d\theta_2 d\phi \implies \\ L_1 &= L_2\frac{\cos\theta_2 d\theta_2}{\cos\theta_1 d\theta_1}\,\frac{\sin\theta_2}{\sin\theta_1} = L_2\frac{n_1^2}{n_2^2} \implies \end{aligned}$$

$$\boxed{\frac{L_1}{n_1^2} = \frac{L_2}{n_2^2} = L_0} \iff \boxed{\frac{M_1}{n_1^2} = \frac{M_2}{n_2^2} = M_0,}$$

For
Lambertian Sources

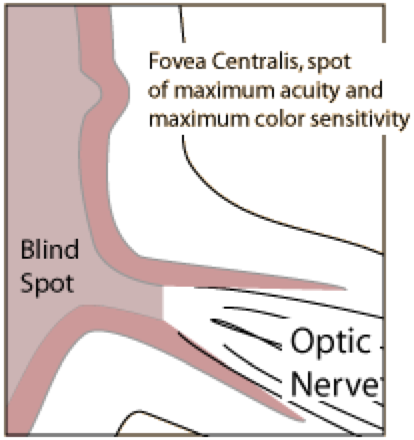
Radiance/Luminance Conservation

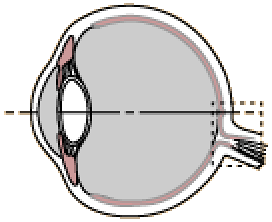


$$\begin{aligned} d^2\Phi_2 &= (1-\rho)d^2\Phi_1 \Longrightarrow \\ L_2 dA\cos\theta_2\sin\theta_2 d\theta_2 d\phi &= (1-\rho)L_1 dA\cos\theta_1\sin\theta_1 d\theta_1 d\phi \Longrightarrow \\ L_2\cos\theta_2\sin\theta_2 d\theta_2 &= (1-\rho)L_1\cos\theta_1\sin\theta_1 d\theta_1 . \end{aligned}$$

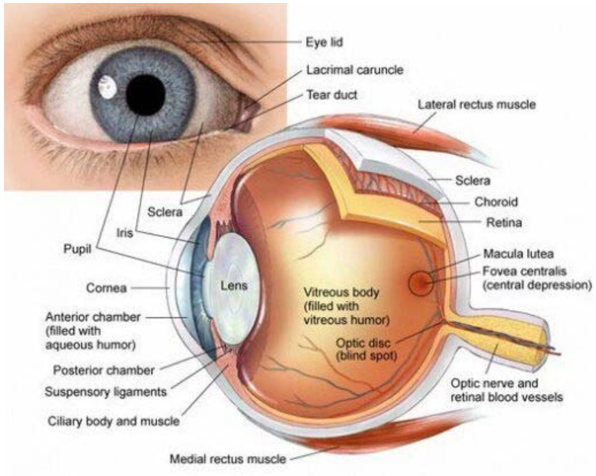
$$\boxed{\frac{L_2}{n_2^2} = (1-\rho)\frac{L_1}{n_1^2}}$$

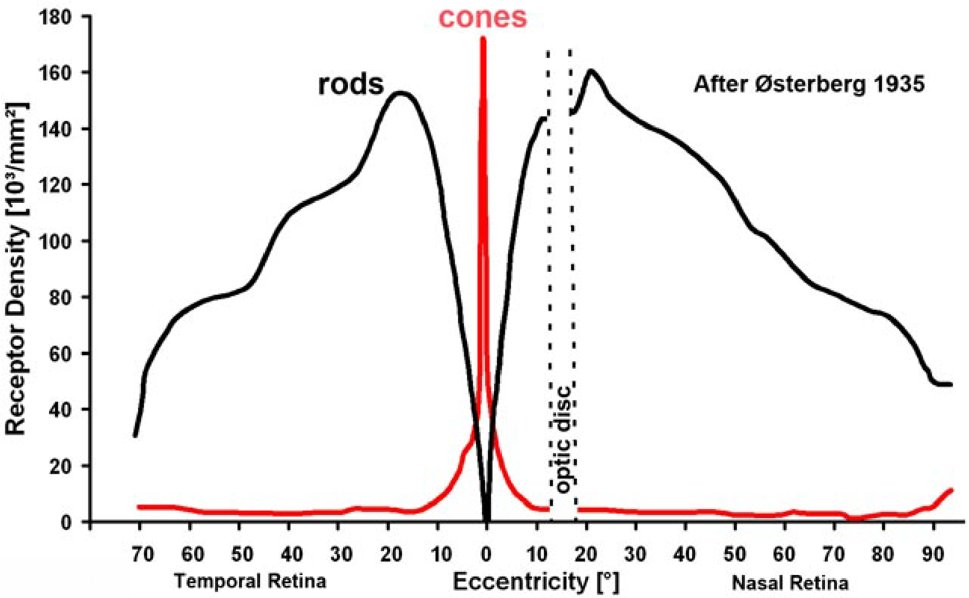
Human Eye – Cones and Rods Photoreceptors



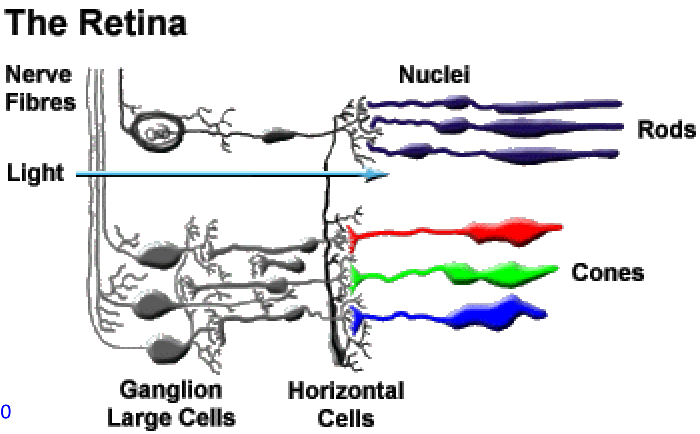


Cross section of human eye



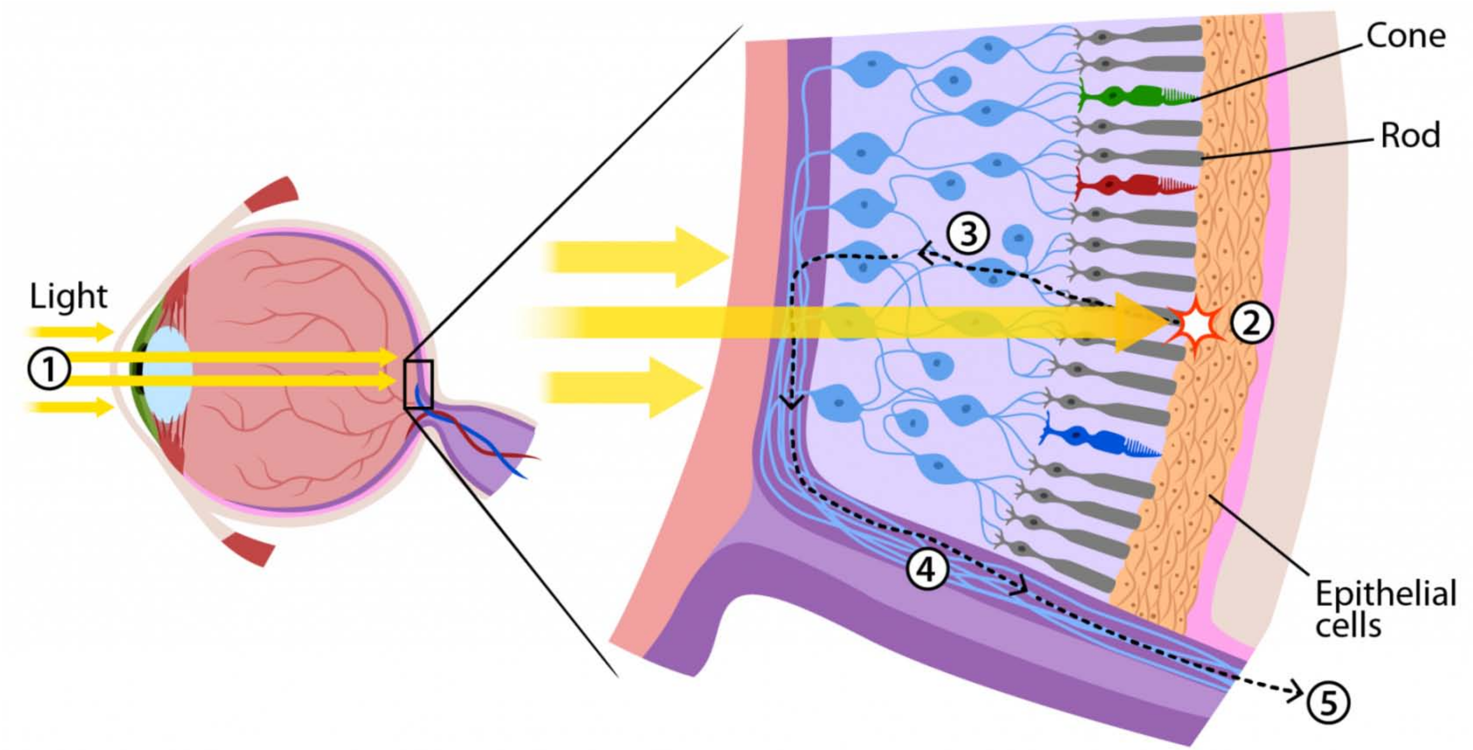


http://webvision.med.utah.edu/imageswv/Ostergr.jpeg



https://www.google.com/url?sa=i&url=https%3A%2F%2Fwww.anatomynote.com%2Fhuman-anatomy%2Fophthalmology-eye-anatomy%2Ffovea-centralis-optic-nerves-and-retinal-blood-vessels-in-eye%2F&psig=AOvVaw1j6cQX1sJT1kfOYlXrElyT&ust=1584866538950000&source=images&cd=vfe&ved=0CAIQjRxqFwoTCOjzqIqWq-gCFQAAAAAdAAAAABAS

Human Eye – Cones and Rods Photoreceptors



https://askabiologist.asu.edu/sites/default/files/resources/articles/seecolor/Light-though-eye-big.png

Human Eye – Cones and Rods Photoreceptors

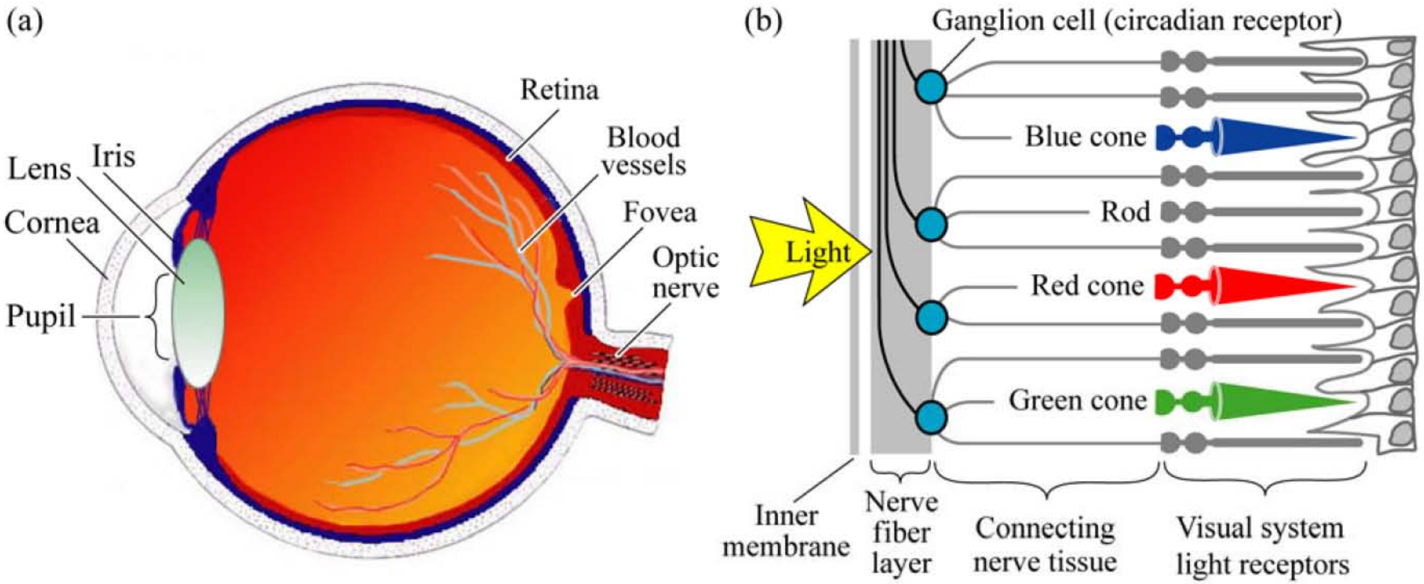
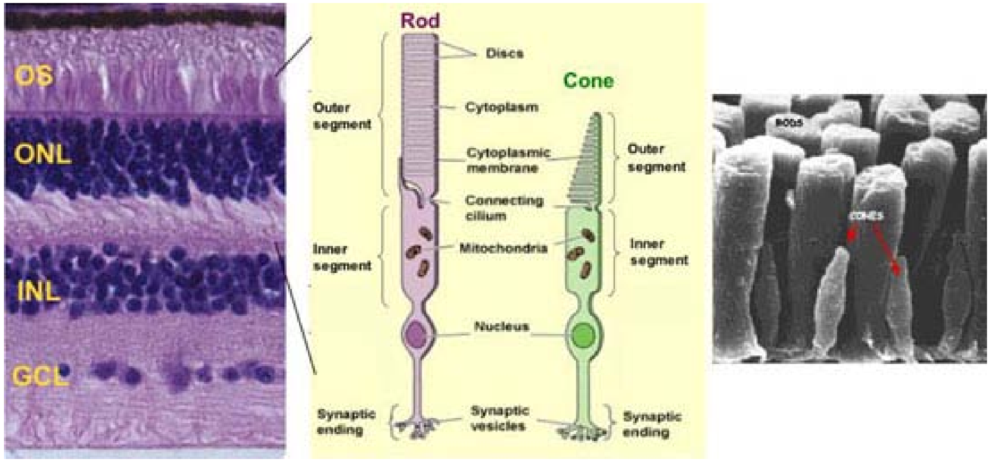


Fig. 16.1. (a) Cross section through a human eye. (b) Schematic view of the retina including rod and cone light receptors (adapted from Encyclopedia Britannica, 1994).

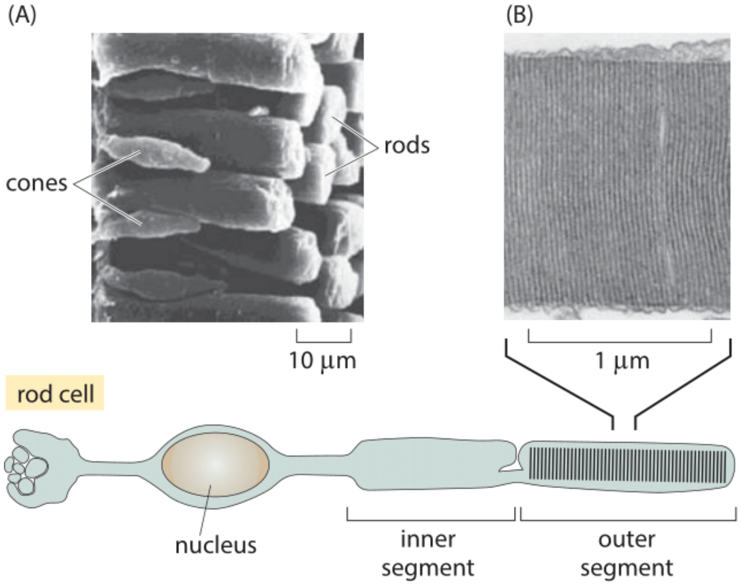
- **Cones**: Provide color sensitivity
- **Rods**: Color-insensitive
- Color perception depends on light level
- Scotopic vision regime: Low-light-level-vision regime
- Photopic vision regime: High-light-level-vision regime

E. F. Schubert, Light Emitting Diodes, 2nd Ed., Cambridge University Press, 2006

Human Eye – Cones and Rods Photoreceptors

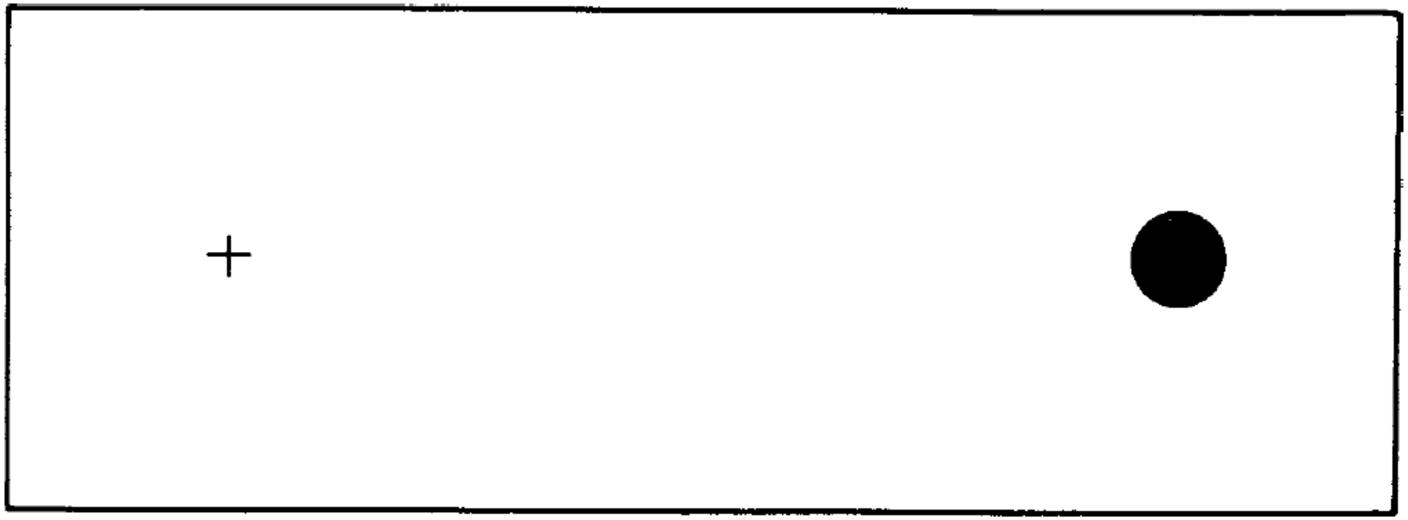


http://vrcore.wustl.edu/portals/chen_shiming/Chen_S_Photo4.jpg



http://book.bionumbers.org/how-big-is-a-photoreceptor/

Rod and Cone photoreceptors in mammalian retina. A) A human retinal section showing three neuronal cell layers: outer nuclear layer (ONL) containing the nucleus of rods and cones; inner nuclear layer (INL) containing the nucleus of bipolar, horizontal and amacrine and Muller glial cells; gonglion cell layer (GCL). B) Diagram of rod and cone structure. C) Scan EM showing the outer segments



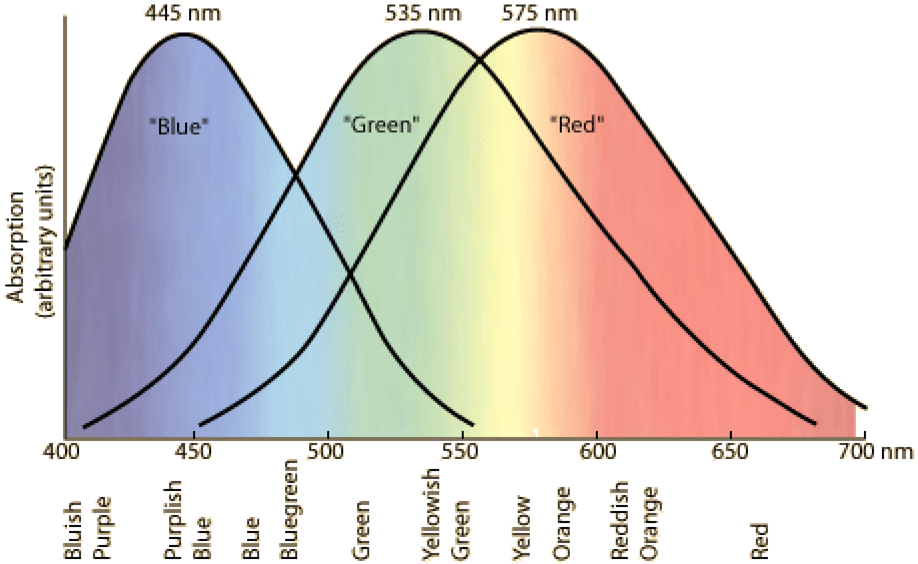
Detection of the Blind Spot

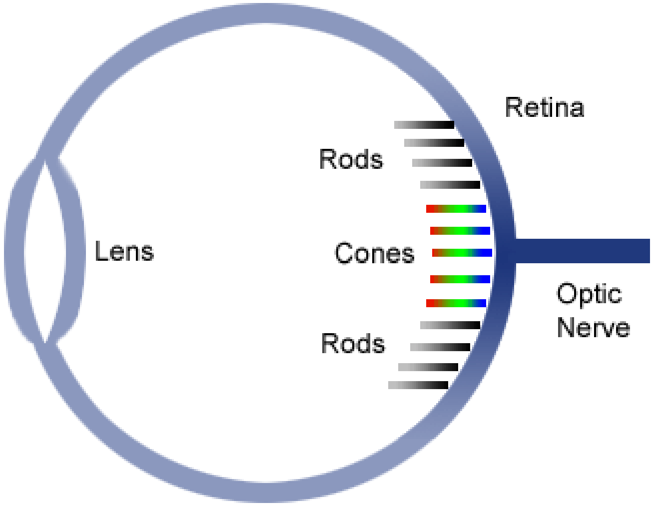
Because no neuroepithelial cell is present in the portion of the retina where the optic nerve penetrates, this portion cannot sense light and is called the blind spot. The blind spot is located at an angle of 15° from the line of sight (optical axis) and is about 5° wide. This can be confirmed readily by a visual experiment using the above figure. If the observer fixates his/her right eye on the cross while closing his/her left eye and adjusting the distance between the eye and the cross to about 20 cm, the solid circle disappears from sight. This occurs because the solid circle is imaged on the blind spot.

N. Ohta and A. R. Robertson, "Colorimetry", J. Wiley & Sons, 2005

The Color-Sensitive Cones

In 1965 came experimental confirmation that there are three types of color-sensitive cones in the retina of the human eye, corresponding roughly to red, green, and blue sensitive detectors. The "green" and "red" cones are mostly packed into the *fovea centralis*. By population, about **64%** of the cones are red-sensitive, about **32%** green sensitive, and about **2%** are blue sensitive. The "blue" cones have the highest sensitivity and are mostly found outside the fovea. The shapes of the curves are obtained by measurement of the absorption by the cones, but the relative heights for the three types are set equal for lack of detailed data.





E. F. Schubert, Light Emitting Diodes, 2nd Ed., Cambridge University Press, 2006

The Color-Sensitivity of Cones and Rods of Human Eye

Number of Cones: 6-7 millions, 12:6:1 to 40:20:1/R-G-B

Number of Rods: 120 millions

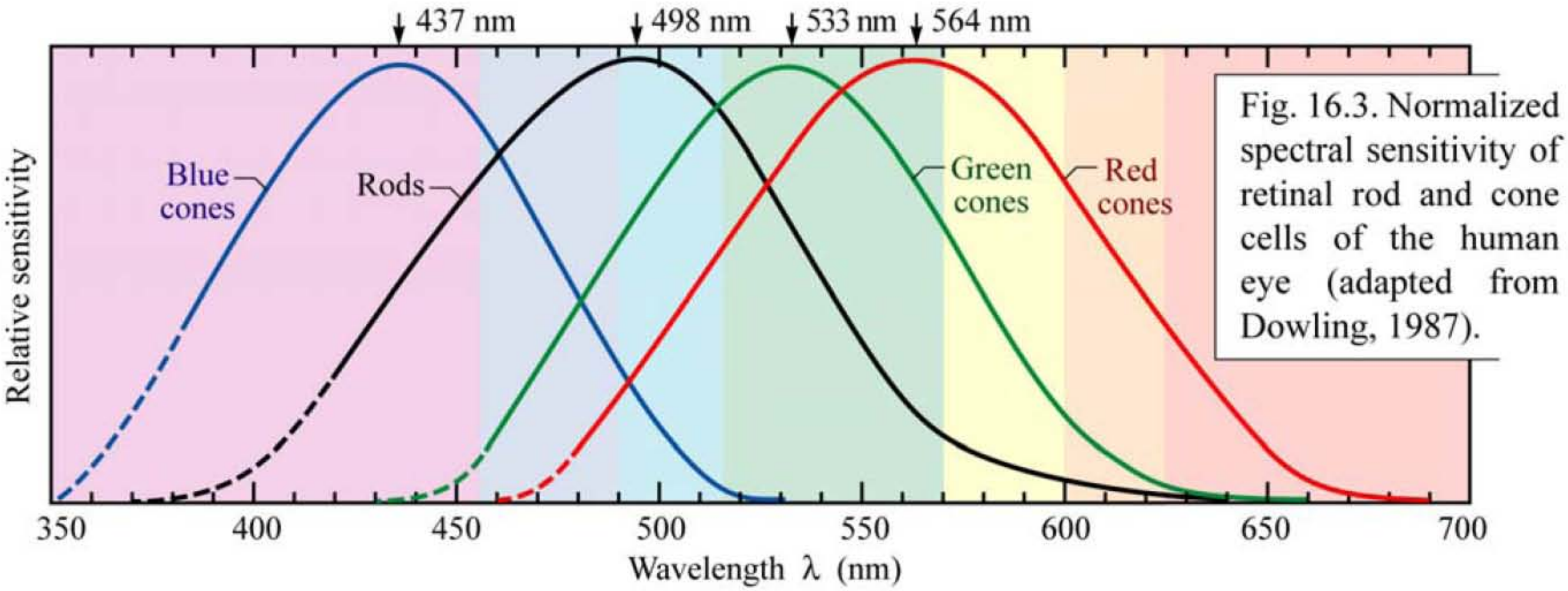
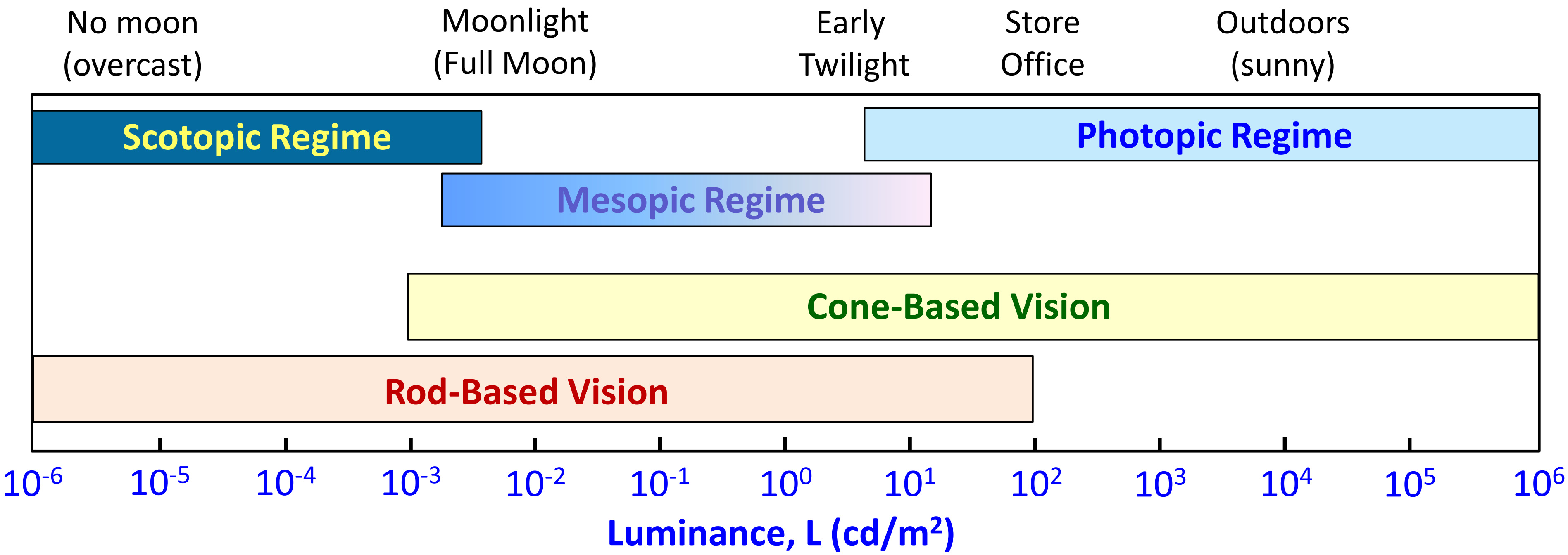


Fig. 16.3. Normalized spectral sensitivity of retinal rod and cone cells of the human eye (adapted from Dowling, 1987).

E. F. Schubert, Light Emitting Diodes, 2nd Ed., Cambridge University Press, 2006

Vision Regimes



Scotopic Vision Regime < 0.003 cd/m^2

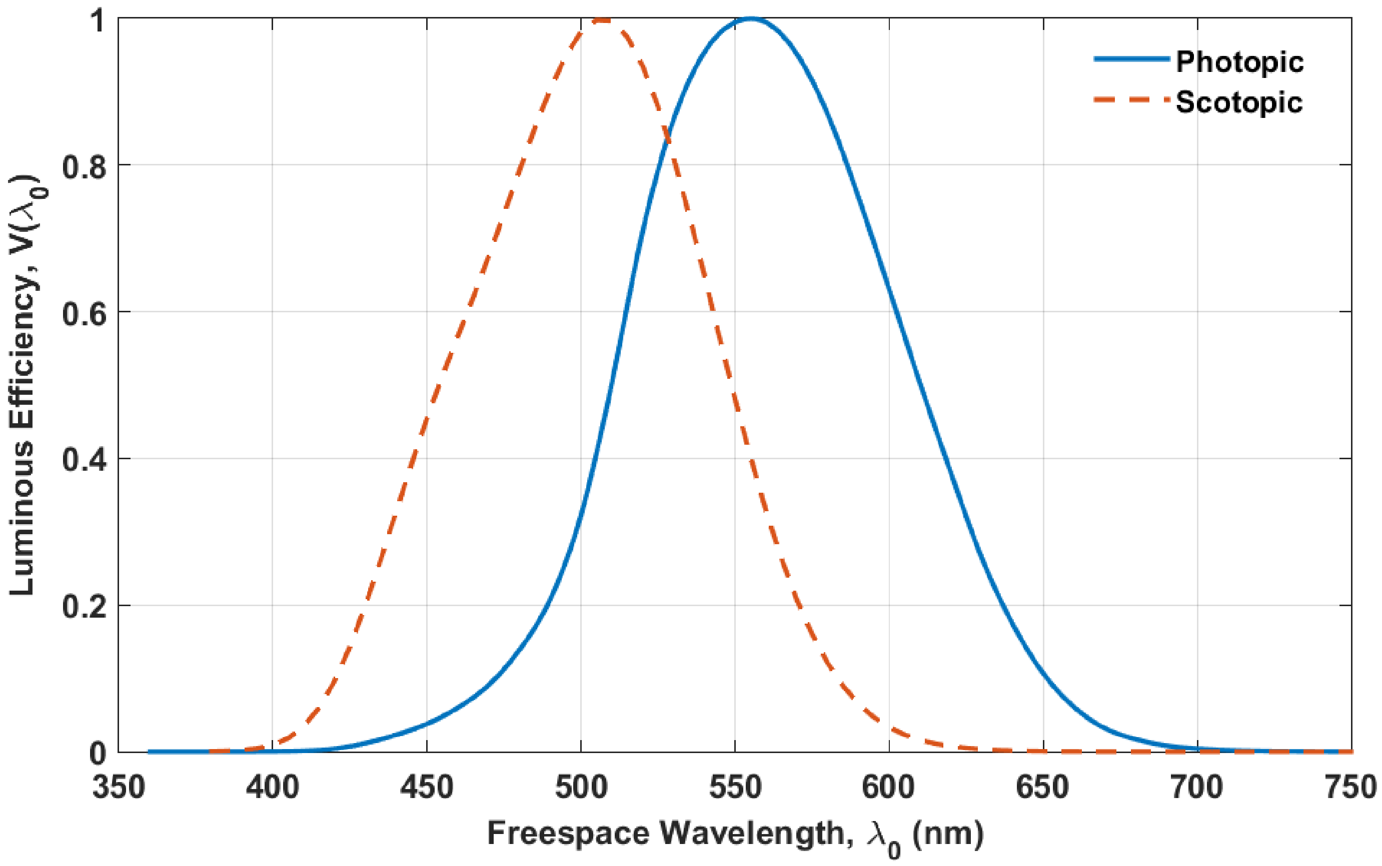
0.003 cd/m^2 < Mesopic Vision Regime < 3 cd/m^2

3 cd/m^2 < Photopic Vision Regime

E. F. Schubert, Light Emitting Diodes, 2nd Ed., Cambridge University Press, 2006

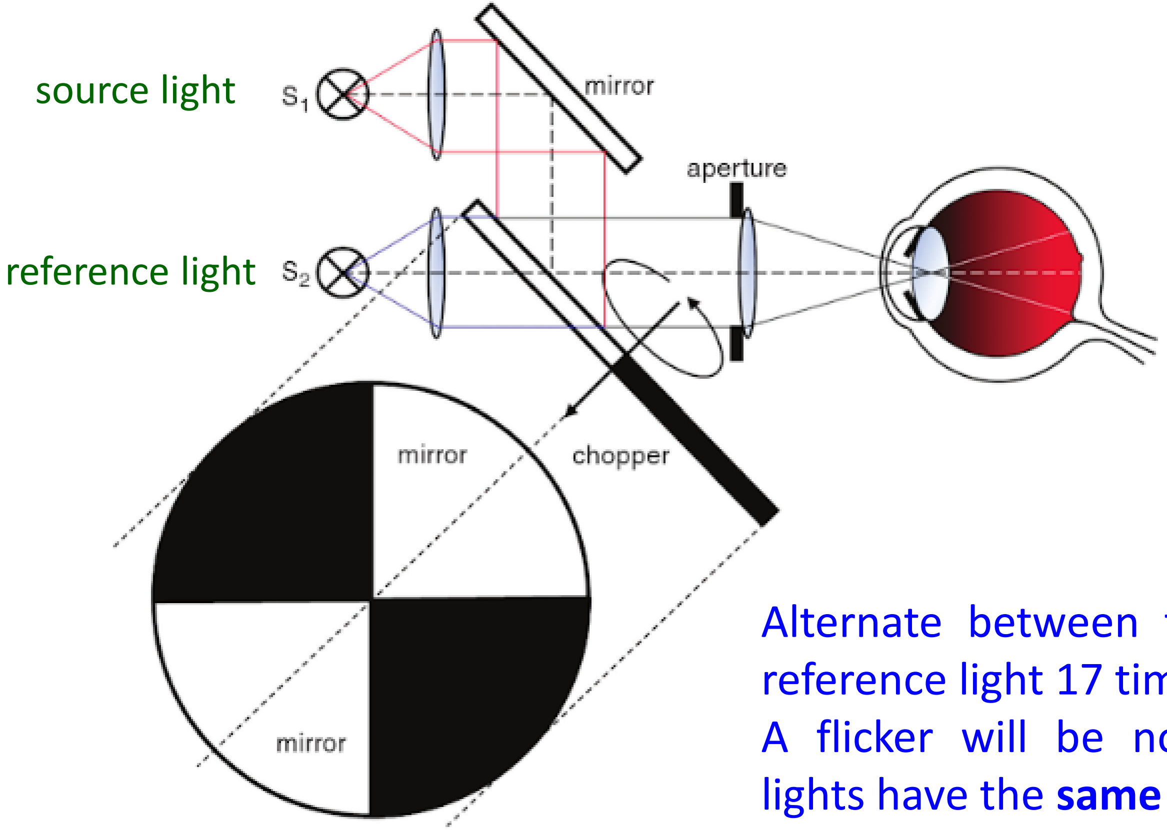
Luminous Efficiency Curve of Human Eye

CIE Standard Curve - 1931



Data from: http://www.cvrl.org

Flickering Photometry Measurements



Viewer gradually increases source radiant power

Alternate between the source light and the reference light 17 times per second (17 Hz).
A flicker will be noticeable unless the two lights have the **same perceived "brightness".**

C. Oleari, "Standard Colorimetry", Wiley & Sons, 2016

History of Photometric Units

- Photograph shows **plumber's candle**
- A plumber's candle emits a **luminous intensity** of 1 candela (cd). The cd is historical origin of all photometric units.

- First definition (now obsolete): The luminous intensity of a standardized candle is 1 cd.

- Second definition (now obsolete): 1 cm^2 of platinum (Pt) at 2042°K (temperature of solidification) has a luminous intensity of 20.17 cd.

- Third definition (current): A monochromatic light source emitting an optical power of (1/683) Watt at $\lambda_0 = 555$ nm into the solid angle of 1 steradian (sr) has a luminous intensity of 1 cd.

E. F. Schubert, Light Emitting Diodes, 2nd Ed., Cambridge University Press, 2006

Luminous and Radiant Powers

Luminous and Radiant Power Relation

$$\Phi_v = K\Phi_e$$

Efficacies

Photopic Vision $K(\lambda_0) = K_m V(\lambda_0),$

Scotopic Vision $K'(\lambda_0) = K'_m V'(\lambda_0),$

$$\boxed{\begin{aligned} \Phi_v &= 683(lm/W)\int_{\lambda_0} \Phi_e(\lambda_0)V(\lambda_0)d\lambda_0, \qquad \text{Photopic Vision,} \\ \Phi_v &= 1700(lm/W)\int_{\lambda_0} \Phi_e(\lambda_0)V'(\lambda_0)d\lambda_0, \qquad \text{Scotopic Vision,} \end{aligned}}$$

Luminous Flux and Efficiency

Luminous flux (Unit: lm)

$$\Phi_{\mathrm{lum}} = 683 \frac{\mathrm{lm}}{\mathrm{W}} \int_{\lambda} V(\lambda)\, P(\lambda)\, \mathrm{d}\lambda$$

Luminous efficacy of radiation (Unit: lm / W)

$$\text{Luminous efficacy} = \Phi_{\mathrm{lum}} / P = \left(683 \frac{\mathrm{lm}}{\mathrm{W}} \int_{\lambda} V(\lambda)\, P(\lambda)\, \mathrm{d}\lambda \right) \Big/ \left(\int_{\lambda} P(\lambda)\, \mathrm{d}\lambda \right)$$

Luminous efficacy of the source (Unit: lm / W)

$$\text{Luminous efficiency} = \Phi_{\mathrm{lum}} / (IV)$$

Caution: Luminous "efficacy" and "efficiency" is being used in literature

E. F. Schubert, Light Emitting Diodes, 2nd Ed., Cambridge University Press, 2006

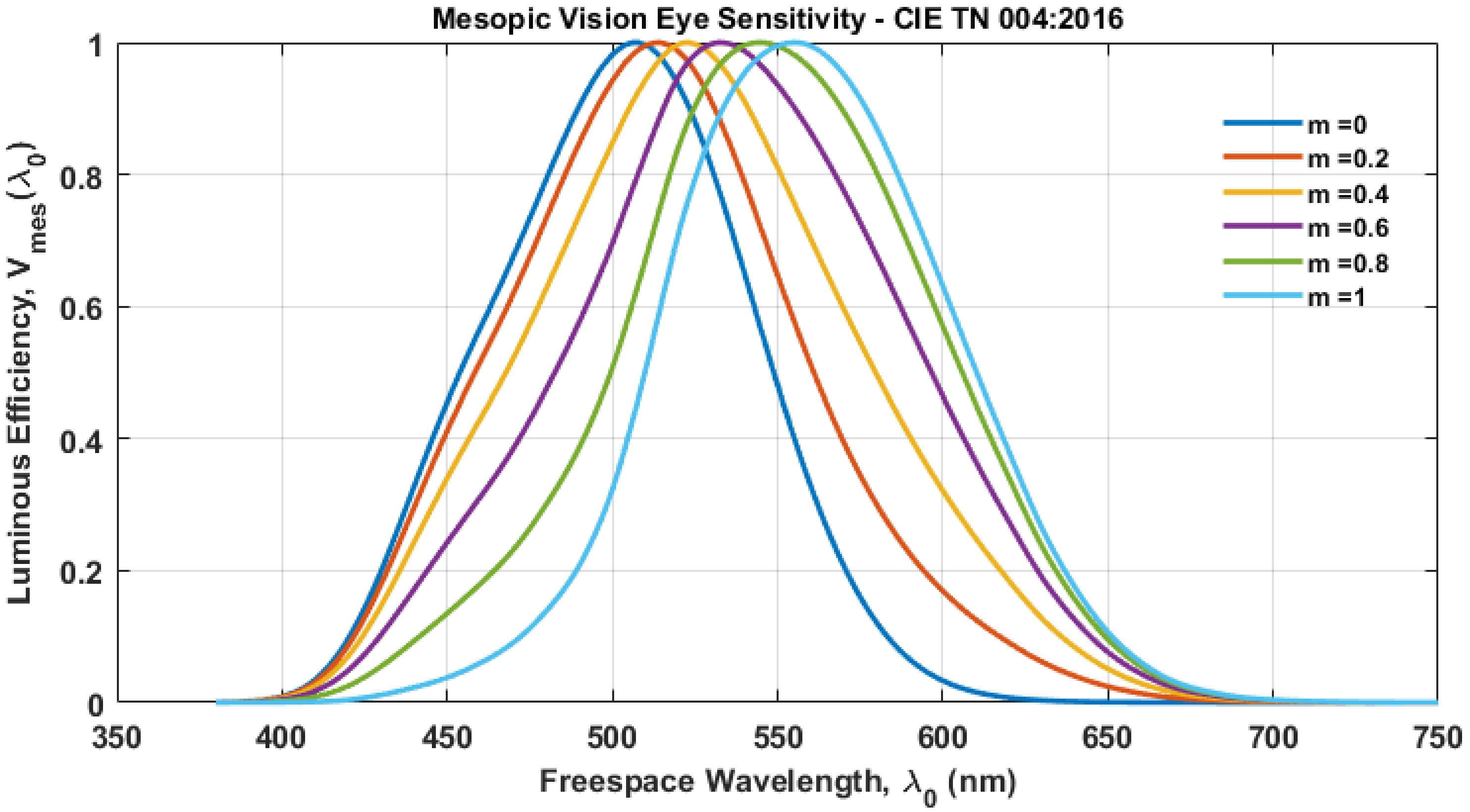
Category	**Type**	**Overall luminous efficacy (lm/W)**	**Overall luminous efficiency**
Combustion	candle	0.3	0.04%
	gas mantle	1–2	0.15–0.3%
Incandescent	100–200 W tungsten incandescent (220 V)	13.8–15.2	2.0–2.2%
	100–200–500 W tungsten glass halogen (220 V)	16.7 – 17.6 –19.8	2.4–2.6–2.9%
	5–40–100 W tungsten incandescent (120 V)	5–12.6–17.5	0.7–1.8–2.6%
	2.6 W tungsten glass halogen (5.2 V)	19.2	2.8%
	tungsten quartz halogen (12–24 V)	24	3.5%
	photographic and projection lamps	35	5.1%
Light-emitting diode	white LED (raw, without power supply)	4.5–150	0.66–22.0%
	4.1 W LED screw base lamp (120 V)	58.5–82.9	8.6–12.1%
	6.9 W LED screw base lamp (120 V)	55.1–81.9	8.1–12.0%
	7 W LED PAR20 (120 V)	28.6	4.2%
	8.7 W LED screw base lamp (120 V)	69.0–93.1	10.1–13.6%
Arc lamp	xenon arc lamp	30–50	4.4–7.3%
	mercury-xenon arc lamp	50–55	7.3–8.0%
Fluorescent	T12 tube with magnetic ballast	60	9%
	9–32 W compact fluorescent	46–75	8–11.45%
	T8 tube with electronic ballast	80–100	12–15%
	T5 tube	70–104.2	10–15.63%
Gas discharge	1400 W sulfur lamp	100	15%
	metal halide lamp	65–115	9.5–17%
	high pressure sodium lamp	85–150	12–22%
	low pressure sodium lamp	100–200	15–29%
Ideal sources	Truncated 5800 K blackbody	251	37%
	Green light at 555 nm (maximum possible LER)	**683**	**100%**

https://en.wikipedia.org/wiki/Luminous_efficacy

Mesopic Vision

$$M(m)V_{mes,m}(\lambda_0) = mV(\lambda_0) + (1-m)V'(\lambda_0)$$

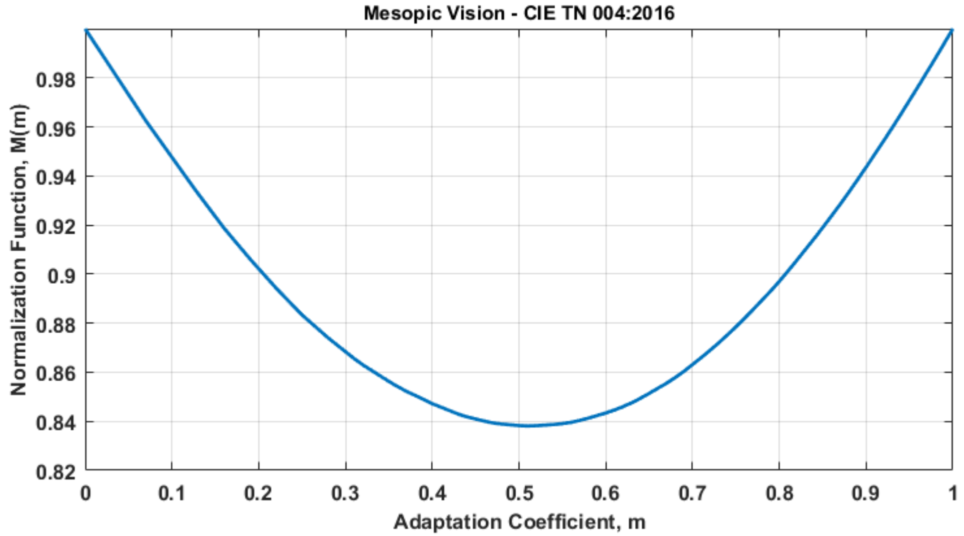
m = adaptation coefficient

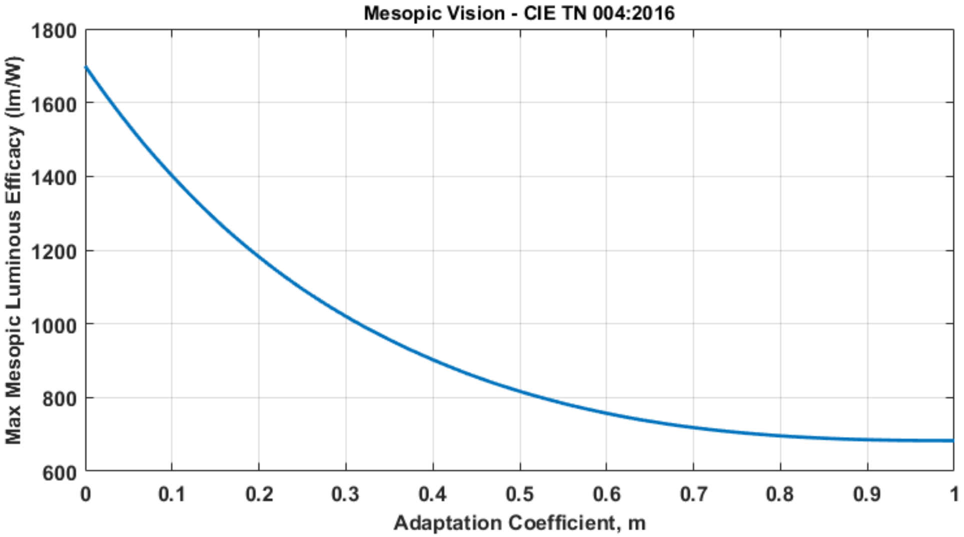


CIE, "The use of terms and units in photometry implementation of the CIE system for mesopic photometry," *Commission Internationale de l'Eclairage Proceedings (CIE)*, 2016. Technical Report 004:2016.

Mesopic Vision

$$M(m)V_{mes,m}(\lambda_0) = mV(\lambda_0) + (1-m)V'(\lambda_0)$$

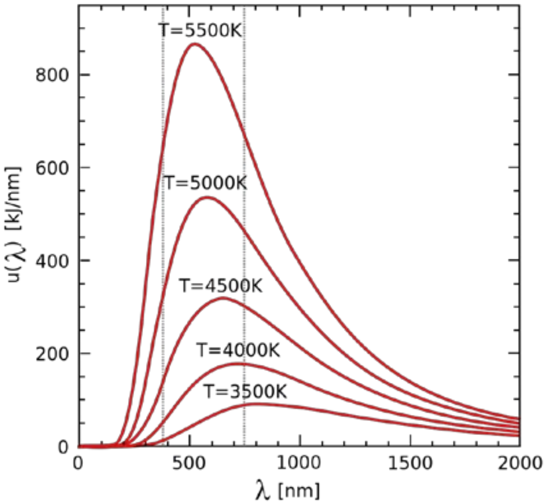




$$K_{mes,m} = \frac{683}{V_{mes,m}(\lambda_0 = \lambda_{0,p})}, \qquad \mathrm{lm/W},$$

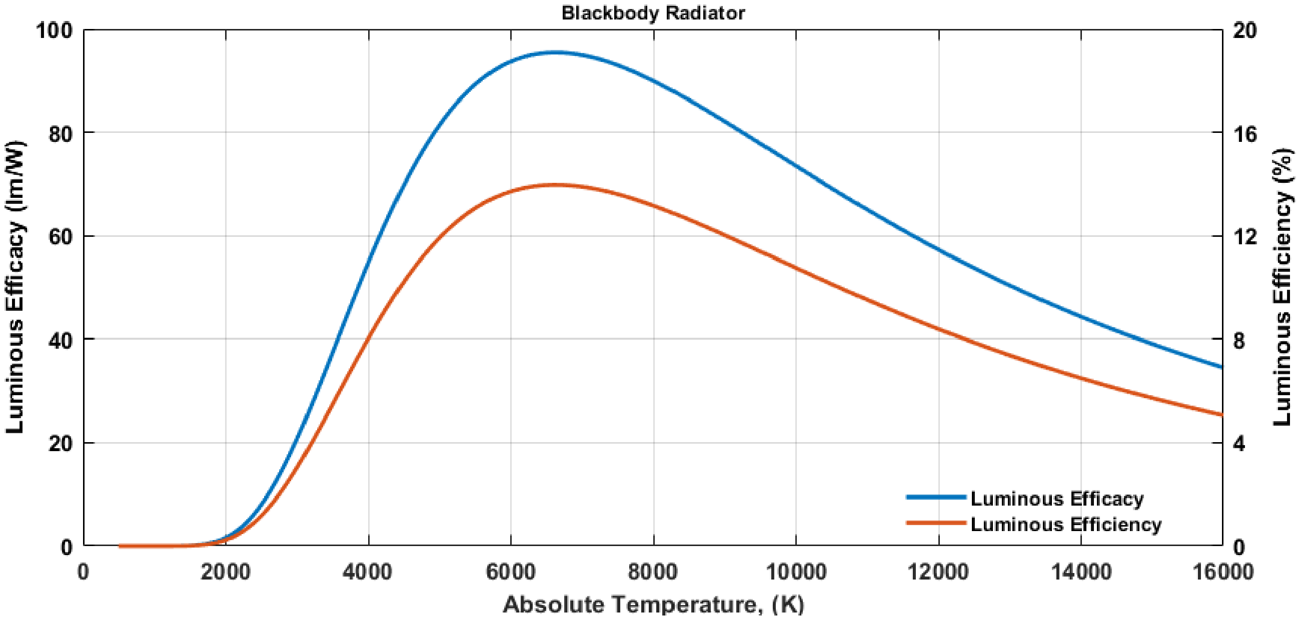
$$\lambda_{0,p} = 555\,\mathrm{nm}$$

Luminous Efficacy/Efficiency of a Blackbody



$$M_{\lambda_0}(\lambda_0) = \frac{2\pi n^2 c}{\lambda_0^4} \frac{hc/\lambda_0}{\exp(hc/\lambda_0 k_B T) - 1}$$

$$\frac{\Phi_v}{\Phi_e} = \frac{683 \int_0^\infty M_{\lambda_0}(\lambda_0) V(\lambda_0) d\lambda_0}{\int_0^\infty M_{\lambda_0}(\lambda_0) d\lambda_0} \quad \left(\frac{lm}{W}\right)$$



ILLUMINATION VALUES

(from I.E.S. Lighting Handbook)

<table>
<tr><th colspan="2">TYPICAL VALUES OF ILLUMINANCE</th></tr>
<tr><th>ILLUMINANCE (footcandles)</th><th>ILLUMINATION SITUATION</th></tr>
<tr><td>0.02</td><td>Full moonlight</td></tr>
<tr><td>50</td><td>Artificial Illuminated Interiors</td></tr>
<tr><td>100</td><td>Sunlight (dull day)</td></tr>
<tr><td>5000-10000</td><td>Sunlight (bright day)</td></tr>
<tr><th colspan="2">RECOMMENDED VALUES OF ILLUMINANCE</th></tr>
<tr><th>ILLUMINANCE (footcandles)</th><th>ILLUMINATION SITUATION</th></tr>
<tr><td>5-10</td><td>Halls, aisles, auto parking areas</td></tr>
<tr><td>10-20</td><td>Stairways, storage rooms, dining rooms, bedrooms, auditoriums</td></tr>
<tr><td>20-50</td><td>Rough assembly, materials wrapping, average workshop, reading usual prints</td></tr>
<tr><td>50-100</td><td>Medium assembly work, kitchens, reading fine print, sewing, writing, workbench, barber shops</td></tr>
<tr><td>100-200</td><td>Drafting rooms, severe visual work, extra fine grading and sorting, difficult inspection</td></tr>
<tr><td>200-500</td><td>Fine bench and machine work, very difficult inspection</td></tr>
</table>

$$1 \text{ footcandle} = 1 \text{ lumen/ft}^2 = 10.7639 \text{ lumen/m}^2 = 10.7639 \text{ lux}$$

Other Luminance (non SI) Units

Name	**Symbol**	**Conversion to SI**
Apostilb	asb	1 asb = $1/\pi$ cd/m^2
Blondel	blondel	1 blondel = $1/\pi$ cd/m^2
Candela per square foot	cd/ft^2	1 cd/ft^2 = 10.764 cd/m^2
Candela per square inch	cd/in^2	1 cd/in^2 = 1550 cd/m^2
Footlambert	fL	1 fL = 3.426 cd/m^2
Lambert	L	1 L = $10^4/\pi$ cd/m^2
Nit	nit	1 nit = 1 cd/m^2
Skot	skot	1 skot = $10^{-3}/\pi$ cd/m^2
Stilb	sb	1 sb = 10000 cd/m^2

Colorimetry Fundamentals

Colorimetry is the field of science and technology that deals with the assessment, quantification and measurement of color as it is perceived by the human eye.

Trichromatic Theory of Color

Every visible color can be reproduced by appropriate mixing of three basic colors (**red, R**), (**green,G**) and (**blue,B**)

CIE Color Matching Functions

$\lambda_R = 700.0\,\mathrm{nm}$ (**red, R**)

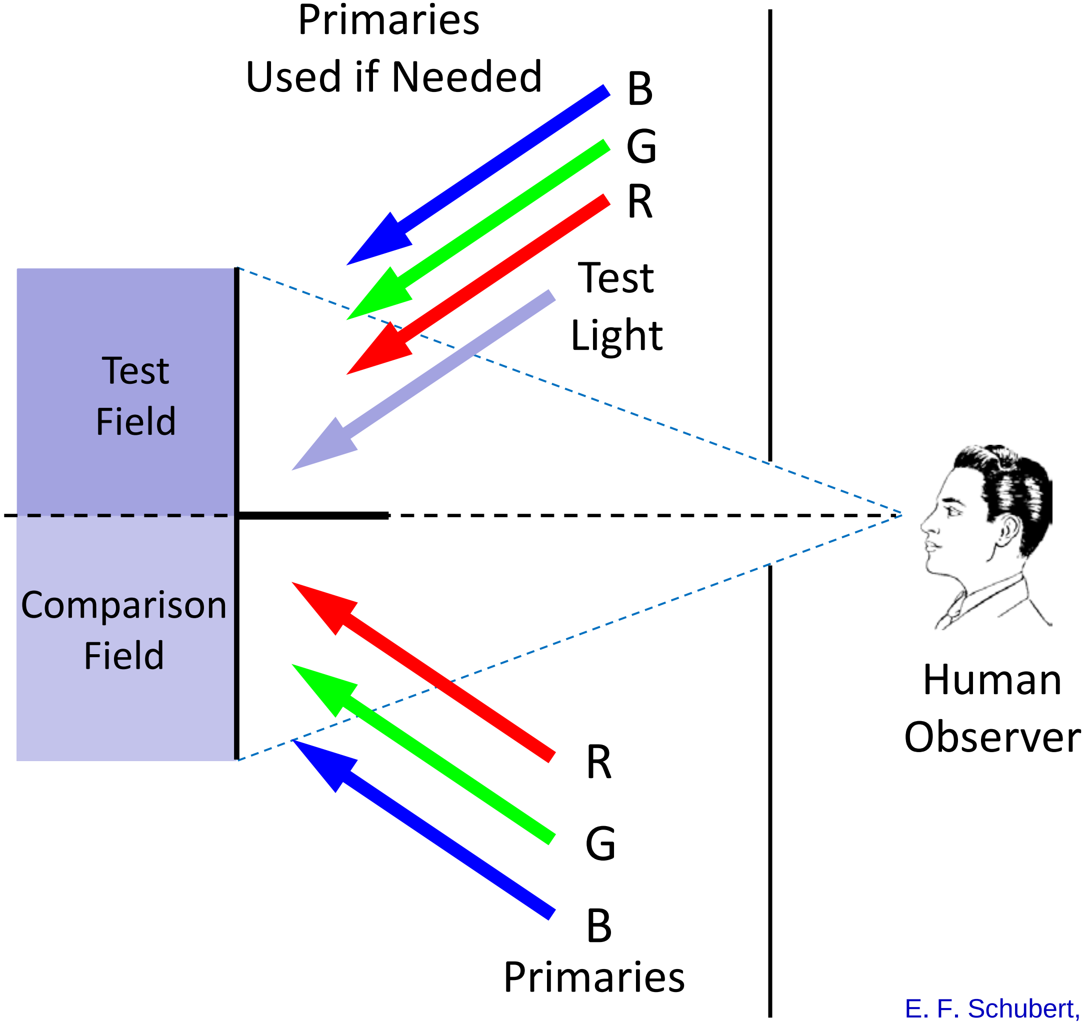
$\lambda_G = 546.1\,\mathrm{nm}$ (**green,G**)

$\lambda_B = 435.8\,\mathrm{nm}$ (**blue,B**)

Color Space

$$[F(\lambda_0)] = \bar{r}(\lambda_0)[R] + \bar{g}(\lambda_0)[G] + \bar{b}(\lambda_0)[B]$$

Color Matching Functions Experiment



Primaries
Used if Needed

B
G
R

Test
Light

Test
Field

Comparison
Field

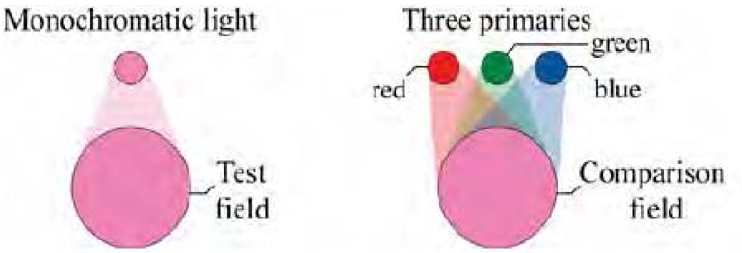
R
G
B
Primaries

Human
Observer

Primaries Used (1931)

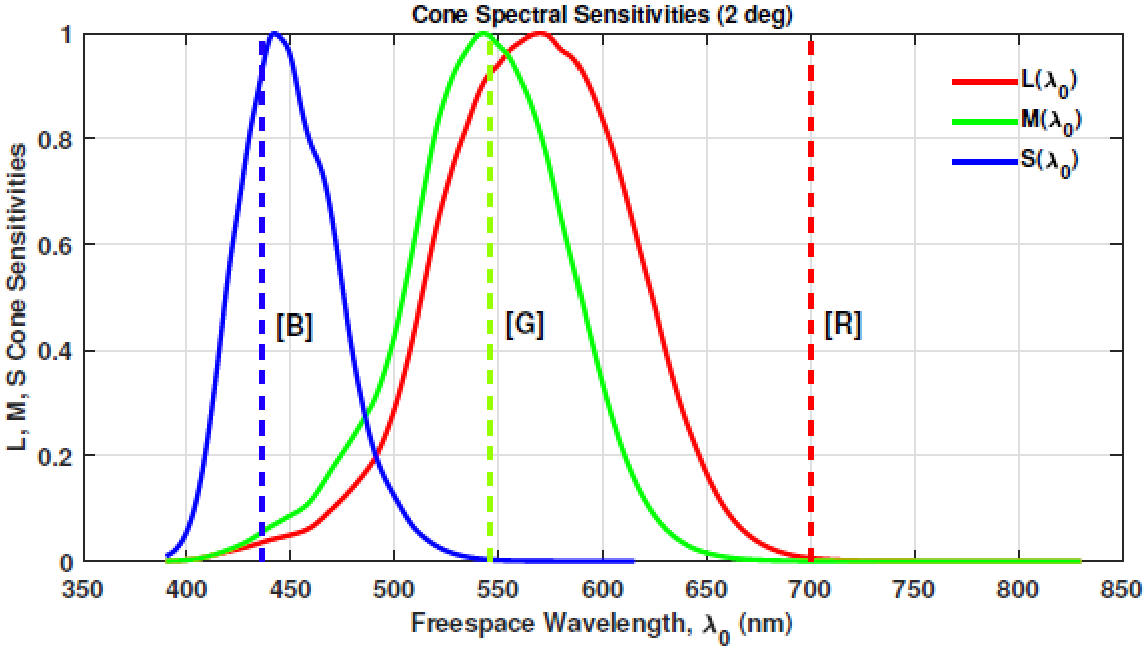
Red: 700nm
Green: 546.1nm
Blue: 435.8nm
(Mercury discharge lamp)

$$[F(\lambda_0)] = \bar{r}(\lambda_0)[R] + \bar{g}(\lambda_0)[G] + \bar{b}(\lambda_0)[B]$$



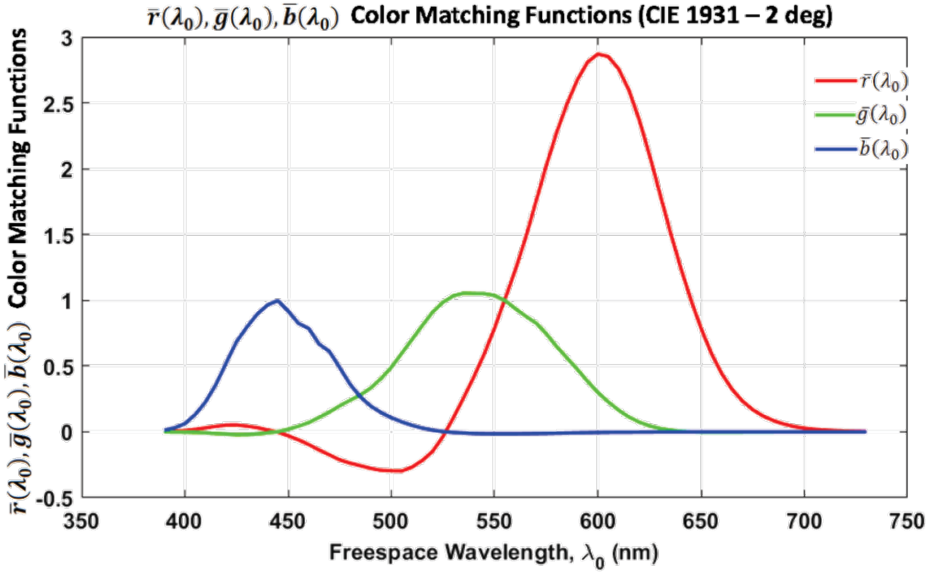
E. F. Schubert, Light Emitting Diodes, 2nd Ed., Cambridge University Press, 2006

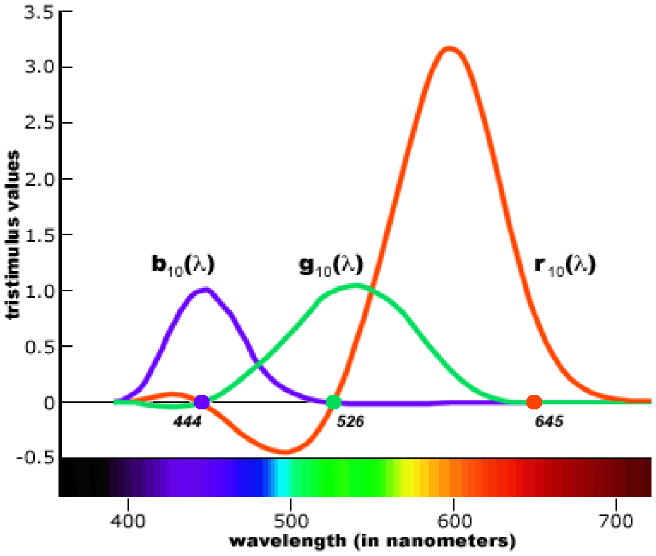
Cone Spectral Sensitivities



Color Matching Functions

$$[F(\lambda_0)] = \bar{r}(\lambda_0)[R] + \bar{g}(\lambda_0)[G] + \bar{b}(\lambda_0)[B]$$





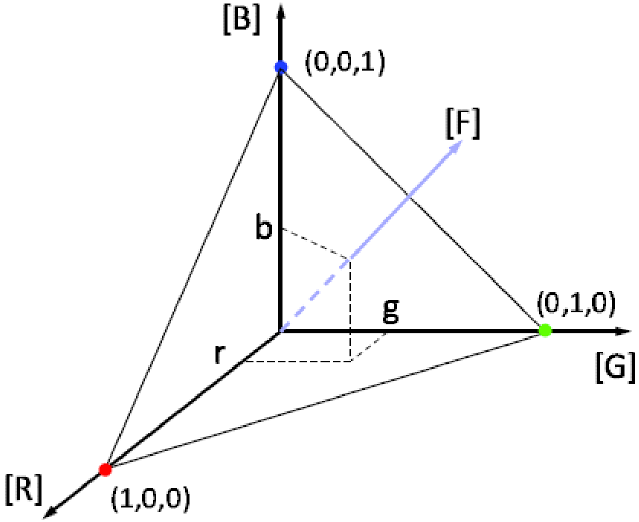
RGB color matching functions

Stiles-Burch 10° color matching functions averaged across 37 observers (adapted from Wyszecki & Stiles, 1982)

RGB Tristimulus Values and Chromaticity Coordinates

Tristimulus Values

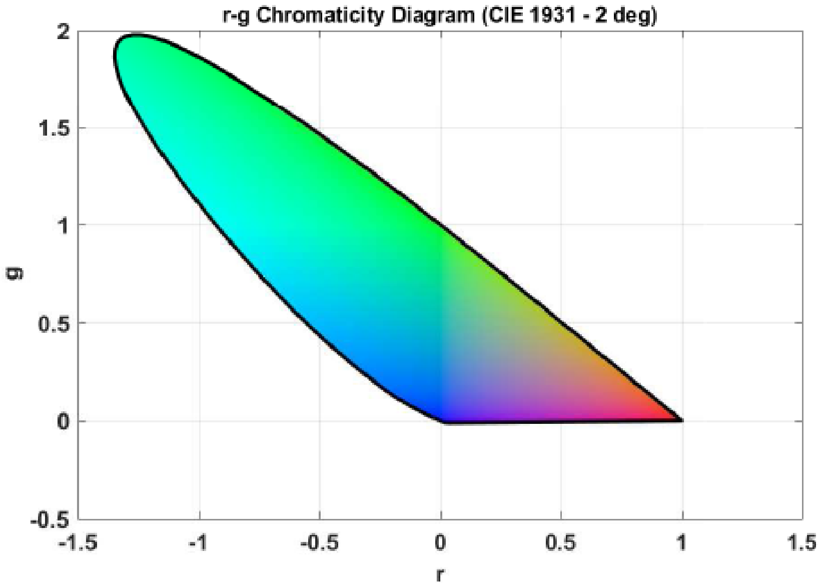
$$R = k \int_{\lambda_0} \bar{r}(\lambda_0) P(\lambda_0) d\lambda_0,$$
$$G = k \int_{\lambda_0} \bar{g}(\lambda_0) P(\lambda_0) d\lambda_0,$$
$$B = k \int_{\lambda_0} \bar{b}(\lambda_0) P(\lambda_0) d\lambda_0,$$



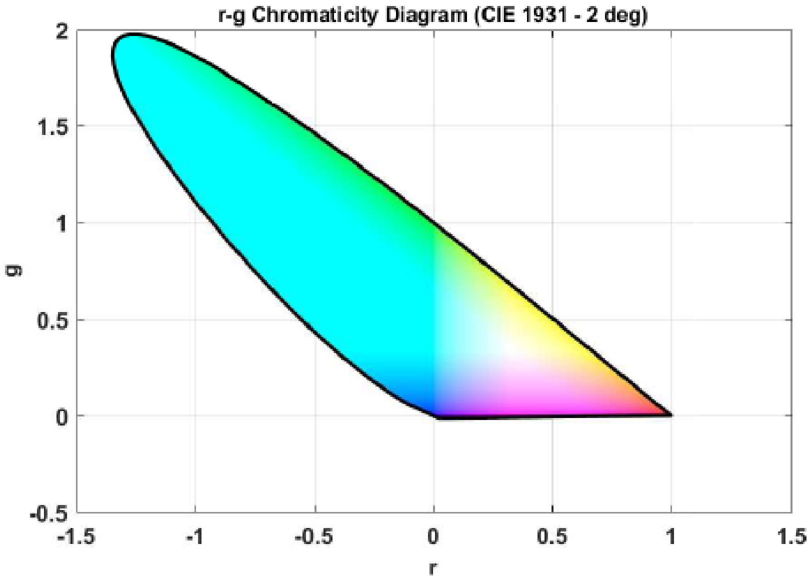
Chromaticity Coordinates

$$r = \frac{R}{R+G+B},$$
$$g = \frac{G}{R+G+B},$$
$$b = \frac{B}{R+G+B},$$

CIE 1931 – r, g Chromaticity Diagram



R = G = B = 1/3



R = G = B = 1

XYZ Color Matching Functions

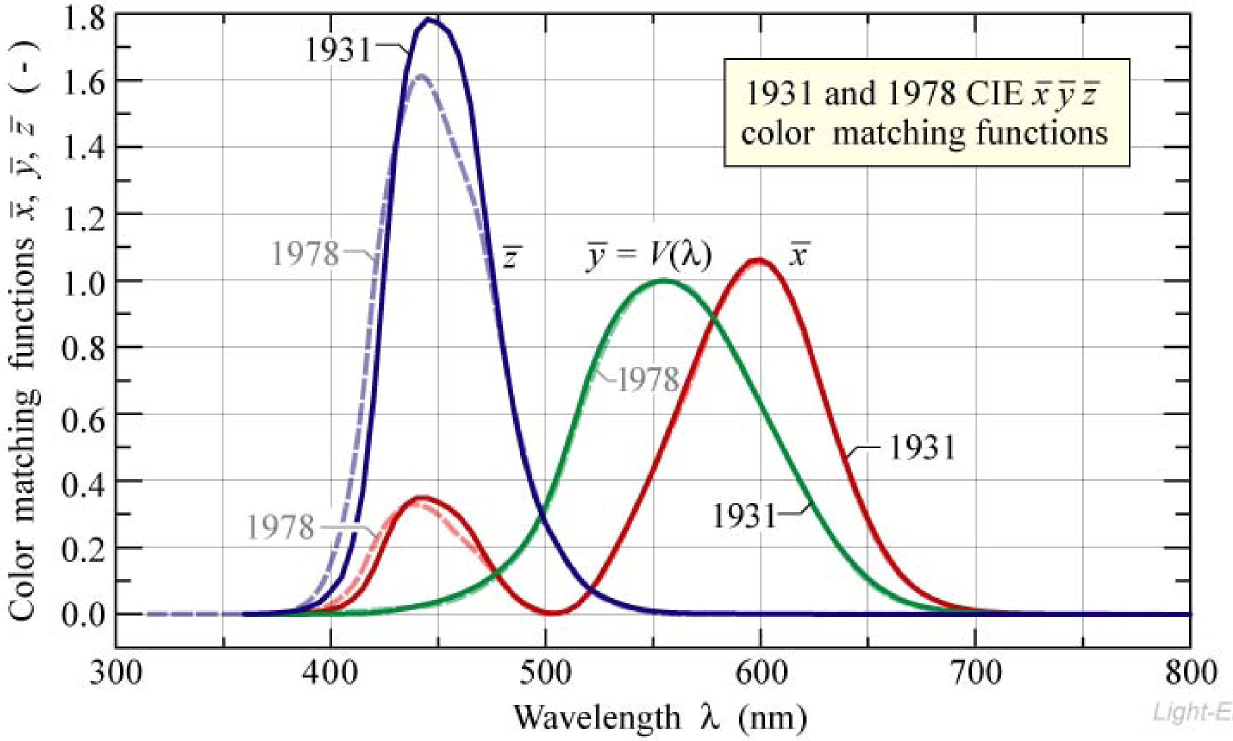
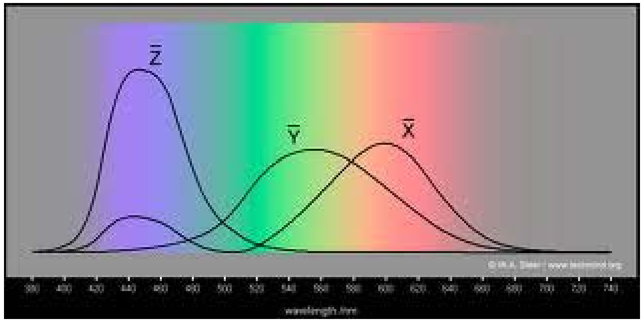


Fig. 17.1. CIE (1931) and CIE (1978) $\bar{x}\bar{y}\bar{z}$ color matching functions (CMFs). The $\bar{y}$ CMF is identical to the eye sensitivity function $V(\lambda)$. Note that the CIE 1931 CMF is the currently valid official standard.

E. F. Schubert
Light-Emitting Diodes (Cambridge Univ. Press)
www.LightEmittingDiodes.org



XYZ Color Matching Functions and Chromaticity Coordinates

X, Y, Z ***Tristimulus*** Values

$$X = k \int_{\lambda_0} \bar{x}(\lambda_0) P(\lambda_0) d\lambda_0,$$
$$Y = k \int_{\lambda_0} \bar{y}(\lambda_0) P(\lambda_0) d\lambda_0,$$
$$Z = k \int_{\lambda_0} \bar{z}(\lambda_0) P(\lambda_0) d\lambda_0,$$

X, Y, Z ***Chromaticity*** Coordinates

$$x = \frac{X}{X+Y+Z},$$
$$y = \frac{Y}{X+Y+Z},$$
$$z = \frac{Z}{X+Y+Z},$$

Chromaticity Coordinates x and y are only needed since:

$$x + y + z = 1$$

Transformation between XYZ and RGB

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} 2.7689 & 1.7517 & 1.1302 \\ 1.0000 & 4.5907 & 0.0601 \\ 0.0000 & 0.0565 & 5.5943 \end{pmatrix} \begin{pmatrix} R \\ G \\ B \end{pmatrix}$$

$$x = (0.49000r + 0.31000g + 0.20000b) / (0.66697r + 1.13240g + 1.20063b)$$
$$y = (0.17697r + 0.81240g + 0.01063b) / (0.66697r + 1.13240g + 1.20063b)$$
$$z = (0.00000r + 0.01000g + 0.99000b) / (0.66697r + 1.13240g + 1.20063b)$$

N. Ohta and A. R. Robertson, *Colorimetry: Fundamentals and Applications*, J. Wiley & Sons, 2005

CIE 1931 – x,y Chromaticity Diagram

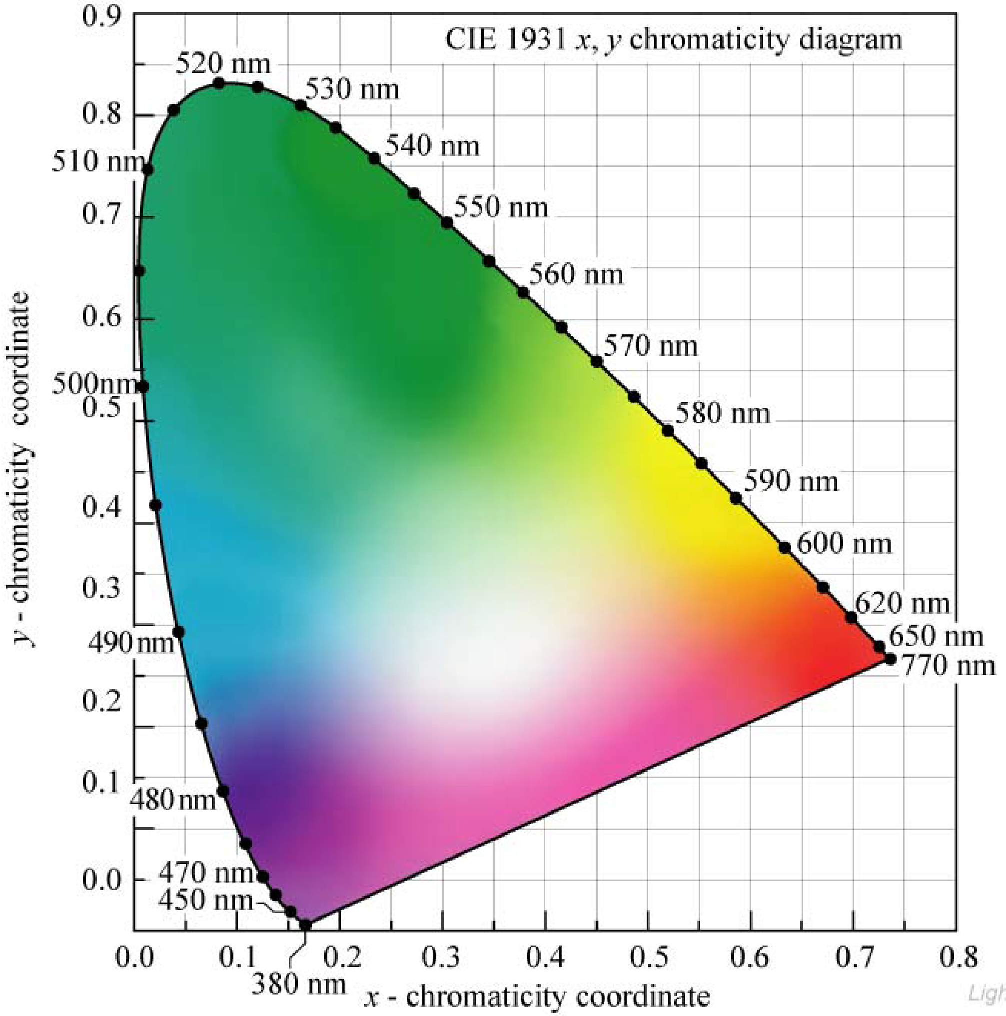


Fig. 17.2. CIE 1931 (*x*, *y*) chromaticity diagram. Monochromatic colors are located on the perimeter and white light is located in the center of the diagram.

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Light-Emitting Diodes (Cambridge Univ. Press)
www.LightEmittingDiodes.org

CIE 1931 – x,y Chromaticity Diagram

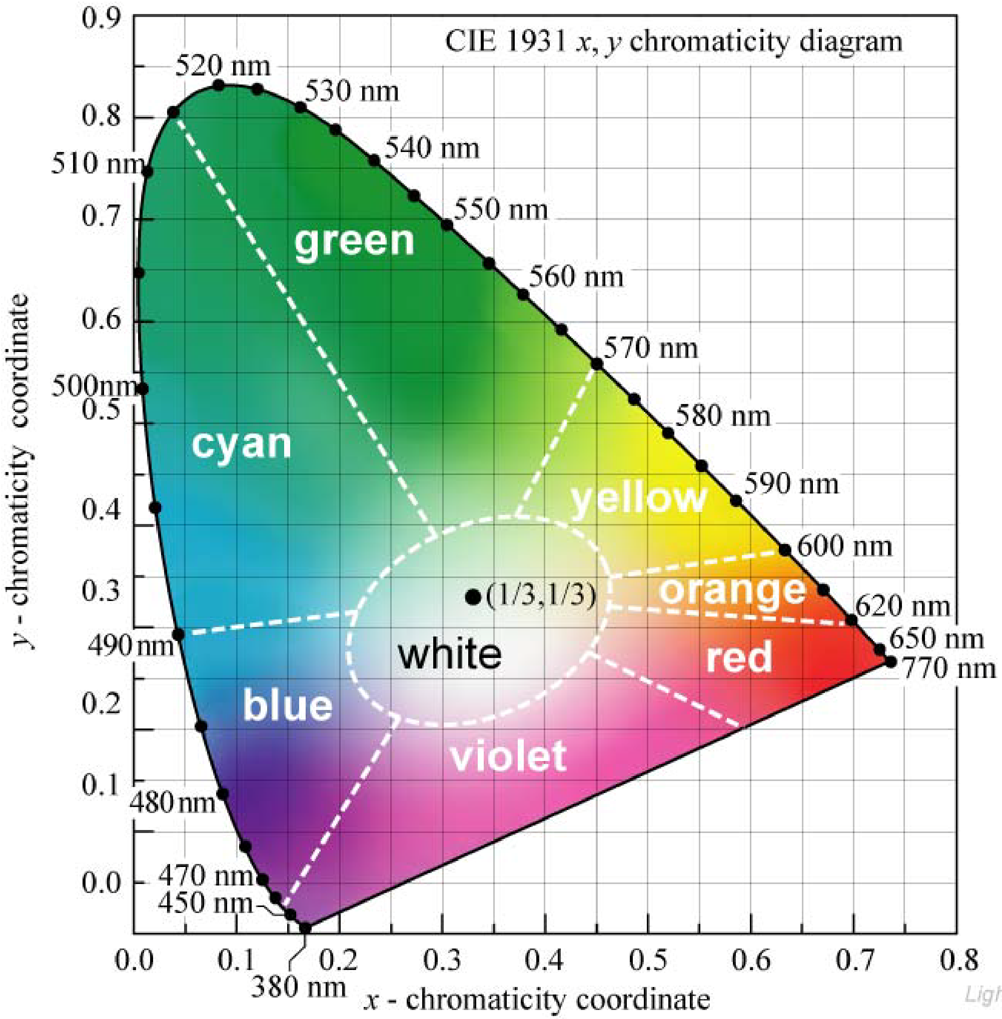
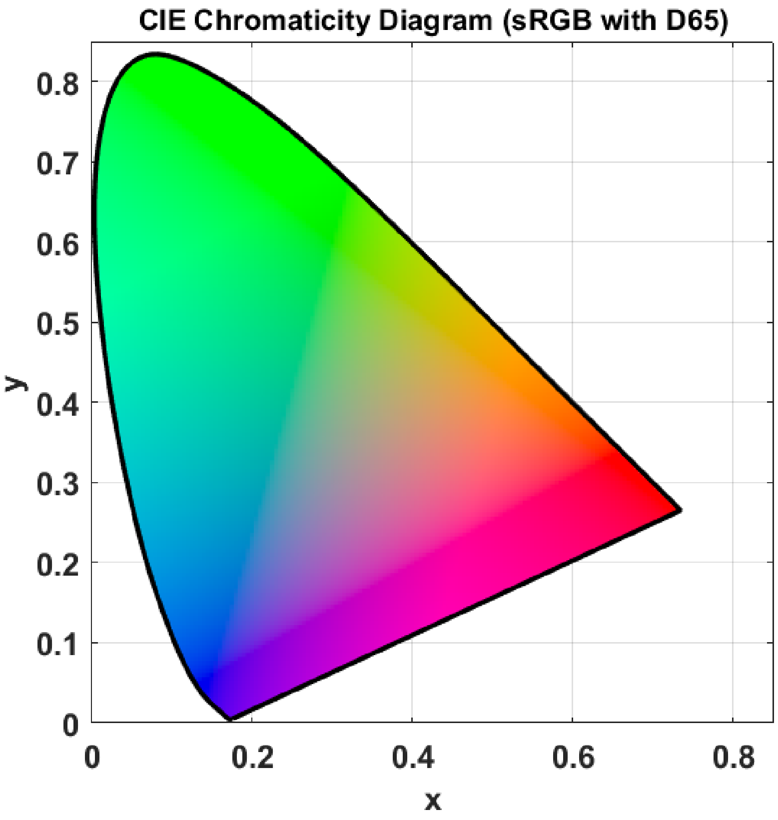


Fig. 17.4. CIE 1931 (*x*, *y*) chromaticity diagram. Monochromatic colors are located on the perimeter. Color saturation decreases towards the center of the diagram. White light is located in the center. Also shown are the regions of distinct colors. The equal-energy point is located at the center and has the coordinates (*x*, *y*) = (1/3, 1/3).

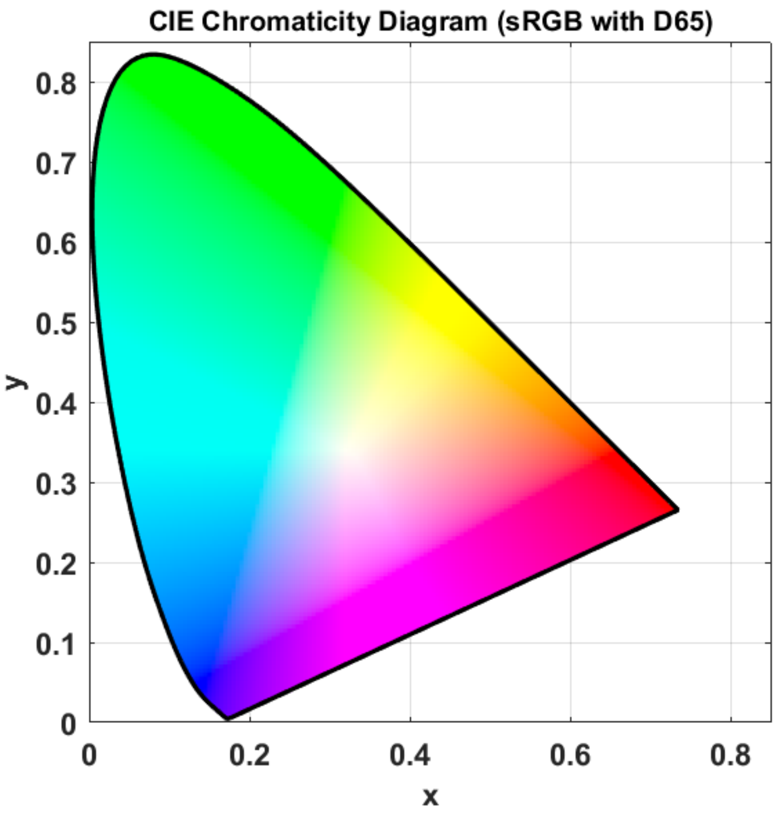
E. F. Schubert
Light-Emitting Diodes (Cambridge Univ. Press)
www.LightEmittingDiodes.org

CIE 1931 – x,y Chromaticity Diagram

(created with MatLab)

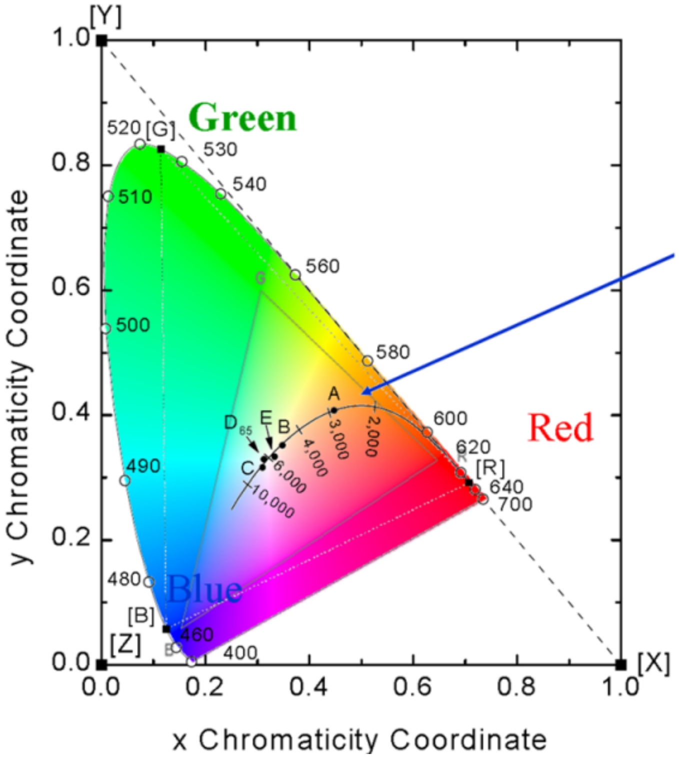


X = x, Y = y, Z = z



X = x/w, Y = y/w, Z = z/w
w = max{x,y,z}

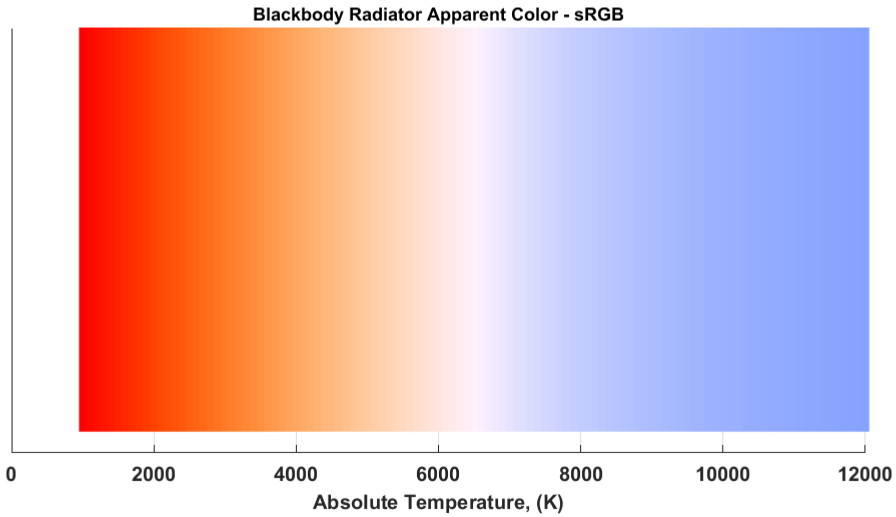
CIE 1931 – x,y Chromaticity Diagram

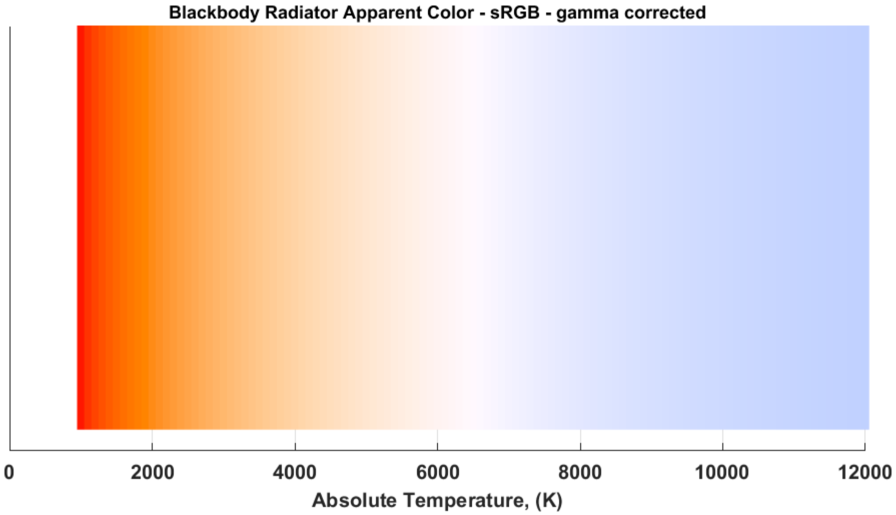


Planckian locus (black body radiation)

- **A (tungsten at 2856 K)**
- **B (direct sunlight, ≈ 4870 K)**
- **C (overcast sunlight ≈ 6770 K**
- **D65 (daylight, ≈ 6504 K)**
- **Point E marks equal energy.**

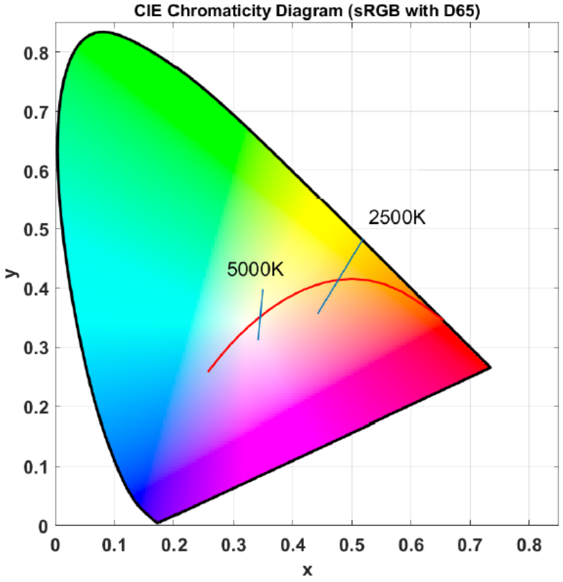
E. F. Schubert, Light Emitting Diodes, 2nd Ed., Cambridge University Press, 2006

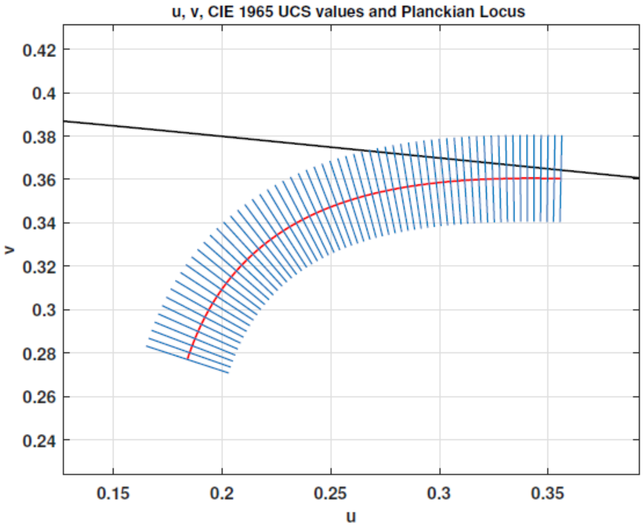

Blackbody Radiator Apparent Color - sRGB

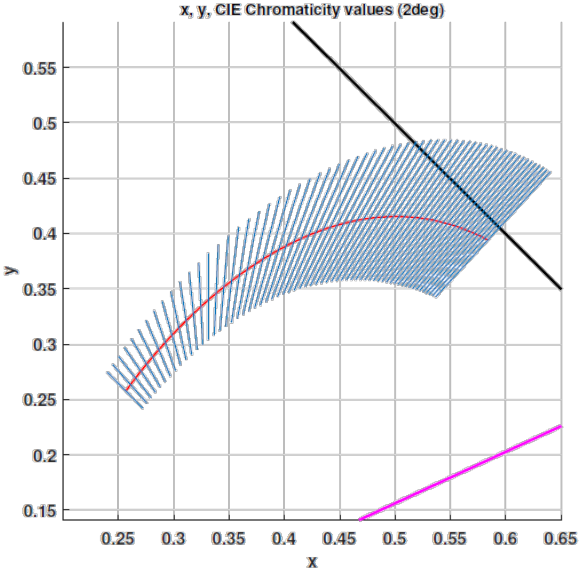

Blackbody Radiator Apparent Color - sRGB - gamma corrected

Isotemperature Lines – Correlated Color Temperature (CCT)

$$\mathrm{CCT} = T_{CCT} = \min_{T}\left\{ d = \left[(u - u(T))^2 + (v - v(T))^2 \right]^{1/2} \right\}$$







Color Mixing

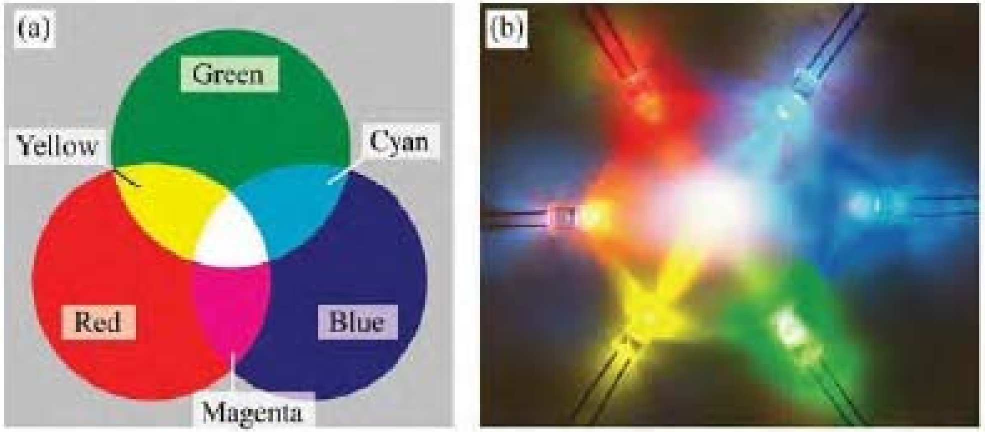
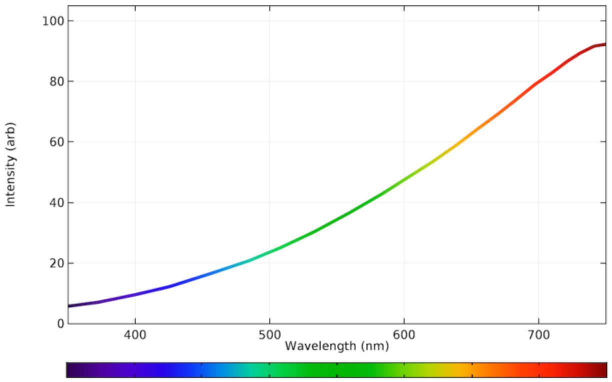


Fig. 19.1. (a) Schematic of additive color mixing of three primary colors. (b) Additive color mixing using LEDs.

E. F. Schubert, Light Emitting Diodes, 2nd Ed., Cambridge University Press, 2006

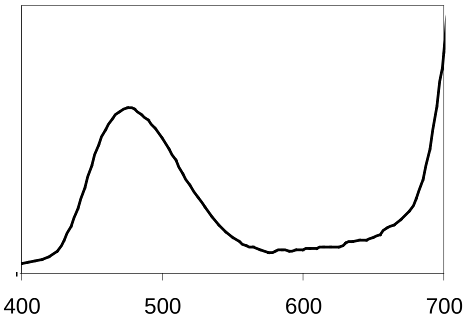
Object's Apparent Color

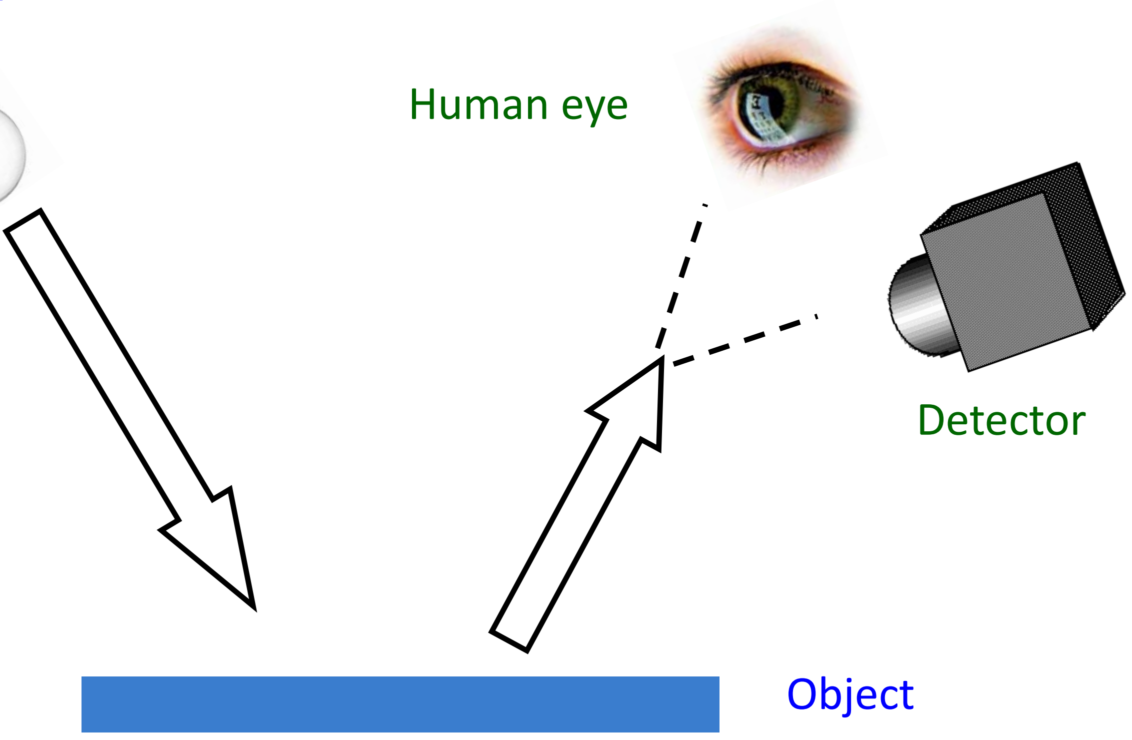
Incadescent Lamp Spectrum, S(λ)



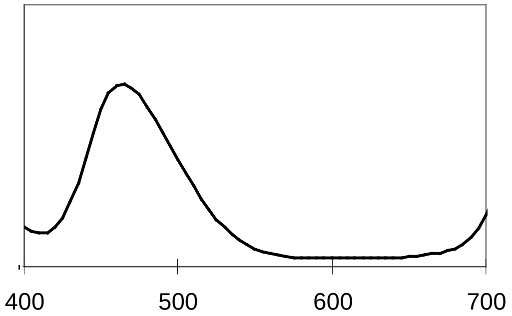
https://cdn.comsol.com/wordpress/2016/01/Emission-spectrum-of-incandescent-bulb.png

Reflected Spectrum, S(λ) × R(λ)





Object's reflectance, R(λ)



From : **Toni Litorja's** presentation, National Institute of Standards and Technology

Additive Colors and Color Mixing

Colored lights are mixed using additive color properties. Light colors are **combining two or more additive colors together which creates a lighter color that is closer to white**. Examples of additive color sources include computer monitors and televisions.

The additive primary colors are **red, green and blue** (RGB). Combining one of these additive primary colors with equal amounts of another one results in the **additive secondary colors of cyan, magenta and yellow**. Combining all three additive primary colors in equal amounts will produce the color white. Remember combing additive colors creates lighter colors, so adding all three primary colors results in a color so "light" it's actually seen as white. Although that may seem strange, if you think of the absence of all light equaling black, it begins to make sense that adding different colors creates white.

Additive Colors Combined in Equal Parts		
Blue + Green	**=**	**Cyan**
Red + Blue	**=**	**Magenta**
Green + Red	**=**	**Yellow**
Red + Green + Blue	**=**	**White**

http://www.colorbasics.com/AdditiveSubtractiveColors/

Additive Color Mixing

Additive Colors Combined in Unequal Parts		
1 Green + 2 Red	=	Orange
1 Red + 2 Green	=	Lime
2 Green + 1 Blue +3 Red	=	Brown

http://www.colorbasics.com/AdditiveSubtractiveColors/

Subtractive Color Mixing

Before TVs and computer monitors, printers and publishers wondered if they could print color pictures using just three colors of ink. Yes, it is possible, but you have to work in reverse of the process of mixing light colors! We see light colors by the process of **emission** from the source. We see pigment colors by the process of **reflection** (light reflected off an object). The colors which are not reflected are absorbed (subtracted).

The subtractive primary colors are **cyan**, **magenta** and **yellow** (CMY). These are the three colors used in printer ink cartridges.

Subtractive Colors
Cyan, Magenta and Yellow

Reflects	**Absorbs**	**Creates**
Blue + Green	Red	Cyan
Red + Blue	Green	Magenta
Green + Red	Blue	Yellow

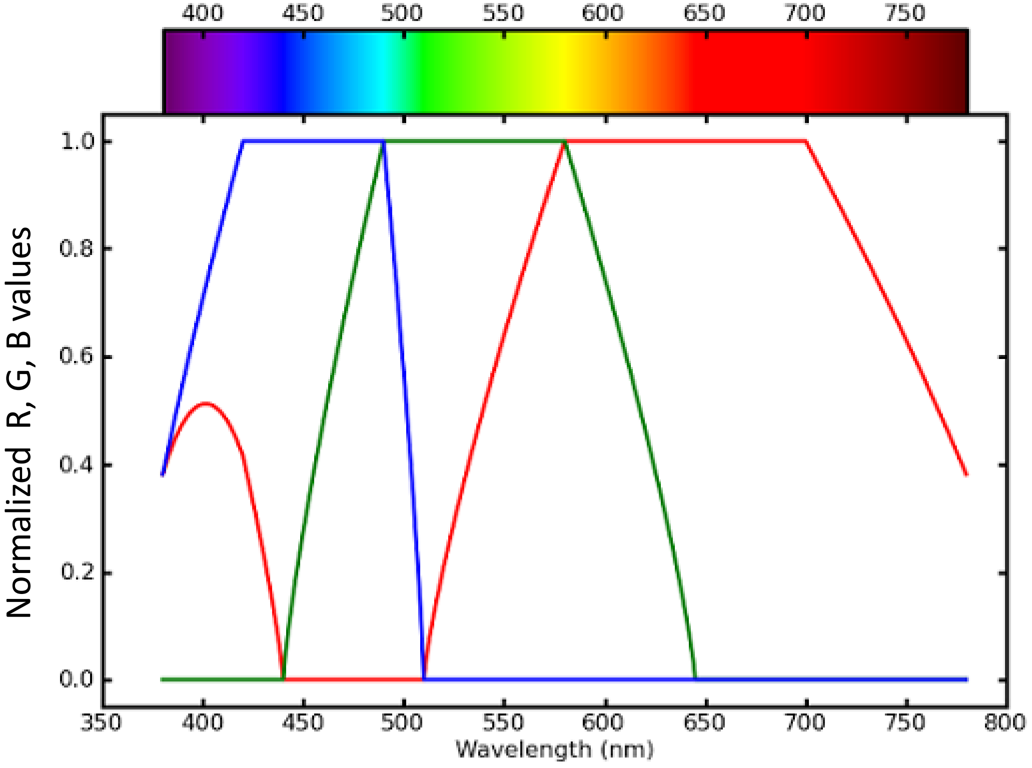
http://www.colorbasics.com/AdditiveSubtractiveColors/

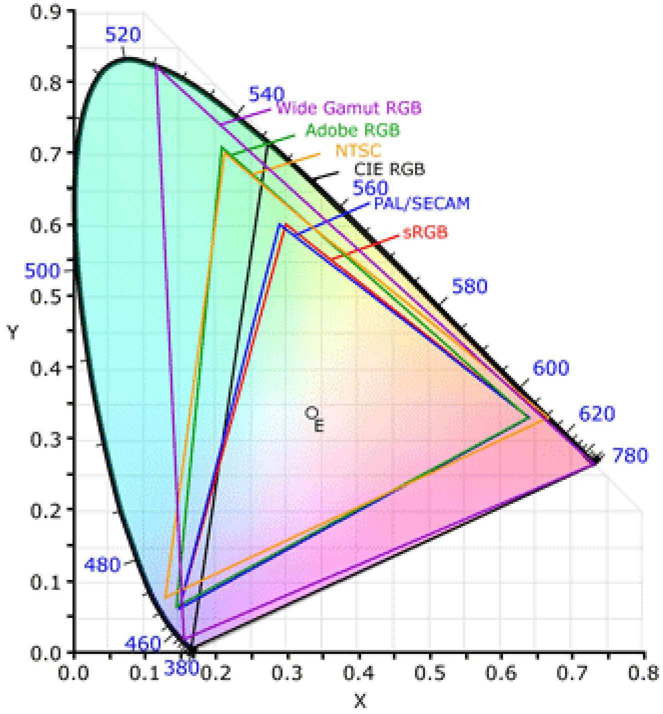
Subtractive Color Mixing

Subtractive Colors Mixing		
Combine	**Absorbs**	**Leaves**
Cyan + Magenta	Red + Green	Blue
Cyan + Yellow	Red + Blue	Green
Magenta + Yellow	Green + Blue	Red
Cyan + Magenta + Yellow	Red + Green + Blue	**Black**

http://www.colorbasics.com/AdditiveSubtractiveColors/

Transform a Visible Wavelength into RGB values



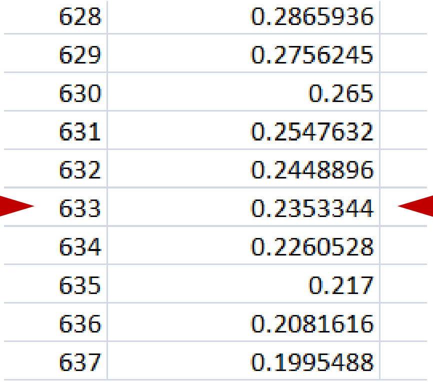


https://www.quora.com/What-is-the-method-to-convert-an-RGB-value-to-a-wavelength

http://www.physics.sfasu.edu/astro/color/spectra.html

Some Simple Photometry Examples

A 50mW He-Ne Laser at 633 nm (red light): what is its luminous power?



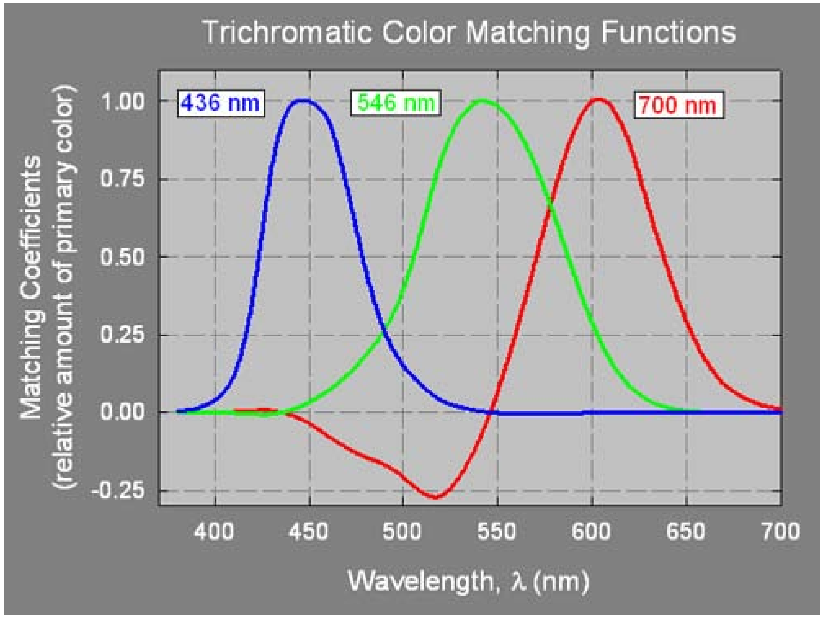
λ_0 (nm)	$V(\lambda_0)$
628	0.2865936
629	0.2756245
630	0.265
631	0.2547632
632	0.2448896
633	0.2353344
634	0.2260528
635	0.217
636	0.2081616
637	0.1995488

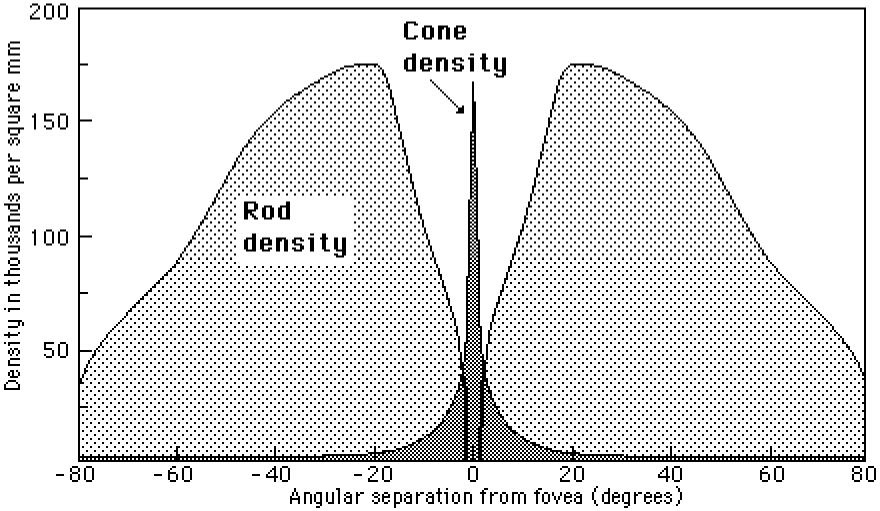
$$\Phi_v = 683\left(\frac{lm}{W}\right)V(\lambda_0)\Phi_e(\lambda_0)$$

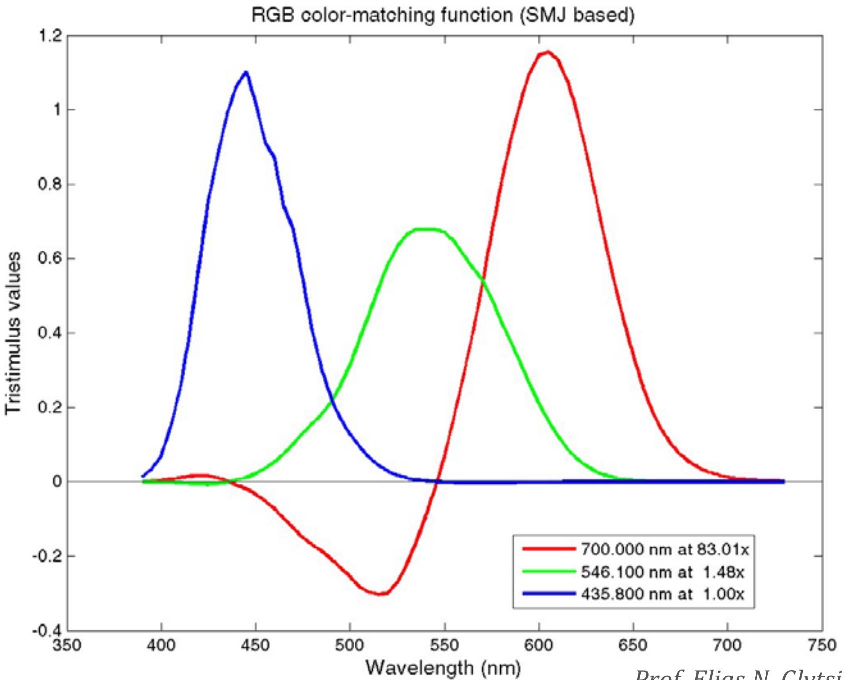
$$\Phi_v = 683\left(\frac{lm}{W}\right)(0.2353344)\times(50\times 10^{-3}\,W) = 8.0367\,lm$$

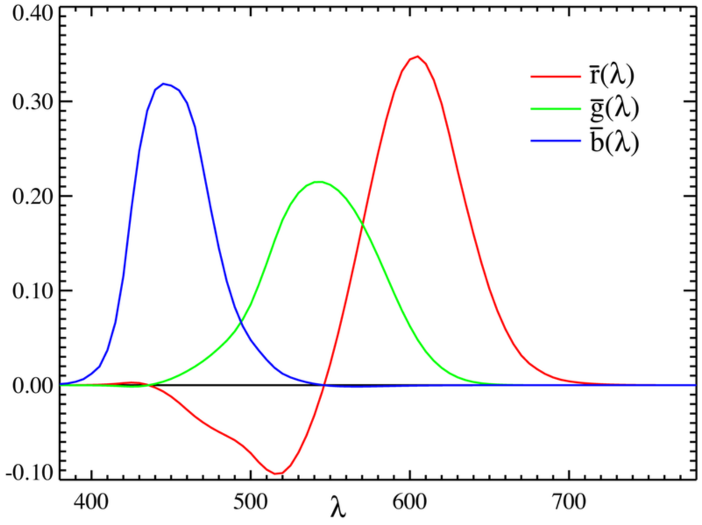
A 1W of white light equally distributed between 0.360μm ≤ λ_0 ≤ 0.750μm: what is its luminous power?

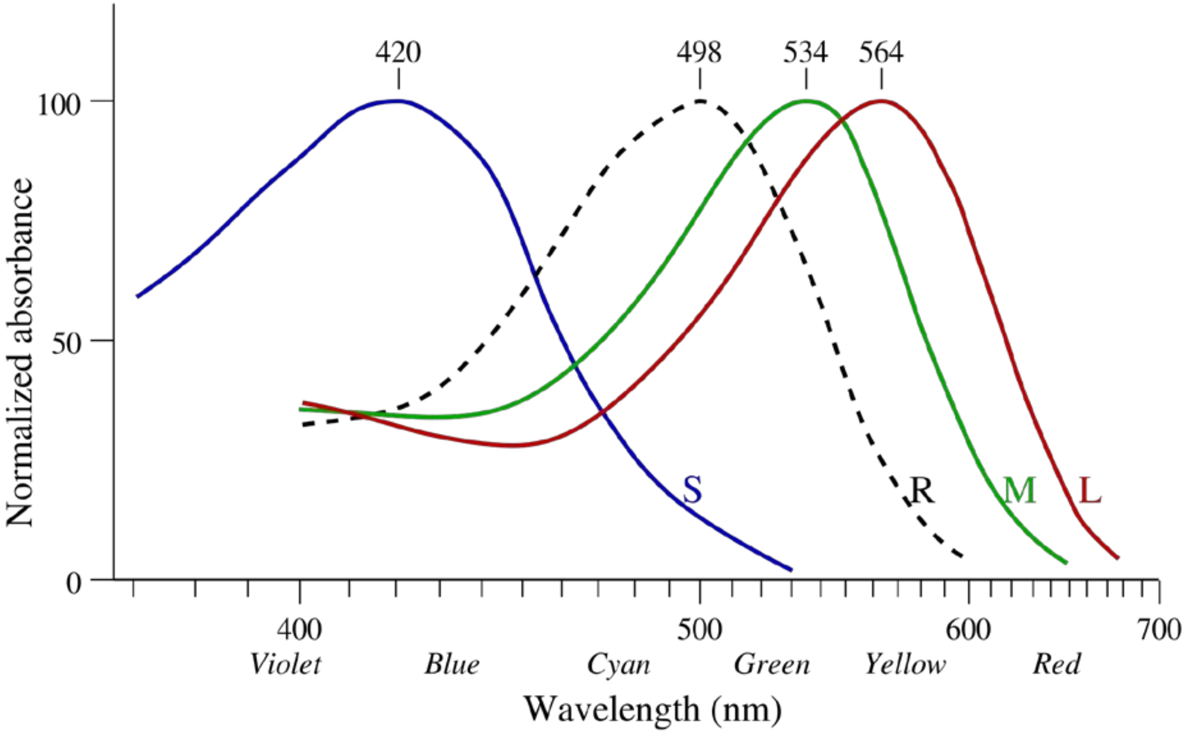
$$\begin{aligned}
\Phi_v &= 683\left(\frac{lm}{W}\right)\int_{\lambda_0} V(\lambda_0)\Phi_e(\lambda_0)d\lambda_0 \\
\Phi_v &\simeq 683\left(\frac{lm}{W}\right)\sum_i V(\lambda_{0,i})\Phi_e(\lambda_{0,i})\Delta\lambda_0 \\
&= 683\left(\frac{lm}{W}\right)\sum_{i=1}^{N}\frac{V(\lambda_{0,i})+V(\lambda_{0,i+1})}{2}\,\frac{1\,W}{\lambda_{max}-\lambda_{min}}\left(\frac{1}{nm}\right)\Delta\lambda_0(nm) \\
&= 243.1340\,lm \qquad \text{for} \quad \Delta\lambda_0 = 1\,nm
\end{aligned}$$


Trichromatic Color Matching Functions
436 nm
546 nm
700 nm
Matching Coefficients
(relative amount of primary color)
1.00
0.75
0.50
0.25
0.00
-0.25
400
450
500
550
600
650
700
Wavelength, λ (nm)

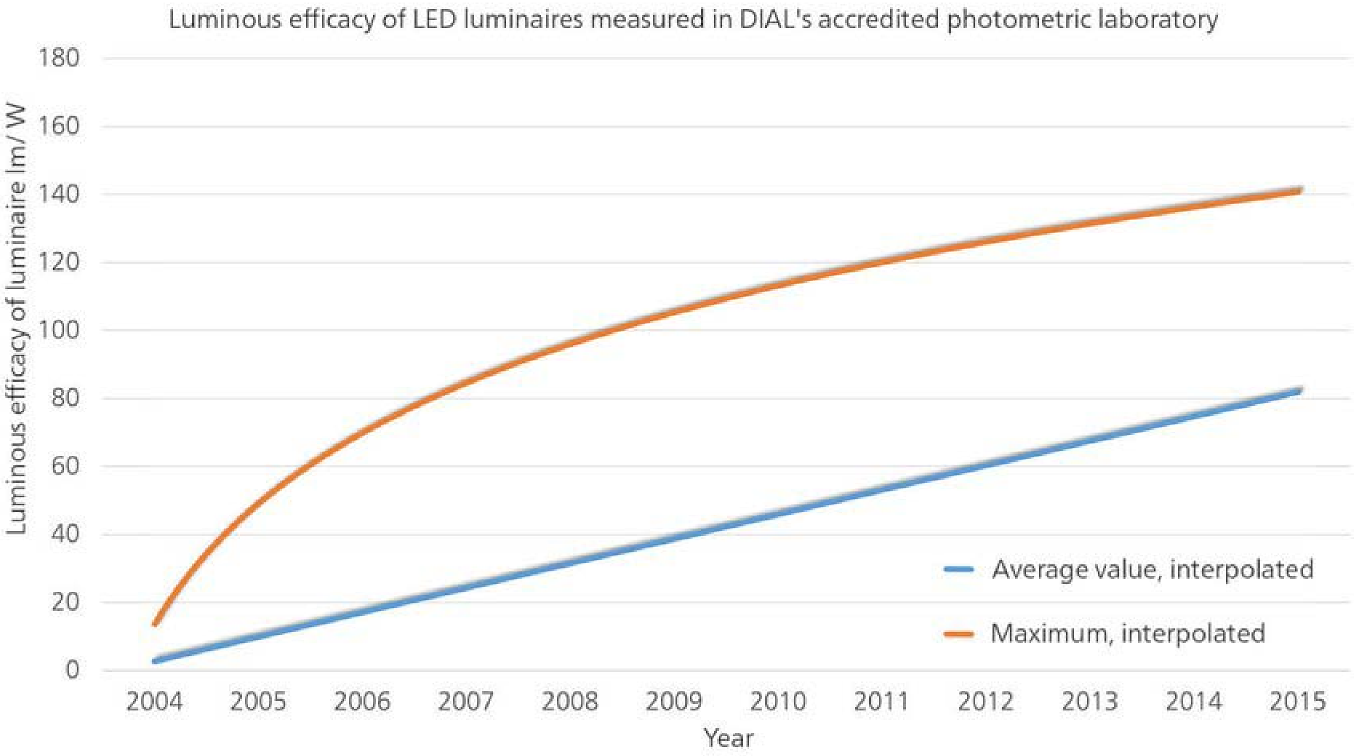

Cone density
Rod density
Density in thousands per square mm
200
150
100
50
-80
-60
-40
-20
0
20
40
60
80
Angular separation from fovea (degrees)


RGB color-matching function (SMJ based)
Tristimulus values
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
350
400
450
500
550
600
650
700
750
Wavelength (nm)
700.000 nm at 83.01x
546.100 nm at 1.48x
435.800 nm at 1.00x

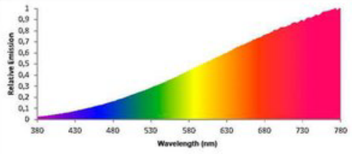
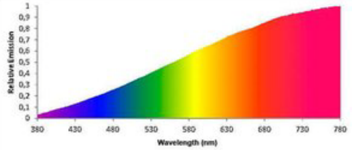
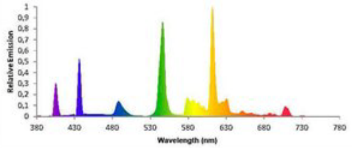
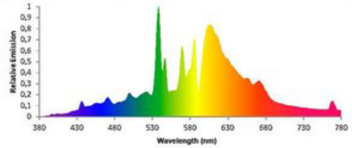
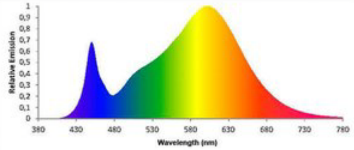
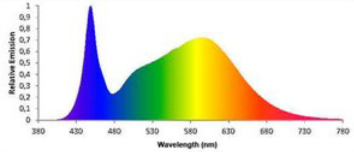
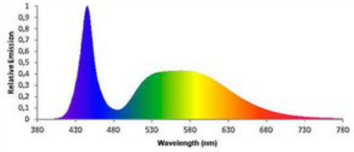

0.40
0.30
0.20
0.10
0.00
-0.10
400
500
600
700
λ
r̄(λ)
ḡ(λ)
b̄(λ)

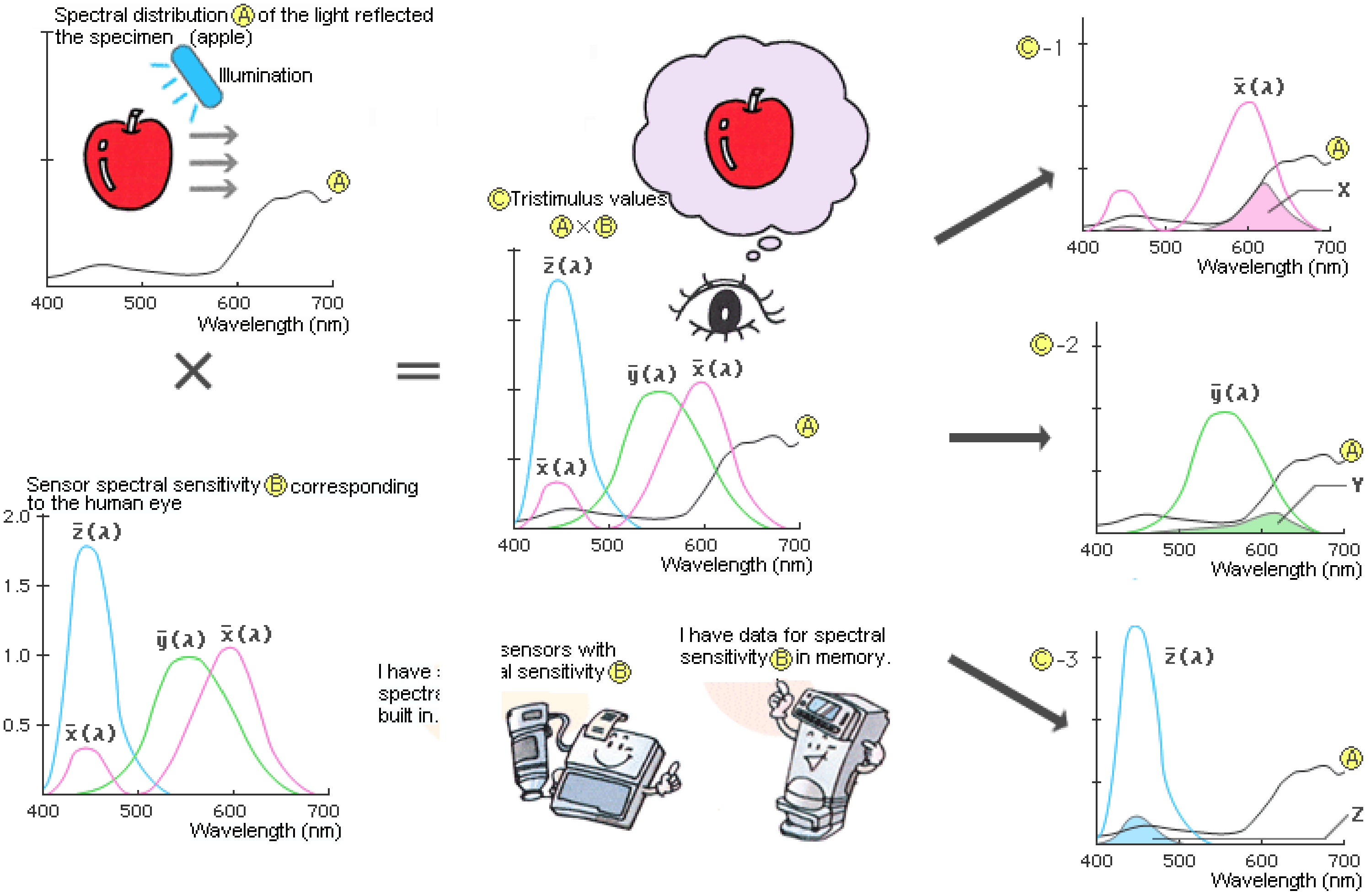


https://briankoberlein.com/2015/04/08/blinded-by-the-light/



https://www.dial.de/en/blog/article/efficiency-of-ledsthe-highest-luminous-efficacy-of-a-white-led/

Description	Spectrum	System power consumption, measured (W)	Lamp or module luminous flux, measured (lm)	System luminous efficacy (lm/W)	Energy conversion efficiency	Theoretical maximum luminous efficacy (lm/W)
High voltage halogen, 120 W		127,4	2.249	17,7	11,9	148,7
Low voltage halogen, 60 W		59,9	1.535	25,6	15,4	166,3
Fluorescent lamp T 5, 54 W, 830		51,3	4.184	81,6	23,7	344,4
Metal halide lamp, 70 W, 830		79,8	7.912	99,2	31,5	314,5
LED, 35 W, 830		34,2	4.739	138,6	42,3	327,6
LED, 35 W, 840		34,5	4.806	139,3	43,7	318,8
LED, 16 W, 750		16,2	2.436	150,5	48,7	309,0



https://www.konicaminolta.com/instruments/knowledge/color/part2/06.html